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Frontispiece

The Hoover Dam power plant, generating 240,000 kilowatts at 287,000 volts Three-phase power is transmitted 266 miles to Los Angeles, California The characteristics of this line are worked out in Chapter XIII (*Courtesy of the U S Bureau of Reclamation*)

INDUSTRIAL ELECTRICITY

VOLUME TWO

ALTERNATING-CURRENT PRACTICE

BY

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PREFACE

This book is a study of alternating-current theory and practice as applied in modern industry. It is Volume II of a course in industrial electricity, the first volume of which is devoted to direct-current practice. It assumes that students are familiar with the material in Volume I or in similar textbooks.

Since students entering technical schools have had little mathematical instruction beyond algebra, the mathematics of this book has been limited to algebra and a few simple trigonometric equations.

It is absolutely necessary that the student get a clear and correct understanding of the great fundamental principles underlying the practice of modern electrical engineering. These principles must be his own, something he is sure of, and not merely so many magic formulas in a textbook, which sometimes work and sometimes don't. For this reason, these principles must be stated and learned in his own language and applied in mathematical terms with which he is thoroughly familiar.

Most basic scientific principles have been discovered by first noting " what happens," then working out " why it happens," and, finally, testing the theory by applying it to several similar conditions. This procedure affords the most natural order by which a student can most readily grasp the real meaning of basic principles and learn how they are to be applied.

Accordingly, each new concept is introduced by stating " what happens," then explaining " why it happens." The explanations are followed by practical examples, worked out in detail with the aid of circuit diagrams and vector diagrams. Finally, numerous problems involving the same principle are included, the problems to be worked out by the students. There are nearly 900 of these problems, covering various fields, so that an instructor can assign different problems to successive classes.

This method, with its 500 diagrams and illustrations of modern machines and apparatus, requires many more pages of text than are usually devoted to the subject. Years of experience, however, have shown that by this procedure students acquire a firm grasp

of the principles and a well-founded confidence in applying them. Moreover, it requires no more of the student's time, and much less of the instructor's, than a method using a more condensed textbook.

I wish to express my appreciation of my long-time friend and collaborator, Frank G. Willson, who passed away just as the manuscript was nearing completion. The book owes much to his sound scholarship and painstaking composition.

Grateful acknowledgment is made to Professor H. B. Dwight for his valuable assistance on the subject of electric transmission, to Mr. James M. Aitken for his capable supervision of the preparation of the cuts, and to Mr. H. Edward Fuller of the Franklin Technical Institute for the checking of the examples and the solution of the problems.

W. H. Timbie

West Newton, Mass.
January, 1949

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CHAPTER I

ALTERNATING CURRENTS: FUNDAMENTAL IDEAS

The supply of electrical energy for industrial and domestic use has grown, in comparatively few years, from an experimental novelty, into a mammoth industry. This energy is now obtained commercially from three main sources. 1. Coal, which is mined in the earth; 2. Gas, or oil, which is taken from driven wells; 3. Water, flowing in natural or artificial waterways. Within a few years it may be obtained from "atomic piles," utilizing the energy released by splitting the atoms of Uranium or kindred metals.

Electric power can be generated either as direct-current or as alternating-current power. In the early days of the industry, generating plants were small; generators were driven by slow-speed reciprocating engines, and energy was distributed over areas in close proximity to the generating plant. For this service, the direct-current generator was satisfactory. But as the industry grew, larger and larger plants became necessary; and energy had to be transmitted longer distances with greatly increased line losses.

To meet this situation, the change was made to the alternating-current system; and to-day, electrical energy is almost universally generated, transmitted and used in this form. This does not mean that alternating current itself is superior to direct current or has supplanted it for commercial use. For instance, many small direct-current plants are operating today with good efficiencies, especially where exhaust steam is used for heating or for some manufacturing process. The direct-current motor is superior to the alternating-current motor for electric traction, for variable speed or multispeed service, and for printing presses, elevators, etc. Direct current is necessary for electrolytic processes, such as electroplating. Also, the manufacture of certain products of the electric furnace, notably aluminum, make use of large quantities of direct current.

1–1. Some Reasons for the Use of Alternating Currents. The possible distance electrical energy can be transmitted and the effi-

ciency of the transmission, both increase as the voltage employed increases. Alternating current can be generated at comparatively high voltages — 2300, 6900 and 13,800 volts being common values. By means of the static transformer, a most efficient and comparatively cheap machine, these voltages can be stepped up to still higher values, such as 66,000, 100,000 and 200,000 volts.*

At these voltages energy can be transmitted comparatively long distances, and then stepped down by transformers to 2300 volts for distribution. At 2300 volts, this energy can be carried on small wires, overhead or underground, to the consumers' premises and again stepped down to 120, 240 or 480 volts for lighting and power. Or it may be used directly for power in 2300-volt induction motors.

The induction motor is cheaper in first cost and maintenance than the d-c motor, and for constant-speed service, is at least equal to the d-c motor in efficiency.

Direct-current voltages, on the other hand, cannot be stepped up or down, except by the use of rotating machines, equipped with commutators, such as d-c motor-generator sets, which are more expensive and much lower in efficiency than the transformer. High voltages are not possible with d-c generators because of the difficulty in commutation. The limit is about 750 volts, although in a few cases, 1500 volts have been successfully generated. For distribution at 120 or 240 volts, large cables are needed to carry direct-current power to the consumers' premises with increased losses due to the necessarily high values of current.

The alternating-current generator can be driven at high speed by the modern turbine and can be built in large units. Generators in capacities of 20,000 to 40,000 kw are quite common in large plants today, and several machines of 100,000 to 200,000 kw are in operation. Speeds may be as high as 3600 rpm for 5000 kw units and 1500 rpm for 40,000 kw generators. These large turbo-alternator units are much more economical of steam, are more efficient and weigh less per kilowatt capacity than slow-speed machines.

The d-c generator cannot be driven at high speed due to commutating difficulties, and is limited in size to about 5000 kw.

Thus alternating-current power can be generated in large stations by large high-efficiency generating units, transmitted greater distances and distributed more efficiently over greater areas than

* The transmission voltage at the Hoover Dam is 287,000 volts, the highest in commercial use today.

direct-current power. Even when d-c energy in any considerable amount is necessary, it is common practice to step down high voltage a-c and convert it to d-c by means of the rotary converter or vacuum tube rectifier.*

2–1. Alternating-Current System for Short Transmission Requiring no Step-up Transformers. An old empirical rule which has some economic basis, and which gives satisfactory results within reasonable limits, states that the proper transmission pressure should be about 1000 volts for each mile in length of line; for instance, 2300 volts may well be used to transmit current within a radius of about two miles from the generating station.

In many stations, 6600 volts or 6900 volts has been adopted as a satisfactory terminal pressure of the generators when transmitting power not over five or six miles. Where transmitting distances are from ten to fifteen miles, generator terminal voltages are usually 11,000 to 14,000 volts. In such cases, the transmission may readily be fed directly from the generators, or from station bus bars to which a number of generators in parallel deliver their output.

When lines are operated at pressures not greatly exceeding this, step-up transformers and transformer substations are usually considered unnecessary. The small distributing transformers which supply individual consumers are attached directly to the transmission line. A typical example of such an alternating-current distribution system is shown in Fig. 1–1. This represents the usual three-phase installation using three wires for each circuit.

"A" The main generator *A* is connected (through a switchboard and switching apparatus not shown) directly to the three-phase transmission line. The a-c generator must have its field magnets excited from a separate source of direct current, usually from a small compound-wound d-c generator, called an exciter. The exciter, *X*, may be attached to the a-c generator or be driven from a separate source.

"B" A three-wire, three-phase transmission line is represented in the figure by the three lines *B*, *B*, *B*. To the transmission line are connected the following service equipment:

* It seems possible at the present time, with the development of the vacuum tube, that electrical energy in the future may be generated and stepped up as high-voltage (a-c), converted to high-voltage (d-c) and transmitted greater distances than is now possible. It may then be converted to alternating voltage and stepped down for commercial use.

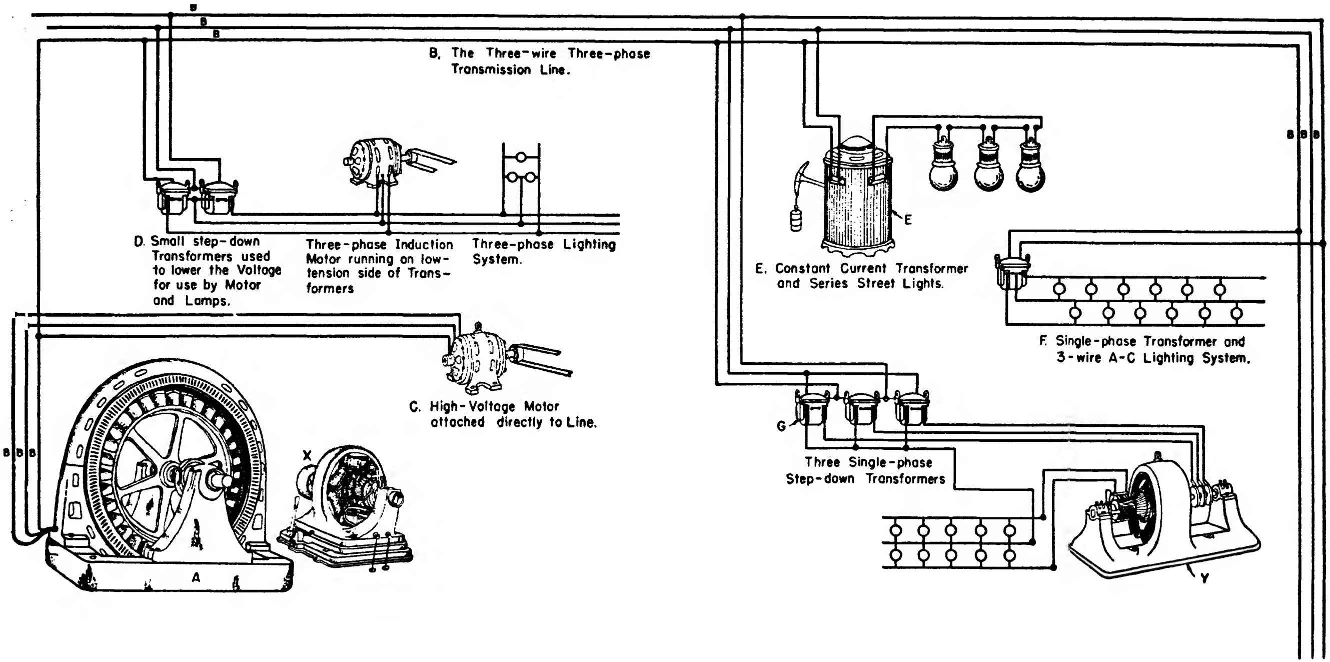


Fig. 1-1. A typical short transmission line. The generator *A* is connected directly to the three-phase line *B*. A three-phase motor *C* takes its power directly from the line wires. Low-voltage motors and lamps receive their power through the step-down transformers *D*. A constant-current transformer delivers power from the line to a series lighting system. A single-phase lighting system is connected to the line through the step-down transformer *F*. A synchronous converter converts power from the line into direct current for a three-wire lighting system.

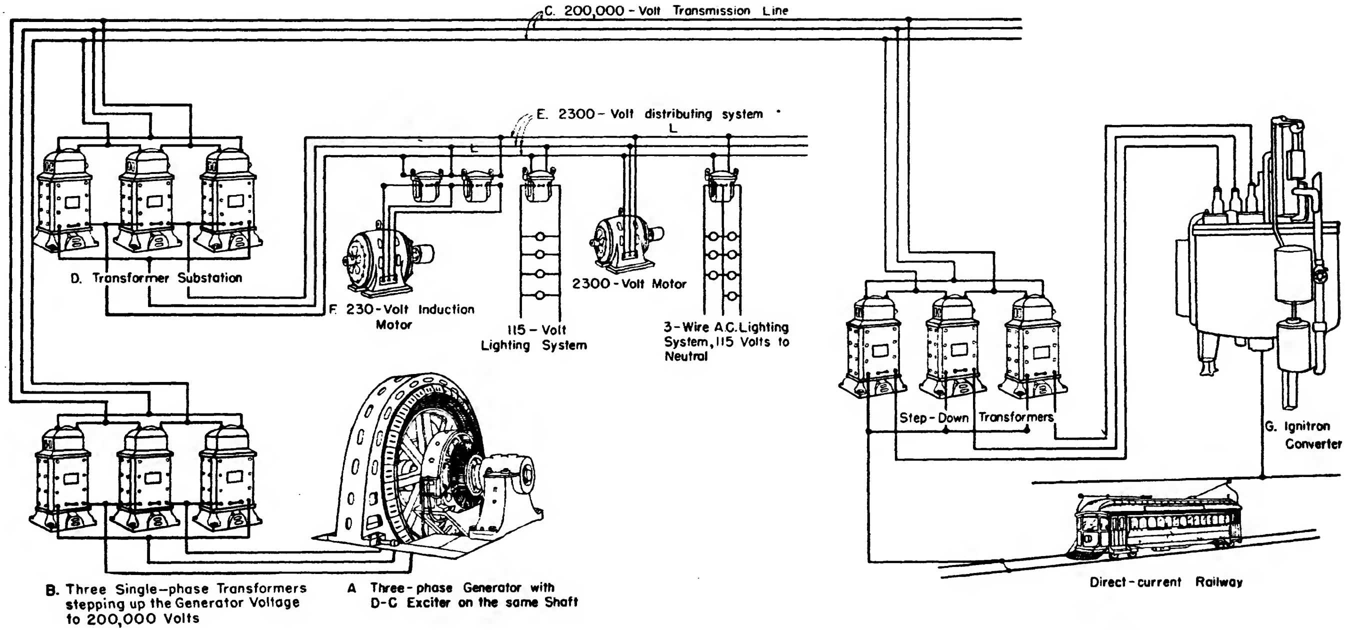


Fig. 2-1. A typical long-distance transmission system. The generator A delivers power at from 6600 to 14,000 volts to the step-up transformers B, which deliver it to the line C at voltages as high as 200,000. The transformer substation D " steps down " the voltage to about 2300 for local distribution over the line E. The small induction motor F requires still another set of transformers to step the voltage down from that of the line E, or, if suitably wound it may be attached directly to the local circuit. In the converter substation G, the ignitron converter takes its alternating-current power from the line C through the step-down transformers, and converts it into direct-current power for operating a city railway and a variety of other uses.

"C" A three-phase alternating-current induction motor, wound to operate properly at the full line-voltage. The motor load is not shown, but it may be either a mechanical or an electrical machine.

"D" A number of single-phase transformers connected to the transmission line, stepping down the voltage to a value suitable for small three-phase motors and incandescent lamps.

"E" A "tub transformer," or constant-current transformer, connected to a series-incandescent circuit for street lighting. This transformer takes power from one phase of the three-phase line as a variable current at constant voltage. It delivers power to the series-lamp circuit as a constant current at a variable voltage, depending upon the number of lamps in the circuit.

"F" A single-phase distributing transformer, connected to one phase of the three-phase line. A three-wire constant-voltage lamp circuit is connected to the low voltage coils of the transformer. Lighting circuits are always single phase. Various lighting circuits are so connected to the transmission-line that the three line-wires are as nearly equally loaded as possible.

"G" A set of three single-phase transformers stepping down the voltage to a value suitable for the low-voltage synchronous converter, *Y*, which is being used to convert the three-phase a-c power into d-c power for distribution on an Edison three-wire power and lighting system.

3–1. Alternating-Current System for Long-distance Transmission where Step-up Transformers are Required. In general, where the transmission of electric power must be made to distances greater than from about five to fifteen miles, or at voltages higher than from 6600 to about 14,000 volts, the use of step-up or central-station transformers and transformer substations is usually employed. A typical example of a long-distance power transmission (switching and protective equipment not shown) is illustrated in Fig. 2–1.

"A" A main central station, generating three-phase a-c power at a voltage of between 6600 and 14,000 volts. Note the d-c exciter erected on the main generator shaft.

"B" A set of step-up or station transformers, so connected to the main generator as to raise the voltage of the station

generator to the value required on the transmission line, — the value depending upon how far the power is to be transmitted.

"C" A three-wire, three-phase transmission line, *C*, *C*, *C*.

"D" A transformer substation, consisting of three single-phase transformers connected to the three-phase high-tension transmission line. These transformers step the line voltage down to perhaps 2300 volts three-phase for distribution over the wires *E*, *E*, *E*.

"F" A variety of appliances may be connected to the distribution system, *E*, *E*, *E*, either through small transformers or directly to the line. The figure shows a small three-phase induction motor, operated through transformers at 230 volts; a system of incandescent lamps at 115 volts; a larger motor, operating directly on the distributing line at 2300 volts; and a three-wire a-c incandescent lighting system operating at 115–230 volts through a small transformer.

"G" A converter substation, consisting of two distinct parts; first, step-down transformers used for reducing the pressure from the high-tension transmission line to a value suitable to drive a synchronous converter; second, the ignitron converter changing alternating current to direct current. In this case the direct-current output of the converter is used for operating a metropolitan street-railway. It might also be distributed for a great variety of purposes.

4–1. An Alternating Current. An alternating current of electricity differs in no respect from a direct current, except that, instead of flowing continuously in one direction, it periodically reverses the direction of its flow. The methods of measuring and computing the values of alternating currents and pressures, however, differ from those used for direct current.

In Vol. I, the flow of direct current has been likened to the flow of water in a river or in a pipe. The flow of an alternating current may be likened to the ebb and flow of the tide in a narrow channel. The tide periodically reverses the direction of the flow of water once about every $6\frac{1}{4}$ hours, or about 4 times a day.

A better idea of this ebb and flow of an alternating current can be gained from a study of the engine-driven pump of Fig. 3–1. As the valveless piston is moved back and forth in the cylinder, the water, which completely fills the system, is made to surge back and

forth through the entire circuit of pipe and cylinder. The number of times the direction of flow changes per minute depends upon the number of revolutions made by the crank shaft driving the piston. Assume the crank shaft to rotate at uniform speed in the direction

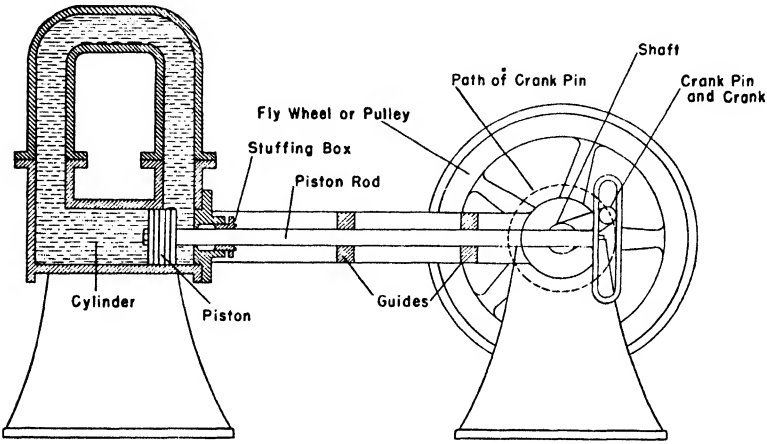


FIG. 3–1. Engine-driven valveless pump with slotted yoke. The water surges back and forth through the pipe.

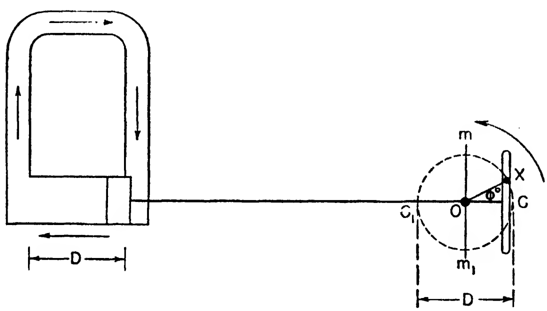


FIG. 4–1. Skeleton diagram of valveless pump. The direction of the piston motion at any instant depends upon the position of the crank at that instant.

marked in Figs. 4–1 and 5–1, which are skeleton diagrams of the pump in Fig. 3–1. It is apparent that the direction of flow in the pipe at any given instant depends upon the position of the crank at that instant.

It is readily seen that as the crank pin passes through the points C and C_1, the piston is at a standstill and no current is flowing. When the crank pin is passing through the points m and m_1, the

piston is moving at the maximum rate and the greatest current is flowing around through the pipe. In Fig. 4–1, the crank pin has just passed the dead center or neutral point C, and the piston is traveling to the left, forcing the water around the circuit in a clockwise direction. In Fig. 5–1, the crank pin has just passed through the neutral point C_1, and the piston has started back, traveling to the right. The direction of the flow of water has also reversed, and is now flowing counter-clockwise around the pipe system. Note that in each revolution of the crank, there are two neutral points (C and C_1) at which the piston is not moving and no current is flowing; and also two maximum points (m and m_1) at which the piston is moving at its fastest rate, and the greatest current is flowing. At all times, however, the crank shaft is revolving at the

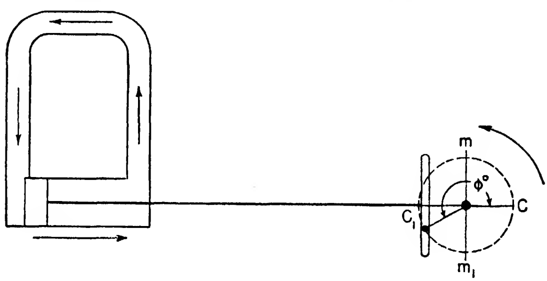


FIG. 5–1. The crank has moved into a new position, such that the piston motion is reversed.

same speed. The flow of water in the above system is a fair picture of what happens in an electric alternating-current system. The current starts flowing in one direction, rises to a maximum value, then dies out and starts flowing in the opposite direction, rising to a maximum value and dying out until it stops again.

This sequence of events is called a **cycle** and is repeated over and over again as the crank rotates. Note again, that in each cycle there are two instants when the current is zero (that is when it stops flowing) and two instants when the current is flowing at the greatest rate. The number of times the current goes through this cycle in a given time is called the **frequency.**

5–1. Relation of Current at any Instant to Crank Position. Referring again to the engine-driven pump of Fig. 4–1, we have seen that the speed and the direction of piston motion at any instant depends upon the position of the crank at that instant. Accordingly, we have said that the amount and the direction of current in

the pipe system at any instant depends upon the position of the crank at that instant.

Let us call OC the "zero" position of the crank. It will be found that when a slotted yoke is used, as in Fig. 3–1, the speed of the piston at any instant is proportional to the sine* of the angle ϕ in Fig. 4–1, which the crank is making at that instant with the zero position.

Thus, when the crank is at O_m, it is at an angle of 90 degrees to the zero position OC, and the piston is now moving at the fastest speed and the greatest clockwise current is flowing. Let us assume this greatest current to be 100 gals. per second. When the crank has just reached the position OX, Fig. 4–1, and is making an angle of only 25 degrees with the zero position, then only a certain fraction of the 100 gals. per second is being forced through the pipe. This fraction will be found to be the sine of the angle 25 degrees, or 0.423 (from a table of sines). Thus the current at this instant would be 0.423 of 100 or 42.3 gals. per second.

This may be stated as a general rule: The current at any instant equals the product of the maximum current times the sine of the angle which the crank is making with the zero position at that instant. This assumes the crank shaft to rotate at constant speed.

This relation may be expressed in the form of an equation:

$$i = I \sin \Phi. \tag{1-1}$$

where i = current at any instant or crank position in gals. per second.
I = maximum current in gals. per second.
Φ = angle which crank makes with zero position.

Example 1. What is the current flow in the pipe system of Fig. 4–1 when the crank is at an angle of 65° with the zero position? Assume the maximum flow to be 100 gals. per sec.

Solution:

$$\begin{aligned} i &= I \sin \phi \\ i &= 100 \sin 65^\circ \\ &= 100 \times 0.906 = 90.6 \text{ gals per second.} \end{aligned}$$

Example 2. Consider a pipe system and pump with a slotted yoke, similar to that in Fig. 3–1, of such size that one inch movement of the piston displaces 3.18 gals of water, the stroke of the piston being 10 inches. The crank pin rotates uniformly at 60 revolutions per minute.

* See Appendix A on Trigonometry.

(*a*) Calculate the distance moved by the piston as the crank pin moves from a position 29° off dead center to a position 31° off dead center (average position 30°).

(*b*) Calculate the distance moved by the piston as the crank pin moves from the 89° to the 91° position (average position 90°).

(*c*) Compute the current flow in gals. per second for each case above.

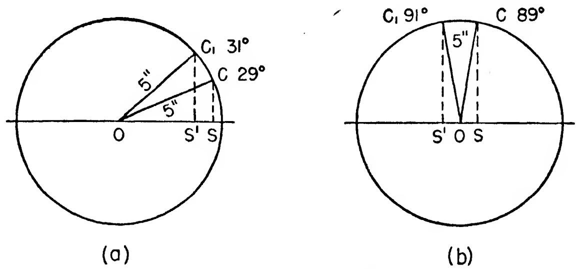


FIG. 6–1. (*a*) Crank pin of Fig. 3–1 at an average position of 30° from dead center. (*b*) Crank pin at an average position of 90° from dead center.

Solution. (*a*) Crank pin rotates in a circle of 5 inches radius, Fig. 6*a*–1, and in moving from 29° to 30° position, the piston must travel the distance OS–OS' or SS'.

$$\begin{aligned} OS &= 5 \cos 29° = 5 \times 0.8746 = 4.3730 \text{ in.} \\ OS &= 5 \cos 31° = 5 \times 0.8571 = \underline{4.2855 \text{ in.}} \\ SS' &= \qquad\qquad\qquad\qquad\quad 0.0875 \text{ in.} \quad \textit{Ans.} \end{aligned}$$

(*b*) Crank pin, moving from 89° to 91° position, must travel the distance O–$S(-OS')$, or SS', Fig. 6*b*–1.

$$\begin{aligned} OS &= 5 \times \cos 89° = 5 \times 0.01745 = \quad 0.0873 \text{ in.} \\ OS &= 5 \times \cos 91° = 5 \times 0.01745 = \underline{-0.0873 \text{ in.}} \\ SS' &= \qquad\qquad\qquad\qquad\qquad\quad 0.1746 \text{ in.} \quad \textit{Ans.} \end{aligned}$$

(*c*) Crank pin revolves at rate of one revolution or 360° per second; equals 2° in 2/360 or $\frac{1}{180}$ second.

Speed of piston in (*a*) = 0.0875 × 180 = 15.75 in. per second.
Rate of flow of water in (*a*) = 15.75 × 3.18 = 50 gals. per sec. *Ans.*

Speed of piston in (*b*) = 0.1746 × 180 or 31.43 in. per second.
Rate of flow of water in (*b*) = 31.43 × 3.18 = 100 gals. per sec. *Ans.*

From Example 2 above, note that in (*a*) the average position of the crank is 30° and the rate of flow is 50 gals. per second, while in (*b*) the average position of the crank is 90° and the rate of flow is 100 gals. per second. The sine of 30° is 0.50 and of 90° is 1.00.

Thus $$\frac{\text{ave. rate of flow in } (a)}{\text{ave. rate of flow in } (b)} = \frac{50}{100} = \frac{0.50}{1.00}$$

Or; the rate of flow of current at any instant in the above system is proportional to the sine of the angle the crank (at that instant) makes with the dead center or zero position.

Therefore:

in position (a), $100 \times \sin 30° = 100 \times 0.50 = \times 50$ gals per sec.

and

in position (b), $100 \times \sin 90° = 100 \times 1.00 = 100$ gals per sec.

This relation between current flow and crank position may be represented by plotting a curve between them as in Fig. 7-1. The horizontal scale, or abscissa, shows crank position in degrees from

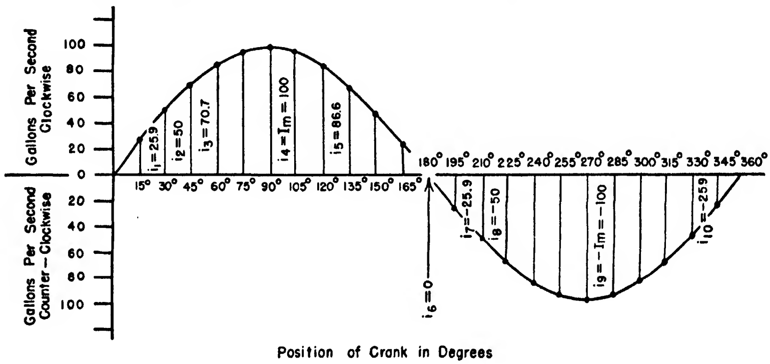


FIG. 7-1. Relation between current and crank position shown by a curve called "The Sine Curve."

the zero position. The different values of the current, in gals. per second, are laid off as ordinates on the vertical scale, reading up for clockwise direction of current in the circuit and down for counter-clockwise.

Assume the maximum current as 100 gals. per second as before. When the crank is at the zero position the current is zero, thus the curve starts at zero.

When the crank reaches the 15° position, the instantaneous value of the current,

$$i_1 = 100 \sin 15° = 100 \times 0.259 = 25.9 \text{ gal per sec.}$$

At the 15° position on the horizontal scale, lay off i_1, upward on the vertical scale to represent 25.9 gal. per sec., clockwise.

When the crank reaches the 90° position,

$$i_4 = 100 \sin 90° = 100 \times 1.00 = 100 \text{ gals. per sec.}$$

At the 90° position on the horizontal, lay off i_4 upward to represent 100 gal. per sec., clockwise. This is the maximum value of the current.

From now on the current grows smaller so that at the 120° position of the crank, for instance, the instantaneous current,

$$\begin{aligned} i_5 &= 100 \sin 120^\circ \\ &= 100 \sin (180^\circ - 120^\circ) \text{ (See Appendix } A) \\ &= 100 \sin 60^\circ = 100 \times 0.866 = 86.6 \text{ gal. per sec.} \end{aligned}$$

Again, i_5 is laid off upward to represent 86.6 gal. per sec. clockwise.

As the crank continues to turn, the current decreases, until at the 180° position, it again becomes zero. This is the position OC_1, Fig. 4–1. This fact is also shown by the equation,

$$\begin{aligned} i_6 &= 100 \sin 180^\circ = 100 \sin (180^\circ - 180^\circ) \\ &= 100 \sin 0^\circ = 100 \times 0 = 0. \end{aligned}$$

But as the crank continues to turn beyond the 180° position, the piston begins to move in the reverse direction and sends a current in the opposite direction, or counter-clockwise through the system.

When, for instance, the crank has reached the 210° position, the instantaneous current,

$$\begin{aligned} i_8 &= 100 \sin 210^\circ \\ &= 100 [- \sin (210^\circ - 180^\circ)] \text{ (See Appendix } A.) \\ &= 100 (- \sin 30^\circ) = 100 (-0.50) = -50 \text{ gals. per sec.} \end{aligned}$$

And i_8 is laid off downward on vertical scale to represent 50 gal. per sec. in the reversed direction or counter-clockwise in the circuit.

At the 270° position of the crank, the current has reached a maximum counter-clockwise value of 100 gal. per sec.

For

$$\begin{aligned} i_9 &= 100 \sin 270^\circ = 100 [- \sin (360^\circ - 270^\circ)] \\ &= 100 (- \sin 90) = 100 \times (-1) = -100 \text{ gal. per sec.} \end{aligned}$$

As the crank continues toward the completion of its cycle, the value of the counter-clockwise current gradually falls off until at the 360° position, OC in Fig. 5–1, it again becomes zero.

If the current at the several intermediate positions of the crank are drawn, and their ends connected by a smooth curve, as in Fig. 7–1, this line is called a **sine curve.** The ordinates, or vertical

distances of the curve give the instantaneous values of the current for the various positions of the crank.

This gives the clearest possible picture of an alternating current, either of water or of electricity. If, instead of the pump and pipe system, an electrical circuit carrying a maximum current of 100 amperes were substituted in the discussion above, the calculations for the instantaneous currents in amperes throughout the cycle would be identical, and the same curve would be plotted. The portion of the curve above the horizontal line indicates current in one direction through the circuit, always considered positive; while that portion below the line indicates current in the reversed direction, or negative current.

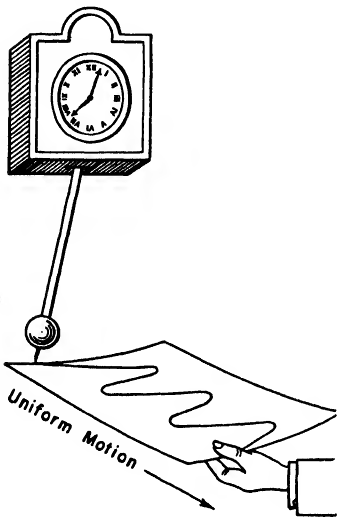


FIG. 8-1. The pendulum traces a sine curve, if smoked paper is moved at uniform speed.

6-1. The Sine Curve a Standard Wave Form. The type of motion represented by a sine curve occurs very commonly in nature. In any natural object which has a periodic motion, such as a swinging pendulum, a vibrating string or the rippling surface of a body of water, we find this form of wave. The sine curve is apparently nature's standard.

Note the curve produced in Fig. 8-1 by allowing a swinging pendulum to trace its motion on a smoked surface which is moved at uniform speed at right angles to the swinging. Figure 9-1 shows the curve of motion of a tuning fork obtained in a similar manner; and Fig. 10-1 represents the cross section of ripples on the surface of water. These all have the form of a sine wave.

Now it is possible to produce alternating currents with an almost endless variety of wave forms, but this natural sine curve has been adopted by engineers as the standard, principally for the following reasons:

First. This form produces the least useless disturbance in the electrical circuit and gives the smoothest, most efficient and most useful current wave.

Second. The mathematical computations, connected with alternating current work, are much simpler with this form of wave.

Prob. 1-1. The maximum value of an alternating current is 16 amperes. Compute the instantaneous values every 10° for one cycle and plot the sine curve.

Prob. 2-1. For every 15° of the cycle, plot the sine curve of a current having a maximum value of 65 amperes.

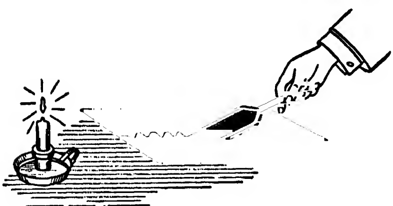


FIG. 9-1. The curve traced by a tuning fork is a sine curve.

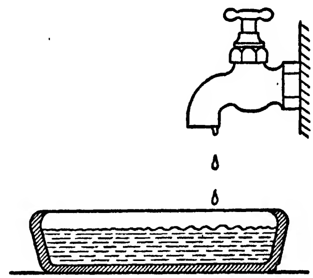


FIG. 10-1. The waves produced on the surface of the water have the form of a sine curve.

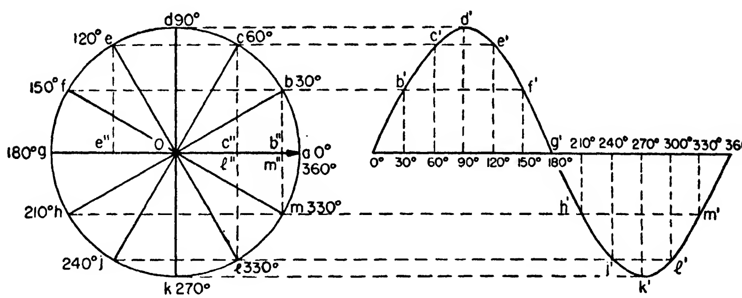


FIG. 11-1. Showing how a sine curve may be constructed from the vertical projections of the rotating radius, *oa*.

7-1. Construction of a Sine Curve. Consider a circle, Fig. 11-1, with a radius, *oa*, rotating counter-clockwise about the point *o*, with its various positions, in this case, 12, measured in degrees from the horizontal. Extend the horizontal line and divide it into equal parts marked in degrees, according to the positions of the rotating radius. At the 30° position of the radius, drop a perpendicular, *bb''*, from the end of the radius to the horizontal line. Lay off this length *bb''* from the 30° point on the horizontal scale

and obtain the point b' on the sine curve. Repeat this for all positions of the radius for a complete revolution, and obtain the points a', b', c', d', e', f', etc. Note that after the 180° position, the perpendicular distances are laid off below the horizontal line. If now a curve is drawn through these points, a sine curve will result.

Note that the radius, oa, equals od, the maximum value of the curve.

The proof of this construction is shown as follows.

In the triangle obb'', $\sin \angle bob'' = \dfrac{bb''}{ob}$,

$$bb'' = ob \sin \angle bob'' \text{ (In this case 30°)},$$

Therefore, $bb'' = ob \sin 30°$.

Since ob equals the maximum value, od, therefore bb'', the instantaneous value at the 30° position, equals the maximum value of the curve multiplied by the sine of the angle. This is the equation of a sine curve.

8–1. Clock Diagrams or Vector Diagrams. So far, we have learned that an alternating current may be represented by a wave form, called a sine curve, which presents to the eye a very definite picture of what is happening at each instant in the circuit. This curve is difficult to draw accurately, is inconvenient and of little use in determining actual values. So, to determine accurately just what current is flowing at any instant, we use the equation of the curve.

$$i = I_m \sin \phi, \tag{2–1}$$

where i = amperes at any instant;

I_m = maximum amperes in the circuit;

ϕ = angle in degrees from the zero position.

But this equation presents no picture to the eye of what is happening in the circuit.

However, a most important fact is to be noted from the preceding article. Since a sine curve may be obtained from the vertical projections of a rotating radius, Fig. 11–1, it is much simpler and more convenient to use the radius itself, called a rotating vector, to represent an alternating current. This is called a clock diagram or vector diagram. This method enables us to

quickly obtain correct numerical values and, at the same time, have a picture before the eye of the events taking place in the circuit. When instantaneous values are under consideration, the length of the vector always represents the maximum value of the wave.

Consider the clock, or vector, diagram of Fig. 12–1. A horizontal line cc_1 and a vertical line mm_1, both of indefinite length are drawn as reference lines. The vector ox, representing the maximum value of the current, is considered as rotating in counterclockwise direction about the point o. In any position, ϕ degrees from the zero position, drop a perpendicular vx, to the reference line cc_1, we thus have at one glance the maximum current ox, and the instantaneous current vx, at ϕ degrees from the zero point of the curve.

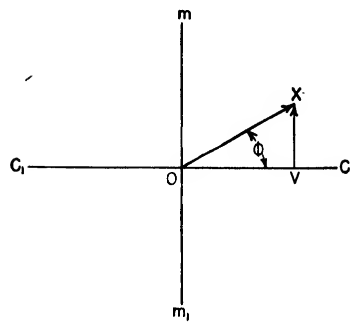


FIG. 12–1. Clock, or vector, diagram of current in a circuit in which ox represents the maximum value of the current and vx the instantaneous current at $\phi°$ from the zero point in the cycle.

For
$$\frac{vx}{ox} = \sin\phi \text{ and } vx = ox \sin\phi$$

or
$$i = I_m \sin\phi$$

Example 3. The maximum value of the current in a circuit is 48 amperes. Find the instantaneous current at the 40°, 90°, 125° and 210° points on the curve.

Solution. Draw ox, Fig. 13–1(a), representing 48 amperes at 40° from the zero position. Then vx represents the current at this instant.

$$vx = 48 \sin 40° = 30.9 \text{ amps.}$$
$$vx = i = 30.9 \text{ amps.}$$

At the 90°, ox is vertical, as in Fig. 13–1(b), and a perpendicular drawn to o equals ox, or $vx = ox = 48$ amps.

At the 125° position, draw ox at 125° from the zero position, as in Fig. 13–1(c).

$$i = 48 \sin 125° = 48 \sin (180° - 125°)$$
$$= 48 \sin 55° = 48 \times 0.819 = 39.3 \text{ amps.}$$

At the 210° position, draw ox as in Fig. 13–1(d). Note that the direction of vx is downward from the horizontal, showing vx, or the instantaneous value, to be negative.

$$i = 48 \sin 210°$$

$$\sin 210° = -\sin (210° - 180°) = -\sin 30°$$

$$i = 48 (-\sin 30°) = 48 \times -0.50 = -24 \text{ amps.}$$

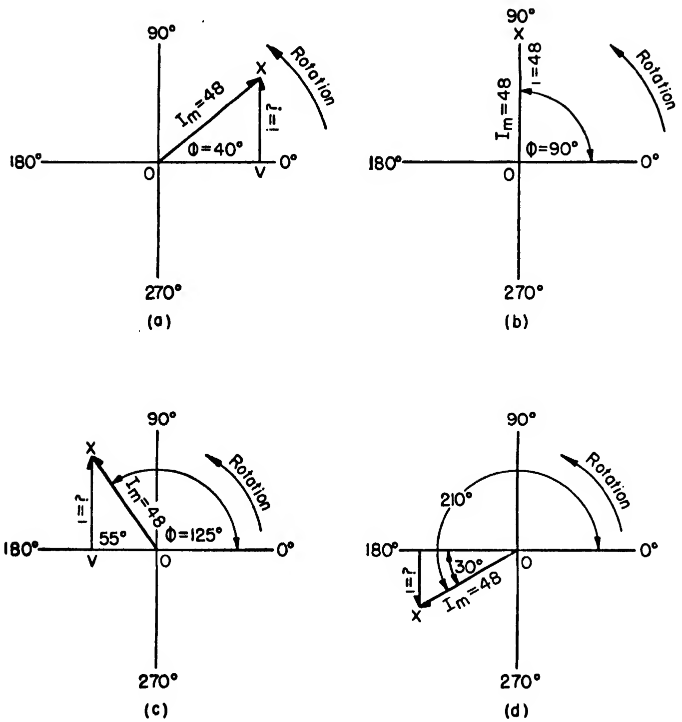


Fig. 13–1. Vector diagrams of current relations in the circuit of Example 3. The vector ox represents the maximum value of the current and the vector xv, the instantaneous value at various points in the cycle.

Figure 14–1 is a rough sine curve, showing the instantaneous current at 210°, corresponding to the vector diagram of Fig. 13–1(d).

In solving the following problems, construct the vector diagram and draw a rough sine curve, indicating the proper point in the cycle, as in Fig. 14–1.

Prob. 3–1. The maximum value of a current is 25 amperes. What current is flowing when it has completed 65° of its cycle?

Prob. 4-1. An alternating current has completed $\frac{1}{8}$ of its cycle and has a value of 15 amperes. What is the maximum value?

Prob. 5-1. What would be the instantaneous current in the circuit of Prob. 4-1 when $\frac{3}{5}$ of its cycle is completed?

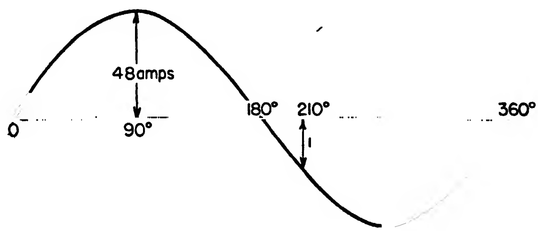


FIG. 14-1. Rough sine curve showing the direction and relative value of the current in Example 3, at the 210° point in the cycle. This corresponds to the vector diagram of Fig 13-1(*d*).

Prob. 6-1. The maximum value of a current is 125 amperes. What point in its cycle is reached when the current is 45 amperes negative and increasing?

Prob. 7 1. At what point in its cycle is the current in Prob. 6-1, if the instantaneous value is 45 amperes negative and decreasing?

9-1. An Alternating EMF. It has already been shown in Vol. I, Chapter X, that when a single coil rotates in a magnetic field, an alternating emf is generated in the coil. To refresh our memory, consider the single coil *BADC*, Fig. 15-1, to be rotating in counterclockwise direction at constant speed in a uniform two-pole magnetic field. The end *B* is connected to the outer collecting ring. This is a very elementary form of alternating-current generator.

In position (*a*), the coil sides are moving parallel to the magnetic flux (not cutting it) and no voltage is induced.

When the coil reaches position (*b*), the coil sides are moving at right angles to the flux and are cutting it at the greatest rate. Therefore, the maximum voltage is being induced in the direction shown by the arrows. Test this by Fleming's Right Hand Rule. (See Vol. I, page 230.)

In position (*c*), after a half revolution, the coil sides are again moving parallel to the flux and no voltage is being induced.

In position (*d*), the coil sides are again generating the maximum voltage. Note, however, that the induced voltage in this

position is in the opposite direction in the coil from that in position (*b*), as shown by the arrows.

And when the coil again reaches position (*a*), after one complete revolution, the voltage has again dropped to zero.

Thus, as the coil is rotated, a wave of voltage is induced, rising from zero to a maximum and then dropping off to zero during a

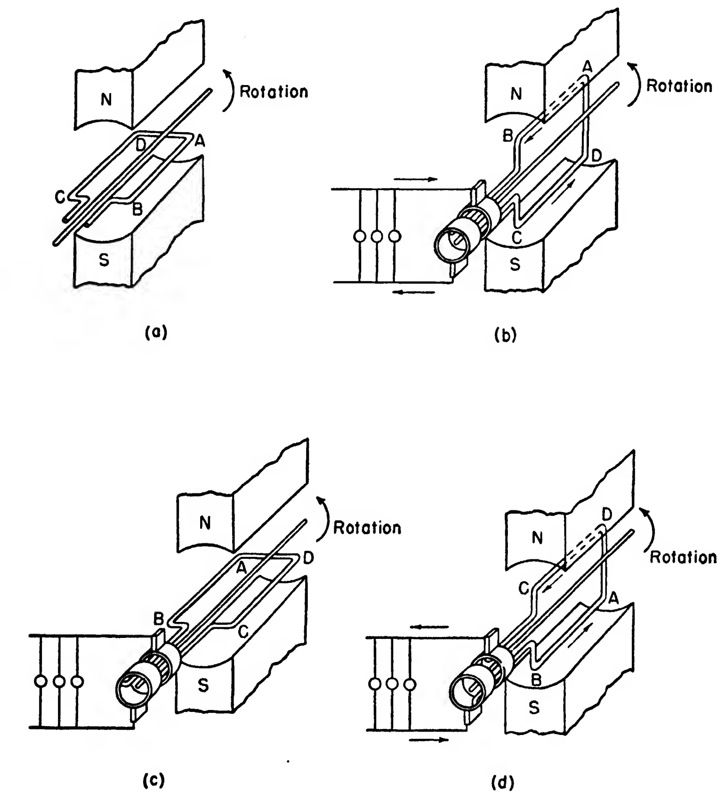


FIG. 15–1. Simple coil used to generate an alternating emf of sine wave form. This emf will set up a sine wave of current through the lamps in the external circuit.

half revolution; then rising to a maximum in the opposite direction and again dropping off to zero during the other half revolution. In one revolution, then, one emf cycle is completed.

Note that the induced voltage in the two coil sides is always in the opposite direction with respect to the axis of the coil, but in the same direction in the circuit; also since the coil sides are at 180°

with respect to each other as they rotate, the same voltage is induced in each coil side at any instant.

If the coil is rotated at constant speed in a uniform, or parallel, magnetic field the exact form of the voltage wave may be determined.

The coil of Fig. 15–1 is shown in cross section in Fig. 16–1 (*a*). Note that the coil is in the zero position and is generating no emf.

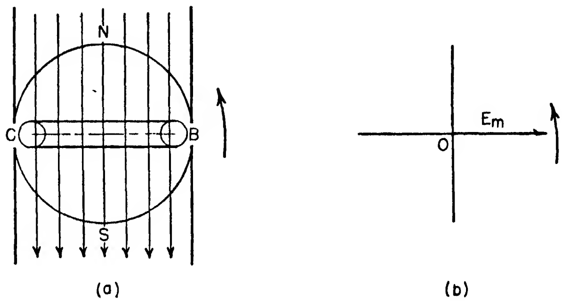


FIG. 16–1. (*a*) The coil *BC* is passing through the zero position and is cutting no magnetic lines. (*b*) The vector diagram of the emf in the coil. The emf at this instant is zero.

Figure 16–1 (*b*) shows the vector diagram in which, in this case, E_m represents the maximum induced emf. Note that E_m is drawn horizontally and that the instantaneous emf is zero.

When the coil reaches the 30° position, Fig. 17–1 (*a*), it is cutting flux and generating an emf in the direction shown. Figure 17–1 (*b*) shows the corresponding position of the vector, E_m, and the instantaneous emf, e_1.

Since $\sin 30° = \dfrac{e_1}{E_m}$

$$e_1 = E_m \sin 30°$$

If it be assumed, for instance, that $E_m = 250$ volts,

then $\quad e_1 = 250 \sin 30° = 250 \times 0.50 = 125$ volts.

When the coil reaches the 60° position, Fig. 18–1 (*a*), it is cutting flux at a faster rate and generating a greater emf. Figure 18–1 (*b*) shows the corresponding position of E_m and the instantaneous emf, e_2.

Thus $\quad e_2 = E_m \sin 60°$

$$= 250 \times 0.866 = 217 \text{ volts.}$$

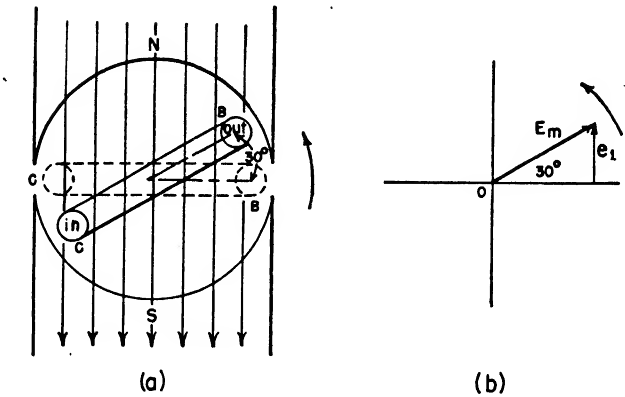


FIG. 17–1. (*a*) The coil *BC* at 30° from the zero position is generating an emf which tends to send a current *out* at *B* and *in* at *C*. (*b*) The vector diagram of the emf in the coil. The line e_1 represents the emf at this instant.

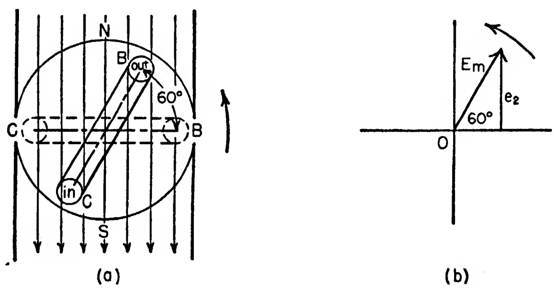


FIG. 18–1. (*a*) The coil *BC* passing through the 60° position. (*b*) Vector diagram showing value of the emf in the coil in this position.

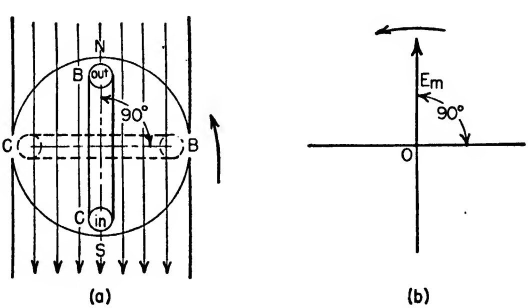


FIG. 19–1. (*a*) The coil *BC* is passing through the 90° position. (*b*) Vector diagram of emf in the coil in this position.

At the 90° position, Fig. 19–1(*a*), the coil is cutting at right angles across the flux and generating the maximum emf. The corresponding position of E_m is shown in Fig. 19–1(*b*). Note that at this instant, e equals E_m or that the instantaneous emf equals the maximum, 250 volts.

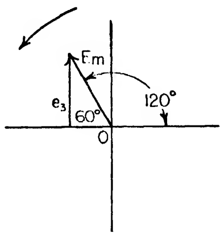


FIG. 20–1. Vector diagram of the emf when the coil has reached the 120° position.

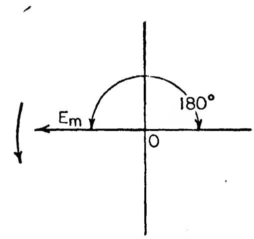


FIG. 21–1. Vector diagram of the emf of the coil in the 180° position.

In the 120° position, the emf has begun to decrease and the vector, E_m, is shown in Fig. 20–1. The induced emf at this instant equals e_3, or

$$e_3 = E_m \sin 120° = E_m \sin (180° - 120°)$$

$$= 250 \sin 60 = 250 \times 0.866 = 217 \text{ volts.}$$

When the coil has reached the 180° position, it has ceased cutting flux and therefore the induced emf has again dropped to zero, as shown by the vector, E_m, in Fig. 21–1.

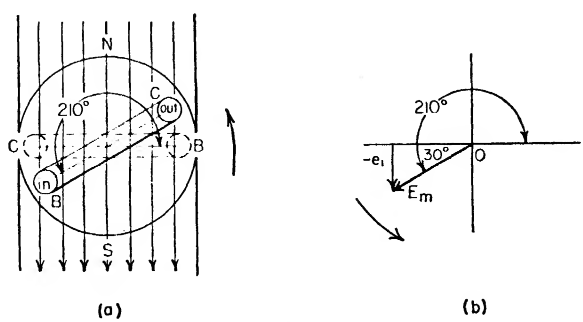


FIG. 22–1. (*a*) The coil *BC* is moving through the 210° position and the emf has been reversed. (*b*) Vector diagram showing instantaneous emf in the coil in this position.

In Fig. 22-1 (a), the coil is moving through the 210° position. It is cutting flux in the opposite direction and the emf induced is reversed, as indicated in the figure. The corresponding position of E_m is shown in Fig. 22-1 (b). Note that the instantaneous emf $-e_1$, is below the horizontal and, therefore, has a negative sign, also indicating that the induced emf has changed direction, Again,

$$e_1 = E_m \sin 210° = E_m [-\sin (210° - 180°)]$$
$$= E_m (-\sin 30°) = 250 \times -0.50 = -125 \text{ volts.}$$

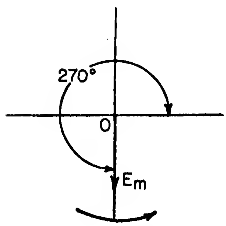


FIG. 23-1. Vector diagram of the emf when the coil is passing through the 270° position.

As the coil continues to rotate, the emf in this reversed direction increases until it reaches a maximum at the 270° position, as seen from the vector diagram of Fig. 23-1. This maximum value is exactly equal to that at the 90° position. The induced emf now begins to decrease, as the coil continues to revolve, until it again becomes zero, just as the coil has completed 360° and is starting the cycle over again. Since each instantaneous value is a point on the emf wave, we plot these values for a sufficient number of positions of the coil and obtain a sine curve of voltage as in Fig. 24-1.

The proof that the coil, rotating at constant speed in a uniform magnetic field, generates a sine wave of emf can be shown from the following construction.

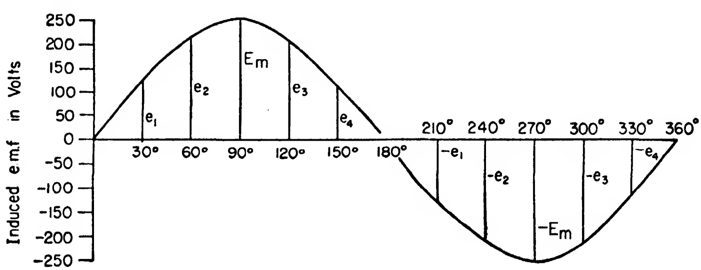


FIG. 24-1. Sine curve showing the values of emf throughout an entire cycle.

Consider Fig. 25-1 in which the small circles A, A' and A'' represent only one coil side of Figs. 15-1 or 16-1. In its rotation, counter-clockwise, the coil side follows the path of the large circle.

The arrows AC, $A'D$ and $A''E$, all equal and drawn as tangents to the large circle, represent the velocity of the coil side and its direction of motion at the instants shown. Since the voltage generated in any coil side, or conductor, is proportional to the rate at which it is cutting the magnetic lines, the conductor in position A is generating no emf, as it is moving parallel to the magnetic lines. The conductor is in a zero position. In position A'', the conductor is moving at right angles to flux, hence is cutting it at the greatest rate and the maximum voltage is being induced. The tangent $A''E$ is proportional to this maximum voltage.

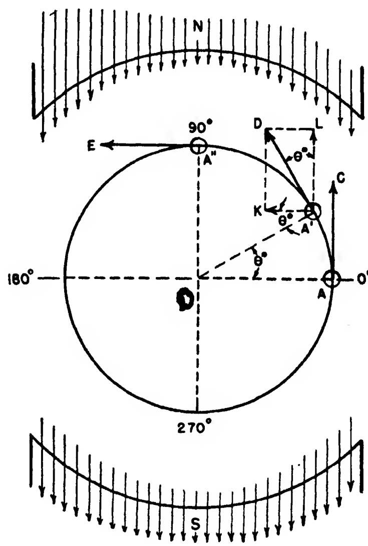


FIG. 25-1. Showing that when a conductor rotates at constant speed in a uniform magnetic field, a sine wave of emf is induced.

In any position, such as A', the conductor, or coil side, is moving at an angle of θ degrees from the zero position. At this instant, its velocity and direction of motion is $A'D$. This may be considered as consisting of two motions, one $A'L$ and the other $A'K$. In other words, the velocity of the conductor may be resolved into two components, one $A'L$, parallel to the flux, and the other, $A'K$, at right angles to the flux. Thus the rate at which the coil side in this position is cutting the flux, and hence the voltage induced at this instant, is represented by the length of the arrow $A'K$.

Now, angle AOA' = angle $OA'K = DA'L$ = angle θ, since their sides are respectively all parallel, or all perpendicular, to each other. The point O is at the center of the circle.

$$\text{Sine angle } \theta = \frac{LD}{A'D} \quad \text{or} \quad LD = A'D \sin \theta.$$

But,

$LD = A'K$; and $A'D = A''E$, the maximum rate of cutting flux;

thus,

$A'K = A''E \sin \theta.$

Hence, the rate at which the conductor, or coil side, at any angle θ degrees from the zero position, cuts the magnetic flux is equal to the maximum rate of cutting, multiplied by the sine of the angle at that instant; or the instantaneous voltage induced in a conductor at θ degrees from the zero position, equals the maximum voltage, multiplied by the sine of θ degrees, and we may write

$$e = E_m \sin \theta, \tag{3-1}$$

where e = instantaneous emf at θ degrees;
E_m = maximum emf,
θ = the angular position of the coil side at this instant.

This, as has been shown, is the equation of a sine curve.

Every modern a-c generator, or "alternator," maintains an emf having approximately a sine wave form; the closer the approximation the better, since irregular wave forms cause trouble in interconnected equipment.

However, the generation of a sine wave of emf is not so readily obtained as indicated in the discussion above, for the modern alternator is not built of a single coil, but of a large number of coils. Also, the uniformity of the field, or the distribution of the flux in the air gaps, is affected both by the teeth on the armature core and by the shape of the pole shoes.

Thus the emf induced in a single coil may differ materially from a sine wave, but a number of coils may be so arranged that the emf across the combination has the standard form.

An "oscillogram," showing the emf curve of a 6600-volt alternator, is shown in Fig. 26–1. Note how closely it follows the form of a sine curve.

Solve as before, using rough sine curve, vector diagram and equation for each problem.

Prob. 8–1. The maximum value of an alternating emf is 2200 volts. What is the instantaneous value when 75° of the cycle have been completed?

Prob. 9–1. What is the instantaneous value of the emf of Prob. 8–1 when it is passing through the 200° point of its cycle?

Prob. 10–1. The instantaneous value of an alternating emf is 1400 volts at 65°. What is the maximum value?

Prob. 11–1. How much of its cycle has the emf of Prob. 10–1 completed when the instantaneous value is 300 volts positive and increasing?

Prob. 12 1. The instantaneous value of an alternating emf is 450 volts 50° after its zero value. What is it at 135°?

Prob. 13-1. The maximum value of an alternating emf is 800 volts. What are the instantaneous values at the following instants: 20°, 80°, 130°, 210°, 310° and 340°?

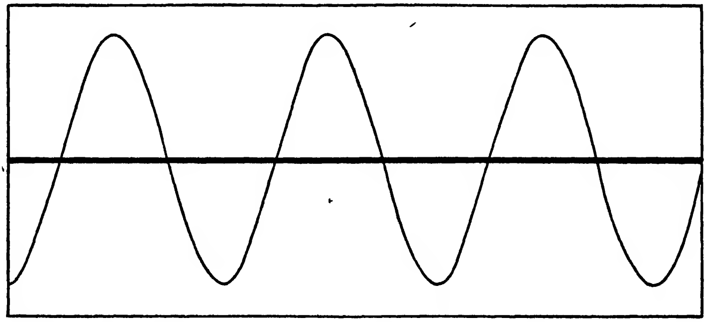


FIG. 26–1. Potential wave, no load, for a 150 kva, 60-cycle three-phase generator, $1\frac{1}{3}$ slots per pole. (*Proc. AIEE.*)

Prob. 14-1. What value and what sign will the instantaneous voltage have when an emf has completed two-thirds of its cycle, if E_m is 2300 volts?

10–1. Alternating-Current Generator: Revolving Field Type. In the generation of an emf, it makes no difference whether the coils cut the flux or the flux cuts the coils. In either case, an alternating emf is set up in the coils.

Small low voltage alternators may be constructed with rotating armatures and stationary fields, but most modern alternators are constructed with rotating fields and stationary armatures. Note in Fig. 27-1 that it is the field poles which revolve. The armature coils are imbedded in the frame of the machine and are cut by the revolving magnetic flux. The revolving-field type has the advantage that the high alternating voltages and large alternating currents are confined to the stationary windings and are not taken off from moving contacts. Note that the only moving contacts are the two small collecting rings, which take the low-voltage direct current from the brushes and conduct it to the field coils. Of course the field coils carry direct current and must be excited from a separate d-c source of constant voltage.

11–1. Frequency. When the coil of the two pole machine of Fig. 15–1 has made one complete revolution, it has passed through 360° in space, or "space degrees." During this time, it has passed

one pair of poles (a north and a south) and has completed one cycle, or 360 " Electrical degrees " or " time degrees," as shown in Fig. 24–1.

(Thus in a two pole machine, one space degree equals one electrical, or time degree/

If the coil is assumed to be rotating at 30 rps, or 1800 rpm, it would complete 30 cycles in one second, or have a frequency of 30 cycles per second. In $\frac{1}{30}$ of a second, the curve would pass

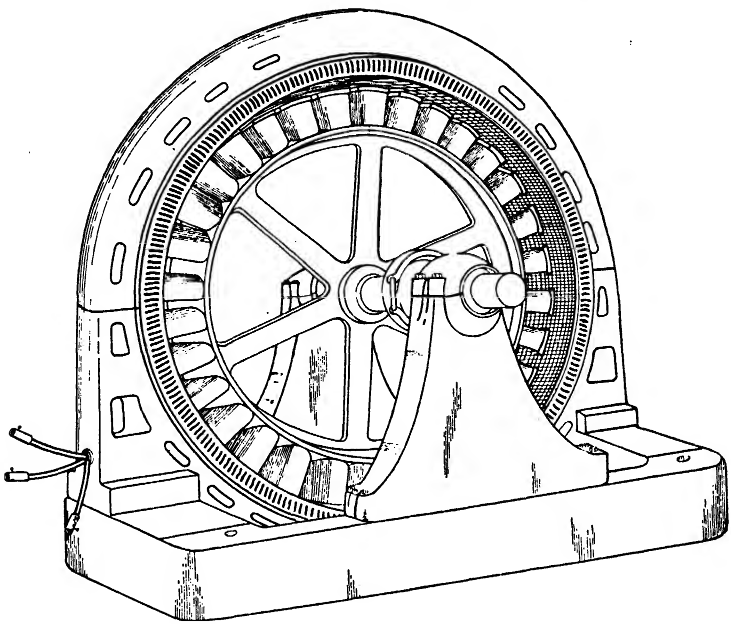


FIG. 27–1. A modern alternator with revolving field poles.

through 360°; in $\frac{1}{60}$ of a second, 180°; in $\frac{1}{120}$ second, 90°, etc. Thus the curve of Fig. 24–1 may be plotted between instantaneous values of emf and time.

One half cycle or 180 time degrees is called one alternation.

In a multipolar alternator, the space degrees and time degrees are not the same. When conductor, *a*, in the four-pole machine of Fig. 28–1 has made one revolution, it has passed through 360° in space. It has also passed two pairs of poles and completed two cycles or 2 × 360° or 720 time degrees. Note that the voltage in conductor, *a*, in passing from position (1) to position (2), rises from zero to a maximum. Thus one quarter of a cycle (90 time degrees)

is completed, while the conductor has moved through only 45 degrees in space. In passing on to position (3), the voltage in the conductor has again become zero and one-half cycle (180 time degrees) has been completed, while the conductor has moved from position (1) to position (3) or 90° in space. In a four pole machine, then, one space degree equals two time degrees.

Similarly, in a six-pole machine, three cycles are completed in one revolution, and one space degree equals three time degrees.

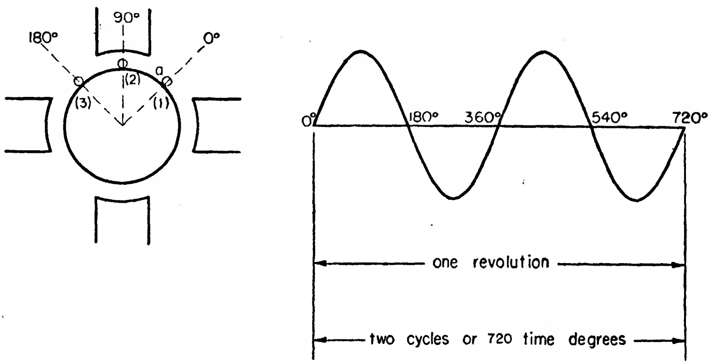


FIG. 28-1. In a four-pole machine the voltage induced in the conductors completes two cycles or 720 time, or electrical, degrees in one revolution.

The relation for machines of any number of poles can readily be seen.

If the conductor in the four-pole machine of Fig. 28-1 also is assumed to be rotating at 30 rps or 1800 rpm, it would pass through 2×30, or 60 cycles in one second, and the frequency would be 60.

The frequency of any alternating current generator, then, is equal to the revolutions per second, multiplied by the number of pairs of poles. Since the speed of rotating machinery is generally expressed in revolutions per minute, the equation for frequency is written

$$f = \frac{\text{rpm}}{60} \times \frac{P}{2}, \quad \text{or} \quad f = \frac{\text{rpm}}{120} \times P \qquad (4\text{-}1)$$

where f = frequency in cycles per sec; P = number of poles.

Example 4. What is the frequency of a 10 pole alternator which is driven at 600 rpm?

Solution. $$f = \frac{600}{120} \times 10 = 50 \text{ cycles per sec.}$$

Frequencies in this country are standardized at 60 cycles and 25 cycles, although some transmission lines in California operate a 50 cycles.

The lower the frequency, the less is the " line drop " in a transmission line. Against this is the fact that the lower the frequency, the greater is the relative amount of iron and copper which must be used in the construction of transformers and induction motors, thus increasing their cost. Moreover, incandescent lamps " flicker " when operated on circuits much below 60 cycles.

The use of 25 cycles is limited almost entirely to plants supplying power for electric traction.

In the early days of the industry, frequencies of 133 cycles and 40 cycles were used, and today some plants in Europe operate on frequencies as low as 15 cycles.

Prob. 15-1. What is the frequency of a 6-pole alternator if it is driven at 1200 rpm?

Prob. 16-1. How many poles has a 25-cycle a-c generator, if it runs at 1500 rpm?

Prob. 17-1. At what speed must a 12-pole alternator run to produce 60 cycles per sec?

Prob. 18-1. A small synchronous motor has 2 poles and is to run at 7200 rpm? What must be the frequency of the circuit?

Prob. 19-1. How many poles must a 25-cycle alternator have, if it is to be run at the same speed as a 60-cycle machine having 24 poles?

12-1. The Average Value of a Sine Curve. Since half of the instantaneous values of a sine curve are negative and half are positive, and since the negative values are exactly equal to the positive values, the average for a complete cycle of values must be zero. But the average of a half cycle, or one alternation, is not zero. Nor is the actual average value of the emf of an alternating current generator zero. It would be just as reasonable to say that the actual average value of the pressure exerted by the piston of the water pump in Fig. 3-1 is zero, just because the pressure alternates with equal values in opposite directions. The actual average value in both cases is the average of all the instantaneous values, regardless of signs, which indicate direction only.

The average value of a sine curve equals 0.636 of the maximum. This average may be easily found by taking a number of instantaneous values at equal intervals and finding their average. Since

each half cycle or alternation of the curve is the same, it is necessary to take values for a half cycle only, as in the example below.

Example 5. Let the maximum value of a sine wave of emf be 10 volts. Compute the instantaneous values every 5° from 0° to 180° as shown in Table I, Art. 14–1. This will give 36 instantaneous values. Their sum will be approximately 229.

$$\text{Average value} = \frac{229}{36} = 6.36 \text{ volts}$$

Thus the average value of a sine curve equals 0.636 of the maximum value. In the form of an equation,

$$\text{Ave } e = 0.636E_m. \tag{5–1}$$

or

$$E_m = \frac{1}{0.636} \text{ Ave } e = 1.57 \text{ Ave } e$$

It is also apparent that the average and maximum values of a sine wave of current have the same relation or

$$\text{Ave } i = 0.636I_m;$$

and

$$I_m = 1.57 \text{ Ave } i. \tag{6–1}$$

It should be noted here that this ratio, 0.636 to 1.00, holds true only when the wave shape is a sine curve. For instance, in a peaked wave such as that in Fig. 29–1, the average value can readily be shown to be only 0.50 the maximum.

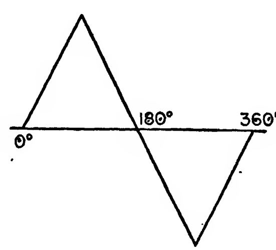


FIG. 29–1. A peaked wave of emf or current.

13–1. To Find the Maximum Value of an Alternating EMF. In Volume I, Chapter X, we learned that in a d-c generator, the total number of lines cut per second by a conductor determined the average value of the voltage induced in the conductor. Thus

$$\text{Ave } E = \frac{\text{lines cut per second}}{} \tag{7–1}$$

Now, if the several conductors which make up a winding are bunched or concentrated* there is practically the same voltage induced in each conductor at any instant. The average voltage induced in the winding is, then, merely the voltage induced in any

* See Article 23, Chap. VI, on Armature Windings.

one conductor, multiplied by the number of conductors in series in the winding.

Of course, in an armature where the conductors making up a winding are **distributed*** over the core, all are cutting flux at different rates at any instant. The average voltage, induced in such a winding, clearly is not the voltage induced in one conductor multiplied by the number of conductors in series.

Thus the average voltage, induced in an alternator using an armature with a concentrated winding, may be found by use of the above equation, if the number of series conductors in the winding is taken into consideration.

And, if the emf is a sine curve, the maximum value may be found by multiplying this average by the constant 1.57, as computed in Art. 12 above. This is about the only case in which the average value of an emf is ever used.

Example 6. What is the maximum voltage generated in a drum armature, consisting of 300 series conductors, concentrated winding, which has a speed of 1200 rpm? Each conductor cuts twice through a field of 1,500,000 lines during each revolution.

Solution:

$$\text{Ave } E = \frac{\text{lines cut per sec} \times \text{no. of series conductors}}{10^8}$$

$$= \frac{1{,}500{,}000 \times 2 \times 300 \times 1200}{10^8 \times 60} = 180 \text{ volts}$$

$$E_m = 1.57 \times 180 = 283 \text{ volts}$$

Prob. 20-1. A bipolar a-c generator with a drum armature, concentrated winding, has a speed of 2400 rpm. The field of each pole has 2,500,000 lines. Number of series conductors on the armature is 600. (*a*) What is the maximum value of the emf generated? (*b*) What is the frequency?

Prob. 21-1. A 4-pole a-c generator has a drum-wound armature with 2000 series conductors, concentrated winding. Speed is 2700 rpm. Flux per pole is 2,400,000 lines. (*a*) What is the average emf? (*b*) The maximum emf? (*c*) What is the frequency?

Prob. 22-1. In an alternator, one set of coils containing 200 concentrated conductors is being cut 8 times per second by 10^7 lines. What maximum emf is being induced in the coils?

Prob. 23-1. It is desired to generate a maximum alternating emf of 10,000 volts. The number of concentrated conductors in series in the armature is 1000. There are four poles and the machine is run at 2400 rpm. What is the flux per pole?

* See Article 28, Chap. VI, on Armature Windings.

Prob. 24–1. A 6-pole a-c generator has 800 concentrated conductors in series on the armature. Flux per pole is 4×10^6 lines. It is desired to generate a voltage of 7550 maximum. (*a*) At what speed must the machine run? (*b*) What is the frequency?

Prob. 25–1. What would be the average voltage and the frequency of the generator in Prob. 24, if the speed were reduced one half and the flux doubled?

14–1. Alternating Current Ampere: Effective Value. It might seem, at first thought, that the average value of a sine wave of alternating current should be used to measure alternating current in amperes. But, if the wave is considered over a complete cycle, there is just as much negative as positive current. If a direct-current ammeter is used to measure this current, it will read zero.

Also, an ampere is defined as that **steady** rate of flow which will deposit a standard amount of silver from a standard solution in one hour. But an alternating current is not a steady current, and neither will it deposit any silver from a solution, since nearly all it deposits in one half cycle it takes off during the other half. Therefore, the value of an alternating current cannot be determined by its average value.

However, the value of a current can be determined by its heating effect. We have learned that if a given current is put through a coil of wire of given resistance for a certain time, a definite amount of heat will be generated, which can be computed. (See Vol. 1, Chapter IV.) This is true regardless of the direction of the current through the coil. The heat generated varies with the square of the current, or is proportional to I^2R, and the rate at which heat is generated at any instant is proportional to i^2 at that instant.

Therefore, the heating effect of an alternating current during one complete cycle, or any period of time is proportional to the square of all the instantaneous values during that time, and the average of these squared values gives the square of a steady current which would produce the same heating. Thus the square root of this average gives the steady current in amperes which would produce the same heating during the cycle.

Note that we do not average the instantaneous values and then square them. This average heating effect can be found only by squaring the instantaneous values first, and then averaging them.

This current is known as the " effective value " and, for a sine curve, is equal to 0.707 of the maximum value. It is often called the " square root of mean squares " or the rms (root mean square)

value. This effective value is the current that a-c ammeters indicate, and is the value by which alternating currents are measured, unless it is definitely specified otherwise.

The value of an ampere of alternating current may thus be defined as that current which, flowing through a given resistance, will produce heat at the same rate as an ampere of direct current.

The method described above of determining the effective value of an alternating current can be clearly shown by the example below.

Example 7. Assume that 10 amperes is the maximum value of the sine curve (a) in Fig. 30–1. Since all half cycles of the curve are

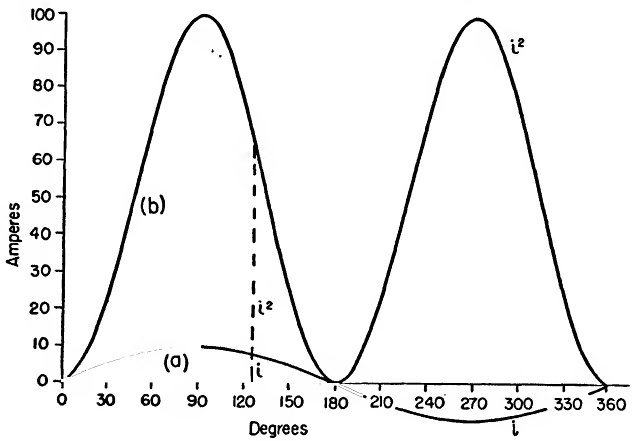


FIG. 30–1. Curve (a) is a sine curve of current, having a maximum value of 10 amperes. Curve (b) is a sine squared curve, plotted from the squares of the instantaneous values of the sine curve (a).

identical, this instantaneous value can be found for each 5° from 0° to 180°, as in Example 5, shown in Table I. This will give 36 instantaneous values. If each of these values is squared, also shown in Table I, and these squared values are plotted at corresponding instants over the cycle, a " sine squared " curve with a maximum of 100 amperes will result, as shown by curve (b) in Fig. 30–1. Note that when instantaneous values are negative, their square becomes positive and both loops of this second curve are above the horizontal. From Table I, the sum of the squared values equals 1800 approximately.

$$\text{Effective value} = \sqrt{\frac{1800}{36}} = \sqrt{50} = 7.071$$

Since the effective value is approximately 0.707 of the maximum this relation may be expressed by the equation

$$I = 0.707 I_m.$$

or
$$I_m = \frac{1}{0.707} I = 1.414I;$$

But
$$1.414 = \sqrt{2}$$

Accordingly, the equation is often written,

$$I_m = \sqrt{2}I \tag{8-1}$$

or
$$I = \frac{I_m}{\sqrt{2}}$$

where I_m = maximum value of the current,
I = effective value of the current.

It should be definitely understood that this ratio of 0.707 between effective and maximum values holds true only for a sine

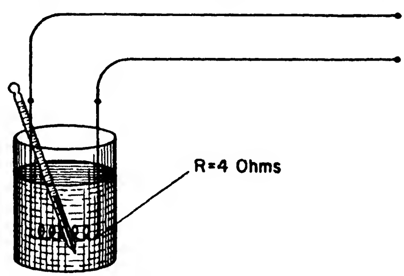


Fig. 31–1. The alternating current will heat the 4-ohm coil and raise the temperature of the liquid.

curve. For instance, the ratio between effective and maximum values for the peaked wave of Fig. 29–1, computed by the method described above, is approximately only 0.578.

Prob. 26–1. The maximum of a sine wave of alternating current is 24 amperes. Find the instantaneous values every 15° for one cycle of this current and plot the curve. From these instantaneous values compute the effective value of this current.

Prob. 27–1. The effective value of a sine curve of alternating current is 250 amperes. What is the instantaneous value at the 30° instant in the cycle?

That the heat, generated by an alternating current is equal to a steady or direct current of the same value as the effective value of the a-c wave, can be shown by the solution of the following problem.

Prob. 28–1. A sine wave of current of 10 amperes maximum value and 25 cycles frequency is sent through a 4 ohm wire immersed in water in a calorimeter, as in Fig. 31–1. (*a*) On fine coordinate paper, plot the

current curve, Fig. 32–1, using instantaneous values every 15° throughout the cycle. (*b*) Compute from the heat equation ($H = .24I^2Rt$), the heat in calories throughout the cycle. (*c*) Determine the steady or direct current which will generate the same heat as in (*b*).

Suggestion. The rate in calories per second at which heat is generated by each instantaneous current (i) thus drawn, would equal $.24i^2R$ or $.24 \times 4i^2$. Compute these values and plot the curves H and H_1. Note that the square of a negative i is positive and H_1 is above the line.

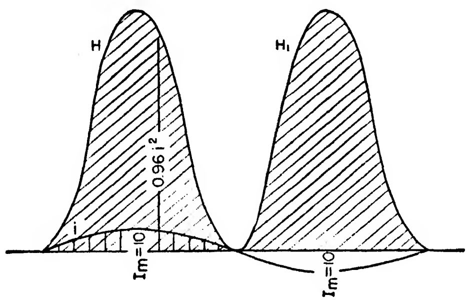


FIG. 32–1. The area of the loops, H and H_1 represent the heat given off in one cycle in Prob. 29–1.

TABLE I

AVERAGE AND EFFECTIVE VALUES OF A SINE CURVE MAXIMUM VALUE = 10

Angle in degrees	10 sin θ	(10 sin θ)^2	Angle in degrees	10 sin θ	(10 sin θ)^2
5	0.872	0.76	95	9.962	99.20
10	1.740	3.03	100	9.850	97.00
15	2.590	6.70	105	9.660	93.30
20	3.420	11.60	110	9.400	88.30
25	4.226	17.90	115	9.063	82.10
30	5.000	25.00	120	8.660	75.00
35	5.736	33.00	125	8.192	67.10
40	6.430	41.40	130	7.660	58.75
45	7.071	50.00	135	7.071	50.00
50	7.660	58.75	140	6.430	41.40
55	8.192	67.10	145	5.736	33.00
60	8.660	75.00	150	5.000	25.00
65	9.063	82.10	155	4.226	17.90
70	9.400	88.30	160	3.420	11.60
75	9.660	93.30	165	2.590	6.70
80	9.850	97.00	170	1.740	3.03
85	9.962	99.20	175	0.872	0.76
90	10.000	100.00	180	0.000	0.00

Summation (10 sin θ) = 229.064 Summation (10 sin θ)^2 = 1800.28

36 instantaneous values. Average value $= \frac{229.064}{36} = 6.36$

$$\text{Effective value} = \sqrt{\frac{1800.28}{36}} = \sqrt{50} = 7.071$$

Since the horizontal axis represents time ($\frac{1}{25}$ of a second), the area included under these loops represents, and is directly proportional to the heat generated by all the instantaneous currents during one complete cycle. This is true from the fact that the area of the loops equals the average length of ordinates, multiplied by the abscissas. The ordinates equal $.24i^2R$ and the abscissas are time units. Thus the calories per small square may be calculated; and from the number of small squares under the loops, the calories generated in one cycle may be computed.

15–1. Effective Value of an Alternating EMF. If an alternating emf wave is a sine curve, it is obvious, from the preceding article, that the effective value is also 0.707 of the maximum value. And this relation is expressed by the equation,

$$E = 0.707E_m = \frac{E_m}{\sqrt{2}}$$

or

$$E_m = \sqrt{2}E = 1.414E \tag{9-1}$$

where E_m = maximum value of the emf;

E = effective value of the emf.

Alternating-current voltmeters indicate effective values.

The torque of the voltmeter, at any instant, is proportional to the square of the current in the coils of the instrument, and so is proportional to the square of the voltage on the instrument at that instant. Therefore, the average torque is proportional to the average of the instantaneous voltages squared over the cycle. The scale is graduated to indicate the square root of this average, which, as we have seen, is the effective value of the voltage. Thus the indications of the voltmeter are proportional to the effective value. This is true, regardless of the shape of the emf wave.

It is important to note here that when we speak of an alternating current of so many amperes and an alternating emf of so many volts, it is always **effective current** and **effective voltage** that is

understood. These values are always used in the measurement of current and voltage, since the scales of a-c ammeters and voltmeters are graduated to read in terms of effective values.

The effective values of current and voltage may also be used in

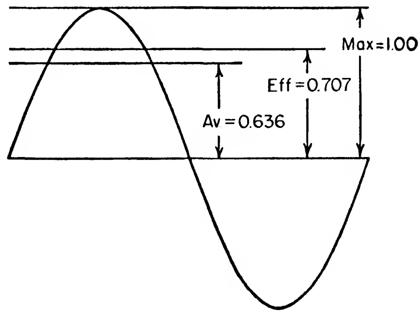


FIG. 33–1. Relation of maximum, effective and average value of a sine curve.

vector diagrams instead of maximum values, as will be shown in the next chapter.

The relation of the maximum, effective and average values of a sine curve is clearly shown in Fig. 33–1

$$\text{The ratio of maximum to effective} = \frac{1.00}{0.707} = 1.414;$$

$$\text{The ratio of maximum to average} = \frac{1.00}{0.636} = 1.57;$$

$$\text{The ratio of effective to average} = \frac{0.707}{0.636} = 1.11.$$

The ratio, of effective to average value, is called the **form factor.** The form factor is used principally in the computation of the effective values of a sine wave of current or voltage when the average value is known. The ratio of the maximum value to the effective value is called the **crest** or **peak factor,** and is of much greater importance because it indicates the greatest instantaneous voltage which is being applied to various parts of the apparatus. This maximum, rather than the effective value is the voltage which the insulation must be able to withstand. The value, 1.41, is that of a sine-curve only.

Prob. 29–1. What is the effective value of a sine curve of alternating emf, if the instantaneous value at 50° is 500 volts?

Prob. 30–1. What would a voltmeter indicate if put across the terminals of the machine in Prob. 21–1.

Prob. 31–1. If a voltmeter were put across the terminals of the machine in Prob. 22–1, how much would it indicate?

Prob. 32–1. At what speed must the generator in Prob. 23–1 run to generate 10,000 volts?

Prob. 33–1. At what speed must the generator in Prob. 23–1 run to generate 7550 volts?

Prob. 34–1. What is the maximum value of an alternating current of 32 amperes?

Prob. 35–1. An electric flat iron draws an alternating current of 4.5 amperes. What maximum current flows through the iron?

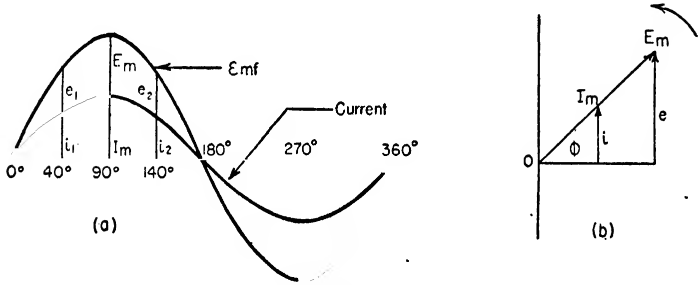


Fig. 34–1. (*a*) Sine curves of voltage and current in phase. (*b*) Vector diagram of voltage and current in phase.

Prob. 36–1. It requires about 150 amperes d-c to thaw out a frozen water pipe in a given time. What value of alternating current would be required to thaw the pipe in the same time? What is the maximum value of this current?

16–1. Phase Relations of EMF and Current. Whenever an alternating emf sends a current through a circuit, this current is also alternating at the same fundamental frequency as the emf wave. If the emf is a pure sine curve, the current curve, under normal conditions, also will have the same form. Thus the equation and vector diagram will be similar. Therefore, the curves of voltage and current in a circuit can both be drawn on the same axis as in Fig. 34–1(*a*). The vector diagrams can also be drawn on the same pair of axes, as in Fig. 34–1(*b*).

In a-c circuits, there are three possible "phase relations" between emf and current.

(*a*) The current may be "in phase" with the voltage curve, as in Fig. 34–1.

(*b*) The current may "lag behind" the voltage curve, as in Fig. 35–1.

(*c*) The current may "lead" the voltage curve, as in Fig. 37–1.

17–1. EMF and Current in Phase. When the emf and current are "in phase," both current and voltage curves reach a maximum at the same time and pass through their zero values at the same instant, as shown in Fig. 34–1 (*a*). Also at any instant throughout their complete cycle, they are in the same phase. This is shown also by the vector diagram of Fig. 34–1 (*b*), in which the two vectors, E_m and I_m, representing the maximum values of the voltage and current respectively are always in the same position with respect to each other as they rotate. Thus in Fig. 34–1 (*b*), each vector is at $\phi°$ with respect to the horizontal axis, and the value of both the voltage and the current at this instant may be written as,

$$e = E_m \sin \phi°,$$

$$i = I_m \sin \phi°.$$

At the 40° position, as indicated in Fig. 34–1 (*a*), these values are

$$e = E_m \sin 40°,$$

$$i = I_m \sin 40°.$$

18–1. Ohm's Law in A-C Circuits. If a resistor offers 1 ohm resistance to the flow of direct current, it will also offer 1 ohm resistance to the flow of alternating current, provided the resistor has no inductance and the alternating current flowing through it produces no eddy currents, nor "skin effect."*

When one alternating volt is put across such a resistor, one alternating current ampere will flow.

We have learned from Ohm's law for d-c circuits that the current through a resistor (at any instant) equals the voltage across the resistance (at that instant) divided by its resistance, or $i = \dfrac{e}{R}$. This is also true in an a-c circuit containing resistance only.

* At high frequencies, the resistance of a conductor is greater than that determined by direct-current measurement. This is due to the fact that, as the frequency increases, there is an increasing tendency for the current to flow only in the outer part of the conductor cross section. This has the effect of decreasing the useful area of the conductor, thereby increasing its resistance. This is called "skin effect."

Thus one maximum volt will force one maximum ampere through a resistance of one ohm, or

$$I_m = \frac{E_m}{R}. \tag{10-1}$$

Similarly, one effective volt will force one effective ampere through one ohm resistance, or

$$I = \frac{E}{R}. \tag{11-1}$$

Also when the a-c voltage passes through the zero value, the current will be zero.

Thus the current in a circuit, containing resistance only, is " in phase " with the voltage, as shown in Fig. 34-1.

It should be noted here that Ohm's law, applied to both direct and alternating current circuits, deals with voltage, current and resistance only, and holds true where these three quantities above are concerned. It does not apply to parts of circuits containing inductance and capacitance.

Example 8. A lamp having 55 ohms resistance only is put across an a-c emf, the effective value of which is 110 volts. (*a*) What maximum current does the lamp take? (*b*) What is the current through the lamp when the voltage is at the 60° point in its cycle? (*c*) What effective current does the lamp take?

Solution. (*a*) $$E_m = \frac{E}{0.707} = \frac{110}{0.707} = 156 \text{ volts},$$

$$I_m = \frac{E_m}{R} = \frac{156}{55} = 2.83 \text{ amp.}$$

(*b*) $$e = E_m \sin \phi$$

$$= 156 \sin 60° = 156 \times 0.866 = 135 \text{ volts.}$$

$$i = \frac{e}{R} = \frac{135}{55} = 2.45 \text{ amp.}$$

or by the equation,

$$i = I_m \sin \phi$$

$$= 2.83 \sin 60° = 2.83 \times 0.866 = 2.45 \text{ amp.}$$

(*c*) $$I = \frac{E}{R} = \frac{110}{55} = 2 \text{ amp.}$$

Prob. 37-1. The effective voltage on a circuit containing 30 ohms resistance only is 115 volts. What is the effective current?

Prob. 38-1. (*a*) At what phase angle is the voltage in Prob. 37-1 when its instantaneous value is 95 volts positive and increasing? (*b*) What is the value of the current at this instant? Show vector diagram of both voltage and current.

Prob. 39-1. A maximum voltage of 340 volts is impressed on a resistance of 85 ohms. Draw a vector diagram, showing the voltage and current at 240°. What is the value of both voltage and current at this instant?

Prob. 40-1. An 8-ohm resistor and a 12-ohm resistor are in series across a 120-volt circuit. What is the voltage across each resistor?

Prob. 41-1. At the instant the voltage across the 8-ohm resistor is 20 volts, what is the voltage across the 12-ohm resistor? What is the current in the circuit at this instant?

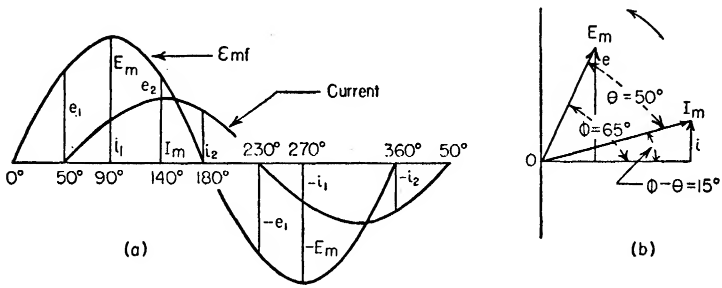


FIG. 35-1. (*a*) Sine curves in which the current lags 50° behind the voltage. (*b*) Vector diagram of voltage and current in (*a*).

19-1. Lagging Current. The current in an a-c circuit may lag the voltage, as in Fig. 35-1. Note in Fig. 35-1 (*a*) that electrical degrees, or time, is measured from left to right and that the current curve lags 50° behind the voltage curve; that is, the voltage curve has reached a value e_1 at 50°, while the current curve is zero at 50°. Also, the voltage curve reaches its maximum at the 90° point, while the current curve reaches its maximum at the 140° point, or 50° later. Thus when the voltage curve has become zero again at 180°, the current curve does not reach its zero value until 50° later and thus has a value of i_2. The current, in this case, is said to be lagging 50° " out of phase " with the voltage.

Figure 35-1 (*b*) shows how this same phase difference is represented in a vector diagram. Since the current reaches its maximum value 50° later than the voltage, the vector I_m lags 50° behind the vector E_m. (Vectors are rotating counter-clockwise.) Thus when vector E_m has reached the 65° point in its cycle, the vector I_m has reached only the $65° - 50°$ or 15° point in its cycle.

The values of voltage and current at this instant are respectively,

$$e = E_m \sin 65°$$
$$i = I_m \sin (65° - 50°) = I_m \sin 15°.$$

The general equations for lagging current are

$$e = E_m \sin \phi,$$
$$i = I_m \sin (\phi - \theta),$$

where ϕ = phase angle of voltage in degrees,
θ = difference in phase between E_m and I_m.

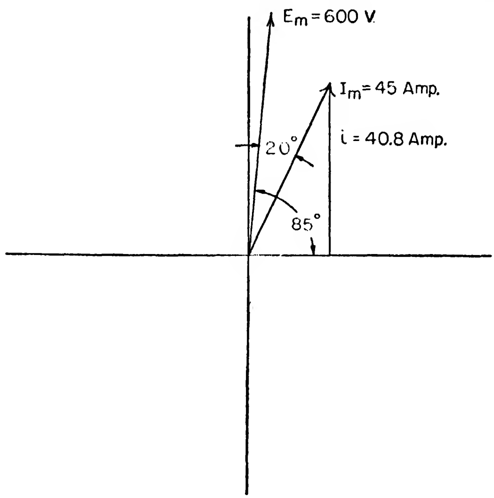


FIG. 36–1. Voltage is at 85° point in its cycle. Current lags 20° behind the voltage.

It will be shown in Chap. IV that inductance in an a-c circuit is a cause of lagging current.

Example 9. In an a-c circuit, the current lags 20° behind the voltage. The maximum value of the voltage is 600 volts, and of the current is 45 amperes. What is the instantaneous value of the current when the voltage is at its 85° phase?

Solution. Draw the vector diagram of Fig. 36–1. When the voltage is at 85°,

$$i_1 = I_m \sin (\phi - \theta),$$
$$= 45 \sin (85° - 20°),$$
$$= 45 \sin 65° = 40.8 \text{ amperes.}$$

Show vector diagrams in the solution of the following problems.

Prob. 42–1. In Example 9, what instantaneous values will the current and voltage have when the voltage is at the 135° point in its cycle?

Prob. 43–1. In an a-c circuit, the current lags the voltage by 45°. The current has an instantaneous value of 65 amperes when the voltage is at its maximum positive value. (*a*) What is the maximum value of the current? (*b*) Its effective value?

Prob. 44–1. (*a*) What is the maximum value of the voltage in Prob. 43–1, if the instantaneous value is 750 volts when the current is at its maximum positive value? (*b*) The effective value of the voltage?

Prob. 45–1. The maximum value of an a-c voltage is 2300 volts and the maximum value of the current is 60 amperes. If the instantaneous value of the current is 20.5 amperes when the instantaneous value of the voltage is 1762 volts, what is the phase difference between current and voltage?

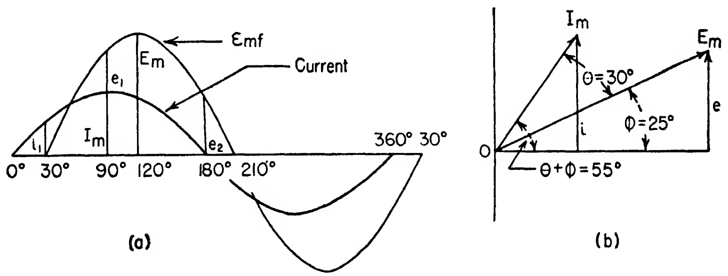


FIG. 37–1. (*a*) Sine curves in which the current leads the voltage by 30°. (*b*) Vector diagram of voltage and current in (*a*).

20–1. Leading Current. The current may lead the voltage, as in Fig. 37–1. Note that the current curve in Fig. 37–1(*a*) has reached a value i_1 at 30°, while the voltage is at zero. Also while the current reaches its maximum value at the 90° point, the voltage does not reach its maximum until the 120° point or 30° later. And so on throughout the cycle. The vector diagram of Fig. 37–1(*b*) shows the current vector I_m is 30° ahead of the voltage vector E_m. Thus when the voltage E_m has reached the 25° point in its cycle, the current vector I_m has reached the 25° + 30°, or 55° point in its cycle. The values of voltage and current at this instant are

$$e = E_m \sin 25°,$$

$$i = I_m \sin (25° + 30°) = I_m \sin 55°.$$

The general equations for a leading current are

$$e = E_m \sin \phi, \tag{12-1}$$

$$i = I_m \sin (\phi + \theta), \tag{13-1}$$

where $\theta =$ difference in phase between E_m and I_m.

It will be shown in Chapter IV that capacitance in an a-c circuit is a cause of leading current.

Example 10. In a certain a-c circuit, the current leads the voltage by 40°. The effective current is 100 amperes. What is the instantaneous current when the voltage is at the 15° point in its cycle?

Solution. The maximum value of the current from the equation

$$I = 0.707 I_m,$$

$$I_m = \frac{100}{0.707} = 141 \text{ amperes.}$$

From a vector diagram similar to Fig. 37*b*–1,

$$\begin{aligned} i &= I_m \sin (\phi + \theta) \\ &= 141 \sin (15° + 40°) \\ &= 141 \sin 55° = 115.5 \text{ amperes.} \end{aligned}$$

Prob. 46–1. When the voltage in Example 10 is at its maximum negative value, what is the instantaneous value of the current?

Prob. 47–1. A current of 25 amperes leads a voltage of 300 volts by 70°. When the instantaneous voltage is 100 volts, negative and increasing, what is the instantaneous value of the current?

Prob. 48–1. What is the instantaneous voltage in Prob. 47–1 when the current is zero?

SUMMARY OF CHAPTER I

ELECTRICAL ENERGY is obtained from coal, oil or gas and water by means of prime movers in the form of steam engines or turbines, gas or oil engines, and water wheels. It may be generated either as direct or as alternating current.

ALTERNATING CURRENT is the term used to distinguish an electric current, which flows back and forth in a circuit, from a direct current which always flows in the same direction.

ALTERNATING-CURRENT SYSTEMS are used because higher voltages may be employed for transmission; thereby increasing the efficiency of the system. In an a-c system, the high-efficiency static transformer may be used to step up the voltage for transmission, and also to step down the voltage for commercial use.

Also, a-c generators may be built in larger sizes than is possible with d-c generators, and are driven by large turbines which are much more efficient than are the prime movers used to drive comparatively small d-c generators.

SHORT TRANSMISSION SYSTEMS for transmitting power from five to six miles or less consist of a-c generators of 6600 to 6900 volts, connected directly to the line. At the receiving end, induction or synchronous motors may be connected directly to the line. By the use of step-down transformers, low voltage for lighting, for small motors and for converters may be obtained.

LONG TRANSMISSION SYSTEMS are those which transmit power comparatively long distances (more than fifteen miles). Generator voltages are from 6600 to 14,000 volts. This is "stepped up" by station transformers, oftentimes as high as 200,000 volts, and then supplied to the line. At points along the line, where power is to be delivered, it is either stepped down by transformer substations to around 2300 volts for distribution over small areas, and then stepped down by transformers to even lower voltages, and fed to converters for distribution as direct current.

THE ACTION OF WATER in a Closed-Pipe System, operated by a valveless pump driven by an engine with a slotted yoke, gives a fair picture of the action of an alternating electric current in a closed circuit. The water current, flowing through the system at any given instant, depends upon the speed of the piston at that instant, which in turn depends upon the position of the crank.

The speed of the piston, and thus the amount of the water current at any instant, is proportional to the sine of the angle which the crank is making with the "dead center," or zero position.

The flow of water (or of electricity) in a system has a wave of values, rising from zero to a maximum in one direction; then decreasing in value to zero and reversing direction; then increasing in value to a maximum in this reverse direction and again dropping off to zero.

A CYCLE is said to be completed when an alternating current has passed through one complete set of values in both directions, or when 360° of the curve have been completed.

The Equation for the current flowing at any instant is

$$i = I_m \sin \phi,$$

where i = instantaneous current,
I_m = maximum current,
ϕ = angle from the dead center or zero position at any instant.

This is the equation of a SINE CURVE. This form of wave has been adopted by engineers as the standard, because it simplifies mathematical computation, and design of appliances.

Since a sine curve may be constructed from the vertical projections of a Rotating Radius or Vector, a Vector diagram may be used to visualize an alternating current. When instantaneous values are to be determined, the Vector represents the maximum value of the wave.

Alternating-current generators are constructed to generate an alternating emf wave, very closely approximating the sine curve. This sine curve goes through the same cycle of values as the water system above, and is represented by the same equation and the same vector diagram.

The equation usually is written

$$e = E_m \sin \phi,$$

where e = the instantaneous voltage,
E_m = the maximum voltage,
ϕ = the degrees of the cycle completed at the instant chosen.

Most modern alternators are built with a rotating field, thus confining the high-voltage armature circuits to stationary windings. The fields are separately excited with low-voltage direct current, conducted through brushes to collector rings on the rotating field structure.

By FREQUENCY is meant the number of cycles completed in one second. The conductors of a two-pole generator, rotating at one revolution per second, would pass two poles, or 360° of the emf wave, in one second, and have a frequency of one cycle per second. In a four-pole machine, rotating at the same speed, the conductors would pass four poles, or 720° of the emf wave, in one second, and have a frequency of two cycles per second.

The equation for frequency of an alternator is

$$f = \frac{\text{rpm}}{120} \times P,$$

where f = frequency in cycles per second,

P = number of poles in the machine.

Average Value of a Sine Curve of emf or current, if signs are neglected is 0.636 of the maximum value. In an equation

$$E_{av} = 0.636E_m,$$

or

$$E_m = \frac{1}{0.636} E_{av} = 1.57E_{av}.$$

For a generator with concentrated armature winding,

$$\text{Av emf} = \frac{\text{no. of lines cut per sec} \times \text{no. of conductors in series}}{10^8}$$

$$E_m = 1.57E_{av}.$$

AN ALTERNATING-CURRENT AMPERE is based upon its heating value. It will produce the same heat as an ampere of direct current. This is known as the effective value of the current, and is equal to the square root of the average of the squares of all the instantaneous values. This is the value always meant unless otherwise specified. For a sine

curve, it is 0.707 of the maximum value. The equation is

$$I = 0.707I_m = \frac{I_m}{\sqrt{2}},$$

or

$$I_m = \frac{1}{0.707} I = 1.414I,$$

where

I = effective current in amperes,
I_m = maximum current in amperes.

ALTERNATING CURRENT VOLTAGES are given in terms of their effective value, since a-c voltmeters indicate this value

Thus

$$E = 0.707E_m,$$

or

$$E_m = 1.414E,$$

where

E_m = maximum emf in volts,

E = effective emf in volts.

The Ratio of the maximum to the effective value is called the CREST or PEAK factor. For a sine wave this has the value 1.41.

The Ratio of Effective to Average Value of an alternating current or voltage wave is called the FORM FACTOR. For a sine curve, the form factor equals $\frac{0.707}{0.636}$ or 1.11.

OHM'S LAW is applicable to those parts of a-c circuits having resistance only, that is to those parts in which there is no inductance or capacitance. The current at any instant in such part of a circuit is equal to the voltage across that part at that instant, divided by the resistance of that part, or

$$i = \frac{e}{R},$$

Similarly,

$$I_m = \frac{E_m}{R},$$

and

$$I = \frac{E}{R}.$$

Whenever an alternating emf sets up a current, this current is also an alternating current of the same fundamental frequency as the emf. If the emf wave is a sine wave, the current wave will have a similar form and be expressed by a similar equation. In a-c circuits, the current may be " in phase " with the voltage; or " lag " behind the voltage; or " lead " the voltage. This applies to a circuit as a whole and to each part separately.

PROBLEMS ON CHAPTER I

Prob. 49–1. A 12-pole 60-cycle alternator has 9×10^5 lines per pole. There are 300 conductors concentrated in series on the armature. What maxmum emf does the machine generate?

Prob. 50–1. If the machine in Prob. 49–1 were run as a 25-cycle generator with the same flux per pole, what maximum emf would it generate?

Prob. 51–1. At what speed must the generator in Prob. 49–1 be run to generate a 40-cycle emf?

Prob. 52–1. What would be the effective voltage of the generator in Prob. 51–1?

Prob. 53–1. Construct a table of possible speeds for a line of 60-cycle generators having various numbers of poles up to 40.

Prob. 54–1. Construct a similar table to that in Prob. 53–1 for a line of 25-cycle generators.

Prob. 55–1. A 60-cycle alternator, having 24 poles with 3×10^6 lines per pole, generates an average emf of 2000 volts, sine wave form. How many conductors are there in series on the armature, the winding being concentrated?

Prob. 56–1. What is the maximum voltage of the generator in Prob. 55–1?

Prob. 57–1. At what speed must the generator of Prob. 55–1 be driven to generate a 40-cycle emf?

Prob. 58–1. What would be the maximum voltage of the generator in Prob. 57–1, assuming the same flux per pole and sine wave form?

Prob. 59–1. By how many lines must the fields of each pole of the generator in Prob. 57–1 be increased to raise the maximum emf to that of the generator in Prob. 55–1?

Prob. 60–1. In the 24-pole alternator of Prob. 55–1, how many degrees of the emf wave are completed in $\frac{1}{40}$ of a revolution?

Prob. 61–1. An emf wave has instantaneous values, positive and negative throughout the cycle, as given below. (*a*) Plot the curve and find the effective value. (*b*) What is the peak factor? (*c*) What is the form factor?

0° and 180° = 0 volts	50° and 130° = 50 volts
10° and 170° = 5.5 volts	60° and 120° = 67.5 volts
20° and 160° = 13 volts	70° and 110° = 87 volts
30° and 150° = 23 volts	80° and 100° = 109 volts
40° and 140° = 35 volts	90° and 270° = 130 volts

Prob. 62–1. A distorted current wave has the following instantaneous values, positive and negative throughout the cycle. (*a*) Plot the curve and find the effective value. (*b*) What is the peak factor? (*c*) What is the form factor?

0° and 180° = 0 volts	50° and 130° = 90 volts
10° and 170° = 34 volts	60° and 120° = 94.5 volts
20° and 160° = 57 volts	70° and 110° = 97.5 volts
30° and 150° = 73 volts	80° and 100° = 99.5 volts
40° and 140° = 84 volts	90° and 270° = 100 volts

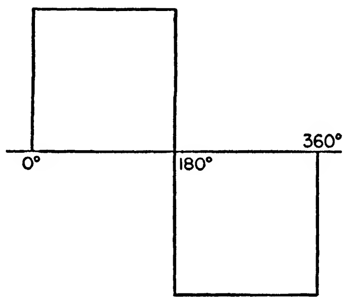


FIG. 38–1. Flat-topped curve.

Prob. 63–1. (*a*) For a wave form of the shape shown in Fig. 38–1, what is the ratio of maximum to effective value? (*b*) The ratio of maximum to average value? (*c*) What is the form factor?

Prob. 64–1. How many calories of heat are generated in one minute in a 40-ohm resistance by the passage of 150 amperes of alternating current?

Prob. 65–1. How many amperes of direct current would be required to generate the same amount of heat in the same time in the resistance of Prob. 64–1?

CHAPTER II

THE USE OF VECTORS IN COMPUTING A-C CURRENTS AND PRESSURES. SERIES, PARALLEL AND POLYPHASE CIRCUITS

It has been shown in Chapter I that the emf wave of the modern alternator closely follows the sine wave form (see Fig. 26–1). The wave of current follows approximately the same form, except in special cases. Accordingly, in practice, all ordinary computations of currents are based on the assumption of sine wave form.

Thus, when two or more emfs are combined in a series a-c circuit, the addition, or combination of these voltages, is essentially the addition or combination or two or more sine waves. The same may be said of the combination of two or more currents in a parallel circuit.

Since, as shown in Chapter I, a sine curve may be represented by a rotating vector, the combination of pressures, or of currents, may also be shown by a combination of vectors.

The methods of computing the numerical value of combinations of emfs and of currents are discussed in this chapter.

1–2. Alternating-Current Emfs in Series: in Same Phase. Consider Fig. 1–2, which shows two coils, AB and A_1B_1, on an armature rotating counter-clockwise in a parallel magnetic field. Note that the direction of the induced emf in each coil, at the instant shown, is from A to B, according to Fleming's right hand rule. Each coil generates a sine wave of emf and these two emfs are "in phase"; that is, the emf of each coil passes through the zero value at the same instant, rises to a maximum value in the same direction at the same instant and drops to zero at the same instant, etc.

Assume that the two coils have a different number of turns; although, for the sake of clearness, only one turn per coil is indicated in the figure. Also assume that coil AB generates 25 volts maximum, while coil A_1B_1 generates 15 volts maximum. The sine curves, representing these two emfs, are shown in Fig. 3–2(a).

Now, if the two coils are connected in series end B to A_1, as indicated in Fig. 2–2, their emfs will be in the same direction through

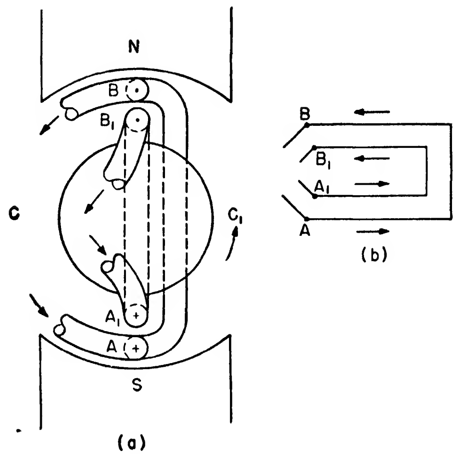


FIG. 1–2. The sine curves of emf, induced in the coils AB and A_1B_1, are "in phase" with each other. That is, as the armature rotates, the emfs in both coils reach a maximum value in the same direction at the same instant.

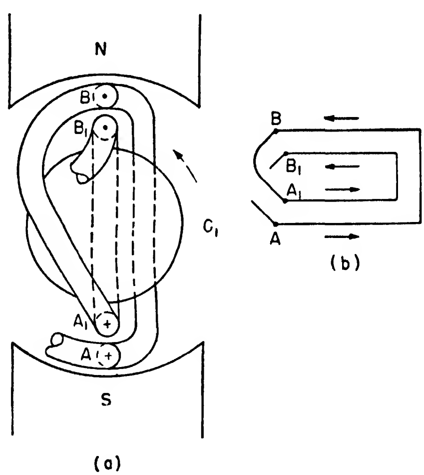


FIG. 2–2. The coils AB and A_1B_1 are connected in series so that their emfs add together. The resultant emf of the series combination is also a sine curve, shown as curve R, in Fig. 3–2, the maximum value of which is equal to the arithmetic sum of the maximum emfs induced in the two coils. The three curves are all "in phase."

the circuit, Fig. 2–2(*b*). And the maximum voltage across AB_1 will be the sum of the maximum emfs, induced in the two coils, or 25 + 15 equals 40 volts.

Also, at any instant during the cycle the sum of corresponding instantaneous values of the two curves will be equal to the voltage across the two coils in series at that instant. Thus, at any instant xx_1, Fig. 3–2, representing in this case the 40° point in the cycle of

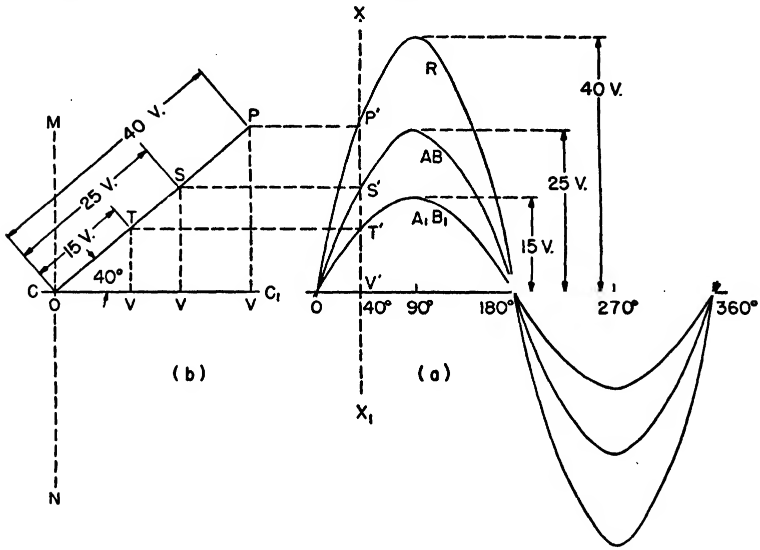


FIG. 3–2. The curves of emf in the individual coils AB and A_1B_1, and also their resultant, R, can be represented by a combination of vectors. At the instant xx_1, the 40° point in the cycle of each of the three curves, they are represented by the vectors, OS, OT, and OR, which are equal respectively to the maximum values of the emfs of coils AB, A_1B_1, and their resultant R. Note in the diagram that the vectors are all "in phase" as are the curves they represent and $OS + OT = OP$.

each wave, the emf of coil $AB = S'V'$; the emf of coil $A_1B_1 = T'V'$; and $S'V' + T'V' = P'V'$, the emf of the two coils in series at this instant. Other points may be determined in similar manner and the resulting curve, R, is also a sine curve of 40 volts maximum value.

Now a sine curve as we have seen, can be represented by a vector whose length is equal to the maximum value of the curve, rotating counter-clockwise about a point* and the perpendicular

* Counter-clockwise rotation of vectors is accepted as standard by international agreement.

dropped from the end of this vector to the horizontal reference axis, represents the instantaneous value of the curve for this position of the vector.

Accordingly, a vector diagram, Fig. 3–2(*b*), representing the emf of each coil and the resulting emf of the two coils in series at the instant xx_1 of Fig. 3–2(*a*) may be constructed as follows. From point O, Fig. 3–2(*a*), the intersection of the reference axes mm_1 and cc_1, lay off OS equal to 25 volts, maximum of curve AB, at 40° to the horizontal and $SV = S'V'$. Also, lay off OT equal to 15 volts maximum of coil A_1B_1 at the same angle and $TV = T'V'$. And finally, at the same angle, lay off OP of such length that

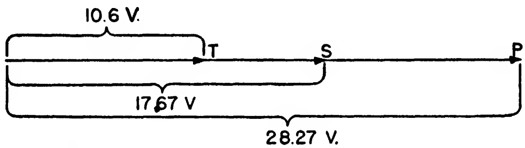


FIG. 4–2. The polar vector diagram for Fig. 3–2, in which the vectors represent the effective value of the respective curves. Note that all vectors are "in phase" and are drawn from the point O. Also $OT + OS = OP$.

$PV = P'V'$. Now OP is equal to the 40 volts maximum of the resulting curve. Note that these three vectors are all "in phase," as are the sine curves they represent.

Also, note that the vector diagram of Fig. 3–2(*b*) represents only one instant in the cycle and the length of the vectors represents maximum values of the emfs. But we know from Chapter I that **effective** values are used in practice, and that effective values are 0.707 of the maximum. Therefore, the so-called **space** vector diagram similar to Fig. 3–2(*b*) may be drawn, using **effective** values and without any reference to the axes mm_1 and cc_1. This does not change the **relative** length of the vectors, but has no relation to the **time** diagram of Fig. 3–2(*b*) and therefore does not indicate any instantaneous values.

Accordingly, in Fig. 4–2, we lay out OS from O in any convenient direction representing the effective emf of coil AB as 25×0.707 or 17.67 volts; also, from O lay out OT in the same direction, representing the emf of coil A_1B_1 as 15×0.707 or 10.6 volts; and OP, the effective value of the resultant emf of the two coils in series as 40×0.707 or 28.27 volts. Note that $17.67 + 10.6 = 28.27$ volts. This is called a **polar space** vector diagram, since all vectors are laid off from the same point O.

The effective value of the resultant emf also may be determined by another form of diagram, called a **topographic space** vector diagram, as shown in Fig. 5–2. Note that OS is laid off as before, representing the effective emf of coil AB, and the " tail " of the vector representing the emf of coil A_1B_1 is laid off from S, or from the " head " of OS, as ST. The total length of the two vectors, OT, now represents the 28.27 volts effective value of the resultant emf of the two coils in series.

Two important facts are to be noted from the discussion above. The first is, that when two or more emfs of the same frequency and in the same phase are connected in series, their resultant is the

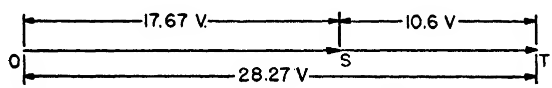


FIG. 5-2. The topographic vector diagram for Fig. 3–2, representing effective values. Note that the " tail " of vector ST, representing effective emf of curve A_1B_1, is laid out from the " head " of vector OS, representing effective emf in AB. Vector OT is the effective emf of the resultant curve, and $OS + ST = OT$.

arithmetic sum of the individual emfs. The second is that instead of graphically adding the sine curves themselves, the effective value of the resultant emf may be determined, either by means of a **polar** vector **space** diagram or by a **topographic** vector **space** diagram.

Prob. 1–2. (*a*) What is the value of the emf in each coil of Fig. 2–2 at the instant xx_1 represented by $T'V'$ and $S'V'$ of Fig. 3–2? (*b*) What is the value of the resultant emf $P'V'$ at this instant?

Prob. 2–2. Two coils in series in the same phase generate equal voltages. The instantaneous emf of a series combination of the two coils in the 160° position is 225 volts. What effective emf is generated in each coil?

Prob. 3–2. Two coils in the same phase on an alternator are connected in series: a voltmeter across each coil reads respectively 17 and 26 volts. What does a voltmeter across a series combination read? Show both polar and topographic vector diagrams for these emfs.

Prob. 4–2. Two coils on an alternator are in series. A voltmeter across one of them reads 115 volts, across the other 230 volts, and across the combination 345 volts. What is the phase relation between these emfs?

2–2. Alternating-Current Emfs in Series: In Opposite Phase (180°). If the coils of Fig. 1–2 are reconnected B to B_1 as in Fig. 6–2, the emf of each coil will reach a maximum at the same instant as before, but the two coils now are so connected that their emfs oppose each other in the circuit, as indicated in Fig. 6–2(*b*). Thus the voltage of coil A_1B_1 is reversed with respect to that of AB and the voltage across AA_1 is the difference between the two emfs. The sine curve, representing the emf of A_1B_1 must now be drawn

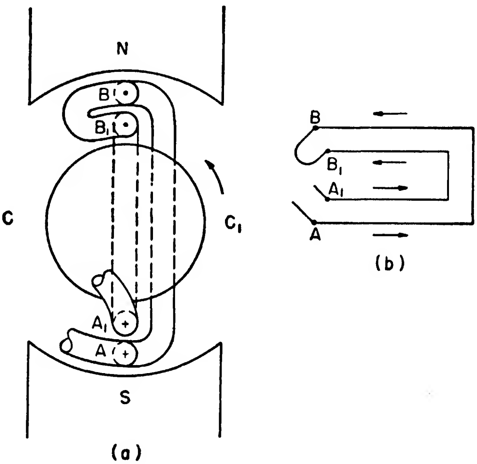


FIG. 6–2. The two coils of Fig. 2–2 are connected in series with that of A_1B_1 reversed. The emf of coil A_1B_1 is reversed in the series circuit, and the two emfs oppose each other or are in the "opposite phase" or 180° from each other. The sine curve, representing the emf of A_1B_1 is now drawn reversed with respect to that of coil AB, as shown in Fig 7–2, and the resulting emf is a sine curve R whose maximum value is equal to the arithmetical difference of the maximums of the two coil emfs.

reversed with respect to that of AB, or at 180° from it, as shown in Fig. 7–2.

Again, let it be assumed that the maximum emf, induced in AB, is 25 volts and that in A_1B_1 is 15 volts. The maximum value of the resulting curve now will be 25 − 15, or 10 volts.

Also, at any instant xx_1 in Fig. 7–2, the instantaneous value of the curve AB equals $S'V'$; of curve A_1B_1 equals $T'V'$, a negative value; and $S'V' - T'V' = P'V'$, the emf of the two coils in series at this instant. Other points on the resulting sine curve may be found in similar manner.

Note here, that in the figure, xx_1 represents the 60° point in the cycle of coil AB, which is also the 180° + 60° or 240° point in the cycle of A_1B_1, since the curves are 180° apart. It is also the 60° point in the cycle of the resulting emf, R.

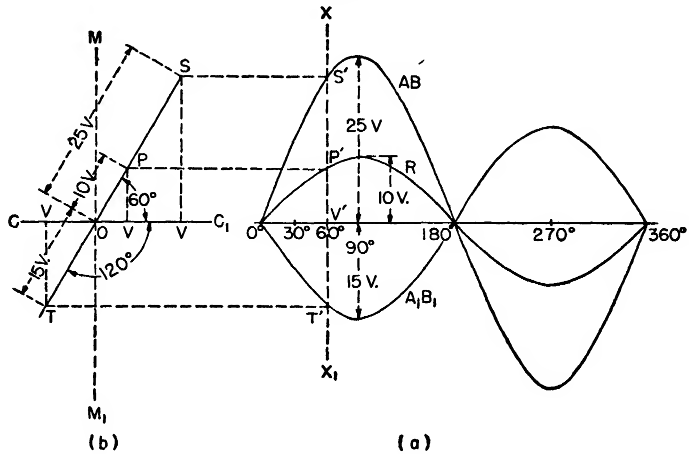


FIG. 7–2. Shows the emf curves of coils AB and A_1B_1 of Fig. 6–2 and also the resultant curve R of the series combination, together with the vector diagram representing these curves at the instant xx_1. Vectors represent maximum values. Vector OT, the emf in coil A_1B_1, is drawn at 180° from OS, the emf in coil AB; and OP, the resultant emf is equal to the arithmetical difference of the coil emfs, or $OS - OT = OP$.

The vector diagram, representing the emfs at this instant, is drawn as follows. From O, Fig. 7–2(b), draw OS equal to the 25 volts maximum of curve AB at 60° with the horizontal axis and $SV = S'V'$; from O draw OT equal to 15 volts maximum of

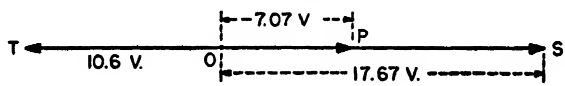


FIG. 8–2. Polar vector diagram of effective emfs in Figs. 6–2 and 7–2. Note that all vectors are drawn from O; OT being drawn 180° from OS. $OS - OT = OP$.

A_1B_1 at 180° from OS, or at 180° − 60° or 120° from the horizontal and $TV = T'V'$; also from O draw OP at 60° to the horizontal of such length that $PV = P'V'$, and OP will be equal to the maximum value of the resulting emf curve, R.

Using effective values as in Art. 1, the space polar vector diagram is drawn, Fig. 8–2, by laying out the vector OS in any convenient

direction, representing the 17.67 volts of coil AB, as before. Now, the vector OT, representing 10.6 volts of coil A_1B_1, is laid off from O, but at 180° from OS, as shown. And the difference of these two vectors, OP, is laid off from O, in the direction of the larger, representing 10×0.707, or 7.07 volts, effective value of the resultant emf. Note that $17.67 - 10.6 = 7.07$.

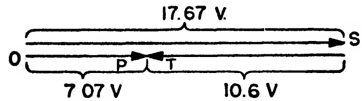


FIG. 9–2. Topographic vector diagram of effective emfs in Figs. 6–2 and 7–2. The "tail" of vector ST, the emf of coil A_1B_1 is laid off from the head of OS, the emf of AB, but at 180° from it. OP equals the effective value of the resultant emf, or $OS - ST = OP$.

The topographic vector diagram, Fig. 9–2, is drawn by laying off OS, as before. The "tail" of the vector OT is now laid off from S, the "head" of vector OS, and at 180° from it, shown as ST in the figure. The vector OP now represents the resulting emf of 7.07 volts.

Thus, when two emfs of sine wave form are combined in series in the "opposite phase," or at 180° from each other, the resulting emf is equal to their arithmetical difference.

Prob. 5–2. (a) What is the value of the emf in each coil of Fig. 6–2 at the instant xx_1, represented by $S'V'$ and $T'V'$ of Fig. 7–2? (b) What is the value of the resulting emf, $P'V'$ at this instant?

Prob. 6–2. What would be the effective voltage across the two coils connected in phase, as in Fig. 2–2, if the maximum voltage of one is 22 volts, and of the other 30 volts? Show both polar and topographic vector diagrams, using effective values.

Prob. 7–2. What is the effective voltage across the two coils of Prob. 6–2, if connected in the opposite phase, as in Fig. 6–2? Show both polar and topographic diagrams, using effective values.

Prob. 8–2. What is the effective emf across two coils, joined in series in the same phase, if the maximum emf of one is 40 volts and of the other 200 volts? Show both types of vector diagrams.

Prob. 9–2. Two coils are joined in series in the opposite phase. The instantaneous emf of one is 100 volts at the 30° phase; of the other, 100 volts at the 60° phase. What is the maximum voltage across the combination?

Prob. 10–2. What is the effective voltage across each coil and across the combination in Prob. 9–2? Show both types of diagrams.

Prob. 11–2. What would be the effective voltage in Prob. 9–2, if the coils were connected in the same phase? Show diagrams.

Prob. 12–2. The maximum emf of three coils on an armature is 24 volts each. What would be the effective voltage across the combination, if they were joined in series and were all in phase with one another? Show diagrams.

Prob. 13–2. Two coils in series and in phase with each other give a total voltmeter reading of 40 volts across the combination. When the connections to one of the coils is reversed, the resultant voltage is zero. What is the effective voltage of each coil? Show diagrams.

Prob. 14–2. Two coils, joined in series and in phase, give a voltmeter reading of 235 volts. When one of them has its connections reversed, the total voltage becomes 25 volts, as shown by voltmeter. What is the effective voltage of each coil? Show diagrams.

Prob. 15–2. Three coils, A, B, and C in series, on an alternator give a total voltmeter reading of 110 volts. When connections to A are reversed, the voltmeter again indicates 110 volts. When the connections to A are restored as originally, and those of B reversed, the total voltage becomes 330 volts. When B is restored as originally and C reversed, the total voltage again becomes 110 volts. What is the emf of each coil? Construct both polar and topographic vector diagrams to represent the relations between these emfs, as originally connected, and as reconnected after each change.

3–2. Alternating-Current Emfs in Series in Quadrature (at 90°). The two-coil generators represented in the preceding articles are examples of very elementary a-c machines with **concentrated** windings. In the commercial alternator, the coils are distributed over the surface of the armature and the induced emfs are "out of phase" with one another — that is, the emfs in the different coils do not all reach a maximum value at the same instant.

Let us now consider the case where two coils, AB and A_1B_1, are placed on an armature at 90° from each other, as in Fig. 10–2. The armature is rotated counter-clockwise, and the emf induced in each coil is a sine wave. Assume the maximum induced emf in coil AB is 15 volts, and in coil A_1B_1 is 10 volts. In the figure, coil AB is in the 30° position in its cycle, while A_1B_1 is just 90° ahead of AB, or in the 120° position in its cycle. Note that at the instant shown, the emf induced in each coil is in a direction from A to B (Fleming's right hand rule). Thus, as the armature rotates, the wave of emf induced in A_1B_1 is just 90° ahead of that induced in AB. The curves representing these emfs are plotted in Fig. 12–2. Note again that time (electrical degrees) is measured from left to right and the emf cycle of coil AB starts 90° after, or behind that of A_1B_1. That is, the curve of AB is zero, when that of A_1B_1 is at a

maximum, or 10 volts. These two coils are thus said to be "in quadrature," or 90° (electrical) "out of phase" with each other.

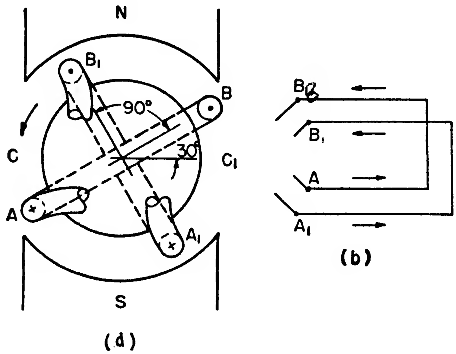


FIG. 10–2. The two coils are placed at 90° to each other on the armature. As they rotate counter-clockwise, the sine curve of emf of coil A_1B_1 reaches a maximum value 90° before that of AB, and the two curves are 90° "out of phase," or in quadrature, as shown in Fig. 12–2.

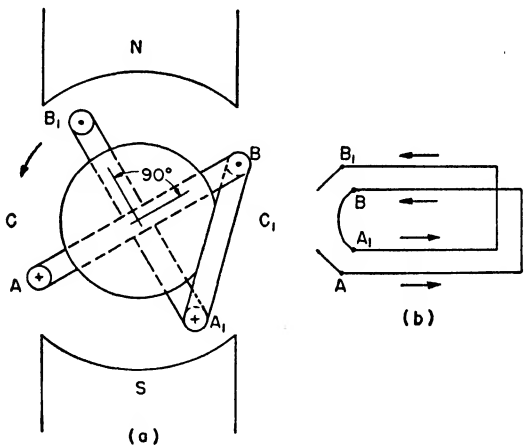


FIG. 11–2. The two coils of Fig. 10–2 are connected in series. The resultant emf of the series combination is also a sine curve, shown as R in Fig. 12–2, the maximum value of which is less than the arithmetic sum of the maximums of the coil emfs.

Now if the two are joined in series, end B to end A_1, as indicated in Fig. 11–2(a and b), the resultant voltage across AB_1, at any instant, will be the algebraic sum of the emfs of the two coils at that instant. Thus at any instant xx_1, Fig. 12–2, the voltage of coil

AB is $S'V'$ and that of A_1B_1 is $T'V'$. And $S'V' + T'V' = P'V'$, the emf of the two coils in series at this instant, and a point on the resulting emf curve, R.

Also, at the instant yy_1, the voltage of coil AB equals $F'H'$ and that of A_1B_1 equals $E'H'$ — a negative value. Therefore, $F'H' - E'H' = -G'H'$ (since $E'H'$ is greater than $F'H'$). This is another point — a negative one — on the resulting curve, R. At this instant, the emfs of the two coils are in opposite directions in the circuit AB_1 due to the fact that, as the armature rotates, the emf of coil A_1B_1 reverses direction 90° before that of AB.

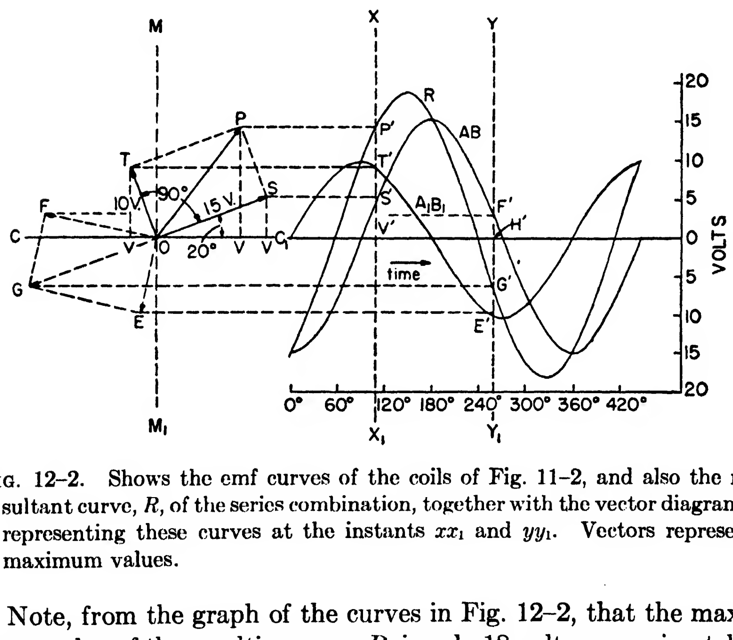


FIG. 12–2. Shows the emf curves of the coils of Fig. 11–2, and also the resultant curve, R, of the series combination, together with the vector diagrams, representing these curves at the instants xx_1 and yy_1. Vectors represent maximum values.

Note, from the graph of the curves in Fig. 12–2, that the maximum value of the resulting curve R, is only 18 volts approximately. This is less than the arithmetical sum of the maximum of the two curves. Also note that the resulting emf R is leading, or ahead of the curve AB by approximately 30°.

It is evident that this method of adding emfs which are " out of phase " is cumbersome and inaccurate, so again we resort to the use of vectors.

We, therefore, construct the vector diagram as before, for any instant, as xx_1 on the sine curve of Fig. 12–2. Note that this instant represents the 20° point in the cycle of the emf of AB, which is also the 90° + 20°, or 110° point in the cycle of the emf of

A_1B_1. From O lay off OS at 20° from the horizontal equal to 15 volts (maximum of AB), and SV will equal $S'V'$; from O lay off OT at 90° ahead of OS, or at 110° from the horizontal, equal to 10 volts (maximum of A_1B_1) and TV will equal $T'V'$. Now complete the rectangle and draw the diagonal OP and PV will equal $P'V'$. The vector OP when scaled off will be found to be equal to the maximum value of the resultant curve, R. Also the three vectors, OS, OT, and OP have the same relative position to each other as do the

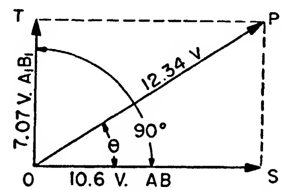


FIG. 13–2. Polar vector diagram of effective emfs of Figs. 11–2 and 12–2. Note that all vectors are drawn from point O, and OT, representing the emf of coil A_1B_1, is drawn 90° leading OS, the emf of coil AB. The resulting voltage across the two coils in series is OP. $OP = OS \oplus OT$.

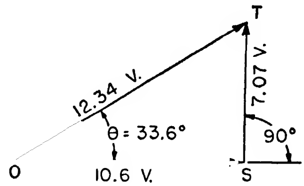


FIG. 14–2. Topographic diagram of effective emfs of Figs. 11–2 and 12–2. The "tail" of vector ST, emf of coil A_1B_1, is laid off from the "head" of vector OS and leading it by 90°. Resulting emf across the two coils in series is OT. $OT = OS \oplus ST$.

sine curves they represent. That is, OT is rotating counterclockwise 90° ahead of OS, with OP some 30° ahead of OS.

The vector diagram for the instant yy' is also shown in Fig. 12–2.

Using effective values, as already discussed in the previous articles, the vector diagram is constructed without reference to the curves, as follows. Draw OS, Fig. 13–2, in any convenient direction, representing 15 × 0.707, or 10.6 volts, effective emf of coil AB; from O draw OT leading OS by 90°, representing 10 × 0.707, or 7.07 volts, effective emf of coil A_1B_1; complete the rectangle and draw the diagonal OP, which is the effective value of the resultant emf of the two coils in series. This is the **polar vector diagram,** since all vectors are drawn from the common point O, and is a space diagram since it does not indicate instantaneous values of voltage.

The **topographic** vector diagram is shown in Fig. 14–2. The vector OT is drawn as before, but the "tail" of the vector ST,

representing the emf of coil A_1B_1, is connected to S, the " head " of vector OS, and 90° ahead of it; complete the triangle, and vector OT is the resultant emf of the two coils.

Solution of the Vector Diagram. Figure 14–2 forms a right-angle triangle. In such a triangle, the hypothenuse, OP, equals the square root of the sum of the squares of the other two sides. Thus,

$$OT = \sqrt{\overline{OS}^2 + \overline{ST}^2} \tag{1-2}$$

$$= \sqrt{10.6^2 + 7.07^2} = 12.34 \text{ volts,}$$

the effective value of the resulting emf.

The angle θ, Fig. 14–2, which the resultant makes with OT may be determined, for

$$\text{Tangent } \theta = \frac{ST}{OS} \tag{2-2}$$

$$= \frac{7.07}{10.6} = .666$$

From Table I, 0.666 is the tangent of an angle of 33.6°.

Thus, the emf of the two coils in series reaches its maximum value 33.6 electrical degrees or $\frac{33.6}{360}$ of a cycle before that of coil AB.

In the same manner, the resultant of any two a-c emfs, acting at right angles, or differing in phase by 90°, may be found, together with the phase difference between the resultant and either of the two emfs.

Since the resultant voltage, R, is neither the arithmetical sum nor the arithmetical difference of the emfs of the two coils, AB and A_1B_1 in series, we can **not** write, $R = AB + A_1B_1$. However, in order to express the correct relations, that is, the **vector** relations of the emfs of AB and A_1B_1, we write the following equation:

$$R = AB \oplus A_1B_1 \tag{3-2}$$

The sign $\oplus$ means that we add **vectorially** the quantities between which it stands. Thus, the above expression means that " R " equals the **vector** sum of AB and A_1B_1.

In mathematics, there are two general types of quantities, known as " Scalar " and " Vectors."

Scalar quantities are those which have magnitude only, and

are added algebraically. Examples of scalar quantities are mass, volume, area, dollars, temperature, population, etc.

Vector quantities are those which have direction as well as magnitude. A force is a vector quantity. The sum of two forces is determined by their relative direction, as well as by their magnitude, and is computed by a force diagram, as used in mechanics.

Alternating currents and voltages are, therefore, vector quantities, because they have direction with respect to each other, as well as magnitude.

Write the equation and construct both polar and topographic vector diagrams for each of the following problems.

Prob. 16–2. An emf of 75 volts is joined in series with one of 50 volts of the same frequency, which lags behind the former by 90°. (*a*) What is the resultant emf across the combination? (*b*) What is the difference in phase between the resultant and each of the other emfs?

Prob. 17–2. The resultant of two emfs of the same frequency is 220 volts. The first is 120 volts. (*a*) What is the emf of the second, if it leads the first by 90°? (*b*) By what angle is the resultant out of phase with each of the two other emfs?

Prob. 18–2. (*a*) What is the maximum value of the three emfs of Prob. 17–2? (*b*) At the instant the resultant emf is zero, what is the value of each of the other two emfs?

Prob. 19–2. When two emfs of the same frequency, but with a phase difference of 90°, are joined in series, the resulting emf is 440 volts. If the lagging emf has a value of 220 volts, what is the other emf?

Prob. 20–2. What is the phase difference between the emf across the combination, and each of the other two emfs in Prob. 19–2?

Prob. 21–2. At the instant the resultant emf in Probs. 18–2 and 19–2 reaches its maximum positive value, what will be the value of the other two emfs?

4–2. Alternating Current Emfs in Series at 60° Phase. In Fig. 15–2(*a*) the coils AB and A_1B_1 are placed at 60° to each other on an armature, rotating counter-clockwise, as before. At the instant shown in the figure, the induced emf is in a direction from A to B through each coil, as indicated. Coil AB is in the 30° position, while coil A_1B_1 is in the 90° position, or 60° ahead of AB. Therefore, as the armature rotates, the emf induced in A_1B_1 leads that induced in AB by 60°.

Let it be assumed that each coil generates a sine wave of 15 volts maximum. The sine curves representing the two emfs are

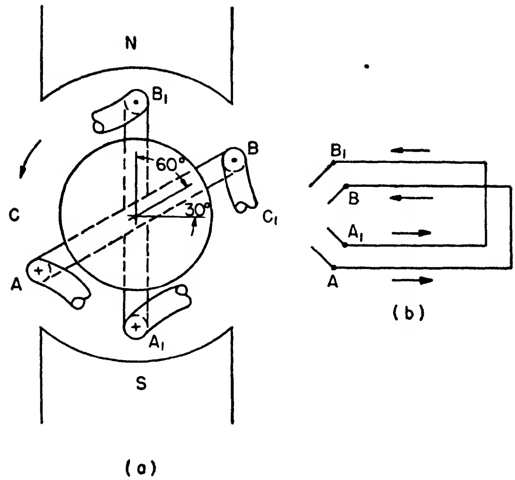


FIG. 15-2. The two coils are placed at 60° to each other on the armature. As they rotate counter-clockwise, the sine curve of emf of coil A_1B_1 reaches a maximum value 60° before that of AB, and the two curves are 60° "out of phase," as shown in Fig. 17-2.

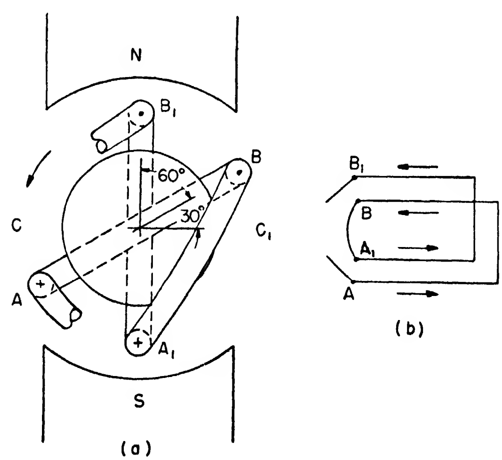


FIG. 16-2. The two coils of Fig. 15-2 are connected in series. The resultant emf of the series combination is shown as curve R in Fig. 17-2, the maximum value of which is less than the arithmetic sum of maximum emfs of the two coils.

plotted to scale in Fig. 17–2. Again note that time (electrical degrees) is measured from left to right and the emf cycle of coil AB starts 60° after, or behind that of A_1B_1. The emfs in these coils are thus 60° " out of phase " with each other.

Now if the two coils are joined in series, end B to end A_1, as indicated in Fig. 16–2, the voltage across the two coils in series or from A to B_1, at any instant, will be the algebraic sum of the emfs induced in each coil at that instant. Thus, at the instant represented by xx_1, in Fig. 17–2, the curve AB is at the 20° point in its

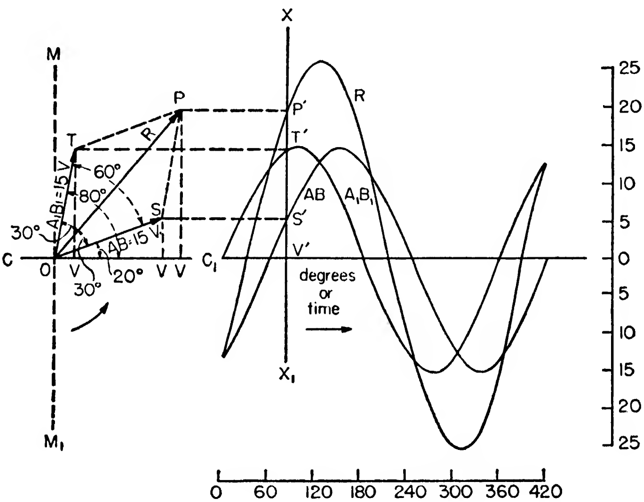


FIG. 17–2. Shows the emf curves of the coils in Fig. 16–2 and also the resultant curve, R, of the series combination, together with the vector diagram, representing these curves at the instant xx_1. Vectors represent maximum values.

cycle and the emf at this instant is $S'V'$, while the curve A_1B_1 is at the 20° + 60°, or 80° point in its cycle and the emf at this instant is $T'V'$. The emf of the two coils in series at this instant equals $S'V' + T'V'$, or $P'V'$, and P' is a point on the resulting curve R. Other points may be obtained in similar manner.

Note, from the graph of Fig. 17–2, that here again the maximum value of the resulting curve is less than the arithmetical sum of the maximum emfs of the two coils. The instant xx_1 represents the 50° point in the cycle of the resulting curve, and therefore, in this case, it lags 30° behind that of A_1B_1 and leads AB by the same angle. These relations can also be represented by a vector diagram, as follows.

From O, Fig. 17–2, lay off OS at 20° to the horizontal, equal to the 15 volts maximum of curve AB, and $SV = S'V'$; also from O, lay off OT at 20° + 60°, or 80° to the horizontal (leading OS by 60°) equal to the 15 volts maximum of A_1B_1, and $TV = T'V'$. Complete the parallelogram and draw the diagonal OP. Thus, $PV = P'V'$ and OP will be found to be equal to the maximum value of the resultant curve R. From the construction of the diagram, it is seen that OP differs 30° in phase position from both OT and OS; and these three vectors are seen to be rotating counter-clockwise in the same relative position to each other as

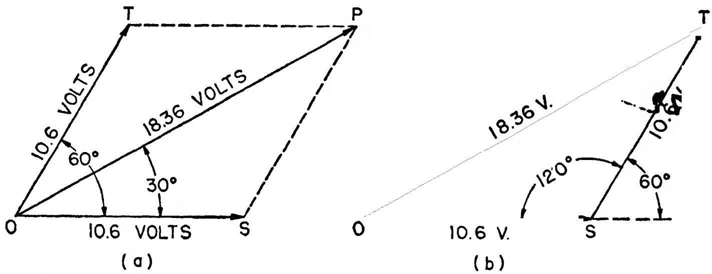


FIG. 18–2. (a) Polar vector diagram of effective emfs of Figs. 16–2 and 17–2. All vectors are drawn from point O. Vector OT, the emf of coil A_1B_1, is drawn 60° leading OS, the emf of coil AB. OP is the resultant emf of the two coils in series. (b) Topographic diagram of effective emfs of Figs. 16–2 and 17–2. The " tail " of ST, emf of coil A_1B_1, is laid off from the " head " of OS and leading it by 60°, the phase position being determined by the outside angle. The resultant emf is OT.

are the sine curves they represent. Note that the vector OP is the diagonal of a parallelogram, the value of which depends not only upon the length of the vectors OT and OS, but also upon their relative position with respect to each other — that is, upon the angle between them.

Since, as has already been explained, the vectors in Fig. 17–2 represent maximum values and the vector diagram represents only one instant in the cycle of these three emfs, a similar diagram, using effective values, is drawn without reference to any particular instant in the cycle of these emfs. The effective value of the emf of each coil is 15 × 0.707, or 10.6 volts. Accordingly, in Fig. 18–2(a), draw OS in any convenient direction, representing 10.6 volts, effective emf of coil AB; from O, draw OT, representing 10.6 volts, effective emf of coil A_1B_1 leading OS by 60°; complete the parallelogram and draw the diagonal OP. Vector OP is equal to

the effective value of the resultant emf wave. This is the **polar** diagram.

The **topographic** diagram is shown in Fig. 18–2(*b*). Again note in this type of diagram, the " tail " of the vector ST, representing the emf of coil A_1B_1, is drawn from the " head " of vector OS and 60° ahead of it; and vector OT, drawn from the " tail " of OS to the " head " of ST, represents the resultant emf. Also note particularly that the **outside** angle represents the phase relation of the vectors OS and ST.

Solution of the Polar Vector Diagram (see Appendix A, for the rule for determining the diagonal of a parallelogram).

In Fig. 18–2(*a*),

$$\begin{aligned} OP &= OS \oplus OT; \\ \overline{OP}^2 &= \overline{OS}^2 + \overline{OT}^2 + 2 \times OS \times OT \cos 60° \\ &= 10.6^2 + 10.6^2 + 2 \times 10.6 \times 10.6 \times 0.5 \\ &= 112.36 + 112.36 + 112.36 = 337.08; \\ OP &= \sqrt{337.08} = 18.36 \text{ volts.} \end{aligned}$$

Thus, 18.36 volts is the effective value of the resultant emf of the two coils in series.

Solution of the Topographic Diagram (see Appendix A for the relation of the sides and angles of an oblique triangle).

In Fig. 18–2(*b*),

$$\begin{aligned} OT &= OS \oplus ST; \\ \overline{OT}^2 &= \overline{OS}^2 + \overline{ST}^2 - 2 \times OS \times ST \cos 120° \\ &= 10.6^2 + 10.6^2 - 2 \times 10.6 \times 10.6 \times (-0.5) \\ &= 112.36 + 112.36 + 112.36 = 337.08; \\ OS &= \sqrt{337.08} = 18.36 \text{ volts.} \end{aligned}$$

Note here the change in signs and the use of the cosine of the interior angle. This angle is opposite that side of the triangle, the value of which is to be determined.

Of course, the solution of either type of diagram gives identical results. It is obvious that the resultant voltage of any two emfs may be obtained by carefully drawing Figs. 18–2(*a*) or 18–2(*b*) to scale; but this method is cumbersome and inaccurate, compared with the algebraic solutions illustrated above.

Example 1. An emf of 25 volts is leading another of 75 volts by a phase difference of 80°. When these emfs are connected in series: (*a*) What is the value of their resultant? (*b*) What phase angle does the resultant make with the 75 volt emf?

Solution. (*a*) Draw the vector diagram of Fig. 19-2.

$$OC = \sqrt{25^2 + 75^2 + 2 \times 25 \times 75 \cos 80°}$$

$$= \sqrt{625 + 5625 + 652.5} = \sqrt{6902} = 83 \text{ volts.} \quad \textit{Ans.}$$

(*b*) Angle $OAC = 180° - 80° = 100°$.

In triangle AOC, $\dfrac{\sin \measuredangle AOC}{\sin 100°} = \dfrac{25}{83}$.

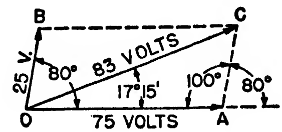


FIG. 19-2. OC is the resultant of OA and OB at a phase difference of 80°.

See Appendix A for law of sines.

$$\frac{\sin \measuredangle AOC}{0.985} = \frac{25}{83}.$$

$$\sin \measuredangle AOC = \frac{25}{83} \times 0.985 = 0.2965.$$

From Table I, $\measuredangle AOC = 17° 15'$ (leading) *Ans.*

Show both polar and topographic diagrams for each of the following problems, and solve.

Prob. 22-2. Two coils are joined in series. The emf of one coil is 48 volts, leading the other of 70 volts by 65°. What is the voltage across the combination?

Prob. 23-2. Two coils, having emfs of 60 and 80 volts respectively, are joined in series with a phase difference of 45°. If the emf of the 60-volt coil leads that of the other, (*a*) what is the voltage across the combination? (*b*) what phase angle does the resultant make with the emf of the 60-volt coil?

Prob. 24-2. What would be the resultant voltage across the two coils in Prob. 23-2, if the emf of the 60-volt coil was lagging 50° behind that of the other? What would be the phase position of the resultant voltage with respect to that of the 60-volt coil?

Prob. 25-2. A coil, generating 90 volts, is joined in series with another generating 120 volts, and lagging 100° behind the first. (*a*) What is the voltage across the combination? (*b*) By what angle does the resultant voltage lag behind the first?

Prob. 26-2. Two coils generate emfs of 110 and 150 volts respectively. When joined in series, the combined voltage is 220 volts. What is the phase difference between the two coils?

Prob. 27-2. If the voltage across the two coils in Prob. 26-2 had been 260-volts, what must the phase difference have been?

Prob. 28–2. If the voltage across the series combination of the two coils of Prob. 26–2 had been 40 volts, what would the phase difference have been?

Prob. 29–2. The emfs of two coils have a phase difference of 40°. The value of one emf is 86 volts. What must be the value of the other emf, in order that the voltage across the two in series may be 110 volts?

5–2. Series Arrangement of More Than Two Coils. When more than two coils, whose emfs are out of phase with each other, are joined in series, the mathematical operation of determining the resultant requires several additional steps. Methods of solving the vector diagrams of such circuits are described in this section.

The resultant voltage may be obtained by means of: (*a*) A **polar diagram**; by finding the resultant of any two emfs, and then combining this resultant with the third emf, and so on until all the emfs have been included. (*b*) A **topographic diagram**; as in (*a*) by the **method of triangles.** (*c*) A **polar diagram**; by the method of **90° components.** (*d*) A **topographic diagram**; by the method of **90° components.**

The above methods are illustrated in the examples below.

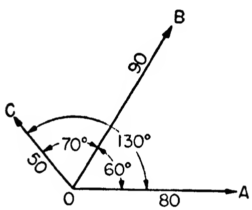


FIG. 20–2. Polar vector diagram of three emfs, *OA*, *OB*, and *OC*, connected in series.

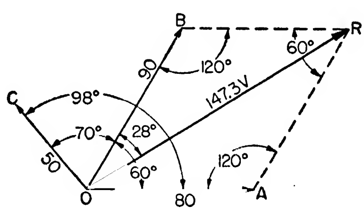


FIG. 21–2. Polar diagram of Fig. 20–2, showing resultant *OR* of the vectors *OA* and *OB*.

Example 2. Three coils, *A*, *B*, and *C*, generating emfs of 80, 90, and 50 volts respectively, are joined in series. The emf of coil *B* is leading that of *A* by a phase difference of 60°, and the emf of *C* is leading that of *A* by a phase difference of 130°, or leading that of *B* by 70°. What is the resultant voltage across the three coils?

The general equation is, $E_{\text{total}} = 80 \oplus 90 \oplus 50$.

Solution: (*a*) **Polar diagram and Cosine Law.** Figure 20–2 shows the polar diagram in which vector *OA* represents the emf of coil *A*; vector *OB*, that of coil *B*; and vector *OC*, that of coil *C*. In Fig. 21–2, vectors *OA* and *OB* are combined into their resultant *OR*, and

$$OR = \sqrt{80^2 + 90^2 + 2 \times 80 \times 90 \cos 60^\circ} = 147.3 \text{ volts.}$$

To combine the resultant OR with the third vector OC, the phase angle between them must be determined.

First find angle BOR in Fig. 21-2.

In parallelogram $OBRA$, $\measuredangle\ BOA = 60°$

Thus $\measuredangle\ OBR = 120°$

and $$\frac{\sin BOR}{\sin 120°} = \frac{BR}{OR} = \frac{80}{147.3} \quad \text{(Law of sines).}$$

$$\text{Sine of } \measuredangle\ BOR = \frac{80}{147.3}\sin 120° = 0.542 \times 0.866 = 0.472.$$

From Table I, $\measuredangle\ BOR = 28°$.

Angle between OR and $OC = \measuredangle\ ROC = 70° + 28° = 98°$.

In Fig. 22-2, OR is combined with OC into their resultant OR_1, the total voltage across the three coils in series.

$$OR_1 = \sqrt{50^2 + 147.3^2 + 2 \times 50 \times 147.3 \cos 98°} = 148.8 \text{ volts.} \quad \textit{Ans.}$$

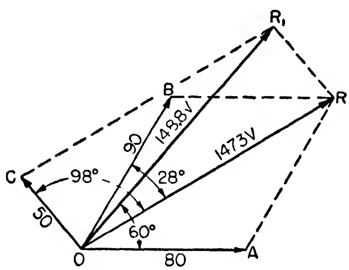


Fig. 22-2. Polar diagram showing how OR, the resultant of OA and OB, is combined with OC to obtain OR_1, the resultant of the three emfs, OA, OB, and OC, in series.

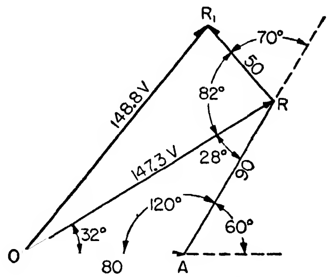


Fig. 23-2. The topographic diagram showing how the resultant OR_1 of the three emfs, OA, OB, and OC in series, is obtained by the method of triangles.

(By application of the law of sines, the phase angle between the resultant OR_1 and any one of the three original emfs may be found.)

Solution: (*b*) **Method of Triangles.** In Fig. 23-2, OB of Fig. 20-2 is laid off from end A of OA as AR. Note that the 60° angle is the outside angle and measured in counter-clockwise direction, leading. The inside angle is thus 120° as shown. The resultant of these two emfs, OR, is drawn from O to R and has the same value as OR in Fig. 21-2. Thus in the triangle OAR,

$$OR = \sqrt{80^2 + 90^2 - 2 \times 80 \times 90 \cos 120°} = 147.3 \text{ volts.}$$

Now lay out OC from R as RR_1, in Fig. 23-2, at an angle of 70° leading $AR(OB)$. Note again that the 70° angle is on the outside, in counter-

clockwise direction, leading. Also, again note that the "tail" of one vector must be attached to the "head" of the preceding vector. The line drawn from O to R_1 is the resultant of the three emfs.

To compute the value of OR_1, we first find angle ROA.

$$\frac{\sin \measuredangle\ ROA}{\sin 120°} = \frac{90}{147.3} \quad \text{(Law of sines).}$$

$$\text{Sin} \measuredangle\ ROA = \frac{90}{147.3} \times \sin 120° = 0.611 \times 0.866 = 0.529.$$

From Table I, $\measuredangle\ ROA = 32°$.

Therefore, $\measuredangle\ ORA = 180° - 120° - 32° = 28°$.

(Sum of interior angles in a triangle equals 180°.)

Now in triangle ORR_1,

$$\measuredangle\ ORR_1 = 180° - 70° - 28° = 82°.$$

and

$$OR_1 = \sqrt{50^2 + 147.3^2 - 2 \times 50 \times 147.3 \cos 82°} = 148.8 \text{ volts.} \quad \textit{Ans.}$$

This value checks with that computed by method (*a*) above.

It is convenient to use this method when the values of the resultant and the several emfs are known, or are measured in an actual circuit, and the phase angles are to be determined.

Solution (*c*) Polar Diagram, 90° Component Method. The three vectors of Fig. 20-2 are again drawn in the polar diagram of Fig. 24-2.

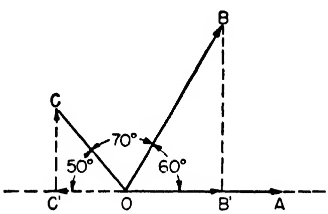


FIG. 24-2. The three emfs, OA, OB, and OC, are resolved into 90° components, parallel to, and at 90° with vector OA.

We now resolve each vector into **components,** parallel to and at 90° with vector OA. By drawing a perpendicular from B to OA, vector OB may be considered to consist of the two vectors OB', with an action **to the right,** and $B'B$ at 90°, with an action **up.** Similarly, by drawing a perpendicular from C to OA (projected), the vector OC becomes equivalent to the two vectors OC', with an action **to the left,** and $C'C$, with an action **up.** The vector OA acts to the **right only** and by an amount represented by the length of OA. (Note the arrow heads on the vector components in the figure.) We now have a total of five vectors all acting parallel to or at 90° with OA.

Thus, we have a total action parallel to OA equal to

$$OA + OB' - OC';$$

and a total action at 90°, or up, equal to

$$B'B + C'C.$$

The resultant, $OR_1 = \sqrt{(OA + OB' - OC')^2 + (B'B + C'C)^2}$,

but
$$\begin{aligned} OA &= 80; \\ OB' &= OB \cos 60° = 90 \cos 60°; \\ OC' &= OC \cos 50° = 50 \cos 50°; \end{aligned}$$
and
$$\begin{aligned} B'B &= OB \sin 60° = 90 \sin 60°; \\ C'C &= OC \sin 50° = 50 \sin 60°. \end{aligned}$$

Therefore, the numerical value of the resultant,

$$\begin{aligned} OR_1 &= \sqrt{(80 + 90 \cos 60 - 50 \cos 50)^2 + (90 \sin 60° - 50 \sin 50°)^2} \\ &= \sqrt{(80 + 90 \times 0.5 - 50 \times 0.64)^2 + (90 \times 0.866 + 50 \times 0.766)^2} \\ &= \sqrt{(80 + 45 - 32.2)^2 + (77.9 + 38.3)^2} \\ &= \sqrt{92.8^2 + 166.2^2} = 148.8 \text{ volts. } \textit{Ans.} \end{aligned}$$

The polar diagram of Fig. 25–2 shows the total action to the **right** as OP, equal to 92.8 volts, and OQ, the total action **up,** equal to 116.2 volts and OR_1, equal to 148.8 volts.

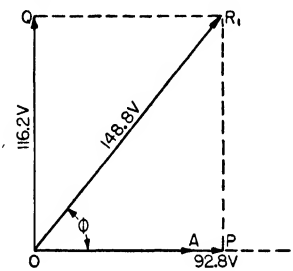


Fig. 25–2. The summations of the horizontal and vertical components of Fig. 24–2 are OP and OQ, respectively, in a polar diagram. The resultant of the three emfs is OR_1.

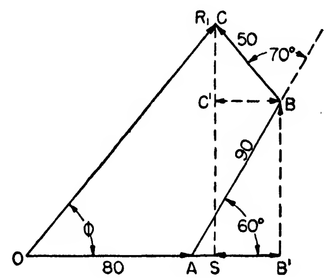


Fig. 26–2. Topographic diagram of the three vectors, OA, AB, and BC, resolved into their horizontal and vertical components to obtain OR_1, the resultant of the three emfs in series.

The phase angle ϕ, which determines the position of OR_1 with respect to vector OA, is easily determined, for

$$\text{Tan } \phi = \frac{116.2}{92.8} = 1.25, \text{ tangent of } 51° \text{ (leading in this case);}$$

$$\text{Cos } \phi = \frac{92.8}{148.8} = 0.624, \text{ cosine of } 51°;$$

$$\text{Sin } \phi = \frac{116.2}{148.8} = 0.7809, \text{ sine of } 51°.$$

Solution (*d*) **Topographic Diagram 90° Component Method.** Applying the same method as in (*c*) above, to the topographic diagram, we

construct Fig. 26-2, similar to Fig. 23-2. In this case, we draw the perpendicular from B to OA (extended), thus separating the vector AB into two components, AB', with an action **to the right,** and $B'B$ at 90′, with an action **up.** Similarly, a perpendicular CS is drawn from C to OA and BC' is drawn parallel to OA from B. Vector BC is now separated into two components, BC', with an action **to the left,** and $C'C$ at 90°, with an action **up.**

The total action parallel to OA equals

$$OA + AB' - BC' = OS$$

The total action at 90° or up (in this case) equals

$$B'B(= SC') + C'C = SC$$

$$OR_1(OC) = \sqrt{OS^2 + SC^2}$$

But $OS = 80 + 90 \cos 60° - 50 \cos 50°$, or 92.8 volts, as before,

and $SC = 90 \sin 60° + 50 \sin 50°$, or 116.2 volts, as before;

Then $OR_1 = \sqrt{92.8^2 + 116.2^2} = 148.8$ volts. *Ans.*

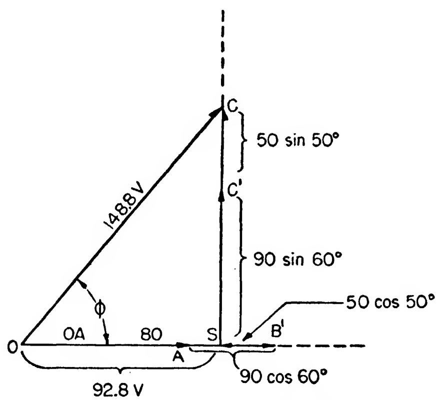


FIG. 27-2. The summation of the horizontal and vertical components of Fig. 26-2 in a topographic diagram are OS and SC. The resultant of the three emfs is OC.

Figure 27-2 is a topographic diagram for this case, showing how the five components of the three vectors form a right angle triangle, the hypothenuse of which is the resultant, or vector sum, of the three voltages.

The solutions illustrated above show that the numerical value of the resultant of a complex diagram is more readily obtained by the use of 90° components. It should also be noted that in using the 90° component methods, as in (*c*) above, the vectors of Fig. 25-2 may be resolved into components, parallel to, and at 90° with any one of the several vectors. Or they may be resolved into components with respect to any other reference line drawn at a known angle with any one of the original vectors.

In the solution of the following problems, show both polar and topographic vector diagrams.

Prob. 30–2. Three coils, A, B, and C, are joined in series and each generates 110 volts. The emf of coil B leads that of A by 90° and the emf of C leads that by B by 30°. (*a*) Determine the resultant voltage by means of the method shown in Fig. 26–2. (*b*) Compute the phase angle between the resultant emf and that of coil A, noting whether it leads or lags.

Prob. 31–2. Three coils, each generating 100 volts, are joined in series. The emf of coil B leads that of A by 45° and the emf of C leads B by 30°. (*a*) Solve for the resultant emf by the method of triangles, as in Fig. 26–2. (*b*) Compute the phase angle between the resultant and the emf of coil A, noting whether it leads or lags.

Prob. 32–2. Solve Prob. 31–2 by the 90° component method, as in Fig. 24–2.

Prob. 33–2. In a series arrangement of three coils, coil A generates 60 volts; coil B, 120 volts, leading that of A by 75°; and coil C, 100 volts, leading A by 120°, and B by 45°. What is the resultant voltage and the phase angle (leading or lagging) between it and the emf of coil C? Solve by the method of Fig. 26–2.

Prob. 34–2. Solve Prob. 33–2 by the method of 90° components, as in Fig. 24–2.

Prob. 35–2. What would be the resultant emf, in Prob. 33–2, and its phase angle with the emf of coil C, if the emf of coil B lags 105° behind that of coil A? Compute by the method of Fig. 24–2.

Prob. 36–2. What is the resultant of a series arrangement of four coils, A, B, C and D, generating emfs of 100, 80, 70 and 50 volts respectively. The emf of coil B leads that of A by 90°, emf of C leads B by 60° and emf of D lags 75° behind A. Compute the resultant by the method of Fig. 24–2, resolving the vectors into components parallel and at 90° to the emf of coil A. What is the phase angle (leading or lagging) between the resultant and the emf of coil A?

Prob. 37–2. Solve Prob. 36–2 by resolving the vectors into components, parallel and at 90° to the emf of coil D. Compute the phase position of the resultant with the voltage of coil D, and compare its position in this problem with that in Prob. 36–2.

Prob. 38–2. Each of three coils generates an emf of 200 volts. The emf of the second leads that of the first by 120°, and the third lags behind the first by the same angle. What is the resultant emf across a series combination of the three coils?

6–2. Reversing Coil Connections Reverses Phase Relations. It was shown in Art. 1 that when the two coils, AB and A_1B_1, of Fig. 2–2, Art. 1–2, are joined in series, their emfs combine in phase

with each other, as shown in Fig. 3–2. It was also shown in Art. 2 that when these coils are reconnected with connections to coil A_1B_1 reversed, as in Fig. 6–2, Art. 2–2, the emf in coil A_1B_1 is reversed, or displaced 180° from that in coil AB, as shown in Fig. 7–2. Thus,

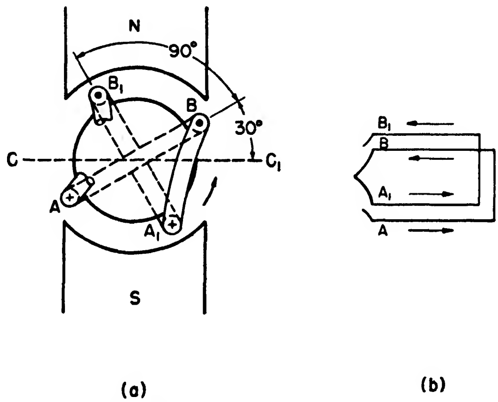


FIG. 28–2. Two coils at 90°, connected in series B to A_1. The emf of coil A_1B_1 leads that of AB by 90°.

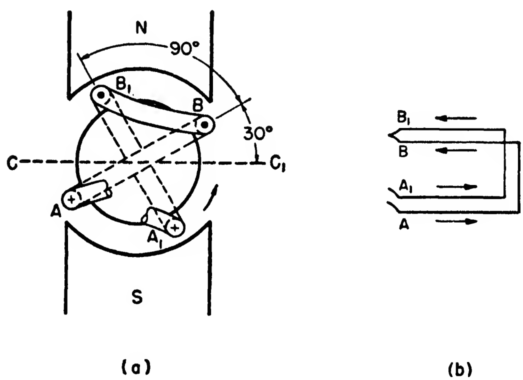


FIG. 29–2. The connections to coil A_1B_1 of Fig. 28–2 are reversed and connected end B_1 to B. The emf of coil A_1B_1 now combines at 90° behind that of coil AB.

reversing the connections to a coil causes its emf to combine in the circuit at 180° from its former position. This is true regardless of the relative phase position of the coils on the armature.

Consider Fig. 28–2, which is the same as Fig. 11–2, Art. 3–2. A_1B_1 is shown, rotating 90° ahead of AB and the two coils are in series; end B of coil AB connected to end A_1 of coil A_1B_1. The

vector diagram for this relation is shown in Fig. 30–2 in which OA represents the effective emf of coil AB; and OA', the emf of A_1B_1, leading OA by 90°. Then OR represents the resulting emf of the two coils.

Now, if the connections to coil A_1B_1 are reversed, and end B_1 is connected to end B of coil AB, as in Fig. 29–2, the emf of A_1B_1 will combine with that of AB in reversed sense, or at 180° from its former position in Fig. 28–2. We, therefore, draw OA' in Fig. 30–2, reversed from its former position, as shown by the dash line OA'. The emf of coil A_1B_1 now lags 90° **behind** OA, and the resultant emf is now OR'. Note, however, that OR' has the same numerical value and differs only in its phase relation. Thus, if the emfs of the two series connected coils are at 90° to each other, it will make no difference in the **numerical** value of the resultant emf which two ends are connected.

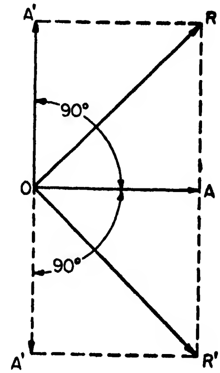


FIG. 30–2. Vectors OA and OA' represent two emfs in series at 90°. Reversing one emf in the circuit does not change the value of the resultant OR, which becomes OR'.

But suppose the two coils are displaced on the armature by 60°, as shown in Fig. 16–2, Art. 4–2. The emf of coil A_1B_1 leads that of AB by 60°, and end B of AB is connected to end A_1 of A_1B_1. The vector diagram for this connection is again shown in Fig. 32–2, in which OA represents the emf of coil AB and OA' that of A_1B_1. Now if connections to coil A_1B_1 are reversed and end B_1 is connected to end B of AB, as in Fig. 31–2, the emf of coil A_1B_1 combines with that of AB in reversed sense, or at 180° from its former position. We, therefore, draw the vector OA', in Fig. 32–2, 180° from its former position, as shown by the dash line OA'. The emf of coil A_1B_1 now lags behind that of AB by 120°, and the resultant emf is OR'. This is numerically less than with the former connection. Thus, if the emfs of two series connected coils differ in phase by an angle other than 90°, both the value of the resultant emf and its phase relation are changed, when the connections to one coil are reversed.

From both the connections of the coils on the armature and the vector diagrams, it can be seen that:

Reversing a coil which generates an emf, changes its phase relations to other coils in the circuit by 180°.

On the actual machine, there is usually no convenient way of telling by inspection the angles between the emfs of two or more coils. The usual method is to connect the coils in series at random,

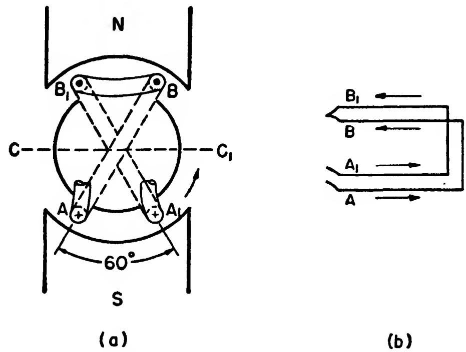


FIG. 31–2. Similar to Fig. 16–2, but with the connections to coil A_1B_1 reversed, so that the two emfs combine at 120°.

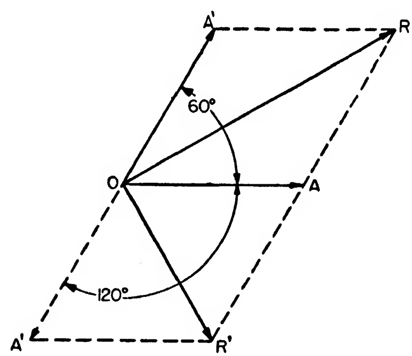


FIG. 32–2. Vector OR is the resultant of two emfs, OA and OA', at 60°. When OA' is reversed, it lags 120° behind OA and the resultant is OR', changed both in value and phase position.

and measure the emf of each coil and the emf of the series combination; and from the voltmeter readings determine the phase angle.

Example 3. The emf of each of two coils, as read by a voltmeter, is 220 volts. When the coils are connected in series at random, the resulting emf is 381 volts. (*a*) What is the phase angle of the emfs of the two coils? (*b*) If the connections to one of the coils is reversed, what will be the phase angle between the emfs, and what will be the resulting voltage?

Solution: (*a*) The vector diagram, Fig. 33–2(*a*), is constructed to comply with the voltmeter readings, and we solve for angle ϕ.

By cosine law for parallelograms,

$$381^2 = 220^2 + 220^2 + 2 \times 220 \times 220 \cos \phi;$$

$$\text{Cos } \phi = \frac{381^2 - 220^2 - 220^2}{2 \times 220 \times 220} = 0.50.$$

From Table I, $\phi = 60°$. *Ans.*

(*b*) Now if we reverse one coil as *OB* the vector diagram will be, as in Fig. 33–2(*b*). The angle between the emfs of the two coils is now $180° - 60°$, or $120°$. *Ans.*

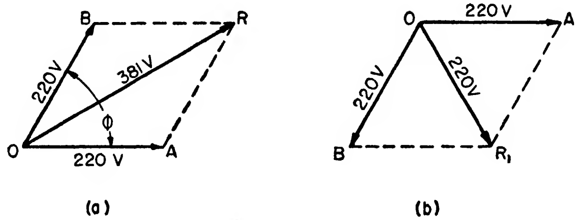


FIG. 33–2. (*a*) The resultant emf of two series connected coils, each generating 220 volts, is 381 volts. To determine the angle between them. (*b*) Coil *OB* is reversed in its connection to *OA*. To determine the resultant voltage.

The resultant emf will now be,

$$OR_1 = 220^2 + 220^2 + 2 \times 220 \times 220 \cos 120°$$

$$= \sqrt{220^2 + 220^2 - 220^2} = \sqrt{220^2} = 220 \text{ volts.}$$

Show both polar and topographic vector diagrams in the solution of the following problems.

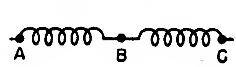


FIG. 34–2. Two coils connected in series. To determine the phase angle between them.

Prob. 39–2. Two coils, *AB* and *BC*, of Fig. 34–2, are connected in series and a voltmeter, placed across the circuit, or from *A* to *C*, indicates 110 volts; while the voltmeter, reading across *A* to *B*, is 440 volts, and across *B* to *C* is also 440 volts. What is the phase angle between the emfs of the two coils?

Prob. 40–2. If the connections to one of the coils in Prob. 39–2 is reversed, what will a voltmeter across the two coils indicate, and what will be the angle between their two emfs?

Prob. 41–2. If the voltage across the circuit from *A* to *C* in Fig. 34–2 were 440 volts, while a voltmeter across each of the two coils also

indicated 440 volts, what would be the phase angle between the emfs of the two coils?

Prob. 42–2. If the connections to one of the coils in Prob. 41–2 is reversed, what would be the voltage across the combination, and the phase angle between the emfs of the coils?

Prob. 43–2. When two coils, AB and $A'B'$, are connected, as in Fig. 35–2, the combined emf across them is 240 volts. The emf of each coil is 180 volts. Find the phase angle between them.

Prob. 44–2. In Prob. 43–2, if end A' of coil $A'B'$, in Fig. 35–2, were connected to end B of coil AB, what would the combined emf be? What would be the phase angle of the emfs of the two coils?

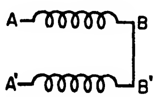


FIG. 35–2. To determine the phase angle between the emfs of the series connected coils.

FIG. 36–2. Three series connected coils.

Prob. 45–2. In Prob. 43–2, if end B' of coil $A'B'$ were connected to end A of coil AB, what would be the combined emf and the phase angle between the emfs of the two coils?

Prob. 46–2. A series circuit consists of three coils, as in Fig. 36–2. The following voltmeter readings are taken under steady conditions of operation. Across, $AB = 100$; $BC = 100$; $CD = 100$; $AC = 173.2$; $BD = 100$; $AD = 100$. What are the phase relations between these emfs?

Prob. 47–2. Solve Prob. 46–2 on the basis of the following voltmeter readings: $AB = 100$; $BC = 100$; $CD = 100$; $AC = 100$; $BD = 100$; $AD = 173.2$.

Prob. 48–2. Solve Prob. 46–2 on the basis of the following voltmeter readings: $AB = 100$; $BC = 100$; $CD = 100$; $AC = 173.2$; $BD = 173.2$; $AD = 200$.

Prob. 49–2. Solve Prob. 46–2 on the basis of the following voltmeter readings: $AB = 100$; $BC = 100$; $CD = 100$; $AC = 100$; $BD = 100$; $AD = 0.0$.

7–2. Phase Angle of Emfs Depends upon Both Polarity and Position of Coils. In preceding figures in this chapter, representing coils placed on a rotating armature, the direction of the instantaneous induced emfs, according to Fleming's right hand rule, is indicated. Since, in each of these figures, the beginning of the emf cycle in each coil is measured from the position CC_1, these coils are generating emfs in the first, or positive, half-cycle of the sine wave.

Thus, in these figures, the mark ⊕ and ⊙ in the circles (and also the arrows on the conductors) indicate definite directions of induced emf. So the positive direction of emf in the coils marked AB is from A to B; and in the coils marked A_1B_1 is from A_1 to B_1. Of course, as the armature rotates, the induced emf in each of the coils reverses during the second half-cycle and becomes negative, but a direction from A to B, through each of the coils, may be taken as the positive direction.

Thus in Fig. 15–2, Art. 4–2, coil A_1B_1 is at the 90° position in its cycle, and its emf has reached a positive maximum value in a

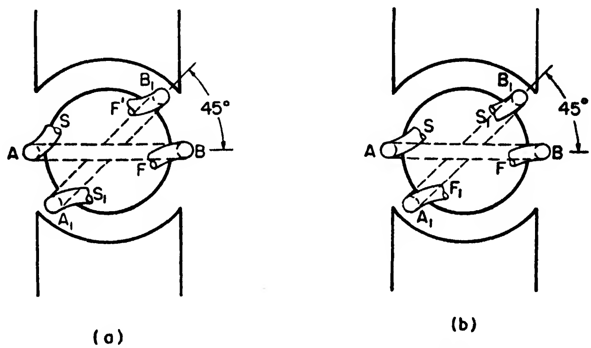


FIG. 37–2. (*a*) Assuming a positive direction through coil AB to be from S to F, and through coil A_1B_1 to be from S_1 to F_1, the emf of coil A_1B_1 leads AB by 45°. (*b*) Assuming the positive direction through coil A_1B_1 to be reversed (note reversal S_1F_1), the emf of coil A_1B_1 *lags behind* that of AB by 135°.

direction through the coil from A_1 to B_1. As the armature turns 60° farther in its revolution, coil AB also reaches the 90°, or corresponding position in its cycle, and its emf is now a positive maximum value in a direction from A to B; that is, in the **same** or **corresponding** direction through this coil. Therefore, with respect to a positive (or the same) direction through both coils, the emf of coil A_1B_1 is leading that of AB by 60°. And if the coils are connected in series B to A_1, as in Fig. 16–2, so that their emfs both add together in a positive (the same, or corresponding) direction in the circuit, as shown by the arrows in Fig. 16–2, their emfs will combine at 60°.

This idea also can be further illustrated by considering Fig. 37–2. The armature is represented as rotating counter-clockwise; but the direction both of the magnetic field and the instantaneous emf

in the coils is not indicated. If it be assumed, in Fig. 37–2(*a*), that in coil AB a direction of emf from A to B, or from S to F, through the coil is positive, and in coil A_1B_1 a direction from A_1 to B_1, or from S_1 to F_1, is also positive, then the voltage of coil A_1B_1 leads that of AB by 45°. This is so, because coil A_1B_1 goes through its sequence of positions 45° earlier than coil AB, as already explained.

On the other hand, if it be again assumed that the positive direction of emf in coil AB is from A to B, or S to F, Fig. 37–2(*b*), while the positive direction of emf in coil A_1B_1 is now assumed from B_1 to A_1, or from S_1 to F_1, then the emf in coil A_1B_1 lags behind that of AB by 180° − 45°, or 135°, or the emf in coil AB is 135° ahead

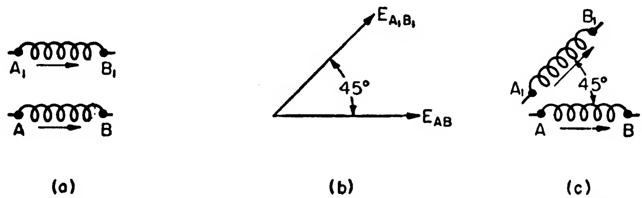


FIG. 38–2. (*a*) The positive direction of emf in the coils is indicated by the arrows. (*b*) The vectors with their subscripts indicate that emf of coil A_1B_1 leads that in AB by 45° with respect to a positive direction through each coil from A to B. (*c*) Also indicates that emf of coil A_1B_1 leads that of AB by 45°.

of that of A_1B_1. Again, this is so, because coil AB goes through its sequence of positions 135° earlier than does coil A_1B_1.

Thus, in the above figures, we must first assume a positive (or corresponding) direction of voltage through each coil (**not direction of instantaneous emfs**), and then their phase relation is clearly established from the position of the coils on the armature.

8–2. Coil Diagrams and Vector Notation. The definite phase position, at which the emfs in the coils on the armature of Fig. 37–2 combine with each other in the electric circuit, can be shown by **coil diagrams** and **vector diagrams properly marked.**

The coils may be represented, as in Fig. 38–2(*a*), in which the arrows represent a **positive direction** of the emf in each coil. The phase relation of these emfs is indicated by the vector diagram of Fig. 38–2(*b*) which shows the emf of coil A_1B_1, leading that of coil AB by 45°. The coil diagram may also be indicated, as in Fig. 38–2(*c*), which shows clearly that, with respect to the positive direction through the coils as shown by the arrows, A_1B_1 leads AB by 45°.

Three facts should be carefully noted here.

First, the arrows under the coils of Figs. 38–2(*a*) and 38–2(*c*) are intended to indicate the direction of the positive maximum instantaneous voltage in each coil. They do **not** indicate the direction of the instantaneous voltages in the two coils at the same instant.

Second, the notation, or lettering of the vectors, in Fig. 38–2(*b*), $E_{A_1B_1}$ and E_{AB}, is intended to convey the idea that the emf of coil A_1B_1 leads that of AB by 45° (in this case) with respect to a positive direction through each coil from A to B. This will be further illustrated below.

Third, a vector diagram, such as that of Fig. 38–2(*b*), shows only two things. The length of the vectors represents only the magni-

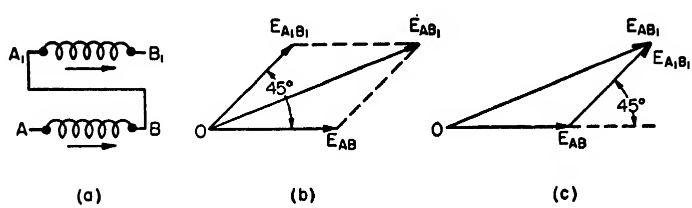


FIG. 39–2. (*a*) The coils are connected in series with the positive direction of their emfs in the same direction in the circuit. (*b*) The polar diagram indicates that the emfs combine at 45° to produce the resultant emf E_{AB_1} from A to B_1. (*c*) The corresponding topographic diagram.

tude of the induced voltages, while the angle between them indicates only the time interval which exists between the maximum positive instantaneous values of their respective sine curves. The vectors in themselves do **not** show how the coils are connected in the electrical circuit. This is also further explained in the following paragraphs.

The coils of Fig. 38–2 may be connected in series in the electrical circuit in four different ways, and their corresponding phase relations and resulting voltage shown by corresponding vector diagrams, as illustrated below.

(*a*) When the two coils are connected B to A_1, as in Fig. 39–2(*a*), the positive direction of their emfs is in the same direction in the electric circuit, as shown by the arrows, and, therefore, combine with the emf of A_1B_1, leading AB by 45°. Thus the vectors E_{AB} and $E_{A_1B_1}$ of Fig. 39–2(*b*) are added, as they stand, to obtain the resultant E_{AB_1}; the emf across the two coils A to B_1, Fig. 39–2(*c*), shows the topographic diagram for this connection. Note that we

take the emf of coil AB, and to it add vectorially the emf of coil A_1B_1, in that sequence, and we can write,

$$E_{AB} \oplus E_{A_1B_1} = E_{AB_1}$$

(*b*) When the coils are connected B to B_1, as in Fig. 40–2(*a*), the positive direction of the emf in coil A_1B_1 is reversed **in the circuit** with respect to coil AB, as shown by the arrows. Thus in Fig. 40–2(*b*), vector $E_{A_1B_1}$ is reversed, or drawn at 180° to its former position in the diagram, and becomes $E_{B_1A_1}$ (note reversal of subscripts). It now combines in reversed sense, or lagging 135° behind E_{AB}. The resulting voltage A to A_1 across the two coils is

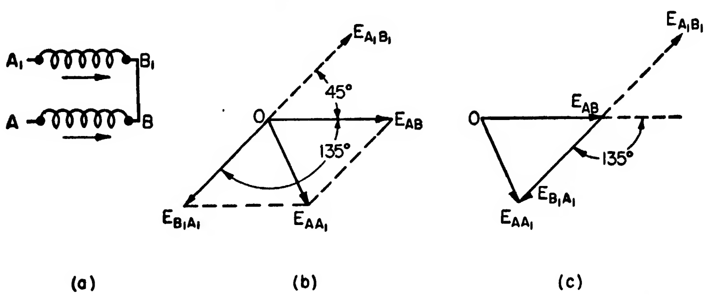


FIG. 40–2. (*a*) The coils connected in series with the positive direction of emf in coil A_1B_1, reversed in the electric circuit. (*b*) The polar diagram indicates that emf of coil A_1B_1, combines reversed with that of AB to produce the resultant emf E_{AA_1} from A to A_1. (*c*) The corresponding topographic diagram.

now E_{AA_1}. The topographic diagram is shown in Fig. 40–2(*c*). We now write,

$$E_{AB} \oplus E_{B_1A_1} = E_{AA_1}$$

(*c*) When the coils are connected B_1 to A, as in Fig. 41–2(*a*), the positive direction of the emf in both coils is reversed through the circuit from B to A_1 (considering the emf of coil AB first, and combining with it the emf of coil A_1B_1 in the same sequence, as in cases (*a*) and (*b*) above). Thus both emfs combine in reversed sense from that in Fig. 39–2.

Therefore, in the vector diagram, Fig. 41–2(*b*), the resultant emf, B to A_1, is the vector sum of E_{AB} reversed and $E_{A_1B_1}$ reversed, which become E_{BA} and $E_{B_1A_1}$ respectively. The resultant emf is now E_{BA_1}. This relation is also shown in the topographic diagram, Fig. 41–2(*c*). Note that the two emfs again combine at 45°, but

with the emf of coil AB (reversed), **lagging behind** that of A_1B_1 (reversed). Also the resultant emf, E_{BA_1}, is numerically equal to the emf E_{AB_1} but at 180° from its position in case (*a*) above. We may, in this case, write

$$E_{BA} \oplus E_{B_1A_1} = E_{BA_1}.$$

(*d*) And finally, when the coils are connected A to A_1, Fig. 42–2(*a*), the positive direction of the emf of only coil AB is re-

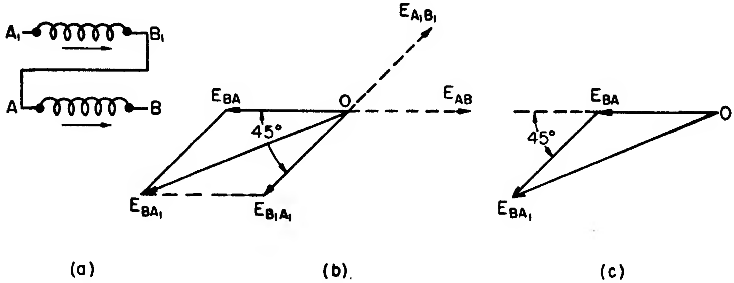


FIG. 41–2. (*a*) The emf from B to A_1 is that of a series connection with the positive direction of emf in both coils reversed in the electric circuit. (*b*) The polar diagram shows both vectors reversed to produce the resultant E_{BA_1} from B to A_1. (*c*) The corresponding topographic diagram.

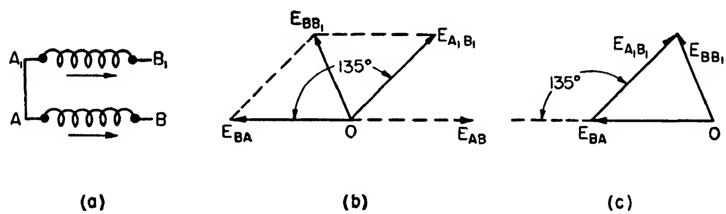


FIG. 42–2. (*a*) The coils in series with the positive direction of emf in coil AB, reversed in the electric circuit. (*b*) The polar diagram indicates that emf of coil AB combines, reversed, with that of A_1B_1 to produce E_{BB_1}, the emf from B to B_1. (*c*) The corresponding topographic diagram.

versed through the circuit from B to B_1 (using the same sequence as before), as shown by the arrows. Thus, in Fig. 42–2(*b*) the vector E_{AB} is reversed and becomes E_{BA}, leading $E_{A_1B_1}$ by 135°. The resultant emf is now E_{BB_1}. Figure 42–2(*c*) is the topographic diagram. Note that the resultant emf, E_{BB_1} is numerically equal to, but at 180° from the resultant emf, E_{AA_1} in case (*b*) above.

Reversal of Sequence in the Electric Circuit. In the four cases above, the direction through the electric circuit has been in

sequence through coil AB and then coil A_1B_1. Thus in (*a*), Fig. 39–2, the direction through the circuit is taken from A to B_1, and the resultant emf is shown in the vector diagrams.

If now, we reverse the sequence, and take the direction through the circuit of Fig. 39–2(*a*) from B_1 to A, the positive direction of the emfs of both coils is reversed. Therefore, in the diagram of Fig. 43–2, both vectors $E_{A_1B_1}$ and E_{AB} are reversed and become $E_{B_1A_1}$ and E_{BA} respectively, and their vector sum, or the resultant **emf** from B_1 to A becomes E_{B_1A}. Note the change in subscripts. This resultant emf is numerically equal to the resultant in Fig. 39–2, but at 180° from it.

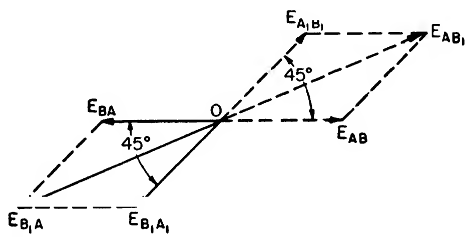


FIG. 43–2. With sequence of coils of Fig. 39–2(*a*) reversed, the resulting emf, E_{B_1A}, from B_1 to A is that of both vectors reversed.

Similarly in case (*b*), Fig. 40–2, the emf through the circuit from A_1 to A is equal to that through the circuit from A to A_1, but displaced by 180° from it. This is also true with respect to the other connections of the coils.

Thus reversing the order or sequence, through the electric circuit does not change the numerical value of the resultant emf, but does displace its position by 180°.

In the solution of the following problems show both diagram of coil connections and vector diagrams. Use vector notation, as explained in the above paragraphs.

Prob. 50–2. Two coils, AB and A_1B_1, generate 50 and 75 volts respectively. The emf of coil A_1B_1 leads that of AB by 75° with respect to a positive direction through the coils, as indicated in Fig. 44–2. When the coils are joined in series B to A_1, what will be the relative phase positions of the emfs of the two coils and the resultant voltage?

Prob. 51–2. Answer the questions in Prob. 50–2, if the coils are joined in series B to B_1.

Prob. 52–2. Answer the questions in Prob. 50–2, if the coils are joined A to A_1.

Prob. 53–2. Each coil in Fig. 45–2 generates 120 volts. With respect to a positive direction through the coils, as indicated, the emf of coil A_1B_1 leads AB by 90°, and A_2B_2 leads A_1B_1 by 30°. (*a*) When the coils are joined in series, B to A_1 and B_1 to A_2, what is the resultant voltage? (*b*) What is its phase position with respect to the emf of coil AB?

Prob. 54–2. Answer the questions of Prob. 53–2, when the coils are joined in series B to B_1 and A_1 to A_2.

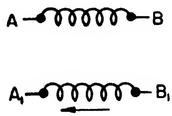


FIG. 44–2. The emf of coil A_1B_1 leads that of AB by 75° with respect to a positive direction of emf through the coils, as shown by the arrows.

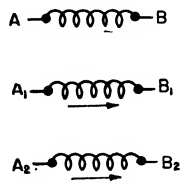


FIG. 45–2. The positive direction of emf through the coils is shown by the arrows.

Prob. 55–2. Answer the questions of Prob. 53–2(*a*) when the coils are joined in series A to A_1 and B_1 to B_2. (*b*) When the coils are joined in series B to A_1 and B_1 to B_2.

9–2. Single-Phase Circuits and Machines. When the coils on the armature of an a-c generator are connected inseries so that they form a single winding, brought out to two terminals, the machine is called a **single-phase** alternator. These terminals are connected to collector rings, if the machine is of the revolving armature type, or are brought out through the frame of the machine, if the field revolves. In either case, the two terminals of the winding are connected to a two-wire, or **single-phase circuit.**

A single-phase alternator, supplying power to lamps, and to a single-phase motor is indicated in Fig. 46–2. Note that the circuit supplies power to only one winding in the motor.

The flow of power in such an a-c circuit is not steady, but is pulsating. In fact, as will be shown in Chapter III, instantaneous values of power drop to zero twice in each cycle, and may even be negative, or reversed, during parts of each cycle. The operation of motors, particularly of large size, connected to such a circuit is not entirely satisfactory. Moreover, single-phase motors require special starting devices.

Because of the above facts, alternator armatures are generally constructed with **two or more separate windings, called phases.** All of these windings generate equal emfs which are displaced in phase position from one another on the armature. Power is supplied from these alternators, over circuits of more than two wires,

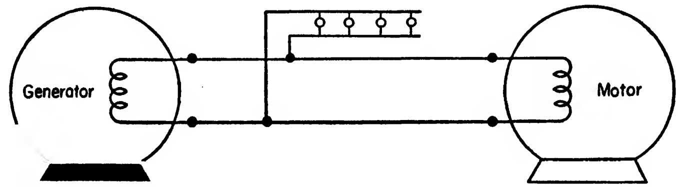


FIG. 46–2. A single-phase generator supplying power to both lamps and a single-phase motor.

to motors having two or more circuits in their windings. Such alternators and motors are called " polyphase " machines, and the circuits " polyphase " circuits.

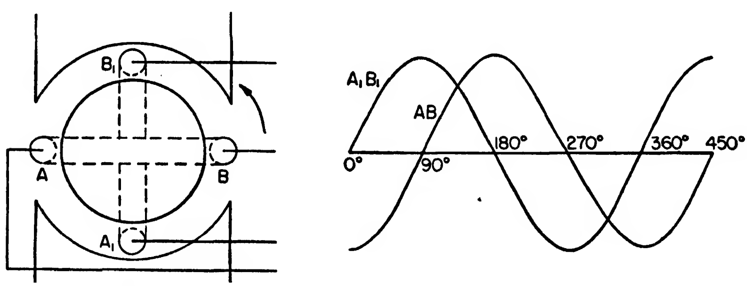


FIG. 47–2. (*a*) A two-phase generator. (*b*) The emf curves of the two phases are displaced 90°.

10–2. Two-Phase Circuits and Machines. When the armature of an a-c generator consists of two (or four) windings, displaced 90 electrical degrees from each other, the machine is called a " two-phase," or " quarter-phase," alternator. Two such windings, or phases, on a revolving armature are represented by coils AB and A_1B_1, in Fig. 47–2(*a*). Assuming a positive direction of emf through each coil from A to B and counter-clockwise rotation, the emf of coil A_1B_1 leads that of AB by 90°. The emf of coil A_1B_1 is thus a maximum when that of AB is zero, as indicated by the curves in Fig. 47–2(*b*).

Figure 48–2 represents a two-phase alternator, supplying power over a two-phase four-wire line to lamps and to a two-phase motor. Note that there are two separate windings in the motor, as in the generator, and each motor winding is supplied by a separate single-phase circuit, there being no electrical connection between

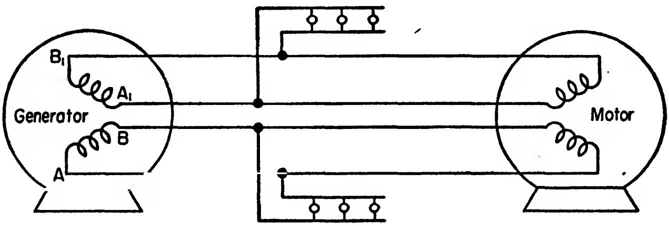


FIG. 48–2. Two-phase generator, supplying power to lamps and to a two-phase motor over a two-phase, four-wire system.

the phases. This four-wire circuit is, of course, more complicated than a two-wire circuit, but the flow of power is constant, not pulsating, as in a single-phase circuit.

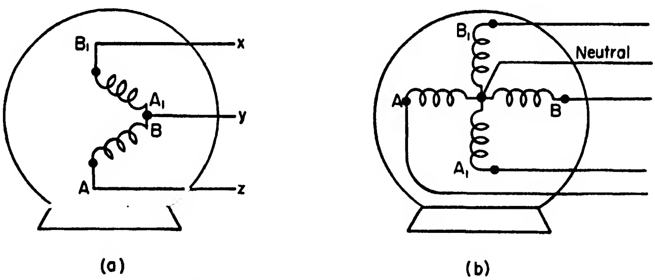


FIG. 49–2. (*a*) Two-phase generator, connected to supply a two-phase, three-wire system. (*b*) Windings of a two-phase generator, interconnected to supply a two-phase, five-wire system.

By joining the two phases together, B to A_1 as in Fig. 49–2(*a*), the alternator may be connected to a two-phase, three-wire circuit. The single-phase lamp loads are connected across each phase, as before, but the motor windings must be connected together. This arrangement reduces the number of wires and simplifies the circuit, but the voltage across the outside wires is now the vector sum of the two windings, or phase, at 90°, and equal to 1.41 times the voltage per phase.

When the mid points only of the two phases are connected together, as in Fig. 46–2(*b*), a five-wire, two-phase circuit consist-

ing of four wires and a " neutral " may be obtained. The voltage between the neutral and any of the other four wires is half that of one phase, or one winding. Because of its complexity this circuit has been seldom used.

Two-phase circuits and machines are little used today, except in older plants, having been supplanted by the three-phase system.

Prob. 56–2. If the voltage across each phase, in Fig. 49–2(*a*), is 120 volts, what is the voltage between the outside wires x and z?

Prob. 57–2. If 460 volts are desired across the outside wires in Fig. 49–2(*a*), what must be the voltage across each phase?

Prob. 58–2. A two-phase generator, 2300 volts per phase, is connected, as in Fig. 48–2, to a four-wire system for distribution of power, with all wires equally insulated. If one of the wires is " grounded, " what voltage is brought to bear upon the insulation, separating each of the other three wires from ground?

Prob. 59–2. The two-phase generator of Prob. 58–2 is connected to a three-wire system for distribution of power, as in Fig. 49–2(*a*). If the common wire of the two phases becomes connected to ground, to what maximum voltage is the insulation between each of the other wires and ground subjected?

Prob. 60–2. Solve Prob. 59–2 on the supposition that it is one of the " outside " wires of the three-wire system (not the common wire), which became connected to ground.

Prob. 61–2. Show by vector diagram that in reality **three phases** (not " three-phase "), but three emfs differing in phase with respect to one another, may be obtained from any three-wire system connected to a two-phase generator. Specify the voltages and phase relations of these three phases, using a 2300-volt, two-phase generator.

Prob. 62–2. If the two-phase generator of Prob. 57–2 is connected to a five wire system, as in Fig. 49–2(*b*), what is the voltage between the neutral and each of the other four wires?

11–2. Three-Phase Generator. Consider an armature on which three coils A_1B_1, A_2B_2, and A_3B_3 have been placed, as in Fig. 50–2(*a*), and note this figure carefully. If we assume the positive direction of the emf in each coil to be from S (start) to F (finish), also shown by the arrows; then the emf of coil A_3B_3, from S_3 to F_3, leads that of coil A_1B_1 from S_1 to F_1 by 60°, and the emf of coil A_2B_2, from S_2 to F_2, leads that of coil A_3B_3 from S_3 to F_3 also by 60°. The curves of Fig. 50–2(*b*), representing the emfs in these coils, show their phase relations with respect to each other. And

the diagram of Fig. 50–2(c) with proper vector subscripts also shows the emfs in the coils to be at 60° with each other.

However, if we consider the positive directions of emf, in coils A_1B_1 and A_2B_2 to be from S_1 to F_1 and from S_2 to F_2, as in Fig.

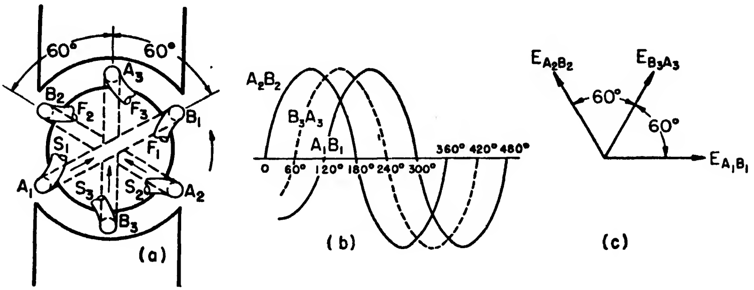


FIG. 50–2. (a) Assuming a positive direction of emf in the three coils to be from S_1 to F_1, S_2, to F_2, and S_3 to F_3, the coils differ in phase by 60°. (b) The emf curves of the three coils are 60° apart. (c) Shows the phase position of the three emfs under the assumption in (a).

50–2(a), but assume the positive direction of the emf in coil A_3B_3 to be in the opposite direction, as reversed, the new relations of the

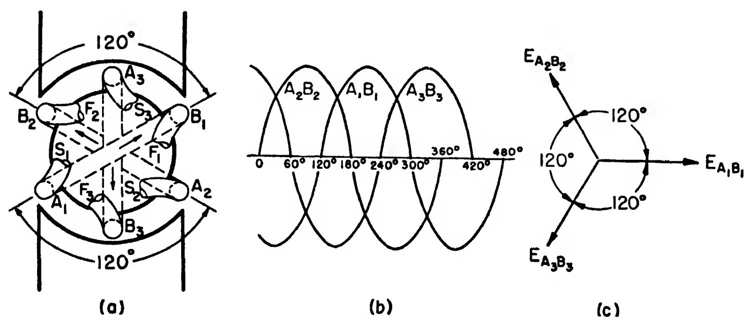


FIG. 51–2. (a) Assuming the positive direction of emf in coil 3 to be reversed, the positive direction in the three coils is from S_1 to F_1, S_2 to F_2, and S_3 to F_3, the emfs all differ in phase by 120°. (b) The emf curves of the coils are 120° apart. (c) Polar diagram, showing the position of $E_{B_3A_3}$ of Fig. 39–2(c) reversed. The three emfs all differ by 120° — a three-phase machine.

coils can be shown in Fig. 51–2(a). Note that the "start" and "finish," S_3 and F_3, of coil A_3B_3, are reversed in position, as is the direction of the arrow, representing positive direction of emf. Therefore, both the curve and the vector, representing this emf, are reversed in Figs. 51–2(b) and 51–2(c). From both curves

and vector diagram, it is seen that the emf of coil A_3B_3 now lags 120° behind that of A_1B_1, or leads A_2B_2 by 120°, while A_2B_2 leads A_1B_1 by 120°. The emfs in the three coils are thus all displaced from each other by a phase angle of 120°.

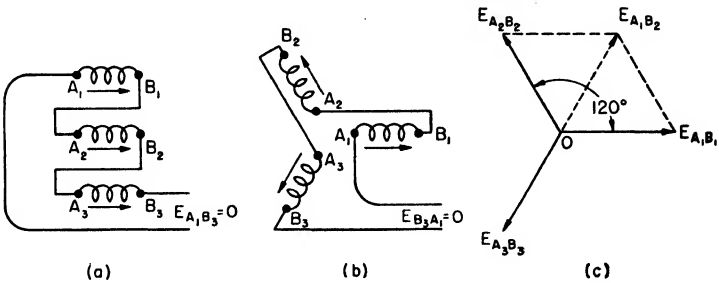


FIG. 52–2. (*a*) Showing the three phases of the machine in Fig 51–2(*a*), with the positive direction of their emfs in the same direction in the series circuit. (*b*) Another arrangement of the diagram for the connection of the phases as in (*a*). (*c*) The polar vector diagram for (*a*) and (*b*), showing that the vector sum of the three emfs, so connected, is zero.

A three-phase alternator is one that generates three equal emfs, displaced 120° from one another; thus the generator of Fig. 51–2 is a three-phase machine.

If the three coils or phases of Fig. 51–2(*a*) are joined in series B_1 to A_2 and B_2 to A_3, as in the coil diagrams of Figs. 52–2(*a*) and

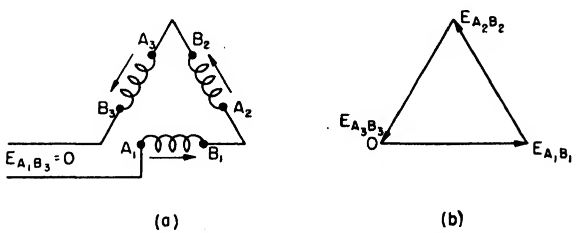


FIG. 53–2. (*a*) The same connection of the phases, as in Fig. 52–2, in another form of coil diagram. (*b*) The corresponding topographic vector diagram.

52–2(*b*), so that their emfs all combine in the same, or positive direction through the electric circuit, the resultant voltage from A_1 to B_3 is **zero**. This is shown by the polar vector diagram of Fig. 52–2(*c*). Vectors $E_{A_1B_1} \oplus E_{A_2B_2} = E_{A_1B_2}$. Since $E_{A_1B_1}$ and $E_{A_2B_2}$ differ in phase by 120°, $E_{A_1B_2}$ is numerically equal to either of them and differs in phase from each by 60°. (See Example 3,

part *b*, Art. 6–2.) Thus $E_{A_1B_2}$ and $E_{A_3B_3}$ are equal and differ in phase by 180°, so their sum is zero. Therefore, we may write,

$$E_{A_1B_1} \oplus E_{A_2B_2} \oplus E_{A_3B_3} = 0. \tag{4–2}$$

Figure 53–2 shows another arrangement of a coil diagram for the same connection of the phases, together with the corresponding topographic vector diagram. Note in Fig. 53–2(*b*) that the "head" of $E_{A_3B_3}$ closes on the "tail" of $E_{A_1B_1}$, and the resultant emf $E_{A_1B_3}$ is again seen to be zero.

12–2. Three-Phase Generator, Open-Delta Connection. It has been shown that when two coils, generating equal voltages, are so connected in series that their emfs combine at a phase difference of 120°, the resulting voltage will be that of one coil.

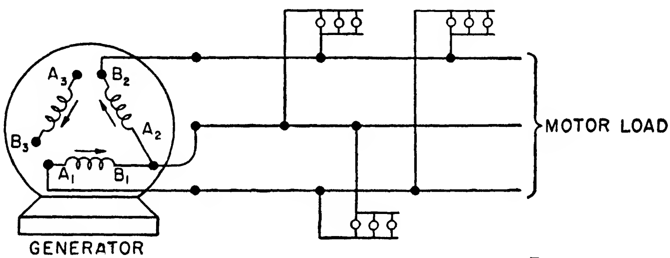


Fig. 54–2. Two phases of a three-phase generator, connected in "open delta" and supplying a three-phase, three-wire system.

Accordingly, if two phases of a three-phase generator are thus joined in series, the third phase being entirely disconnected, and leads are brought out to a three-wire circuit, as illustrated in Fig. 54–2, the voltage between any pair of line wires will be that of one phase of the generator. Such a circuit is called "three-phase." Single-phase circuits for lights, etc., may be taken from any two of the line wires and a three-phase motor may be operated from all three lines.

This connection of the armature windings is called an "open-delta" connection. Alternating current generators are very rarely connected in this manner, but transformers, however, are often so connected. (See *D*, Fig. 1–1.)

A constant flow of power is delivered by an open-delta connection.

Prob. 63–2. The three-phase generator of Fig. 55–2 has both ends of each phase of the armature brought out to terminals. It is desired

to connect the machine in open delta, and, on testing out the ph with a voltmeter, the following data are obtained:

$$\begin{aligned}
\text{From } A_1 \text{ to } B_1 &= 230 \text{ volts.}\\
\text{From } A_1 \text{ to } A_2 &= 0.\\
\text{From } A_1 \text{ to } A_3 &= 0.\\
\text{From } A_1 \text{ to } B_2 &= 0.\\
\text{From } A_1 \text{ to } B_3 &= 0.\\
\text{From } A_2 \text{ to } B_2 &= 230 \text{ volts.}\\
\text{From } A_2 \text{ to } A_3 &= 0.\\
\text{From } A_2 \text{ to } B_3 &= 0.\\
\text{From } A_3 \text{ to } B_3 &= 230 \text{ volts.}
\end{aligned}$$

(*a*) Name the two terminals of each phase. (*b*) State which connections of the terminals you would try in order to produce an open-delta arrangement. (*c*) Show by vector diagram what voltage there should be across an open delta arrangement, and what voltmeter readings you should obtain to prove you have made the proper connection.

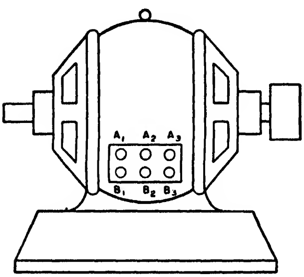


FIG. 55-2. A special generator with both terminals of each phase brought out.

Prob. 64-2. If you measured the voltage across the series connection of phases, as you had connected them in Prob. 63-2, and found it to be 398 volts, how would you change the connections?

Prob. 65-2. Suppose that one of the phases which you had intended to use in the open-delta arrangement of Prob. 63-2 were found to be faulty. State how you would connect the third phase in its place.

Prob. 66-2. (*a*) Connect the third phase of the generator, described in Prob. 63-2, in series with the other two phases, after the latter have been connected in correct open delta and calculate the resultant total voltage and the entire series. (*b*) Then reverse the connections to the third phase, and again calculate the total voltage. Draw both polar and topographic diagrams to illustrate each connection.

Prob. 67-2. Repeat the work of Prob. 66-2, starting with the incorrect open delta, described in Prob. 64-2.

13-2. Three-Phase Generator, Closed-Delta Connection. If the ends of the three phases of a three-phase generator are brought out separately to six terminals, they may be connected to a six-wire, three-phase circuit, as indicated in Fig. 56-2. Each pair of wires may supply a single-phase load or may be connected to a separate motor winding in a three-phase motor, as shown. But a

six-wire circuit is somewhat complicated and is seldom used in practice, except in certain transformer connections.

However, it was shown, in Art. 11, that if these three generator windings, or phases, are properly connected in series at 120°, the sum of their emfs is zero. Accordingly, we can connect the three phases B_1 to A_2, B_2 to A_3, and close the circuit B_3 to A_1 as indicated in Fig. 57–2. This is **called a closed-delta connection.**

At first thought, this appears to be a short-circuit, but since the sum of the three voltages is zero, no current will circulate through the winding. This is somewhat similar to the condition when two d-c generators are connected in parallel. If the polarity of the machines is correct, no current will circulate between the two.

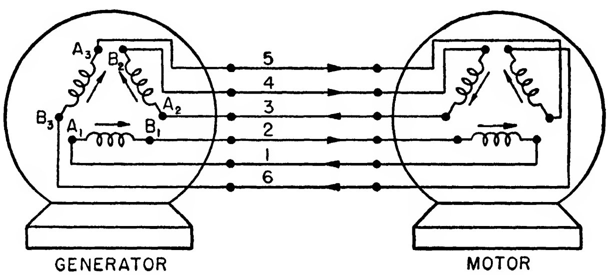


FIG. 56–2. A three-phase generator, supplying power over a three-phase, six-wire system to a three-phase motor.

The junction point of each pair of phases can now be brought out to one wire of a **three-wire, three-phase circuit.** Since the terminals, B_1 and A_2 of phases one and two are joined together, lines 2 and 3, of Fig. 56–2, can be replaced by a single line wire. Similarly, one line wire can be substituted for lines 4 and 5, and also for lines 6 and 1. It is usual practice to join the terminals of the phases together inside the machine and bring out three lead wires only. In case of a rotating field machine, the leads are brought out through the frame; if the armature revolves, the three leads are connected to three collector rings mounted on the shaft.

The voltage across the terminals, or between line wires, of a delta-connected generator is equal to the voltage of one phase of the armature winding, called the **phase-voltage.** The three windings of three-phase motors, connected to a three-wire, three-phase system, must also be interconnected. Figure 57–2 also indicates a delta connection in the motor.

The name " delta " has been given to the method of connecting

the phases just described, since the diagrammatic representation of this connection resembles the Greek letter Δ (delta).

Incorrect Delta Connection. In connecting the phases in a closed delta connection, great care must be used to see that they

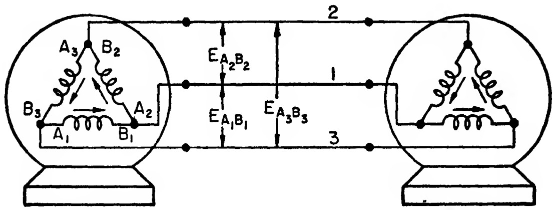


FIG. 57–2. A delta-connected three-phase generator, supplying power over a three-phase, three-wire line to a delta-connected motor.

are properly connected. Otherwise, when the machine is operated, a circulating short-circuit current will damage the windings. For instance, if the end B_2 of coil A_2B_2, Fig. 58–2, is connected to B_3 by mistake, the voltage of A_3B_3 will combine in reversed sense with that of the other two phases, and the voltage across A_1A_3 will **not** be zero. **If these terminals are now connected, a short-circuit current will circulate through the three windings.**

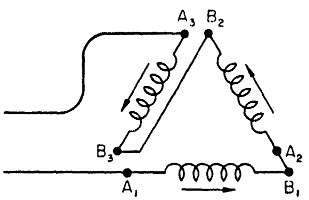


FIG. 58–2. Incorrect series arrangement of the three-phases to form a delta connection.

In the solution of the following problems, construct both coil and vector diagrams, properly marked to show the relations among the various emfs.

Prob. 68–2. If the voltage, induced in each winding of the armature of the three-phase generator in Fig. 55–2 is 230 volts, show, by numerical solution of a polar diagram, that the voltage across A_1 B_3 is zero, when properly joined in series for a closed delta connection.

Prob. 69–2. If, in attempting to make a closed delta in Prob. 68–2, the phase A_1B_1, Fig. 55–2, were connected oppositely by mistake, compute the resultant emf of the series, which would act to produce an internal circulating current through the windings, when the connections of the closed delta are completed.

Prob. 70–2. Repeat the solution of Prob. 68–2 with phase A_2B_2, only reversed.

Prob. 71–2. Repeat the solution of Prob. 68–2 with phase A_3B_3, only reversed.

Prob. 72–2. Repeat the solution of Prob. 68–2 with phases A_1B_1 and A_2B_2, both reversed.

Prob. 73–2. Repeat the solution of Prob. 68–2 with phases A_2B_2 and A_3B_3, both reversed.

Prob. 74–2. Repeat the solution of Prob. 68–2 with phases A_1B_1 and A_3B_3, both reversed.

Prob. 75–2. Repeat the solution of Prob. 68–2 with phases A_1B_1, A_2B_2, and A_3B_3, all reversed.

14–2. Three-Phase Y (Wye), or Star, Connection. The three coils, or phases, of Fig. 51–2(*a*), Art. 11–2, also may be connected in another arrangement to give three equal voltages, differing in

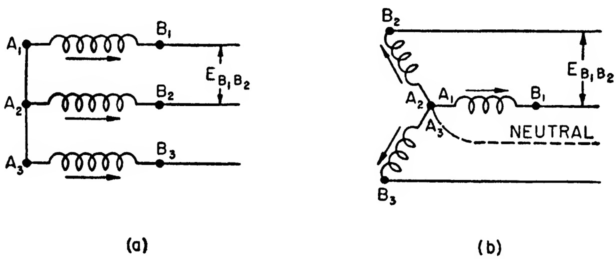


FIG. 59–2. (*a*) A coil diagram of the three phases, connected in "Y." (*b*) Another arrangement of the coils in a "Y" connection.

phase by 120°. The phase terminals B_1, B_2 and B_3 may be joined together, and the other ends A_1, A_2 and A_3 brought out to the generator terminals; or the A ends of the phases may be joined together, and the B ends brought out to terminals and connected to a three-wire, three-phase system, as shown in Fig. 59–2(*a*) and (*b*). This is called a "Y" connection.

It is readily seen from Fig. 59–2 that the voltage across any pair of terminals is that of a series circuit of two phases. For instance, the voltage across the terminals from B_1 to B_2, Fig. 59–2, is that of phases A_1B_1 and A_2B_2 in series. And their emfs are displaced 120° with respect to a positive direction through the coils, as shown by the arrows, and also by the vector diagram of Fig. 60–2. However, these two phases are joined in the common series circuit with the positive direction of their emfs **opposed.** That is, the terminal voltage from B_1 to B_2, Fig. 59–2, is that of coil A_1B_1 **reversed, and**

A_2B_2 **direct.** Therefore, before we can add the vectors $E_{A_1B_1}$ and $E_{A_2B_2}$, of Fig. 60–2, to obtain their resultant, we must reverse vector $E_{A_1B_1}$, so as to show the difference of phase, or time, between

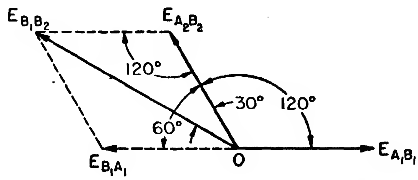


FIG. 60–2. Polar vector diagram of the voltage across the terminals B_1B_2 in Fig. 59–2.

the instants at which these emfs reach their maximum instantaneous values in the same direction in the common circuit. Accordingly, $E_{A_1B_1}$ reversed becomes $E_{B_1A_1}$, and the voltage across

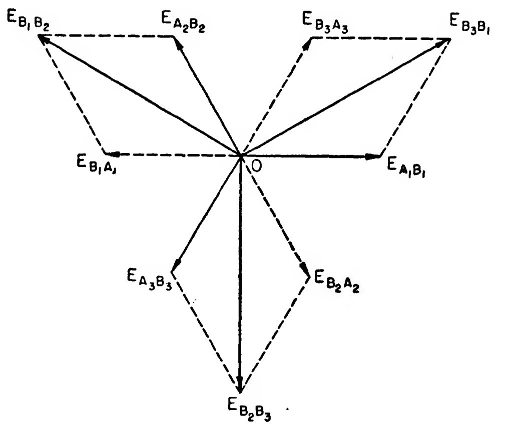


FIG. 61–2. Polar vector diagram of phase voltages and voltage between lines in a Y-connected system.

the terminals B_1B_2 is now the resultant of $E_{B_1A_1}$ and $E_{A_2B_2}$, combining at 60°, or vector $E_{B_1B_2}$.

The numerical value of the resultant in Fig. 60–2 is found to be 1.73, or $\sqrt{3}$, times the voltage of one phase; for by the law of sines (Appendix A)

$$\frac{E_{B_1B_2}}{E_{A_2B_2}} = \frac{\sin 120^\circ}{\sin 30} = \frac{0.866}{0.50} = 1.73 \text{ or } \sqrt{3} \text{ (where } E_{A_1B_1} = E_{A_2B_2}\text{)}.$$

Figure 61–2 shows the complete polar diagram of voltages for the

coils of Fig. 59–2. The voltage across the terminals of B_1 to B_2 equals $E_{A_1B_1}$ (reversed) $\oplus$ $E_{A_2B_2}$, as already explained; terminal voltage B_2 to B_3 equals $E_{A_2B_2}$ (reversed) $\oplus$ $E_{A_3B_3}$; and terminal voltage B_3 to B_1, equals $E_{A_3B_3}$ (reversed) $\oplus$ $E_{A_1B_1}$. Note that we proceed in orderly sequence from one pair of terminals to the next. The diagram of Fig. 61–2 shows that this connection results in three equal terminal voltages displaced 120° from each other, the value of which equals the emf of one phase, multiplied by $\sqrt{3}$.

Thus, in a Y connected generator, the terminal voltage, or voltage between lines, is equal to the phase voltage, sometimes called the Y voltage, times the $\sqrt{3}$.

This connection is also known as " Star " or " Wye." With this arrangement of the phases, a neutral is often brought out from the junction of the three phases and the generator connected to a four-wire, three phase-system, as indicated in Fig. 59–2(*b*). However, in most plants with Y-connected generators, the neutral is generally " grounded."

Incorrect Y Connection. The phases may be connected incorrectly in Y by mistake. For example, in Fig. 62–2, end B_2 of phase A_2B_2 is connected to the common point of the other two phases, instead of end A_2. The resulting voltages across each of three pairs of terminals will not be alike, nor at 120° from one another, as can be shown by the vector diagram.

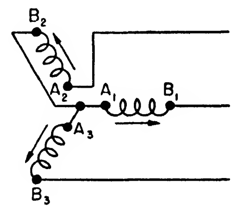


FIG. 62–2. Incorrect connection of the phases to form a Y connection.

Prob. 76–2. Assuming the same positive direction of emf in each coil, as in Fig. 59–2(*a*), construct both a coil diagram and a polar vector diagram of the emfs across each pair of terminals, similar to Fig. 61–2, when the coils are connected A_1 to A_2 to A_3. Show proper notation of all vectors in the diagram. (*b*) Repeat part (*a*) above on the assumption the positive direction of emf in each coil is reversed. State the positions of the emfs across each pair of terminals with reference to those in part (*a*).

Prob. 77–2. In testing out the three-phase alternator of Fig. 63–2 with a voltmeter, the following data were obtained:

From A_1 to B_1 = 230 volts
From A_2 to B_2 = 230 volts
From A_3 to B_3 = 230 volts

(*a*) State what connections you would try in order to produce a Y connected machine. (*b*) Assume a positive direction of emf in each

phase and show by a polar vector diagram properly marked what voltage there should be across each pair of terminals, and what voltmeter readings you should obtain in order to prove that you had made the proper connection.

Prob. 78–2. Assuming the same positive direction through each coil, as in Prob. 77–2, compute, with the aid of a polar vector diagram, the voltage across each pair of terminals, when the phases are connected B_1 to B_2 to A_3.

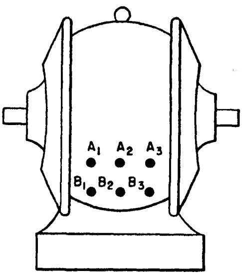


Fig. 63-2. A three-phase generator with both terminals of each phase brought out.

Prob. 79–2. Assuming the same positive direction of emf, repeat Prob. 77–2, when the phases are connected B_1 to A_2 to B_3.

Prob. 80–2. Assuming the same positive direction of emf, repeat Prob. 77–2, when the phases are connected A_1 to B_2 to B_3.

15–2. Vector Addition of Currents in Parallel Circuits. In Chapter I, Art. 16–1, it was shown that, when an a-c voltage is impressed upon a single circuit, an alternating current will flow, which has the same fundamental frequency as the voltage. It was also stated that the current may be in phase, lag behind, or lead the impressed voltage. Figure 64–2(a) shows the polar diagram of a current in phase with the voltage while Fig. 64–2(b) shows the same diagram when the current is lagging behind the voltage of an angle of $\theta°$, in which the vectors represent effective values.

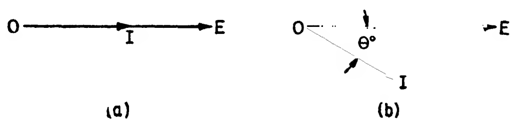


Fig. 64–2. (a) Polar vector diagram of current and voltage in phase. (b) Polar diagram of current, lagging $\theta°$ behind the voltage.

Likewise, in a parallel circuit, the current in each branch has the same fundamental frequency as the impressed voltage. However, the manner in which the currents in the branches of an a-c circuit combine differs from that in a d-c circuit, as shown below.

In a d-c system, two currents may unite at a junction point in the circuit, and the resulting current in the common wire may be either the arithmetical sum or difference of the two currents.

For instance, the two d-c generators of Fig. 65–2(a) are operating

in parallel to supply power to a common load. One generator supplies 16 amperes, while the other supplies 10 amperes. These two currents unite at P, and 26 amperes flows over the common

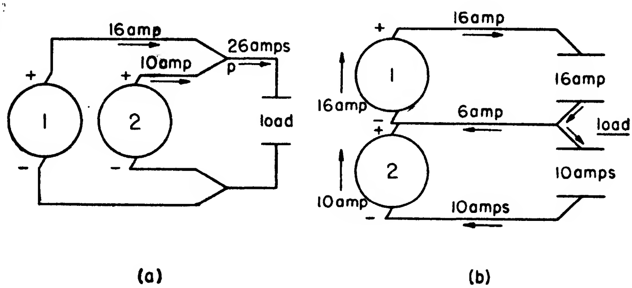


FIG. 65–2. (*a*) Two d-c generators in parallel, supplying a common load. (*b*) Two d-c generators, supplying an unbalanced three-wire system.

wire; or 16 amperes + 10 amperes equals 26 amperes. While in the Edison three-wire system of Fig. 65–2(*b*), the two generators are supplying 16 amperes and 10 amperes to their respective loads; but in this case, the current in the common, or neutral, wire is the arithmetical difference of the two currents, or 16 amperes − 10 amperes equals 6 amperes. (See Vol. I, Chap. XV.)

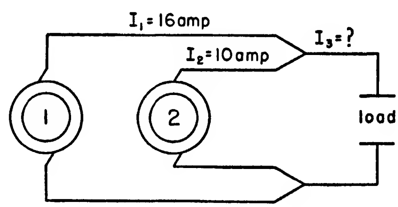


FIG. 66–2. Two a-c generators in parallel, supplying a common load.

However, in an a-c system, two such currents, having effective values of 16 and 10 amperes, may unite to produce a resulting current of any value from 26 amperes to 6 amperes. For instance, the two a-c generators of Fig. 66–2 might be operating in parallel at the same voltage and frequency, but with the emf of generator No. 2 leading that of generator No. 1 by a phase difference of 60°.

Actually, if two alternators were put in parallel at any such angle as 60°, a circulating current would flow between them large enough to injure their armatures. They do, however, operate in parallel at a phase angle of approximately 5°. The larger angle is here chosen for clearness in the illustration. If the first generator supplies 16 amperes, and the second 10 amperes, effective value, at this phase difference to a common load, the maximum values of these currents are 16 × 1.41, or 22.56 amperes, and 10 × 1.41, or

14.1 amperes, respectively. The sine curves representing these currents are shown at a phase difference of 60° in Fig. 67–2, and the resulting current curve can be determined from the algebraic sum of corresponding instantaneous values of these two curves. But since they are both sine curves, the effective value of the resulting current in the common load may be found by vector diagram, as in Fig. 68–2. Note that the 10 amperes of alternator No. 2 is leading

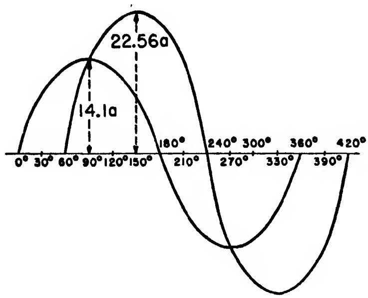


FIG. 67–2. The currents from the two generators in Fig. 66–2 are 60° out of phase with each other.

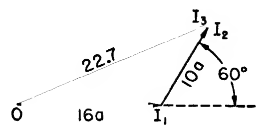


FIG. 68–2. Vector diagram, showing the resultant current in the common load of Fig. 66–2.

that of No. 1 by 60°. From the diagram, the value of the resultant current, I_3, may be computed as follows:

$$I_3 = \sqrt{(16 + 10 \cos 60°)^2 + (10 \sin 60°)^2}$$
$$= \sqrt{(16 + 5)^2 + (8.66)^2} = 22.7 \text{ amperes.}$$

Again, a resistor and an induction coil may be placed in parallel in an a-c circuit, as in Fig. 69–2(*a*). Current through the coil is A_1, lagging behind the voltage, as stated in Art. 16, Chap. I; and current through the resistor is A_2, in phase with the voltage. The total current, A_3, supplied to the two branches is thus the vector sum of A_1 and A_2, or $A_1 \oplus A_2 = A_3$, as shown by the diagram in Fig. 69–2(*b*).

In the solution of each of the following problems, show a vector diagram.

Prob. 81–2. An induction coil, carrying 18 amperes, is put in parallel with a resistor, carrying 9 amperes, as in Fig. 69–2(*a*). The current in the coil lags 80° behind that in the resistor circuit. How much current does the line feeding them carry?

Prob. 82-2. In the parallel circuit of Fig. 70-2, a-c ammeter A_3 reads 46 amperes; A_2 reads 28 amperes; and A_1 reads 24 amperes. What is the difference in phase between the currents in A_1 and A_2?

Prob. 83-2. What is the phase difference between the currents in A_1 and A_3?

Prob. 84-2. If the current through A_1, in Fig. 70-2, were 90° ahead of that through A_2, and these ammeters each indicated 20 amperes, what would ammeter A_3 indicate?

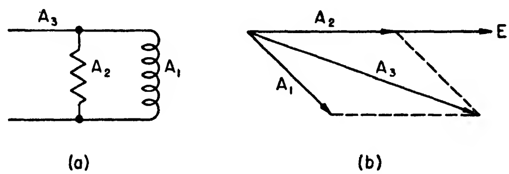


FIG. 69-2. (*a*) A resistor and a coil in a parallel circuit. A_3 is the vector sum of the currents A_1 and A_2 in the branches. (*b*) Vector diagram of the currents in (*a*).

Prob. 85-2. If the current through ammeter A_1 in Prob. 84-2 were 120° behind that through A_2 and the ammeters each indicated 20 amperes, what would ammeter A_3 indicate?

Prob. 86-2. The current in one branch of a parallel circuit is 25 amperes, in phase with 115 volts impressed on the circuit, and in the

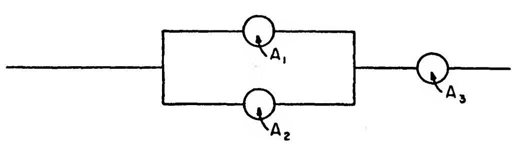


FIG. 70-2. Ammeters A_1 and A_2 measure the current in the parallel branches of the circuit. Ammeter A_3 measures the resultant current in the line.

other branch is 15 amperes, lagging behind the voltage by 45°. (*a*) What is the total current supplied to the circuit? (*b*) What is the phase position of the total current with respect to the voltage?

Prob. 87-2. An a-c circuit consists of three branches. Through one branch flows a current of 10 amperes, in phase with the voltage. Through the second flows a current of 12 amperes, lagging 30° behind the current in the first. Through the third flows a current of 16 amperes, leading the current in the first by 50°. What is the current through the circuit feeding the combination?

Prob. 88-2. In Prob. 87-2, what is the phase angle (leading or lagging) between the total current and the voltage on the circuit?

16–2. Current in the Phases and Line Wires of a Delta Connected Machine. Assume the three phases, A_1B_1, A_2B_2, and A_3B_3, of the generator of Fig. 71–2 are connected in delta, as shown, and are feeding the line wires 1, 2, and 3, to which are attached three equal lamp loads, also connected in delta. This is similar to Fig. 57–2, Art. 13–2, in which the generator supplies a motor. In a delta connected generator, as we have previously seen, the voltage, across any pair of terminals or between any pair of line wires, is the same as the voltage across each phase of the generator winding. Assume that each phase in Fig. 71–2 generates 230 volts, and supplies a current of 10 amperes to the terminals. The current through each set of lamps will also be 10 amperes. This is the usual case of a "balanced load."

Let us see how the current in each line wire compares with the current in each phase of the generator, and in each group of lamps.

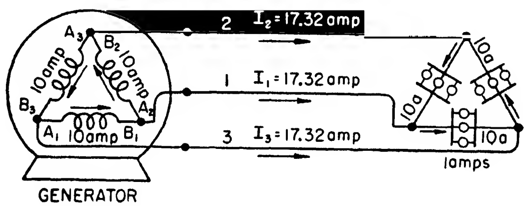


FIG. 71–2. Each phase of the delta-connected generator delivers 10 amperes to a group of lamps, also connected in delta.

In Fig. 57–2, the positive direction of the emf in each phase of the generator is indicated. This is also the positive direction of the **current** in each phase, and is indicated by the arrows in Fig. 71–2. Thus the currents in the phases, or "phase currents," differ in phase position with each other by 120°; so the current in phase A_2B_2 leads that in phase A_1B_1 by 120°, and both currents flow in line 1.

If the generator in Fig. 71–2 were connected to a six-wire system, as in Fig. 56–2, Art. 13–2, in which the positive direction of the currents is shown by the arrows, 10 amperes would flow in each line wire. Note, however, that the positive direction of the current in line 3 is reversed with respect to that in line 2; and that both of these currents in the delta connection of Figs. 57–2 and 71–2, flow in line 1. Therefore, the currents in phases A_1B_1 and A_2B_2 combine with each other, in line 1, but with the positive direction of current in A_2B_2 **reversed.** Thus they combine with each other at

60°, as shown in the diagram of Fig. 72–2(*a*). Note that the current $I_{A_2B_2}$ combines reversed, as $I_{B_2A_2}$, with current $I_{A_1B_1}$ to equal $I_{B_1A_2}$ in the line, indicated as I_1 in Fig. 72–2(*a*).

The currents in lines 2 and 3 are obtained in similar manner.

In any three-phase system, each line wire may be considered as a return wire for the currents flowing in the other two line wires; the current, flowing out from the generator along one line wire, must be equal in value and of opposite phase to the vector sum of the currents, flowing out from the generator along all the other line wires of the system. In fact, we merely apply Kirchhoff's first law

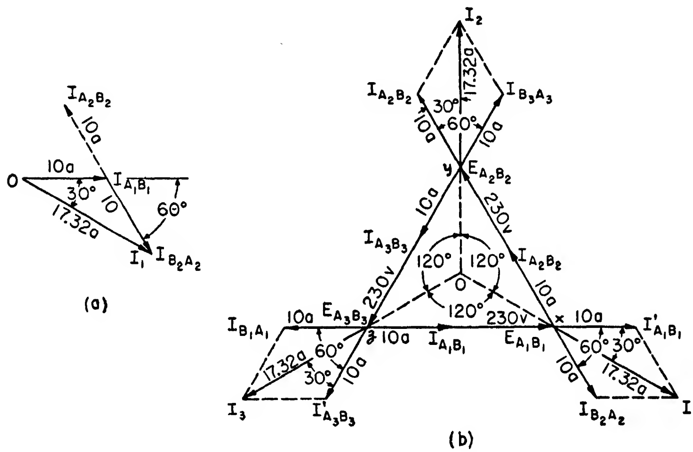


FIG. 72–2. (*a*) Topographic vector diagram. Shows how the currents, delivered by phases A_1B_1 and A_2B_2 of Fig. 71–2, combine to produce I_1 in line 1. (*b*) Complete vector diagram of the phase currents and line currents of Fig. 71–2.

(see Vol. I, Chap. XII) to the point where line 1, Fig. 71–2, joins the phases A_1B_1 and A_2B_2, which states that " whenever any number of conductors join at a point, the sum of the currents flowing away from that point must be equal to the sum of the currents flowing toward that point." This rule holds algebraically, or arithmetically, with regard to direct currents, or with regard to instantaneous values of alternating currents. It also applies **vectorially** to effective and maximum values of alternating currents.

Thus **the vector sum of the currents in all the wires of a three-phase** system is zero.

The complete vector diagram of voltages and currents in the delta connected system of Fig. 71–2 is shown in Fig. 72–2(*b*).

Vectors $E_{A_1B_1}$, $E_{A_2B_2}$, and $E_{A_3B_3}$ are drawn in **counter-clockwise sequence** in a topographic diagram, each representing the 230 volts of the respective phases. The vectors $I_{A_1B_1}$, $I_{A_2B_2}$ and $I_{A_3B_3}$ representing 10 amperes in each phase, are also laid out in the **same** direction **on** their respective voltage vectors, as shown; since the current in the lamps is in phase with the voltage across them.

To determine the current in line 1: vector $I_{A_1B_1}$ is redrawn from point x, parallel to its former position, as $I'_{A_1B_1}$, and vector $I_{A_2B_2}$ is reversed, for the reasons explained above, and shown as $I_{B_2A_2}$. Note that these two currents now combine in line 1 at 60°.

Solving the small polar diagram at x for the current I, in line 1;

$$I_1 = \sqrt{10^2 + 10^2 + 2 \times 10 \times 10 \cos 60^\circ} = 17.32 \text{ amperes.}$$

To determine current in line 2: vector $I_{A_2B_2}$ is redrawn from point y, as $I_{A_2B_2}$, parallel to its former position, and $I_{A_3B_3}$, the current in the leading phase, is reversed and becomes $I_{B_3A_3}$; so these two currents combine in line 2 at 60°. Solving the small polar diagram at y, the current I_2 in line 2 is also found to be 17.32 amperes.

The current in line 3 is found in similar manner from the construction of the polar diagram at z.

Note that in the generator of Fig. 71–2, the current per phase is 10 amperes, while that in each of the line wires is 17.32 amperes, and $10 \times \sqrt{3}$ equals 17.32. Thus when a three-phase generator is connected in delta and has a "balanced load," the line currents are equal to the phase currents multiplied by $\sqrt{3}$.

In any polyphase machine, when the value of the currents in the various windings is the same, and these currents have the same phase difference with their respective emfs, the load is said to be "balanced."

If the line currents I_1, I_2, and I_3 in Fig. 72–2(*b*) are extended, as shown by the dash lines, they meet in a common point, O, and have a phase difference of 120°. This is clearly shown in Fig. 73–2, and it is apparent that their vector sum is zero.

In Fig. 72–2(*b*), the current in the phases, or windings, are in phase with the phase voltages, since incandescent lamps take a current in phase with the voltage across them; while the line currents differ 30° in phase from these voltages. For instance, the current I_1 in line 1 lags behind the voltage $E_{A_1B_1}$ by 30°, and also lags 150° behind $E_{A_2B_2}$, or leads $E_{A_2B_2}$, reversed, by 30°. There-

fore, in a balanced delta connected system, **when the phase currents and voltages are in phase with each other, the line currents are 30° out of phase with the voltages between lines.**

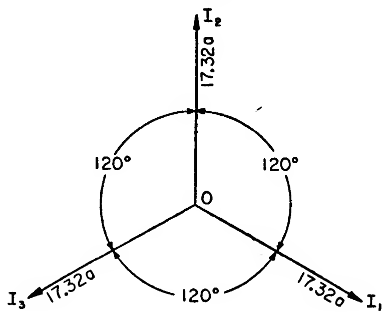


FIG. 73–2. The vector sum of the line currents in the system of Fig. 71–2 is zero.

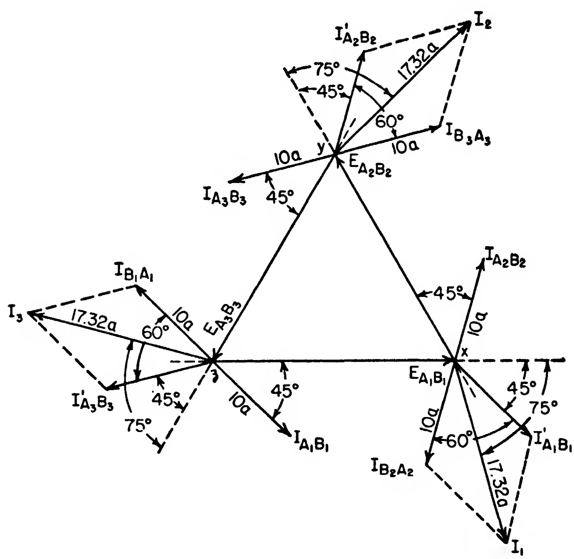


FIG. 74–2. Vector diagram of phase currents and line currents when the currents in the phases lag 45° behind their respective emfs.

Lagging Load. When a generator supplies a motor load, as in Fig. 57–2, Art. 13, the load is balanced, but the phase currents generally lag behind their respective voltages. Assume the currents in the phases are each 10 amperes, as before, but are lagging 45° behind the phase voltages. In Fig. 74–2, the topographic

diagram of voltages is drawn as before, but with the currents $I_{A_1B_1}$, $I_{A_2B_2}$, and $I_{A_3B_3}$, lagging behind their respective voltages by 45°. To obtain the current in line 1 (Fig. 57-2), $I_{A_1B_1}$ is redrawn from point x as $I'_{A_1B_1}$, parallel to its former position as before, and $I_{A_2B_2}$ is reversed and shown as $I_{B_2A_2}$. The diagonal of the small polar diagram at x is I_1. From inspection of the diagram, it is seen that the phase currents $I_{A_1B_1}$ and $I_{B_2A_2}$ again combine at 60°; and I_1, the current in line 1, equals 17.32 amperes. Similar construction at y and z shows the currents in lines 2 and 3 to have the same value. Thus in a balanced load with lagging currents, the ratio of line current to phase current also is equal to $\frac{\sqrt{3}}{1}$.

Unbalanced Loads. In practice, most three-phase loads are balanced, and the current in all three lines is approximately the

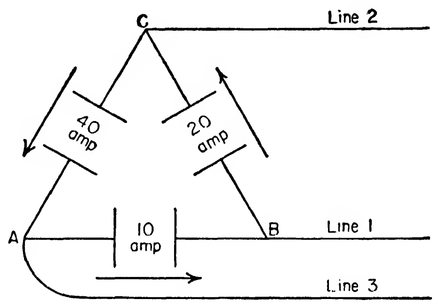


FIG. 75-2. An unbalanced delta connected load.

same. However, unbalanced loads do occur when three single-phase loads, taking different values of current at different phase angles with their respective voltages, are connected to the three-phase system. In such a case, the line currents may differ greatly in value and in phase position with the line voltages. The relations in an unbalanced circuit are illustrated in the example below.

Example 4. Load AB, Fig. 75-2, takes a current of 10 amperes in phase with its voltage, AB; load BC takes 20 amperes, lagging 30° behind its voltage, BC; and load CA takes 40 amperes, lagging 45° behind its voltage, CA. What are the currents in the line wires 1, 2, and 3? The voltages AB, BC, and CA balanced 3-phase voltages.

Solution: The positive direction of voltage and current in each load is assumed in counter-clockwise sequence, as shown by the arrows. Figure 76-2 shows the vector diagram of the voltage, also in counter-clockwise sequence, with current I_{AB} (10 amperes) in phase with volt-

age E_{AB}; current I_{BC} (20 amperes), lagging voltage E_{BC} by 30°; and current I_{CA} (40 amperes), 45° behind E_{CA}.

At point B: Construct the polar diagram with I'_{AB} drawn parallel to I_{AB}, and I_{BC} reversed (I_{CB}). These currents combine at 90°, as shown, and solving for current I_1 in line 1:

$$I_1 = \sqrt{10^2 + 20^2} = 22.4 \text{ amperes.} \quad \textit{Ans.}$$

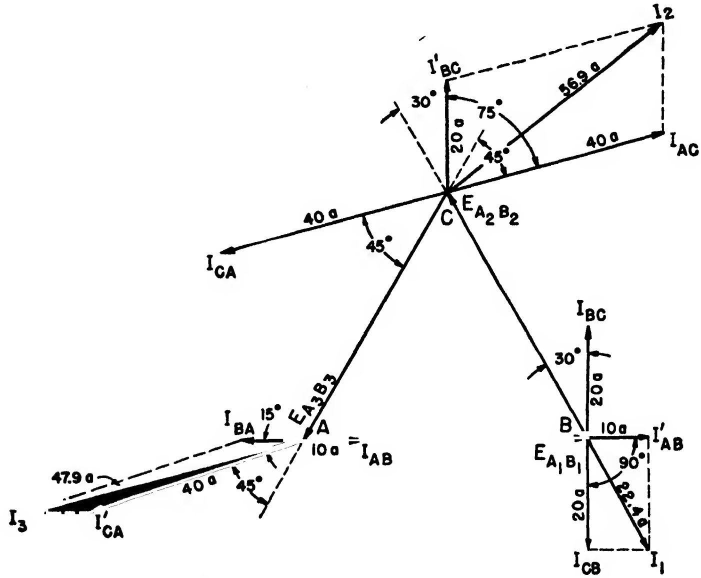


FIG. 76–2. Vector diagram of phase currents and line currents in the unbalanced system of Fig. 75–2.

At point C: Construct the polar diagram with I'_{BC}, drawn parallel to I_{BC}, and I_{CA} reversed (I_{AC}). These currents combine at 75°, as shown, and solving for current I_2 in line 2:

$$I_2 = \sqrt{20^2 + 40^2 + 2 \times 20 \times 40 \cos 75°} = 56.9 \text{ amperes.} \quad \textit{Ans.}$$

At point A: Construct the polar diagram as before, I'_{CA} and I_{AB} reversed (I_{BA}) combine at 15°, as shown, and solving for current I_3 in line 3:

$$I_3 = \sqrt{40^2 + 10^2 + 2 \times 40 \times 10 \cos 15°} = 49.3 \text{ amperes.} \quad \textit{Ans.}$$

Solve the following problems by means of a vector diagram, similar to Figs. 74–2 or 76–2, using counter-clockwise sequence of phases.

Prob. 89–2. The current in each phase of a delta-connected generator is 100 amperes, and is in phase with the emf of each winding. (*a*) What is the current in each line wire? (*b*) What is the phase angle between line current and voltage between lines?

Prob. 90-2. (*a*) What would be the line currents in Prob. 89-2 if the current in each phase were lagging 60° behind the phase voltage? (*b*) What would be the phase angle between line currents and line voltages?

Prob. 91-2. Solve for the line currents in Prob. 89-2, if the phase currents had the following phase differences with their respective voltages.

Current in phase AB differs by 0°;
Current in phase BC is lagging 30°;
Current in phase CA is lagging 60°.

Prob. 92-2. Solve for the line currents in Prob. 91-2, if the current in phase AB were leading its voltage by 30°. Current in each phase is 100 amperes.

Prob. 93-2. Three single-phase loads connected in delta to a 460-volt three-phase line, as in Fig. 75-2, all take currents, lagging 30° behind the line voltage. Current in phase AB is 50 amperes; in BC is 20 amperes; and in CA is 70 amperes. What is the current in each line?

Prob. 94-2. What would be the line currents in Prob. 93-2, if the currents in phases AB, BC, and CA were lagging the line voltage by 25°, 60°, and 75° respectively?

Prob. 95-2. Show that the vector sum of the line currents in Example 4 is zero.

17-2. Currents in a Y-Connected Machine. When a generator is Y-connected, as shown in Fig. 77-2, the coils or phases are in series with the line wires; therefore, the line currents and the phase currents are the same.

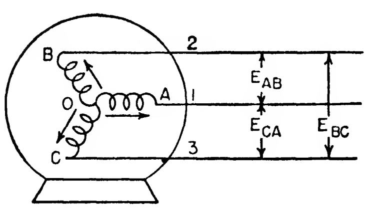


FIG. 77-2. A Y-connected alternator.

Assuming a balanced load, Fig. 78-2 shows the vector diagram for a load in which the phase current is in phase with coil, or phase, voltage. Note in the diagram that the phase currents I_{OA}, I_{OB}, and I_{OC} are in phase with their respective phase voltages, E_{OA}, E_{OB}, and E_{OC}. The voltage between lines 1 and 2, or E_{AB}, is that of E_{OA} reversed and E_{OB} (see Art. 14, Chap. II). The voltages between the two other pairs of terminals are E_{BC} and E_{CA}. From the diagram, it is evident that the current I_{OA}, in line 1, lags 30° behind E_{AC}, the voltage across lines 3 and 1; the current in line 2 lags 30° behind E_{AB}, the voltage across lines 1 and 2, etc. Thus in a Y connection,

when the voltages and currents in the coils are in phase, the line currents differ 30° in phase from the voltage between lines, exactly as in the delta connection. Also note from the vector diagram that the vector sum of the three currents is zero.

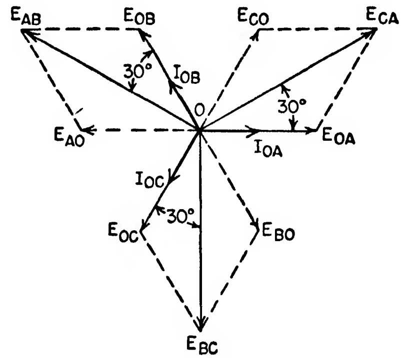


FIG. 78–2. Vector diagram of emfs and currents in a Y-connected system.

18–2. Summary of Delta and Y Connections. The following relations, regarding three-phase circuits and machines, have been discussed in previous articles and are here listed for convenience.

(1) In any three-phase (or polyphase) circuit, the vector sum of the line currents is zero. This is always true in either a balanced or an unbalanced circuit.

(2) **In delta-connected circuits or machines**: (*a*) The voltage between terminals, or between lines, is equal to the phase voltage, or $E_{\text{line}} = E_{\text{phase}}$. (*b*) When the load is balanced, the line current is equal to the phase current multiplied by $\sqrt{3}$, or $I_{\text{line}} = \sqrt{3} \times I_{\text{phase}}$.

(3) **In balanced Y-connected circuits and machines**: (*a*) The voltage between terminals or between lines is equal to phase voltage multiplied by $\sqrt{3}$, or $E_{\text{line}} = \sqrt{3} \times E_{\text{phase}}$. (*b*) The line current is equal to the phase current.

(4) **In both balanced delta and Y connections**: When the phase voltages and currents are in phase with each other, line currents differ 30° in phase from the voltage between lines.

Prob. 96–2. Show a diagram of connections, with the terminals properly marked, for a Y connection of the generator in Fig. 63–2, Art. 14–2. Assume each phase to carry 140 amperes, and to maintain a voltage of 240 volts across the phase terminals. Compute with the aid of a vector diagram, also properly marked:

(*a*) Voltage between each pair of line wires.
(*b*) Current in each line wire.

Prob. 97–2. In the induction motor of Fig. 79–2, the rated voltage between leads is 550 volts, and each lead carries 15 amperes at full load. If the phases of the motor are delta connected:

(*a*) What is the voltage across each phase?
(*b*) What current does each phase of the motor carry?

Prob. 98-2. If the motor of Fig. 79-2 were Y connected, with the same rated voltage between leads and the same current per lead, as in Prob. 97-2:

(*a*) What is the voltage across each phase?

(*b*) What is the current through each phase?

Prob. 99-2. The lamps in Fig. 80-2 are connected in delta and are each rated at 115 volts and 2 amperes. What must be the current per line and the voltage between lines, if the lamps are to operate normally? Show vector diagrams of line voltages and line currents. Current in the lamps is in phase with the voltage across them.

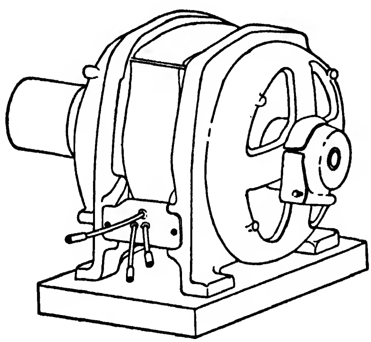


FIG. 79-2. A three-phase induction motor.

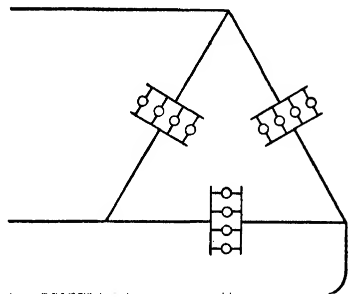


FIG. 80-2. Three lamp loads, connected in delta across a a three-phase line.

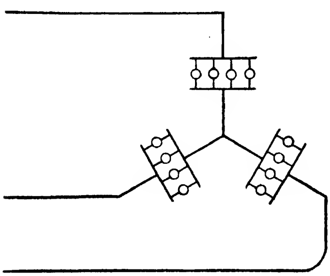


FIG. 81-2. Three lamp loads, connected in Y across a three-phase line.

Prob. 100-2. Answer the question in Prob. 99-2, and show the vector diagram, if the lamps in that problem are connected in Y, as in Fig. 81-2.

Prob. 101-2. A three-phase Y-connected generator is rated at 2300 volts between terminals, and 50 amperes per terminal at full load. (*a*) What is the normal voltage and current per phase? (*b*) If the machine is reconnected, as a delta-connected generator, what will be the terminal voltage and full load current per line?

Prob. 102-2. A 460-volt delta-connected induction motor carries 24 amperes per lead, or line wire, at full load. If this motor is reconnected in Y, what must be the voltage of the circuit from which it should be operated, and what will be the full load current per lead?

SUMMARY OF CHAPTER II

When a-c voltages of sine wave form and the same frequency are combined in a series circuit, the resultant voltage is equal to the combination of their respective sine curves, and the RESULTANT VOLTAGE is also a SINE CURVE.

The arithmetic, or algebraic sum of corresponding instantaneous values of the several voltage waves, equals the corresponding instantaneous value of their resultant wave.

When the curves of the several voltages reach a maximum AT THE SAME INSTANT, the maximum value of their resultant also occurs at the same instant. It is equal to the ARITHMETICAL SUM OR DIFFERENCE OF THEIR RESPECTIVE MAXIMUM VALUES. The latter is also true of the effective values.

When a-c voltages, whose sine curves do NOT reach a maximum at the same instant, combine in a series circuit, the maximum value of the resultant curve is LESS than the arithmetic sum of the maximums of the combining voltages. This is also true with regard to their effective values.

Voltages which do NOT reach a maximum at the same instant are DISPLACED FROM EACH OTHER and are said to be "OUT OF PHASE." In a series circuit, they are represented by vectors displaced by the same angle, called the PHASE ANGLE, and the resultant voltage determined by the solution of a VECTOR DIAGRAM.

Effective values are generally used in vector diagrams.

Vectors are combined either in POLAR or TOPOGRAPHIC vector diagrams.

The voltage across a series circuit equals the vector sum of the voltages across the parts. In the form of an equation, this may be written:

$$\mathbf{E} = \mathbf{E}_1 \oplus \mathbf{E}_2 \oplus \mathbf{E}_3 \oplus \cdots,$$

Where $\oplus$ indicates vector (not arithmetical) addition.

The CURRENT through a series circuit is the same in all parts.

THE PHASE ANGLE BETWEEN EMFS IN THE COILS IN A GENERATOR depends upon the POSITION of the coils on the armature, the POSITIVE DIRECTION of the emf in the coil, and the manner in which the coils are CONNECTED IN THE ELECTRICAL CIRCUIT.

REVERSING THE CONNECTION TO A COIL reverses the position of the emf vector, and causes it to be 180° ahead or behind its original position.

NOTATION OF COIL AND VECTOR DIAGRAMS. The manner in which emfs combine in a series circuit is shown by COIL DIAGRAMS and PROPER NOTATION OF THE VECTORS in a vector diagram. For instance, if the positive direction of emf in a coil is assumed to be from one end marked A to the other end marked B, the vector, representing this emf, is indicated as $\mathbf{E}_{AB}$. If the connection to the coil in the electrical circuit is reversed, the vector is drawn 180° from its original position, and indicated as $\mathbf{E}_{BA}$.

SINGLE-PHASE MACHINES have a single set of coils, or a single winding, on the armature brought out to two terminals and connected to a two-wire or single-circuit.

POLYPHASE MACHINES have two or more sets of coils, or windings, called " PHASES," displaced in phase position from each other, which are connected to circuits of MORE THAN two wires. These circuits are called POLYPHASE-CIRCUITS.

TWO-PHASE GENERATORS have two sets of coils, or phases, on the armature which generate equal emfs at 90° to each other.

IN A TWO-PHASE FOUR-WIRE SYSTEM, the terminals of each phase of the generator are brought out to one pair of line wires, and the voltage across each pair is that of one phase.

IN A TWO-PHASE THREE-WIRE SYSTEM, the two phases of the machine are connected in series and the three terminals are brought out to three line wires. The voltage between the " outside " wires is that of the two phases at 90°, or 1.41 times the voltage per phase.

IN A TWO-PHASE FIVE-WIRE SYSTEM, the mid-points of the two phases are connected, and a " neutral " is brought out from the junction. The voltage between neutral and each of the other four wires equals HALF the voltage across one phase.

THREE-PHASE GENERATOR. When three windings, or phases, generating equal emfs are so placed on a machine that the three voltages have a phase difference of 120° from each other, the machine is called three phase.

IN AN OPEN DELTA, two phases of a three-phase machine are joined in series forming, between the terminals of the series, a third phase. The voltage across this resultant third phase equals the sum of two equal voltages at 120° to each other, and is exactly equal to the voltage across each of the other two phases. The three terminals of the two phases in series may be brought out to a THREE-PHASE, THREE-WIRE SYSTEM.

CLOSED DELTA CONNECTION. When all three phases of a three-phase generator are connected in series at 120°, so that the voltage across any two phases equals the voltage across each, there will be no voltage across the terminals of the three in series. It is, therefore, safe to join these terminals, and although it makes a closed ring, no current will circulate. The three junction points of the three phases are brought out to a THREE-PHASE THREE-WIRE SYSTEM. Since the diagrammatic representation of phases so connected resembles the Greek letter Δ (delta), this method is commonly called the Delta Connection. To distinguish it from the OPEN DELTA, this is often called the CLOSED DELTA CONNECTION.

STAR OR Y CONNECTION. When the corresponding terminals of the three phases of a three-phase machine are so joined to a common (neutral) point, that between any two line terminals are two phases only of the machine, and these are in series, the phases are said to be Star, or Y-connected. The line terminals are connected to a three-phase, three-wire system. The voltage between line terminals is equal to $\sqrt{3}$ times the voltage across each phase. A neutral wire may

be brought out from the junction point of the three phases, making a four-wire, three-phase system. The voltage between this neutral and any one of the three line wires is that of one phase.

THE COMBINATION OF ALTERNATING CURRENTS in the branches of a PARALLEL CIRCUIT is equal to the summation of their respective sine curves. Sine waves of currents in parallel circuits combine in exactly the same way as do voltage waves in series circuits and are represented by similar vector diagrams.

THE CURRENT THROUGH A PARALLEL CIRCUIT is the VECTOR SUM of the currents in the branches. In the form of an equation, this may be written:

$$I = I_1 \oplus I_2 \oplus I_3 \oplus \cdots,$$

where $\oplus$ indicates vector addition.

THE NOTATION OF COIL AND VECTOR DIAGRAMS FOR CURRENTS in parallel circuits is exactly similar to that for voltages in series circuits.

IN A BALANCED THREE-PHASE SYSTEM, THE CURRENTS IN EACH PHASE are equal in value and differ from their respective phase voltages by the same phase angle.

THE CURRENT IN EACH LINE WIRE of a balanced three-phase delta-connected system is the vector sum of the currents in each of two phases and is equal to $\sqrt{3}$ times the current in one phase. The voltage across line wires is the voltage of one phase.

IN ANY THREE-PHASE SYSTEM, the vector sum of all the voltages between line wires is zero. Also the vector sum of all the currents in the line wires is zero.

PROBLEMS ON CHAPTER II

In the solution of the following problems, compute with the aid of vector diagrams properly marked, and show diagram of connections, wherever possible.

Prob. 103-2. Two voltages of 115 and 240 volts are in series at a phase difference of 75°. What is the resulting voltage on the circuit?

Prob. 104-2. Two currents in a parallel circuit have a phase difference of 115°. If one of the currents is 14 amperes and the other 21 amperes, what is the total current in the circuit?

Prob. 105-2. What is the voltage across a series combination of two parts, if the voltage across the first is 85 volts and across the second is 115 volts? The phase difference between the two parts is 40°.

Prob. 106-2. The current in the series circuit of Prob. 105-2 is 10 amperes, which is in phase with the voltage across the first part. What is the phase difference between the current and the voltage across the combination?

Prob. 107-2. A resistor and an induction coil are connected in series and 2 amperes flows in the circuit. The voltage across the resistor is

90 volts and that across the coil is 100 volts. The voltage on the circuit is 150 volts. If the voltage across the resistor is in phase with the current, and that across the coil leads the current: (*a*) What phase angle does the voltage across the coil make with the current? (*b*) What is the phase angle between the impressed voltage and the current?

Prob. 108–2. A group of lamps and an a-c motor are connected in parallel across a 115-volt circuit. The lamps take 12 amperes in phase with the voltage, while the motor takes 24 amperes, lagging 35° behind the voltage. What total current is supplied to the circuit and what is its phase angle (leading or lagging) with the voltage?

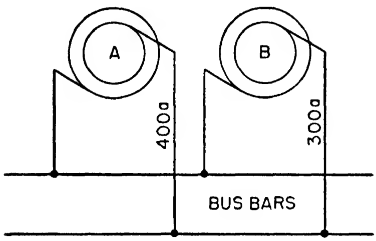


FIG. 82–2. Two single-phase generators, connected in parallel to a common bus.

Prob. 109–2. Two alternators connected in parallel to the same bus bars, as in Fig. 82–2. Each generates 460 volts. Alternator *A* delivers 400 amperes, and alternator *B*, 300 amperes, leading the current in *A* by 15°. What is the total current delivered to the bus bars?

Prob. 110–2. Three voltages of 115, 230, and 345 volts are combined in a series circuit. The first is in phase with the current in the circuit, the second leads the first by 45°, and the third leads the second by 40°. What is the total voltage on the circuit and its phase position with the current in the circuit?

Prob. 111–2. A parallel circuit consists of three branches. Through the first branch flows a current of 12 amperes in phase with the voltage. Through the second flows 20 amperes, lagging 45° behind the voltage. Through the third, flows 16 amperes, leading the voltage by 50°. What is the total current and its phase position with the voltage on the circuit?

Prob. 112–2. (*a*) What is the resultant of the four voltages in Fig. 83–2, and what is its phase position (leading or lagging) with respect to the 60 volt vector?

Prob. 113–2. In a special alternator, built for laboratory purposes, both terminals of each of 6 windings are brought out separately, as in Fig. 84–2. On the assumption that the positive direction of emf in each winding is from *A* to *B*,

A_6B_6 leads A_5B_5 by 30°,
A_5B_5 leads A_4B_4 by 30°,
A_4B_4 leads A_3B_3 by 30°,
A_3B_3 leads A_2B_2 by 30°,
A_2B_2 leads A_1B_1 by 30°.

The emf of each winding is 100 volts. (*a*) Find voltages across all possible series combinations of A_1B_1 and A_2B_2. (*b*) What is the phase difference between the resultant and A_1B_1 in each case?

Prob. 114–2. Find voltages across all possible series combinations of A_1B_1 and A_3B_3 of the generator in Prob. 113–2. State the phase difference between the resultant and A_1B_1 for each case.

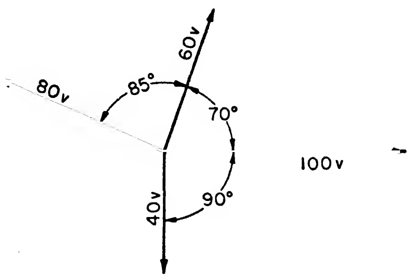


FIG. 83–2. To find the numerical value and phase position of the resultant voltage.

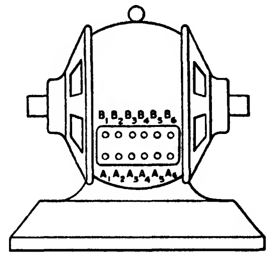


FIG. 84–2. A special a-c generator with both ends of each of six coils brought out to terminals.

Prob. 115–2. Find voltages across all possible series combinations of A_1B_1 and A_4B_4 of generator in Prob. 113–2. State phase difference between resultant and A_1B_1 for each case.

Prob. 116–2. Find resultant emf across the series combination of A_1B_1, A_2B_2, and A_3B_3 of generator in Prob. 113–2 when the windings are connected B_1 to A_2 and B_2 to A_3. State phase difference between resultant and A_1B_1.

Prob. 117–2. What would be the resultant voltage across the series combination of Prob. 116–2, if A_2B_2 is reversed? State the phase difference between the resultant emf and A_1B_1.

Prob. 118–2. If, in the combination of Prob. 116–2, both A_2B_2 and A_3B_3 were reversed, what would be the resultant voltage across the combination, and what would be the phase difference between it and the voltage across A_1B_1?

Prob. 119–2. Find the resultant of the series combination of the windings in Prob. 116–2, if A_4B_4 is added and connected A_4 to B_3. State the phase difference between the resultant and A_1B_1.

Prob. 120–2. What would be the resultant voltage across the series combination of Prob. 119–2, if winding A_2B_2 were reversed?

Prob. 121–2. In the series circuit of Prob. 116–2, what will be the voltage across the combination, if the winding A_3B_3 is reversed?

Prob. 122–2. When the coils are connected, as in Prob. 119–2, what would be the phase difference between the respective resultants of windings A_1B_1 and A_2B_2, and of windings A_3B_3 and A_4B_4?

Prob. 123–2. When all six windings of Prob. 113–2 are connected in series ends A to B, what will be the phase difference between the respective resultants of windings A_1B_1, A_2B_2, and A_3B_3, taken as a group, and of windings A_4B_4, A_5B_5, and A_6B_6, taken as a group?

Prob. 124–2. When all six windings of Prob. 113–2 are connected in series, ends A and B, what will be the phase difference between the respective resultants of the windings 1 and 2, 3 and 4, and 5 and 6?

Prob. 125–2. When the terminals of the three-phase generator of Fig. 63–2, Art. 14–2, are connected A_1 to A_2 and to A_3, it is Y-connected and the voltage across each pair of terminals is 380 volts. Phases are balanced. Show how you would connect the terminals to make a closed-delta connection, and state what the voltage would be across each pair of machine terminals.

Prob. 126–2. (*a*) Show a diagram of connections, marking the coil ends, for two ways in which the generator in Prob. 113–2 can be connected in delta as a three-phase machine. Use but three coils in each case. Compute the voltage across the terminals of the machine so connected.

Prob. 127–2. (*a*) Show a diagram of connections, marking the coil ends, for two ways in which the generator in Prob. 113–2 can be connected in Y, or Star, as a three-phase machine, using but three windings in each case. (*b*) Compute the voltage across the terminals of the machine so connected.

Prob. 128–2. (*a*) Using all six windings of the generator in Prob. 113–2, show a diagram of connections, marking the coil ends, for use as a three-phase delta connected machine. (*b*) Compute the voltage across the terminals of the machine so connected.

Prob. 129–2. The current flowing in each lead wire of generator in Prob. 128–2, when delta connected is 56 amperes. What current flows in each winding of the machine?

Prob. 130–2. (*a*) Using all six windings of the generator in Prob. 113–2, show a diagram of connections, marking all coil ends, for use as a Y-connected machine. (*b*) Compute the voltage across the terminals of the machine so connected.

Prob. 131–2. What current would flow in each line wire in Prob. 130–2, if the current per winding is the same as that found in Prob. 129–2?

Prob. 132–2. If each lamp in Fig. 85–2 takes a current of 5 amperes which is in phase with the voltage across it, what current flows in each line wire? The generator gives correct three phase emfs. Note that

the lamps are connected in delta. Use counter-clockwise sequence of phases.

Prob. 133–2. In place of the lamps in Group III in Prob. 132–2, connect a single-phase induction motor, taking a current of 30 amperes, which lags 40° behind the voltage across it. Compute the current in each line wire.

Prob. 134–2. In place of the lamps in Group II in Prob. 132–2, connect a single-phase synchronous motor, taking a current of 20

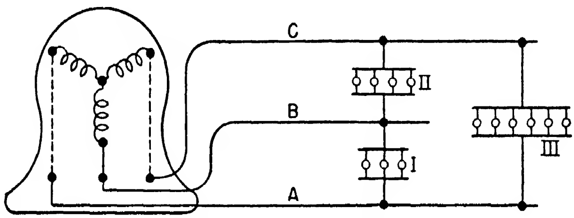


FIG. 85–2. An unbalanced delta-connected load. The current in the lamps is in phase with the voltage across them.

amperes, which leads the voltage across it by 35°. Compute the current in each line wire.

Prob. 135–2. In place of the lamps of Group III, in Prob. 132–2, connect the induction motor of Prob. 133–2. In place of Group II, connect the synchronous motor of Prob. 134–2. Compute the current in each line wire.

Prob. 136–2. If there is available only a 230-volt Y-connected generator to supply a 120-volt three-phase circuit, how may the machine be re-connected to give approximately the desired voltage, and what will be its value?

CHAPTER III

POWER, POWER FACTOR, SINGLE-PHASE CIRCUITS

1–3. Average Power. To find the power (watts), supplied by a d-c generator, we take simultaneous readings of terminal volts and amperes delivered. The product of the volts and the amperes **at any instant** is equal to the power delivered by the generator **at that instant.** If the load is steady, that is, if the current and voltage do not change in value, the rate at which energy is delivered, or the power as measured, is constant. However, most d-c generators supply power to lighting circuits and machines which take a varying amount of current during the day. The **average** power, delivered for a day, can be found by taking instantaneous voltmeter and ammeter readings simultaneously at regular short intervals of time for a cycle of one day of 24 hours. By multiplying these instantaneous readings of pressure and current, the power delivered at these instants is computed. The **average** of these instantaneous values of power (watts) is the average power delivered by the generator during the 24-hour cycle.

Figure 1–3 shows the " log " of the load on a plant. Note that both current and voltage curves are plotted from the readings taken during the 24-hour period. These data are taken from an actual plant used to supply power to a hotel. The corresponding curve of power, computed from the instantaneous values of current and voltage, as found from the other two curves, is also plotted in the same diagram.

Thus the value of the power (p) at the instant 6 o'clock P.M. equals the product of the amperes (i) at 6 P.M. by the volts (e) at 6 P.M. From the curve

$$i = 380 \text{ amperes}$$
$$e = 220 \text{ volts}$$
$$p = 380 \times 220 = 84{,}000 \text{ watts.}$$

Similarly, the instantaneous value of power (p_1), delivered at 10 P.M., equals the product of the amperes (i_1) at 10 P.M. by the

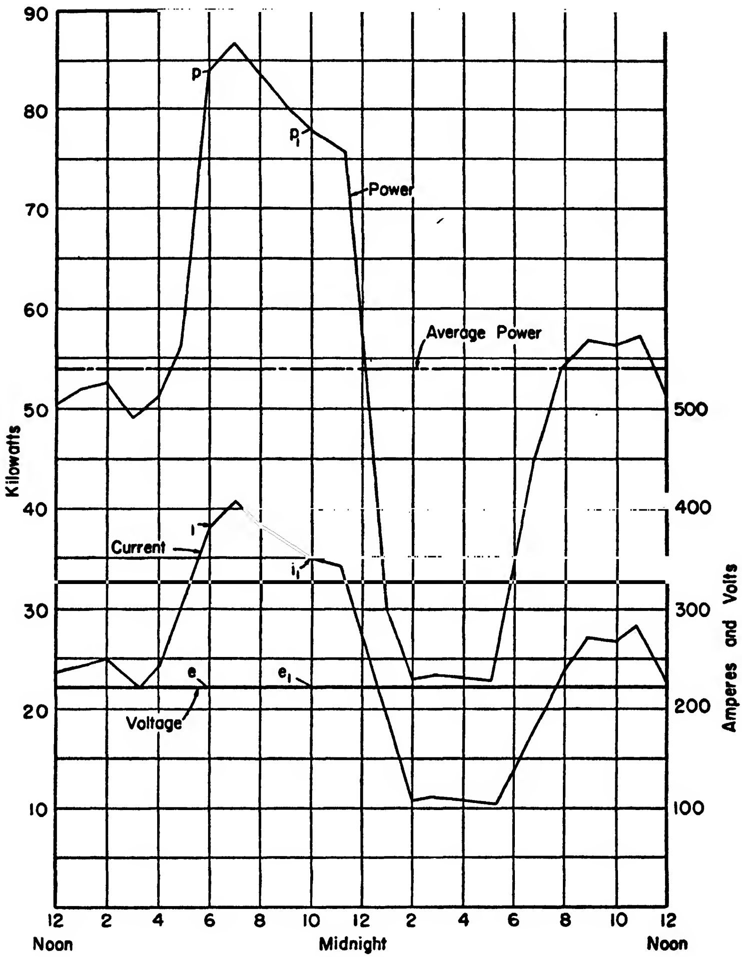


FIG. 1-3. The curve marked power represents the power taken by a hotel from noon to noon. Each point on this curve is a product of the corresponding values on the voltage and current curves for the same hotel. The average power equals nearly 54 kilowatts.

volts (e_1) at 10 P.M., or,

$$i_1 = 355 \text{ amperes},$$
$$e_1 = 220 \text{ volts},$$
$$p_1 = 355 \times 220 = 78{,}100 \text{ watts}.$$

The average of all the instantaneous values of the power, plotted on this power curve at equal short intervals of time, equals

nearly 54,000 watts and represents the average power, delivered by the generator for a cycle of one day of 24 hours.

Also, in each single circuit of an a-c system, the value of both the current and the voltage varies from instant to instant and, therefore, the flow of energy, or the power, also varies from instant to instant during each cycle. Because a cycle in this case repeats itself many times a second, rather than once in 24 hours, and both the volts and the amperes vary through wide ranges during this exceedingly short period of time, does not alter the fact that the power **at any instant** is always equal to the pressure **at that instant,**

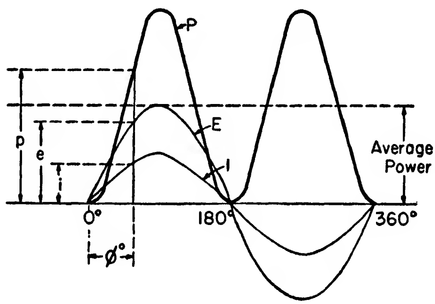


FIG. 2–3. Each point on the power curve P is the product of the corresponding values on the curves of E and I at that instant.

multiplied by the **current at the same instant;** that is $p = ie$. And **the average power** during one cycle, or any number of cycles, is the average of the instantaneous values of power, computed at regular short intervals of time over the cycle.

2–3. Power, Current in Phase with Voltage. Consider, first, an a-c generator which supplies a current in phase with the terminal voltage. From the instantaneous values of amperes and volts, we plot the current wave I and the voltage wave E, as in Fig. 2–3. Points on the power curve P are obtained from the product of corresponding instantaneous values of the current and voltage waves. For example, at the instant $\phi°$, the power (p) equals the product of (e), the value of the volts at that instant, times (i), the amperes at the same instant. Both loops of the power curve are positive, for when values of both current and voltage are negative, their product is positive. This curve is a sine squared curve, and is identical in shape to the i^2R curve of Fig. 30–1.

Note that the power curve touches the zero axis twice in each

cycle of the voltage and current waves. Therefore, the power is **zero** twice in each cycle. Also the frequency of the power curve is **double** that of the circuit frequency.

The average of all these values of power in Fig. 2-3 will be the average power during one cycle, as already stated; and, in this case, is equal to one-half the greatest instantaneous value. This is represented by the dash line, just half the height of the power curve. The half-wave above the line representing average power will just fill the portion below the line.

The greatest instantaneous value of the power curve, or the maximum power, equals the product of the maximum volts times the maximum amperes, or

$$P_{max} = E_m \times I_m \tag{1-3}$$

Therefore $\quad P_{av} = \frac{1}{2}E_m \times I_m$

But $\quad I_m = \sqrt{2}I_{eff} \text{ and } E_m = \sqrt{2}E_{eff}$

Therefore, $\quad P_{av} = \frac{1}{2} \times \sqrt{2}E_{eff} \times \sqrt{2}I_{eff}$

And

$$P_{av} = E_{eff} \times I_{eff}, \tag{2-3}$$

where E_m and I_m = maximum values of volts and amperes

and E_{eff} and I_{eff} = effective values.

Thus, when the voltage and current are in phase, the average power, delivered by a single-phase alternator, equals the product of effective volts times effective amperes.

Example 1. What power does a single-phase alternator deliver when it maintains a terminal voltage of 550 volts and supplies a current of 40 amperes in phase with the voltage?

$$\begin{aligned} P &= EI \\ &= 550 \times 40 = 22{,}000 \text{ watts} = 22\ kw. \end{aligned}$$

Prob. 1-3. Plot the following curves accurately on the largest sheet of co-ordinate paper available, and to as large a scale as the sheet will admit, putting all curves on the same sheet. (*a*) One cycle of a sine curve of a-c voltage of 120 volts effective value. (*b*) One cycle of a sine curve of alternating current of 2 amperes effective value in phase with the voltage. (*c*) The curve, representing the power which this voltage and amperage delivers.

Prob. 2-3. Find the average value of the power in Prob. 1-3 by averaging the instantaneous power taken from the curve every 10 degrees through the cycle. Compare this value of the average power with the values as computed by both the equation, $P = EI$, and $P = \frac{1}{2}E_mI_m$.

Prob. 3–3. A circuit has an alternating current of 2.8 amperes, flowing through it in phase with the pressure, which is 115 volts. How much power is consumed by the circuit?

Prob. 4–3. What is the maximum power consumed by the circuit in Prob. 3–2? The minimum power?

Prob. 5–3. The voltage across the terminals of a transformer is 2300 volts. The transformer takes a current of 14.2 amperes, practically in phase with the voltage. What power does the transformer take?

Prob. 6–3. In testing a single-phase a-c generator which was maintaining a terminal pressure of 2300 volts, it was found, by means of a wattmeter, to have a load of 400 kw. If the current was in phase with the pressure, how many amperes was the generator delivering?

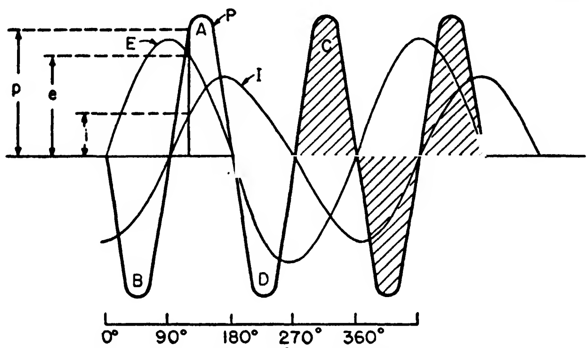


FIG. 3–3. When the current is 90° out of phase with the voltage, the positive power loops equal the negative power loops, and the average power is zero. This figure shows clearly that the frequency of the power curve P is twice that of either the E or I curves.

3–3. Power, Current and Voltage at 90°. It has been stated in Chap. I, and will be shown in Chap. IV, that the current may be out of phase with the voltage. Let us now consider the average power in a single-phase circuit, when the current is out of phase with the voltage by 90°. This is an extreme case.

Figure 3–3 shows curves of voltage and current with the current lagging behind the voltage by 90°. At the 0°, 90°, 180°, 270° and 360° points on the voltage wave, either the current or the voltage waves are zero. Therefore, the power must be zero at these points. Every point on the power curve P has its ordinate equal to the product of corresponding instantaneous values of the current and voltage. Thus from 0° to the 90° point of the voltage wave, the values of the current wave are negative, while those of the voltage

wave are positive. The product of a negative and a positive value is negative. That is, $(-i)$ times $(+e)$ equals $(-p)$. Therefore, the power over this part of the cycle is negative, as shown by the power curve. From 90° to 180°, the values of both current and voltage are positive; therefore, their product is positive, and the power is positive.

This power curve is also a sine squared curve, having twice the frequency of either the voltage or current curve.

From Fig. 3–3, note the fact that during the cycle there are two power loops, B and D, in which the power is **negative,** and two **equal** loops, A and C, in which the power is **positive.** Now positive values of power represent the power being delivered by the generator, since the voltage is acting in the same direction as the current; and negative values represent the power which is being returned to the generator since the voltage is acting in opposition to the current. Since, in Fig. 3–3, the negative loops are of the same size as the positive loops, the circuit returns to the generator, during one quarter cycle, all the power which it received during the preceding quarter cycle; and the power consumed by the circuit is, therefore, zero.

It can be shown that, if the current in a generator is 90° out of phase with the voltage, the current in the armature conductors is in such a direction, during one quarter cycle, that the magnetic action produces a drag, or a counter torque, opposing the rotation, which is generator action. While during the next quarter cycle, the direction of the current is such that it produces a torque in the direction of rotation, which is motor action; the motor action being equal to the generator action over the cycle. At the instant of maximum current and zero voltage, the power is zero and the current in half the armature conductors produces motor action, while that in the other half produces an equal generator action.

Also note that the average of the instantaneous values of the power curve over the cycle is zero. Therefore the average power is zero.

Thus, **when the current** in a single-phase circuit **lags 90° behind the voltage, the average power consumed in the circuit is always zero.** Also, if the **current leads the voltage by 90°, the average power is always zero.**

Prob. 7–3. Following directions in Prob. 1–3, construct the sine curves for an alternating current of 2 amperes, effective value, lagging 90° behind an a-c voltage of 120 volts, effective. Draw the power curve on the same sheet and compute the average power over the cycle.

4–3. Power, Power Factor, Current and Voltage Differing in Phase by θ°. Figure 4–3 shows the wave of current in a circuit, lagging $\theta°$ behind the voltage curve. Again, every point on the power curve has a value equal to the product of corresponding instantaneous values on the current and voltage waves. Thus, the value (p), at any chosen instant, is the product of (e) and (i) at that instant. Note that here, also, whenever either the current or the voltage is zero the power is zero; and that there are two positive power loops, A and C, and two negative loops, B and D. However, it is apparent, the area under the positive loops is greater than that under the negative loops, and the average power is positive. Since the average power is the average of all the

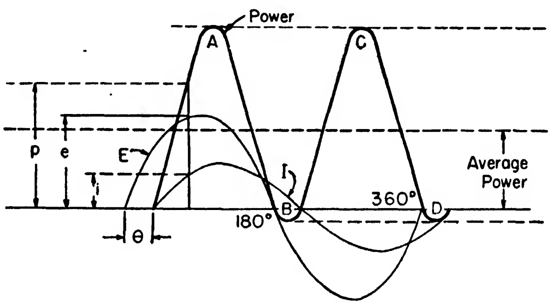


FIG. 4–3. The current I lags the voltage E by $\theta°$, and the average power equals the average of the difference between the positive power loops A and C and the negative power loops B and D.

instantaneous values (both positive and negative) of the power curve, the average power is obviously less than if all values were positive. It is represented by the dash line of Fig. 4–3, half way between the upper and lower lobes of the power curve, and is seen to be less than the value shown in Fig. 2–3, where the current and voltage are in phase. Therefore, the average power is positive, but is less than the product of effective E and effective I.

The **average power** in any single a-c circuit is always equal to the product of effective E and I, **multiplied by a constant which is always equal to the cosine of the angle of phase difference between the current and voltage.** This will be further discussed in following paragraphs.

The cosine of the angle of phase difference is called the **power factor** of the circuit. The power factor can never have a value greater than 1.

The average power in a single a-c circuit, expressed as a general equation, is

$$P = EI \cos \theta, \tag{3-3}$$

where P = average power in watts.
E = effective pressure in volts.
I = effective current in amperes.
θ = angle (electrical degrees) of phase difference between E and I.

Thus, when the current and voltage are in phase, as in Art. 2, the phase difference is 0°. The cosine of 0° = 1.

and $$P = EI \times \cos 0° = EI \times 1 = EI.$$

When the current and voltage are in quadrature, as in Art. 3, the phase difference is 90°. The cosine of 90° is 0,

and $$P = EI \cos 90° = EI \times 0 = 0.$$

Example 2. What power is being delivered by a single-phase alternator, which maintains a pressure of 240 volts across its terminals and supplies a current of 50 amperes, lagging behind the voltage by 25°?

Solution.

$$\begin{aligned} P &= EI \cos \theta \\ &= 240 \times 50 \cos 25° \\ &= 240 \times 50 \times 0.906 = 10{,}872 \text{ watts} = 10.87 \text{ kw.} \end{aligned}$$

Power factor is generally expressed as a percentage. Thus, if the current and voltage differ in phase by about 25°, the term ($\cos \theta$) becomes 0.90 and the circuit is said to have a power factor of 90 per cent.

Prob. 8-3. According to directions in Prob. 1-3, plot curves for current, pressure and power, when an alternating pressure of 120 volts delivers a current of 2 amperes, which lags 20° behind the voltage.

Prob. 9-3. By methods of Prob. 2-3, find the average value of the power delivered in Prob. 8-3. Compare this with the value as computed from the equation, $P = EI \cos \theta$.

Prob. 10-3. What power would the generator in Example 2 deliver, if the voltage and current were in phase?

Prob. 11-3. What power would the generator in Example 2 deliver if the voltage and current differed in phase by 90°?

Prob. 12-3. A 2300-volt alternator delivers 200 kw at a power factor of 60 per cent. What current does it supply?

Prob. 13-3. A 50-hp, 240-volt, a-c motor operates at full load with an efficiency of 85 per cent and a power factor of 85 per cent. What current does it draw from the line?

Prob. 14-3. The motor in Prob. 13-3, at half load, operates at 70 per cent efficiency with a power factor of 60 per cent. What is the current at half load?

Prob. 15-3. (*a*) What is the difference in phase of the current and voltage in Prob. 12-3? (*b*) In Prob. 13-3? (*c*) In Prob. 14-3?

Prob. 16-3. The phase angle between the terminal voltage and the current delivered by an alternator is 36° 10′. If the current is 100 amperes and the power delivered is 37.2 kw, what is the terminal voltage?

5-3. The Wattmeter. Determination of Power Factor. The wattmeter is an instrument used to measure power. One type consists of two stationary air cored coils, *cc*, Fig. 5-3, which carry the line current and, acting together, set up a magnetic field proportional to that current. A movable coil *v*, controlled by the tension of a spring, and carrying a pointer, is mounted in the field

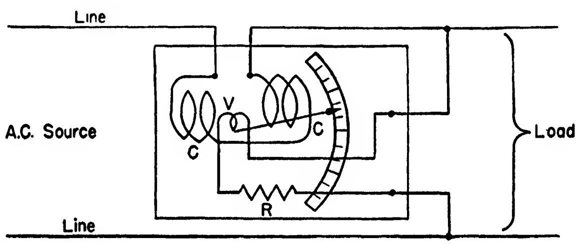


FIG. 5-3. One form of wattmeter for measuring power in an electric circuit.

between the coils *cc*, and, in series with a resistance R, is connected across the circuit, as shown. The torque set up and, therefore, the deflection of the needle, is proportional to the current in the two sets of coils. But the current in coil v is proportional to the voltage on the circuit. Thus the torque exerted on coil v at any instant is proportional to the product of (i) in the stationary, or current, coils times (e) across the movable, or voltage coil at the same instant. But at any instant $p = ie$. Thus the torque at any instant is proportional to the power at that instant. Therefore, the torque on the instrument at each instant throughout the cycle is proportional to the instantaneous values of the power curve. But since the pulsations of the power curve occur at such short intervals of time ($\frac{1}{240}$ of a second in a 60 cycle circuit), the inertia of the moving element prevents the needle from indicating these rapidly changing values; and so the instrument gives a steady deflection, proportional to the average torque, or the average power.

When the current is in phase with the voltage, both the current and voltage are positive during one half cycle, and the torque is in a positive direction. During the next half cycle, both the current and voltage are reversed, thus reversing the current in **both** sets of coils, so the direction of the torque is **not** changed. Therefore, the torque is always in one direction and the deflection of the needle is proportional to half the maximum, or the average, power.

When the current and voltage differ in phase by 90°, the current, or the voltage, reverses direction every quarter cycle, thus reversing the current alternately in the current, or the voltage, coils and reversing the torque. Since the average torque in one direction is

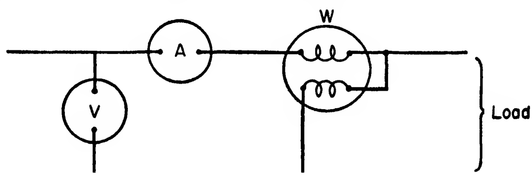


FIG. 6–3. Connections of an ammeter, a voltmeter, and a wattmeter in an a-c circuit to measure power and to determine the power factor.

just balanced by the average torque in the opposite direction, the net torque is zero; so the needle does not move, and the instrument indicates zero power.

When the current and voltage differ in phase by an angle of less than 90° and more than 0°, the torque on the moving coil reverses; but the torque is greater in one direction than in the other, and the instrument indicates the average value, which, as we have seen, is proportional to the average power.

In practice, the phase angle between voltage and current in most a-c circuits is not known, and the power in such circuits can be readily determined only by the use of a wattmeter. In a-c work, it is the standard instrument for measuring power, but is not often used in d-c measurements, since the voltmeter and ammeter give more accurate results. See Vol. I, Chap. III, for caution in the use of the wattmeter. Furthermore, the power factor, and therefore, the phase angle in a single-phase circuit, can be determined by use of a voltmeter, an ammeter, and a wattmeter, connected as shown in Fig. 6–3.

Since $\quad P = EI \cos \theta$

$$\frac{P}{EI} = \frac{\text{watts}}{\text{volts} \times \text{amperes}} = \cos \theta = \text{power factor.}$$

The product of the volts, multiplied by the amperes, or the volt-amperes in a circuit, is called the **apparent power.**

Thus,
$$\text{Power factor} = \frac{\text{True power}}{\text{apparent power}}. \tag{4-3}$$

Example 3. Instruments are connected in a generator circuit, as indicated in Fig. 6-3. The ammeter reads 20 amperes, the voltmeter indicates 230 volts, and the wattmeter shows that the generator is delivering 4 kw. (*a*) What is the power factor of the load? (*b*) What is the phase difference between the current and the voltage?

Solution.
$$\begin{aligned}
\text{True power} &= 4000 \text{ watts.}\\
\text{Apparent power} &= EI\\
&= 230 \times 20 = 4600 \text{ volt-amperes,}\\
\text{Power factor} &= \frac{\text{True power}}{\text{apparent power}}\\
&= \frac{4000}{4600} = \mathbf{0.87 \text{ or } 87 \text{ per cent.}}\\
\text{Power factor} &= \cos\theta,\\
0.87 &= \cos\theta,\\
\theta &= \mathbf{30^\circ \text{ approximately.}}
\end{aligned}$$

Prob. 17-3. A single-phase induction motor takes a current of 26 amperes at 230 volts. The wattmeter in the circuit indicates 5 kw. At what power factor does the motor operate? What is the angle of phase difference between the voltage and the current?

Prob. 18-3. When the motor in Prob. 17-3 operates at no load, the ammeter indicates 12 amperes and the wattmeter shows 1.2 kw. What is the power factor of the motor and phase difference between the current and voltage at no load?

Prob. 19-3. During a working day of 24 hours, the ammeter and voltmeter, connected in circuit with a small single-phase motor driving a drainage pump, indicate 10 amperes and 230 volts respectively. The reading of a watt-hour meter, also connected in the circuit, increases by 44.2 kilowatt-hours during this time. (*a*) Calculate the power factor at which the motor operates. (*b*) Assuming an 80 per cent efficiency for the motor and 60 per cent for the pump, calculate the horse power output of the motor and the total number of gallons of water raised 20 feet from an excavation. 1 gal of water weighs 8.3 lbs.

6-3. Apparent Power, True Power, Reactive Power. Consider a single-phase circuit, supplying a current which lags θ° behind the terminal voltage. The relations in such a circuit are shown by the vector diagram of Fig. 7-3, in which the vector OE represents the voltage on the circuit, and OI the current in the circuit, lagging OE by θ°.

Now it has been shown in Chapter II, Art. 15, that an alternating current may be the resultant of two or more currents in various phase relations to each other. The current in a circuit might be the resultant of two currents at 90°. In the latter case, if the phase angle and the resultant are known, the two components at 90° can readily be found. Therefore, when the current and voltage are out of phase with each other, it is convenient to think of the current in the circuit as made up of two currents, or two components, one in phase with the voltage and the other at 90° to the voltage. Thus in Fig. 7–3, the actual current I, as indicated by the ammeter, is considered as composed of two currents, I_r, in phase with the voltage, and I_x, at 90° to the voltage.

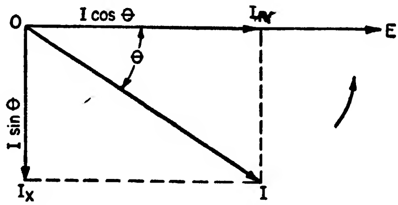


FIG. 7–3. The current in a circuit $\theta°$ out of phase with the voltage. The current, vector I, may be considered as composed of two components, the vector I_R in phase with the voltage, and the vector I_x at 90° to the voltage.

From the figure, it can be seen that

$$I_r = I \cos \theta$$

and

$$I_x = I \sin \theta$$

Also I (the total current in amperes) $= \sqrt{(I \cos \theta)^2 + (I \sin \theta)^2}$.
Thus, in Fig. 7–3, if $I = 100$ amperes and $\theta = 20°$,

$$I_r = 100 \cos 20° = 100 \times 0.94 = 94 \text{ amperes}$$

and

$$I_x = 100 \sin 20° = 100 \times 0.342 = 34.2 \text{ amperes.}$$

Therefore, a current of 100 amperes, lagging 20° behind the voltage, can be thought of as being composed of one current of 94 amperes, in phase with the voltage, and another current of 34.2 amperes, lagging the voltage by 90°.

Since $I \cos \theta$ is in phase with the voltage, it is called the **power component** of the current, or the **power current;** for it is this current, in conjunction with the voltage, which supplies the useful power, or the power which flows in one direction in the circuit and is consumed. It can be called the **working,** or **energy,** current.

Also, since $I \sin \theta$ is at 90° with the voltage, it is called the **reactive component,** or **reactive current,** for this current, in conjunction with the voltage, supplies the power that flows back and

forth in both directions in the circuit, or reacts upon itself. This component of the current has been called the wattless current, since it takes no energy from the generator, but the term is not approved.

The actual current in amperes I, acting in conjunction with the voltage, supplies both the circulating power and the power flowing in one direction in the circuit.

From the explanation above, it is seen that the terminal voltage, multiplied by the power component of the current, or $E \times (I \cos \theta)$, equals the **true power** or **watts** delivered to the circuit. This is the power used to produce torque in motors and supply heat and light, etc., and is expended outside the electrical system. This is the power indicated by the wattmeter.

The terminal voltage, multiplied by the reactive component of current, or $E \times (I \sin \theta)$, equals the circulating power, and is called the **reactive power** or **reactive volt-amperes.** (Circulating power occurs only in circuits which possess the ability to store up energy; receiving it during part of each half cycle and returning it to the generator during the remaining part of the half cycle. This energy is stored in electro-magnetic and electro-static "fields," set up by the current under the action of an electro-motive force. (See Vol. I, Chapters IV and XVI on Inductance and Capacitance.) The wattmeter does not indicate reactive power, since this sets up equal and opposite torques in the instrument during the cycle. Reactive power, or reactive volt-amperes, are called **vars.**

The terminal voltage, multiplied by the actual current in the circuit, or EI, is called the **apparent power** in volt-amperes, as has been previously stated.

Since
$$I = \sqrt{(I \cos \theta)^2 + (I \sin \theta)^2},$$
$$EI = \sqrt{(EI \cos \theta)^2 + (EI \sin \theta)^2}.$$

We, therefore, think of the total volt-amperes, EI, in the circuit, as composed of two components at 90° to each other, of which

$$EI \cos \theta = \text{True power in watts},$$

and
$$EI \sin \theta = \text{Reactive power in vars.*}$$

The relations of these three quantities are shown in the topographic diagram of Fig. 8–3, in which the true power is considered in phase with the power current, or in phase with the voltage

* Volt-amperes-reactive.

vector; the vars in phase with the reactive current, or at 90° to the voltage vector; and the apparent power in phase with the actual current.

In some cases, it may be preferable, or more convenient, to resolve the voltage E into two components, E_r in phase with I and

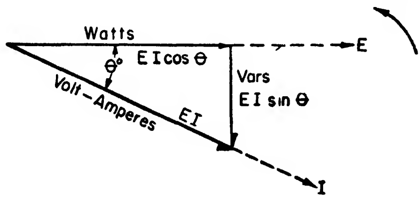


FIG. 8–3. Power diagram for a circuit in which the current is lagging $\theta°$ behind the emf.

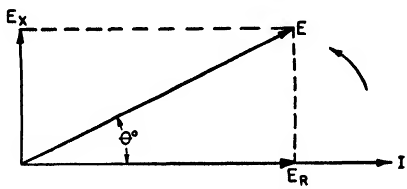


FIG. 9–3. Vector diagram for the same circuit, represented by Fig. 7–3. The voltage vector E may be resolved into two components, E_R in phase with the current I, and E_x at 90° to the current.

E_x at 90° with I, as in Fig. 9–3. This makes no real difference in the result, because

$$E_r = E \cos \theta, \text{ the power component of the voltage,}$$

and $$E_x = E \sin \theta, \text{ the reactive component of the voltage.}$$

Since $$E = \sqrt{E_r^2 + E_x^2} = \sqrt{(E \cos \theta)^2 + (E \sin \theta)^2},$$

$$IE = \sqrt{(IE \cos \theta)^2 + (IE \sin \theta)^2},$$

so that

$$IE \cos \theta = \text{true power in watts,}$$

and $$IE \sin \theta = \text{reactive power in vars.}$$

Strictly speaking, watts and volt-amperes are not vector quantities, since they do not represent sine curves; but because they are obtained by multiplying each of the current vectors of Fig. 7–3 by the same value of effective volts, they have the same

relative value to each other as do the current vectors. We, therefore, combine them in a vector diagram.

In most practical work, power in kilowatts (kw) is used rather than power in watts. Accordingly, we use apparent power in kilo-volt amperes (KVA) and reactive power in reactive-kilo-volt-amperes (RKVA), or kilo-vars (KVARS). This does not affect the relations in the vector diagram.

In Fig. 7–3, the current is shown as lagging behind the voltage. This throws the reactive current, and also the reactive power, 90° behind the voltage. In many circuits, the current **leads** the voltage, as will be shown later. In such cases, the reactive current and

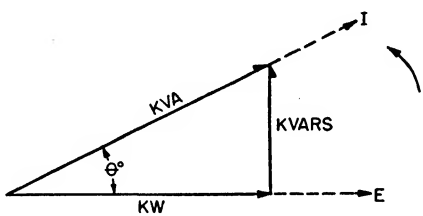


FIG. 10–3. Power diagram for a circuit in which the current leads the emf.

also the reactive power **lead** voltage by 90°, as indicated in the power diagram of Fig. 10–3. In either case, note that **cos θ equals $\frac{\text{kw}}{\text{kva}}$ and sin θ equals $\frac{\text{kvars}}{\text{kva}}$. Therefore, kw equals kva cos θ and kvars equals kva sin θ.**

Example 4. A 2300-volt motor takes a current of 17 amperes, lagging 36° behind the impressed voltage. (*a*) What is the power factor? (*b*) What is the power component of current? The reactive component? (*c*) What power in kw does the motor take? What is the reactive power in kvars? The apparent power in kva?

Solution: (*a*) Power factor = $\cos\theta = \cos 36° = 0.809$ or 80.9 per cent.

(*b*) Power component of $I = I\cos 36° = 17 \times 0.807 = 13.75$ amps.

Reactive component of $I = I\sin 36° = 17 \times 0.588 = 10$ amps.

(*c*) True power $= EI \cos\theta = 2300 \times 13.75 = 31{,}600$ watts, or 31.6 kw.

Reactive power $= EI \sin\theta = 2300 \times 10 = 23{,}000$ vars or 23 kvars.

Apparent power $= EI = 2300 \times 17 = 39{,}100$ va or 39.1 kva.

In the solution of the following problems, show both current and power diagrams, using the voltage as the reference vector.

Prob. 20–3. A single-phase induction motor takes a current of 24 amperes, which lags 30° behind the impressed voltage of 230 volts. (*a*) What is the power factor of the motor at this load? (*b*) How large are the reactive and power components of the current? (*c*) How much power does the motor take? What is the reactive power? The apparent power?

Prob. 21–3. A single-phase synchronous motor takes a current of 24 amperes, which leads the impressed voltage of 230 volts by 20°. (*a*) What is the power factor of this motor under these conditions? (*b*) How large are the reactive and power components of the current? (*c*) How much power does the motor take? What is the reactive power? The apparent power?

Prob. 22–3. If the two motors of Probs. 20 and 21 are connected in parallel across the same 230-volt mains and comprise the total load on the generator: (*a*) What current will flow in the line? (*b*) Compute the reactive component of the line current, and compare it with the reactive components of Probs. 20 and 21. Explain reason for change. (*c*) Compute the power component of the line current. (*d*) What is the power factor of the line? (*e*) Compute the power, supplied by the generator. Use two methods to check. (*f*) Compute the reactive power; the apparent power.

7–3. Power Factor and KVA Rating of A-C Generators. The load capacity of any electrical machine is limited by the temperature rise in its windings. This is primarily due to the heat developed in the armature, or load circuit, of the machine. Therefore, in any generator, the armature current, I, limits the output, and the normal terminal voltage multiplied by this limiting current, or $E \times I$, is the allowable output in volt-amperes.

In a d-c generator, $E \times I$ is the **power output** in watts and the size, or rating, can be expressed in watts or kilowatts.

In an a-c generator, normal terminal voltage, multiplied by this limiting or full load current EI, is not necessarily the power output; for we have seen that power delivered equals $EI \cos \theta$. For a given allowable current then, the maximum steady load, or the kw capacity of an alternator, varies with the power factor. This power factor depends upon the character of the connected load, and cannot be controlled at the generator. Therefore, the size of an a-c generator, or its rating, is generally expressed in terms of **apparent power,** EI, in volt-amperes, or kilo-volt-amperes. This is illustrated in the following example.

Example 5. The full-load current of a 2400-volt, single-phase alternator is 50 amperes: (*a*) What is its kva rating? (*b*) When it is connected to a unity power factor load, what continuous power in kw can it supply? (*c*) At 0.8 power factor, what power can it supply? (*d*) At 0.5 power factor?

Solution: (*a*) $$\frac{2400 \times 50}{1000} = 120 \text{ kva.}$$

(*b*) $$\frac{2400 \times 50 \times 1.00}{1000} = 120 \text{ kw at unity p.f.}$$

(*c*) $$\frac{2400 \times 50 \times 0.8}{1000} = 96 \text{ kw at 0.8 p.f.}$$

(*d*) $$\frac{2400 \times 50 \times 0.5}{1000} = 60 \text{ kw at 0.5 p.f.}$$

Thus, when carrying full load current, the machine in the above example can deliver 120 kw to a unity power factor load, but only 60 kw to load of 0.5 power factor. It can, however, supply 120 kva to loads of **any power factor,** and would be called a 120 kva alternator. To deliver a steady load of 120 kw at 0.5 power factor, the alternator must supply a current of $\frac{12,0000}{2400 \times 0.5}$ or 100 amperes. This is just double the maximum safe current, and would overheat and destroy the insulation on the windings of the machine. When the rating of an alternator is given in kw, the power factor is always specified. This places a definite value on the current the machine can supply.

From the discussion above, it is readily seen that loads, having a low power factor, are undesirable, as they require more current for a given amount of power delivered. This, of course, is because the lower the power factor, the greater is the amount of reactive current which must be supplied. The examples below illustrate this.

Example 6. A 230-volt 50 hp, single-phase motor, at full load, operates at 90 per cent efficiency and 85 per cent power factor. (*a*) What current does it take? (*b*) What is the power current? (*c*) The reactive current?

Solution: $$\frac{50 \times 746}{.90} = 41,500 \text{ watts supplied to motor.}$$

(*a*) $230 \times I \times 0.85 = 41,500;$

$$I = \frac{41,500}{230 \times 0.85} = 212.3 \text{ amps.}$$

(*b*) $212.3 \times 0.85 =$ **180.5 amps, power comp. Ans.**

(*c*) $212.3\sqrt{1^2 - 0.85^2} = 212.3 \times 0.527 =$ **112 amps, react. comp.**

Example 7. If another motor of the same full load rating as that in Example 6 operates at the same efficiency but at 60 per cent power factor, what will be its full load current, reactive component and power current? Power taken from the line = 41,500 watts, as above.

(*a*) $230 \times I \times 0.6 = 41{,}500$ watts

$$I = \frac{41{,}500}{230 \times 0.6} = \textbf{300.8 amperes.}$$

(*b*) $300.8 \times 0.6 =$ **180.5 amp. power comp.**

(*c*) $300.8\sqrt{1^2 - 0.6^2} = 300.8 \times 0.8 =$ **240.6 amps. react. comp.**

In the examples above, the motor with the lower power factor takes a very much greater current, and yet the **power supplied to each is the same**; and the **energy, or power component of current, is the same.** But the **reactive current** in the lower power factor motor is **much greater.**

For the same energy in kwhrs sold, the operating company delivers only 212 amperes to one motor, while it must deliver 300 amperes to the other. In the latter case, the company must install larger wires, transformers, etc., to supply this extra current. This additional current also reduces the load capacity of the generator, for it can supply only so much current, regardless of the power factor.

In practice, customers having loads of very low power factor (generally below 70 per cent) are usually required to pay a higher rate, or more per kwhr, for energy, because of the greater cost of supplying the large amount of reactive current in such loads. The rate may be divided into two parts; — a certain amount per kwhr for the true power, and another amount per kvarhr for the reactive power.

8–3. Power Factor Correction. Certain types of electrical apparatus, such as partially loaded induction motors, may draw **lagging** currents of low power factor. This, as we have seen from the preceding paragraphs, is undesirable.

In such cases, it is often possible to replace some of the induction motors with synchronous motors, which may be adjusted to take currents, **leading** the voltage. Also, an unloaded synchronous motor may be adjusted to take full-load current, leading by a large angle (see Chapter XI, Art. 17); or, condensers, which also

take a leading current, as explained in the next chapter, may be connected across the line.

The combination of these leading currents with the lagging currents of the induction motor load improve the power factor of the system, and may actually reduce the total current drawn from generator. This is illustrated in the example below. The solution of Prob. 22–3 also brings out this fact.

Example 8. The machinery in a certain shop is driven by induction motors, which require a total current of 250 amperes and operate at a combined power factor of 64.3 per cent, or with the current lagging the voltage by 50°, as indicated in Fig. 11–3(*a*). The shop is to be enlarged; and it is decided to drive the additional machines with a synchronous motor. If this motor is adjusted to take 100 amperes, leading the voltage by 80°, what current will the shop require, and what will be the power factor?

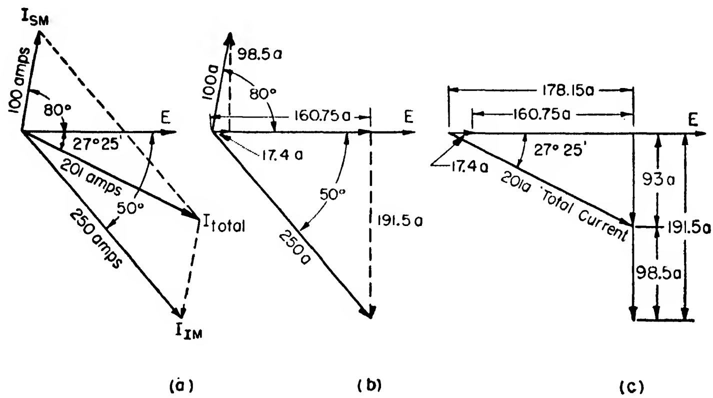


FIG. 11–3. (*a*) Polar vector diagram of currents in Example 8. (*b*) Horizontal and vertical components of the currents in (*a*). (*c*) Topographic diagram of components and total current in (*a*).

Solution: (See Figs. 11–3(*a*), (*b*), and (*c*).)

Power comp. induction motors	$= 250 \cos 50°$	$= 250 \times 0.643$
		$= 160.75$ amps.
Reactive comp. induction motors	$= 250 \sin 50°$	$= 250 \times 0.766$
		$= 191.5$ amps.
Power comp. synchronous motor	$= 100 \cos 80°$	$= 100 \times 0.174$
		$= 17.4$ amps.
Reactive comp. synchronous motor	$= 100 \sin 80°$	$= 100 \times 0.985$
		$= 98.5$ amps.

Total Power component $= 160.75 + 17.4 = 178.15$ amperes.
Total Reactive comp. $= 191.5 - 98.5 = 93$ amperes.

$$\text{Total current} = \sqrt{178.15^2 + \overline{93}^2} = \mathbf{201\ amperes.\ Ans.}$$

$$\text{Power factor of the circuit} = \frac{178.15}{201} = 0.886 \text{ or } \mathbf{88.6\ per\ cent.\ Ans.}$$

Thus, in the example above, while the load on the shop has been increased, the use of the synchronous motor has actually reduced the required current from its former value of 250 amperes to 201 amperes, and increased the power factor. The current now lags the voltage by an angle whose cosine is 0.866 or 27° 25″, and the reactive component has been reduced from 191.5 amperes to 93 amperes.

9–3. Total KVA, Power Factor and Current of a System Consisting of Several Loads in Parallel. It is important to know the size of generator in kva required to supply a system, consisting of several different loads. It is also important to know the power factor. Each of the several loads may have a different power factor, and the power factor of the system may have a value differing greatly from that of any of the individual loads.

In solving a problem of this sort, we may work with currents, or we may work directly with power, whichever we shall find to be simpler. In the latter case, it is unnecessary to know the voltage of the system.

The solution of such a problem is illustrated in the examples below.

Example 9. (*a*) What size generator in kva is required to supply a 500-volt system, comprising the following loads? First; an induction motor, taking 60 kw at 80 per cent power factor (lagging). Second; incandescent lamps, taking 40 kw at 100 per cent power factor. Third; a load, taking 62.5 kva, at 40 per cent power factor (lagging).

(*b*) What will be the power factor of the system?

For simplicity, neglect the voltage drop and losses in the line wires.

Solution: (by use of currents, Fig. 12–3)

$$\text{Load (1) } I_1 = \frac{\text{watts}}{E \cos\theta} = \frac{60{,}000}{500 \times 0.8}$$
$$= 150 \text{ amps. } (0.8 = \cos 36° \, 50' \text{ lag angle}).$$

$$\text{Power Comp. } I_1 = 150 \cos 36° \, 50' = 150 \times 0.8 = 120 \text{ amps.}$$
$$\text{React. Comp. } I_1 = 150 \sin 36° \, 50' = 150 \times 0.6 = 90 \text{ amps.}$$

$$\text{Load (2) } I_2 = \frac{\text{watts}}{E \cos\theta} = \frac{400{,}000}{500 \times 1.00}$$
$$= 80 \text{ amps. } (1.00 = \cos 0°, \text{ lag angle}).$$

$$\text{Power Comp. of } I_2 = 80 \cos 0° = 80 \times 1 = 80 \text{ amps.}$$
$$\text{React. Comp. of } I_2 = 80 \sin 0° = 80 \times 0 = 0 \text{ amps.}$$

$$\text{Load (3) } I_3 = \frac{EI}{E} = \frac{62{,}500}{500}.$$
$$= 125 \text{ amps } (0.4 = \cos 66° \, 20' \text{ lag angle}).$$

$$\begin{aligned}
\text{Power Comp. of } I_3 &= 125 \cos 66° \, 20' \\
&= 125 \times 0.4 = 50 \text{ amps.} \\
\text{React Comp. of } I_3 &= 125 \sin 66° \, 20' \\
&= 125 \times 0.916 = 114.5 \text{ amps.}
\end{aligned}$$

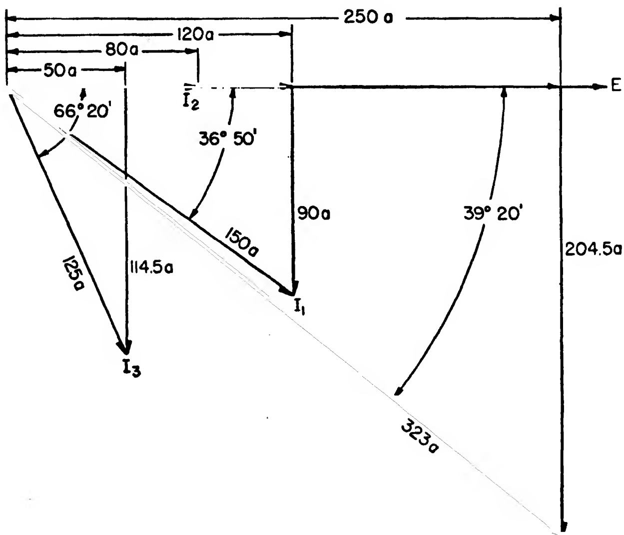


FIG. 12-3. Vector diagram of the three load currents and total generator current in Example 9, together with their "in-phase" and 90° components.

Total Current

$$\begin{aligned}
&= \sqrt{(\text{Total Power Comp.})^2 + (\text{Total React. Comp.})^2} \\
&= \sqrt{(120 + 80 + 50)^2 + (90 + 0 + 114.5)^2} \\
&= \sqrt{250^2 + 204.5^2} = 323 \text{ amperes.}
\end{aligned}$$

$$\begin{aligned}
\text{Total apparent power required} &= 500 \times 323 \\
&= 161{,}500 \text{ volt-amperes} \\
&= \mathbf{16.15 \text{ kva.}} \quad \textbf{Ans.}
\end{aligned}$$

$$\text{(b) Power Factor} = \frac{\text{Power Comp. of total } I}{\text{Total } I} = \frac{250 \text{ amp}}{323 \text{ amp}}$$

$$= \mathbf{0.774} \text{ or } \mathbf{77.4 \text{ per cent.}} \quad \textbf{Ans.}$$

$$0.774 = \cos 39° \, 20' = \text{angle total current lags the voltage.}$$

Example 10. Solution of the problem of Example 9 by the power method. (See Fig. 13–3.)

(*a*) Load (1)

$$\text{Apparent power} = \frac{\text{kw}}{\text{pf}} = \frac{\text{kw}}{\cos\theta} = \frac{60}{0.8} = 75 \text{ kva.}$$

True power = 60 kw (given)
React power = kva $\times \sin\theta = 75 \times 0.6 = 45$ kvars.

Load (2)

$$\text{Apparent power} = \frac{\text{kw}}{\text{pf}} = \frac{40}{1.00} = 40 \text{ kva}$$

True power = 40 kw (given)
React power = kva $\sin\theta = 40 \times 0 = 0$ kvars.

Load (3)

Apparent power = 62.5 kva (given)
True power = kva $\times$ pf $= 62.5 \times 0.4 = 25$ kw
React power = kva $\sin\theta = 62.5 \times 0.916 = 57.25$ kvars.

Total true power output of generator $= 25 + 40 + 60 = 125$ kw.
Total reactive power output of generator $= 57.25 + 0 + 45$
$= 102.25$ kvars.
Total apparent power output of generator $= \sqrt{125^2 + 102.25^2}$
$=$ **161.5 kva Ans.**

$$\text{(b) Power factor} = \frac{\text{True power}}{\text{Apparent power}} = \frac{125}{161.5}$$
$$= 0.774 \text{ or } 77.4 \text{ per cent: } \theta = 39° \, 20'.$$

The power diagram for this circuit is shown in Fig. 13–3.

It is evident that the method shown in Example 10 is much more direct, when dealing with questions of size or power capacity of apparatus, or lines, when pressure, or current, are not under consideration. (Also, two or three phase circuits may be solved by this same identical method.)

Remember that whenever a load takes a leading current, its vector lies above the voltage vector; and its reactive component must be subtracted from the reactive components of loads, taking lagging currents. Also, the kvars of leading loads must be subtracted from the kvars of lagging loads.

Also, note particularly that kws and also kvars are added algebraically, but that kva cannot be combined in this manner, unless the power factors of all individual loads are the same. Total kva in any system is the vector sum of total kw and total kvars, or kva = kw ⊕ kvars.

In the solution of each of the following problems, show the vector diagram.

Prob. 23-3. The machines in a certain shop are arranged in three groups, driven by line shafts, connected to three similar 50-hp a-c motors. One of these operates at $\frac{1}{2}$ load, with an efficiency of 80 per cent, and power factor of 50 per cent. The second operates at $\frac{3}{4}$ load, efficiency 85 per cent, and power factor 75 per cent. The third operates at full load, efficiency 88 per cent, and power factor 90 per cent. Calculate the kw, kvars and kva supplied to each motor and to the entire shop. Calculate also the power factor of the feeder, supplying this shop.

Prob. 24-3. If the half-load motor of Prob. 23-2 were replaced by a synchronous motor, which delivers the same power at the same efficiency,

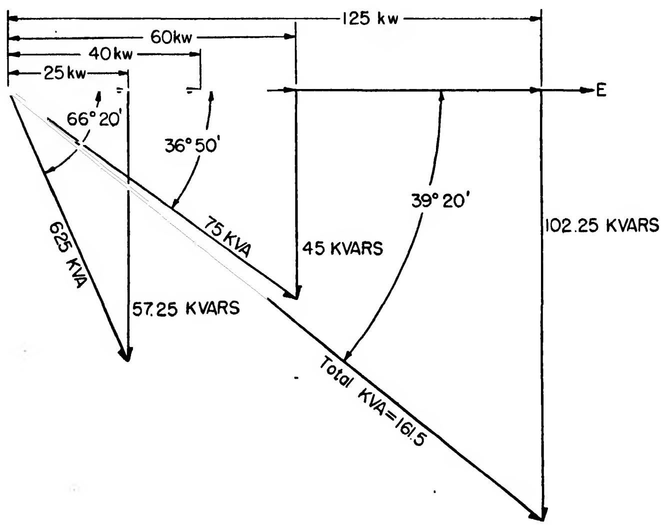


FIG. 13-3. Power diagram for the system in Examples 9 and 10.

but is adjusted to take enough reactive **leading** kvars to compensate, or neutralize the total lagging kvars, taken by the other two motors, calculate: (*a*) The kvars which must be taken by the synchronous motor; (*b*) The kw that must be taken by this motor; (*c*) The required size of the synchronous motor in kva; (*d*) The power factor at which this motor must operate; (*e*) The power factor of the feeder under these conditions.

Prob. 25-3. If the feeder voltage in Prob. 23-3 is 460 volts, and the motors are single phase, calculate the power component, reactive component and total current taken by each motor and by the feeder.

Prob. 26-3. Repeat the calculations called for in Prob. 25-3 for the conditions as given in Prob. 24-3.

Prob. 27-3. An alternator supplies a system consisting of the following loads; 150 kw at 80 per cent power factor, lagging; 125 kw

at 60 per cent power factor, lagging; and 100 kva at 50 per cent power factor, leading. What total kva must the alternator deliver, and what will be the power factor of the system?

SUMMARY OF CHAPTER III

The AVERAGE POWER, delivered when an alternating current and voltage are IN PHASE, equals the product of the effective volts, times the effective amperes. That is,

$$P = EI.$$

WHEN THERE IS A PHASE DIFFERENCE BETWEEN PRESSURE AND CURRENT AT 90°, no power is being delivered. During each cycle, all the energy delivered by the generator is returned to it again; or, there is a motor action in the generator equal to the generator action.

WHEN THE PHASE DIFFERENCE $\theta°$ BETWEEN CURRENT AND VOLTAGE IS LESS THAN 90° AND MORE THAN 0°, the AVERAGE POWER delivered equals the product of EFFECTIVE VOLTS, times EFFECTIVE AMPERES, times the factor COS θ. The equation for power is

$$P = EI \cos \theta.$$

The cosine of the angle of phase difference between E and I, or $\cos \theta$, is called the POWER-FACTOR.

The product of volts times amperes, or EI, is called the APPARENT POWER, and the unit is called VOLT-AMPERES to distinguish it from WATTS, which is the real, or true, power.

$$\text{Power-Factor} = \frac{\text{True Power}}{\text{Apparent Power}}$$

The power-factor is unity, or one, when the current and voltage are in phase; and zero, when there is a phase difference of 90° between them.

The POWER, taken by a circuit carrying alternating current, is MEASURED BY A WATTMETER, one set of coils carrying the current; while another coil, connected across the circuit and movable, carries a current proportional to the voltage. The average torque on the movable coil, and hence its deflection, is proportional to the average power. The wattmeter thus reads $EI \cos \theta$, in that part of the circuit to which it is applied.

POWER AND REACTIVE COMPONENT OF CURRENT. Any alternating current may be considered as made up of two parts, having 90° phase difference. One part, the POWER COMPONENT, is in phase with the voltage, and equals the actual current times the cosine of the angle of phase difference between the current and voltage. POWER COMPONENT $= I \cos \theta$.

The other part lags or leads the voltage by 90°, and is equal to the actual current times the sine of the angle of phase difference between

current and voltage. This is called the REACTIVE COMPONENT of the current.

REACTIVE COMPONENT = I $\sin\theta$.

The current in a circuit, as measured by an ammeter, may be expressed as

$$I = \sqrt{(I\cos\theta)^2 + (I\sin\theta)^2}.$$

TRUE POWER AND REACTIVE POWER. The voltage times the power component of the current, or EI $\cos\theta$, is the TRUE POWER in WATTS. In the power diagram, true power is in phase with the voltage.

The voltage, times the reactive component of current, or EI $\sin\theta$, is called the REACTIVE POWER. This unit is expressed as REACTIVE VOLT-AMPERES, or VARS. In the power diagram, reactive power is in phase with the reactive component of current, or at 90° with the voltage.

APPARENT POWER

$$= \sqrt{(\text{TRUE POWER})^2 + (\text{REACTIVE POWER})^2}$$

APPARENT POWER = VOLT-AMPERES or KILOVOLT-AMPERES (KVA),

TRUE POWER = WATTS or KILOWATTS (KW).

REACTIVE POWER = REACT-VOLT-AMPERES or REACT-KILOVOLT-AMPERES (RKva).

Reactive power is ALSO expressed as VARS or kilo-vars (KVARS).

$$\text{KVA} = \sqrt{(\text{KW})^2 + (\text{KVARS})^2}.$$

THE SIZE, OR RATING, OF A.C. GENERATORS IS GENERALLY EXPRESSED IN APPARENT POWER, OR KVA, since in most a-c work, the terminal voltage, times the full load current, is NOT NECESSARILY the power delivered, as is the case with d-c generators.

LOADS HAVING LOW POWER-FACTOR ARE UNDESIRABLE, since they take relatively large reactive components and, therefore, larger total currents than do loads of high power-factor.

POWER-FACTOR CORRECTION. The power-factor of systems with loads, taking lagging currents of low power-factor, may be improved by adding or substituting loads, taking leading currents. The leading reactive components of these new loads decrease the total reactive component of current and, thereby, raise the power factor of the system.

PROBLEMS ON CHAPTER III

In the solution of the following problems, show either current or power diagrams, using the voltage as the reference vector.

Prob. 28–3. A single-phase motor at full load has a power factor of 85 per cent and draws 15 amperes at 230 volts. What power does it take at this load?

Prob. 29–3. If the motor of Prob. 28–3 has an efficiency of 72 per cent, what is its rating in horsepower?

Prob. 30–3. A 15 hp, 230-volt motor is guaranteed to have a power factor of 86 per cent and an efficiency of 88 per cent at full load. What current does it take?

Prob. 31–3. A 230-volt single phase alternator has a full load rating of 60 kva at 75 per cent power factor. Compute: (*a*) the rated full load current; (*b*) the full load power rating; (*c*) the horsepower required to drive the alternator at rated load, if its efficiency is 88 per cent; (*d*) the maximum power which can be taken from the machine, operating at normal voltage, without exceeding the rated full load current.

Prob. 32–3. A generator is supplying two single phase induction motors in parallel. Each motor takes 7.5 kva at 240 volts, and has a power factor of 80 per cent, lagging current. Compute: (*a*) total kva output of generator; (*b*) total kw output; (*c*) power factor of line; (*d*) current supplied by generator.

Prob. 33–3. In order to improve the power factor of the line in Prob. 32–3, one of the induction motors is replaced by a synchronous motor, which carries the same load and has the same efficiency, but is adjusted to take a leading current at 90 per cent power factor. (*a*) What apparent power in kva does the synchronous motor take? (*b*) What apparent power does the line supply? (*c*) What is the power factor of the line?

Prob. 34–3. An induction motor, taking a lagging current of 20 amperes at 75 per cent power factor, is connected in parallel with a synchronous motor, taking a leading current of 35 amperes at 85 per cent power factor. What current does the line supply?

Prob. 35–3. Does the line current in Prob. 34–3 lead or lag the voltage, and by what angle?

Prob. 36–3. An alternator supplies the following three single phase loads in parallel: (1) 25 kw at 0.8 power factor leading; (2) 100 kva at 0.75 power factor lagging; (3) a 50-hp motor, operating at full load, with an efficiency of 90 per cent and 0.85 power factor lagging. (*a*) What total kva, kvars and kw must the generator supply? (*b*) What is the power factor of the circuit?

Prob. 37–3. If power in Prob. 36–3 is delivered to each of the three loads at 2300 volts, what current does each take; and what total current must the generator supply?

Prob. 38–3. A 500 kva alternator supplies a load of 350 kva at 0.78 power factor lagging. (*a*) How many kw of additional lighting load at unity power factor can it supply without overloading? (*b*) What will be the alternator power factor with this additional load?

Prob. 39–3. (*a*) How many kw of additional load, at 0.85 power factor lagging, can be added to the original 350 kva load on the alternator of Prob. 38–3, without overloading it? (*b*) What will be the power factor of the alternator under this condition?

Prob. 40–3. (*a*) If the additional load on the alternator of Prob. 38–3 had a leading power factor of 0.85, how many kw can be added without overloading the machine? (*b*) What will be the power factor?

CHAPTER IV

CIRCUITS CONTAINING REACTANCE AND IMPEDANCE

In our previous study of direct-current circuits, we have considered the relations in these circuits, when a constant, or unidirectional, emf was impressed upon them. When an alternating emf is impressed, the variation of the voltage, both in value and direction, produces entirely different relations in many of these circuits.

It is the purpose of this chapter to study voltage and current relations in circuits which contain only resistance, inductance or capacitance alone, and in combinations, under the action of an alternating emf.

1–4. Circuits Containing Resistance Only. It was shown in Chapter I, Art. 18, that Ohm's law for d-c circuits applies in the same manner to a-c circuits, which contain resistance only.

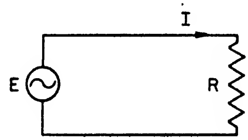


FIG. 1–4. Circuit having resistance only. The RI drop is always equal to the impressed voltage.

When a d-c voltage is impressed upon a circuit, containing resistance only, as in Fig. 1–4, we know that this voltage can force a definite current through the circuit, against the opposition offered by the resistance. We write this in an equation, as $E = IR$, and say that the voltage on the circuit is equal to the RI drop, or resistance drop. Thus the resistance drop may be considered as an opposing action, or a counter-voltage in the circuit.

There is nothing new in this idea; for we know that in starting a d-c motor, a starting rheostat, or " starting box," is used, in order that **the resistance drop may take the place of the counter-emf,** until the motor comes up to speed. Let us call the opposing action in the circuit the " RI " drop, to distinguish it from the IR drop to which it is equal. Thus we can say of the circuit of Fig. 1–4 that a voltage equal and opposite to the opposing " RI " must be impressed, before any current can be forced through the circuit, or $E = RI$.

When an a-c voltage of sine wave form is impressed upon the circuit of Fig. 1–4, an alternating current of the same form will flow, as already shown in Chapter I. At the instant when the voltage e is zero, both the current and the opposing Ri are zero; when e is a maximum, both the current and the opposing Ri are a maximum. Thus, the impressed emf at any instant is always equal to the opposing Ri at that instant; or $e = Ri$.

Therefore, the opposing action Ri is a sine curve 180° displaced from the impressed voltage; while the current of the same form is

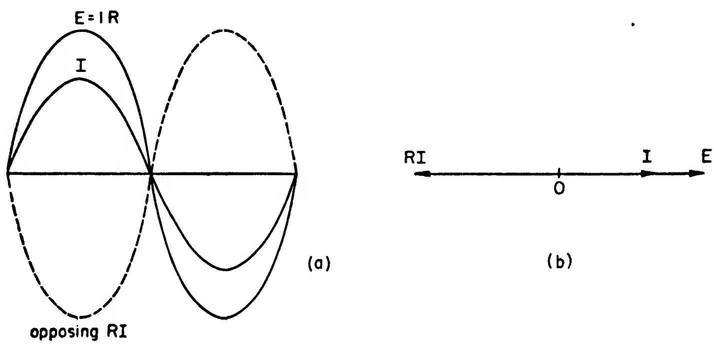


Fig. 2–4. (a) When an emf of sine wave form is impressed on a circuit, having resistance only, a current of the same wave form will flow. At every instant in the cycle, the impressed emf is opposed by an equal IR drop, which may be considered as equivalent to an opposing emf, RI, whose value at every instant is equal to the impressed emf and 180° from it. (b) Vector diagram of the relations in (a).

in phase with the impressed voltage, as shown by the curves in Fig. 2–4(a). When effective values are used, the equation is written as

$$E = RI. \tag{1–4}$$

These relations are also shown in the vector diagram of Fig. 2–4(b).

Since the current is in phase with the voltage, the power factor of a circuit, containing resistance only, is 1 and

$$P = EI = I^2R. \tag{2–4}$$

Thus, it is again seen that an a-c circuit, containing resistance only, behaves exactly like a d-c circuit, and the current flow is determined by Ohm's law.

2-4. Circuit Containing Inductance Only. It was shown in Vol. I, Chapter IX, that when a d-c voltage is impressed upon a circuit having inductance, such as a coil, the current does not im-

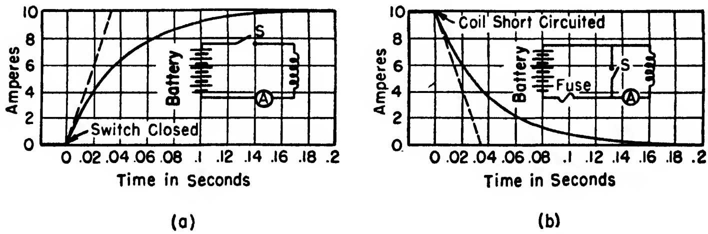


FIG. 3-4. (*a*) Delayed rise of current in an inductive circuit. (*b*) Delayed decay, or decrease, of current in an inductive circuit. In a purely inductive circuit (one having no resistance), the current would rise and fall along the dashed straight lines.

mediately reach its Ohm's law value, but rises gradually, as shown in Fig. 3-4(*a*). And when the voltage is suddenly decreased, or the coil short-circuited, the current decreases gradually, as in Fig. 3-4(*b*). The effect produced is as though when the current attempts to increase, the circles of flux around the turns of the coil, due to the increasing current, spread out in ever widening circles and interlink, or cut, the other turns as indicated in Fig. 4-4; and set up an emf, opposing the rise in current. And when the current attempts to decrease, these circles of flux, shorten their paths, or collapse, and decrease the linkages, or cut the turns of the coil in the opposite direction, and again set up an emf, opposing the decrease of the current.

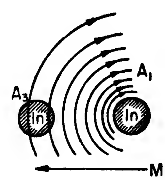


FIG. 4-4. As the current attempts to rise in turn A_1 of a coil, the magnetic lines spread out in ever-widening circles, and sweep across turn A_3 in the direction indicated by the arrow M. This induces an emf OUT in A_3 in opposition to the applied emf.

The action of the flux lines, when the current is increasing, may be thought of as somewhat similar to the ever widening circles of the ripples, which spread out in the water when a pebble is dropped into a pool. When the current is decreasing to zero, the action of the flux lines can be visualized in this illustration, if it is assumed that when the pebble is pulled vertically from the pool, these ripples reverse their movement and collapse at the starting point.

Thus, in a circuit containing inductance, any change in the value of the current sets up an opposing action, or counter-voltage, which is proportional to the **rate of change** of the current. This voltage is called the counter emf of self induction, and in Vol. I has been expressed as,

$$E_{av} = L\frac{I}{t}, \tag{3-4}$$

where L = inductance in henries.
I = total change of current in time t.
t = time in seconds.

In a d-c circuit, inductance shows its presence, only when the voltage on the circuit is changed; and under the action of a steady emf, the ultimate value of the current is determined by the resistance in the circuit.

When an a-c voltage is impressed upon an inductive circuit, such as the coil of Fig. 5–4, the current is continually changing, both in value and direction, and the flux linkages are also continually changing. That is, we can imagine that the flux lines with the rising current, first sweep or spread out, cutting the turns in the coil in one direction, as in Fig. 4–4; as the current decreases, the lines again sweep across, or cut the turns in the coil in the opposite direction, and collapse to zero, as the current falls to zero. As the current reverses and rises again, the flux lines are reversed, and again sweep out, cutting the turns in the coil; and as the current falls again to zero the lines again sweep across the turns and collapse. Thus, the continually changing flux lines set up an opposing emf in the coil, which itself is continually changing in value and direction. Note particularly that the opposing emf is greatest when the current is **changing at the greatest rate.**

When the current in the a-c circuit is increasing in either direction, the inductance retards the rise of current, as in Fig. 3–4(a), and the current does not have time to reach its Ohm's law value before the current begins to decrease. Also the current is retarded when it attempts to decrease. Thus inductance in an a-c circuit both "chokes" the current, thereby reducing its maximum and effective values, and also retards it, and makes it lag behind the impressed voltage. Figure 5–4 shows a "choke coil" used in an a-c line to protect apparatus from a too rapid increase in current, due to short-circuits, or disturbances when lightning strikes the line.

Some a-c circuits contain so little resistance that practically the

only opposition to the impressed voltage and the flow of current is that due to the counter emf of self induction.

For instance, let us assume that an a-c voltage is impressed upon the coil of Fig. 6–4, and that this coil has inductance, but no

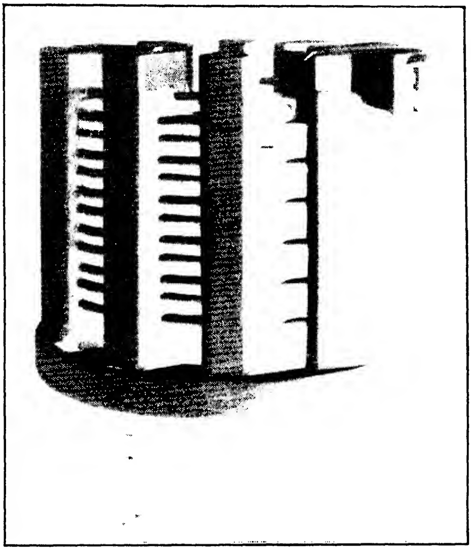


FIG. 5–4. An air-core induction coil, which is connected in series with the armature windings of a-c generators to limit the current under unusual conditions.

resistance. (This is an extreme case, for no actual coil can be wound without some resistance. It is here assumed to bring out more clearly the sole effect of inductance in an a-c circuit.) The waves of impressed voltage, current and induced counter emf in the coil are shown in Fig. 7–4. The current curve lags 90° behind the impressed voltage. Note that when the current is zero, it is rising in a **positive direction** and is changing at its **greatest rate**; since at this point the slope of the curve is the steepest. The magnetic flux through the coil is also **changing at the greatest rate**; therefore, the induced, or counter emf of self induction is the greatest and negative, since it is opposing a positive rise in current. It is shown as $-E'_{\text{max}}$ on the dash curve in the figure. When the current curve reaches the maximum positive point in its cycle, its **rate of change,** and consequently, the

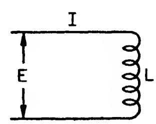


FIG. 6–4. Circuit having inductance only.

rate of change of flux inter-linkages is **zero.** Therefore, the induced counter emf is zero. At the next zero point of the current wave, the **rate of change** of current in a **negative** direction is greatest; therefore, the induced emf is a **maximum** and positive, shown as $+E'_{\max}$. Continuing in this manner, the curve of induced counter emf of self induction is obtained. Note that this curve is also a sine curve and that it **lags** 90° behind the current wave.

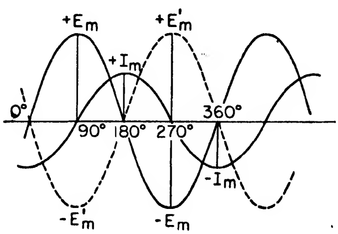


FIG. 7–4. Curve of $+E_{m_1}$ and $-E_m$, represents the impressed voltage on a coil, having inductance only. Curve of $-E'_{m_1}$ and $+E'_{m_1}$, represents the counter emf set up in this coil by the flow of the alternating current $+I_{m_1}$ and $-I_m$. Note that the current curve lags 90° behind the impressed voltage curve.

Since this counter emf is the only voltage opposing the flow of current, it must be equal and opposite to the impressed voltage which causes the current. It can be compared to the opposing RI of Fig. 2–4, as explained in Art. 1. It is also similar to the counter emf in a motor. No current can flow in the motor armature until the impressed voltage is equal and opposite to the counter voltage. Thus, before any current can flow into a circuit containing inductance only, the impressed voltage must be equal and opposite to the counter emf of self induction. The impressed voltage, shown in Fig. 7–4, is thus equal to and 180° from the counter emf. From an inspection of the figure, it is seen that the impressed voltage **leads** the current by 90°.

Thus in a circuit, containing inductance only, the current lags 90° behind the impressed voltage.

If E represents the impressed voltage on the coil of Fig. 5–4, the effective value of the induced counter voltage is also E, since the **average value** of the counter emf equals the inductance times the rate of change of current. Therefore we can write the equation

$$E_{\text{ave}} = L\frac{I_m}{t}, \tag{4–4}$$

where

L = inductance in henries,
I_m = maximum value of the current in amperes,
t = time in seconds required to change from 0 to I_m amperes.

The average rate of change of an alternating current depends upon the maximum value (I_m) which the current attains, and the time (t) required to attain it. In Fig. 8–4, we see that the current makes a change from zero to a maximum, or the reverse, four times during each cycle. During the part of the curve marked,

(1) it rises from zero to a maximum positive value, $+I_m$;
(2) it falls from maximum to zero;
(3) it rises from zero to a maximum negative value, $-I_m$;
(4) it falls from maximum to zero again.

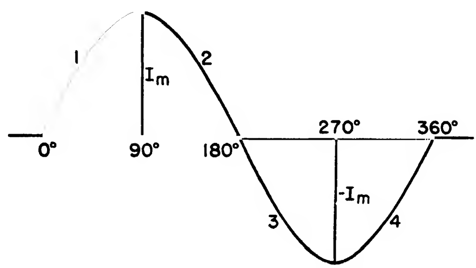


FIG. 8–4. The sine wave of current. Note that it changes between the values of zero and I_m four times during one cycle.

Accordingly, if f = frequency, or the number of cycles per second, then

$$4f = \text{number of changes per second between zero and } I_m.$$

Therefore, if the current changes $4f$ times each second, between the values of 0 and I_m amperes, it must change at an average rate of $\dfrac{I_m}{\frac{1}{4f}}$ or $4fI_m$ amperes per second.

Example 1. What is the average rate of change in current in a 60-cycle circuit, carrying a maximum current of 5 amperes?

Solution: Average rate of change $= \dfrac{5}{\frac{1}{4 \times 60}} = \dfrac{5}{\frac{1}{240}} = 5 \times 240$

$= \mathbf{1200}$ **amperes per sec. Ans.**

3–4. Inductive Reactance. We have learned in Vol. I that average voltage induced in an inductance by a changing current

equals the product of the inductance multiplied by the rate of change of the current. That is,

$$E_{\text{ave}} = L \times \frac{\text{Current change}}{\text{Time taken for change}} \tag{5-4}$$

In the last article we saw that the sine-wave current changed I_m amperes in $\frac{1}{4f}$ second

Thus
$$E_{\text{ave}} = L \frac{I_m}{\frac{1}{4f}}$$

or
$$E_{\text{ave}} = 4fLI_m$$

But for a sine wave

$$E_{\text{ave}} = \frac{E_m}{1.57}$$

Thus
$$\begin{aligned} E_m &= 1.57 \times 4fLI_m \\ &= 6.28fLI_m \\ &= 2\pi fLI_m \end{aligned}$$

Since the effective values of both current and voltage equal 0.707 times the maximum values,

$$E = 2\pi fLI \tag{6-4}$$

In equation (1), the opposing voltage in the circuit E, equals RI, or **ohms** times **amperes.** This is also equal to the impressed voltage, necessary to send the current I through the circuit, and is equal to the resistance, or IR, drop.

Similarly, in equation (5), the counter emf of self induction, E_L, equals X_LI, or ohms times amperes. This is also equal to the impressed voltage necessary to send the current I through the coil, and is equal to the "**inductive reactance drop,**" or "IX_L drop," as it is called, in which

L = inductance in henrys
f = frequency in cycles per sec
E = effective drop across inductance in volts
I = effective current through inductance in amperes

Further the quantity ($2\pi fL$) is called the **inductive reactance** of the circuit and is measured in **ohms.** It is represented by the symbol X_L.

Thus we may write

$$E_L = X_L I \tag{7-4}$$

in which

E_L = the induced emf in volts
I = the current in amperes
X_L = the inductive reactance in ohms

Note the similarity of equations (1) and (7).

Since the current in this circuit lags 90° behind the impressed voltage, the current also lags 90° behind the inductive reactance drop, as shown by the vector diagram of Fig. 9–4. Thus **inductive reactance drop, or IX_L, is always 90° ahead** of the current.

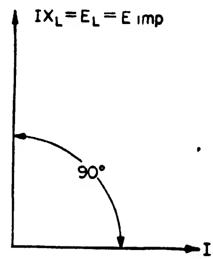


Fig. 9–4. Vector diagram of a circuit, having inductance only. Note that the impressed voltage, or IX_L drop, is 90° ahead of the current.

Note particularly that inductive reactance ($2\pi fL$) varies directly with the frequency, and if equations (4) and (5) are transposed

$$I = \frac{E}{2\pi fL} = \frac{E}{X_L} \tag{8-4}$$

it is apparent that in a circuit, having inductive reactance only, the current is **directly proportional** to the **impressed voltage;** and inversely proportional to both the **frequency** and the **inductance.**

Since the current lags 90° behind the impressed voltage, the average power dissipated in a circuit containing inductance only, is zero; or

$$P = EI \cos \theta = EI \cos 90° = 0$$

and

$$I^2X_L = 0$$

In this case, all the energy is stored in the magnetic field during one quarter cycle, and is returned to the circuit during the next quarter cycle.

Example 2. A coil having 0.03 henry inductance and negligible resistance, is connected across a 120-volt, 60-cycle circuit. (*a*) What is the inductive reactance? (*b*) What current flows in the circuit?

Solution: (a) $X_L = 2\pi fL = 2\pi 60 \times .03 =$ **11.31 ohms. Ans.**

(b) $I = \dfrac{E_L}{X_L} = \dfrac{120}{11.31} =$ **10.61 amperes. Ans.**

Prob. 1–4. If the coil in Example 2 were put across a 120-volt 25-cycle circuit, what would be the reactance and the current in the circuit?

Prob. 2–4. What would be the answers to (a) and (b) in Example 2, if both the impressed voltage and frequency were doubled?

Prob. 3–4. What is the inductance in a coil of negligible resistance, if 5.35 amperes flows when 115 volts at 60 cycles is impressed?

Prob. 4–4. A coil of negligible resistance has 0.15 henry inductance. When connected across a 125-volt a-c circuit, 5.32 amperes flow. What is the frequency of the circuit?

Prob. 5–4. When 240 volts at 60 cycles are impressed upon a coil of 0.182 henry inductance and negligible resistance, 3.5 amperes flow. What voltage at 25 cycles must be impressed to have the coil take the same current?

4–4. Circuit Containing Capacitance Only. It was shown in Vol. I, Chapter XVI, that when an insulator, or a dielectric, is placed between two metal plates, a condenser is formed. It was also shown that when a d-c voltage is applied across these plates, there is an initial rush of current, called the displacement current, which removes electrons from the positive plate and places additional electrons on the negative plate; and so charges the plates to line potential. After this, there is no further flow of current, if the impressed voltage remains constant.

If the impressed voltage is decreased, there is a flow of current in the opposite direction and the charge on the plates is decreased; or if the line is opened and the plates short circuited, this reversed current flows momentarily until the plates are discharged.

This property, which produces a current in the condenser when the voltage across it is changed, is called **capacitance** and is measured in farads. The farad is much too large a unit for practical work; so the capacitance of condensers is generally given in microfarads (one millionth of a farad), or in micro-microfarads (one million-millionth of a farad).

When a change of one volt per second across it produces **a current of one ampere,** the condenser is said to have a capacitance of **one farad.**

Thus, the current in a condenser depends upon the **rate of**

change of voltage across it; and in Vol. I has been expressed by the equation,

$$I_{\text{ave}} = C\frac{E_m}{t}, \tag{9-4}$$

where I_{ave} = the average current in amperes.
C = the capacitance in farads.
E_m = the maximum change in voltage in time t.
t = time in seconds.

When an a-c voltage is applied across the plates of the condenser, as in Fig. 10–4, an alternating current will flow; first charging the plates in one direction as the voltage rises; then discharging them as the voltage falls; and again charging them in the opposite direction, as the voltage rises in a negative direction, etc. Figure 11–4 shows the wave of voltage impressed on the condenser of Fig. 10–4. At the initial point zero value of the emf curve, the impressed voltage is changing in a positive direction at the greatest rate; therefore, the current I is at a maximum positive value. At the maximum point of the voltage curve, the rate of change of voltage is zero; therefore, the current is zero. At the next zero-value position, the

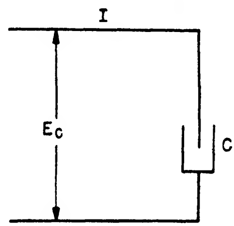


Fig. 10–4. Circuit, having capacitance only.

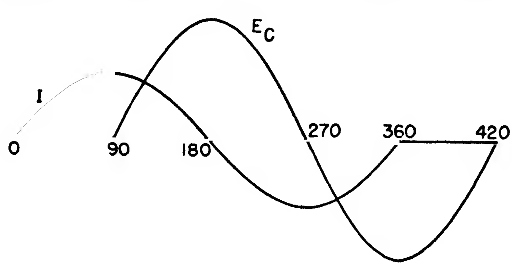


Fig. 11–4. The current in a circuit, having capacitance only, leads the impressed voltage by 90°.

voltage is changing at the greatest rate in a negative direction, and the current is now at a maximum negative value. Continuing in this manner, the current curve is obtained, and it is seen to be 90° ahead, or leading the voltage curve.

Thus, when an a-c voltage of sine wave form is applied to the plates of a perfect condenser, a **sine wave** of current will flow,

leading the applied voltage by 90°. It should be noted that no current of electrons actually flows **through** the condenser; the current is simply a "charging current," which charges the plates, first in one direction and then in the other.*

For an a-c circuit equation (9) can be expressed:

$$I_{\text{ave}} = C\frac{E_m}{t}, \tag{10-4}$$

where

C = capacitance in farads.
E_m = maximum value of the emf in volts.
t = time in seconds required to change the voltage from 0 to E_m in volts.

Since E_m changes from 0 to E_m, or the reverse, four times in one cycle, the average rate of voltage change equals $\dfrac{E_m}{\frac{1}{4f}}$ of $4fE_m$.

Thus $$I_{\text{ave}} = C\frac{E_m}{\frac{1}{4f}} = 4fCE_m.$$

Since
$$I_m = 1.57I_{\text{ave}}$$
$$I_m = 1.57(4fCE_m)$$
$$I_m = 6.28fCE_m = 2\pi fCE_m$$

Multiplying both sides of the equation by 0.707

$$I = 2\pi fCE_c, \tag{11-4}$$

where

E_c and I = effective values respectively of impressed voltage and current

5-4. Capacitive Reactance. If we transpose Eq. 11, we get

$$E_c = \frac{I}{2\pi fC} \quad \text{or} \quad \frac{1}{2\pi fC}I. \tag{12-4}$$

* In Chap. XVI, Vol. 1, we have seen that a charge on the plates of a condenser sets up electrostatic lines of force just as a current in a coil of wire sets up electromagnetic lines of force. We can think of the change in these electrostatic lines as constituting a current through the condenser, just as we think of the change in magnetic lines linking a circuit as constituting a voltage.

The expression $\frac{1}{2\pi fC}$ is called "**capacitive reactance**" and is measured in ohms (where C is expressed in farads). It is universally indicated by the letter X_c.

Thus
$$X_c = \frac{1}{2\pi fC}. \tag{13–4}$$
and we write
$$E_c = X_c I. \tag{14–4}$$

Again note the similarity of equations (1), (7) and (14). Impressed voltage is equal to ohms capacitive reactance times amperes. This is called the capacitive reactance, or IX_c, drop.

The vector diagram for the circuit of Fig. 10–4 is shown in Fig. 12–4. Note that the impressed voltage, or the IX_c drop, lags **90° behind** the current. From the equations above, it is seen that the capacitive reactance varies **inversely** with frequency and capacitance, which is just the reverse of the relations in an inductive circuit. Thus, with a given voltage applied, the current in any condenser will increase as the frequency of the circuit is increased. This is because the higher the frequency, the greater is the rate of change of the impressed voltage.

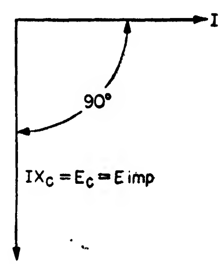


FIG. 12–4. Vector diagram of a circuit, having capacitance only. Note that the impressed voltage, or IX_c drop, is 90° behind the current.

Condensers, therefore, are often used to suppress low frequency currents, and allow the higher frequency currents to flow; while induction, or choke coils suppress the higher frequency currents, and allow the lower frequency currents to pass.

The average power in a circuit of pure capacitance only is zero, since the current is at 90° with the impressed voltage and

$$P = EI \cos 90° = 0.$$

The pure condenser represents another case of reactive power, where all the power delivered to the circuit, during one quarter cycle, is returned to it during the next quarter cycle, as discussed in Chapter IV, Art. 6.

Example 3. A 120-volt, 60-cycle emf is applied across the terminals of a 50 microfarad (.000050 farad) condenser. (*a*) What is the capacitive reactance of the condenser. (*b*) What current will flow?

Solution: (a) $X_c = \dfrac{1}{2\pi fc} = \dfrac{1}{2\pi 60 \times .00005} = \dfrac{1}{0.01884} = $ **53 ohms. Ans.**

(b) $I = \dfrac{E_c}{X_c} = \dfrac{E_c}{\dfrac{1}{2\pi fc}} = 120 \times 0.01884 = 2.26$ amp. *Ans.*

Prob. 6–4. If the frequency of the circuit in Example 3 were 30 cycles, what would be the capacitive reactance and the current in the circuit?

Prob. 7–4. (a) What current will flow in the condenser of Example 3, if the frequency is doubled? (b) What is the capacitive reactance? (c) What relation is there between frequency and current taken by the condenser?

Prob. 8–4. A condenser of 10 microfarads takes a current of 1.85 amperes from 125-volt mains. What is the frequency of the circuit?

Prob. 9–4. (a) What will be the reactance of a 44.3 micro-farad condenser at 60 cycles? (b) At 30 cycles? (c) At 120 cycles? (d) At 5000 cycles? (e) At 0 cycles?

Prob. 10–4. What current will flow in the condenser of Prob. 8–4, when put across a 240-volt, 60-cycle line? What current will flow: (a) If both voltage and frequency are doubled? (b) If the frequency is doubled, and the voltage is reduced one half? (c) If the voltage remains the same, and the frequency is reduced one half?

6–4. Resistance and Inductive Reactance in Series; Impedance. Assume in Fig. 13–4 that an a-c voltage is impressed across a series connection of a resistor R and an air-core coil X_L, containing inductance only. The voltage across this circuit must be the vector sum of the voltage E_R, across the resistor, and E_L, across the coil. Figure 14–4(a) shows the voltage diagram.

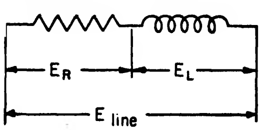


FIG. 13–4. Series circuit, having resistance and inductance.

In Series Circuits, the current is the same in all parts; therefore, in vector diagrams of such circuits, **the current is taken as the reference vector;** and the voltage vectors are drawn with reference to their phase position with the current vector, which may be drawn in any direction.

Accordingly, in Fig. 14–4(a), the current vector is drawn horizontally to the right. The voltage E_R, equal to the IR drop in the resistor, is drawn in phase with the current (see Art. 1). The

voltage E_L, equal to the IX_L drop in the coil, is drawn, **leading** the current by 90° (Art. 2). Figure 14–4(*b*) is the topographic diagram. Since these two voltages are 90° apart, the line voltage is equal to the square root of the sum of their squares, or

$$\begin{aligned} E_{\text{line}} &= \sqrt{E_R^2 + E_L^2} = \sqrt{IR^2 + IX_L^2} \\ &= \sqrt{I^2(R^2 + X^2)} = I\sqrt{R^2 + X^2} \\ &= I\sqrt{R^2 + (2\pi fL)^2} = IZ. \end{aligned} \tag{15–4}$$

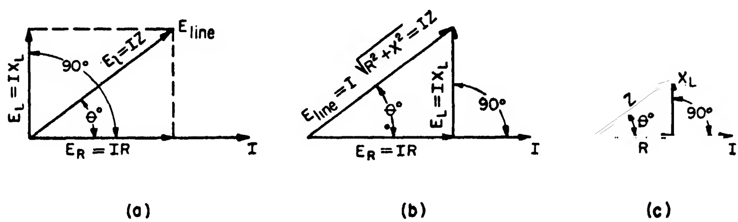


Fig. 14–4. (*a*) Polar vector diagram of voltages in the circuit of Fig. 13–4. Note that impressed voltage, or IZ drop, is the vector sum of the IR drop, in phase with the current, and the IX_L drop, leading the current by 90°. (*b*) Topographic diagram equivalent to that in (*a*). (*c*) Impedance diagram for the circuit in (*a*) and (*b*). Note that Z equals the vector sum of R in phase with the current, and X_L leading the current by 90°.

The expression $\sqrt{R^2 + X^2}$ or $\sqrt{R^2 + (2\pi fL)^2}$ is called the "**impedance**" of the circuit, and is measured in ohms. It is generally indicated by the letter Z.

Transposing equation (13)

$$I = \frac{E_{\text{line}}}{\sqrt{R^2 + X^2}} = \frac{E}{\sqrt{R^2 + (2\pi fL)^2}} = \frac{E}{Z}. \tag{16–4}$$

Equation (16) is similar to the expression of Ohm's law, and the current thus varies directly with the voltage, and inversely with the impedance. The voltage across such a circuit is called the "inductive impedance drop," or the IZ_L drop.

Since Z is numerically equal to the vector sum of R and X_L at 90° or $R \oplus X_L = Z_L$, R, X and Z are all treated as vector quantities and an "impedance diagram" can be constructed, as in Fig. 14–4(*c*). Note that R is drawn in phase with the IR drop, or in phase with the current; X_L, in phase with the IX_L drop, or **leading** the current by 90°; and Z, in phase with the IZ drop, or in phase with the line voltage.

Because of the presence of resistance in the circuit, the diagrams of Fig. 14–4 show that the current lags behind the voltage by an angle θ which is less than 90°, and

$$\text{Tangent } \theta = \frac{IX_L}{IR} = \frac{X_L}{R}.$$

$$\text{Sine } \theta = \frac{IX_L}{IZ_L} = \frac{X_L}{Z_L} \quad \text{or} \quad X_L = Z \sin \theta. \qquad (17\text{–}4)$$

$$\text{Cosine } \theta = \frac{IR}{IZ_L} = \frac{R}{Z_L}, \quad \text{or} \quad R = Z \cos \theta. \qquad (18\text{–}4)$$

Power is dissipated from this circuit, since it contains resistance and

$$P = EI \cos \theta = I^2R$$

Example 4. The circuit of Fig. 13–4 consists of a resistor of 10 ohms and a coil of 0.02 henry inductance and negligible resistance. When 120 volts at 60 cycles is impressed: (*a*) What is the impedance of the circuit? (*b*) The current? (*c*) What is the voltage across the resistor and the voltage across the coil? (*d*) What is the power factor, and by what angle does the current lag behind the impressed voltage? (*e*) What power is dissipated in the circuit?

Solution: $X_L = 2\pi \times 60 \times 0.02 = 7.54$ ohms.

(*a*) $Z_L = \sqrt{R^2 + X^2} = \sqrt{10^2 + 7.54^2} =$ **12.53 ohms. Ans.**

(*b*) $I = \dfrac{E}{Z} = \dfrac{120}{12.53} =$ **9.58 amperes. Ans.**

(*c*) $E_R = IR = 9.58 \times 10 = 95.8$ volts.
$E_L = IX_L = 9.58 \times 7.54 =$ **72.2 volts. Ans.**

$$\sqrt{95.8^2 + 72.2^2} = 120 \text{ volts (check).}$$

(*d*) P.F. $= \cos \theta = \dfrac{R}{Z} = \dfrac{10}{12.53} = 0.798$, so angle $= 37°$.

(*e*) $P = EI \cos \theta = 120 \times 9.58 \times 0.798 = 917$ watts.

$P = I^2R = 9.58^2 \times 10 = 917$ watts (check).

In the example above, the coil is assumed to have inductance only. An actual coil has resistance, as well as inductance, and therefore has impedance. Note Example 5 below.

Example 5. When 100 volts at 25 cycles is impressed on the coil of Fig. 15–4(*a*), the current, as measured by an ammeter, is 5 amperes. The resistance, as measured with direct current, is 4 ohms. (*a*) What

is the impedance? (*b*) The reactance? (*c*) The inductance? (*d*) The power factor? (*e*) And the power consumed by the coil?

Solution: (*a*) $Z = \dfrac{E}{I} = \dfrac{100}{5} = \mathbf{20\ ohms.}$ **Ans.**

(*b*) $Z = \sqrt{R^2 + X^2}$, or $X = \sqrt{Z^2 - R^2} = \sqrt{20^2 - 4^2}$
$= \mathbf{19.6\ ohms.}$ **Ans.**

(*c*) $X_L = 2\pi fL$ or $L = \dfrac{X_L}{2\pi f} = \dfrac{19.6}{2\pi \times 25} = \dfrac{19.6}{157}$

$= \mathbf{0.125\ henry.}$ **Ans.**

(*d*) $\text{PF} = \cos\theta = \dfrac{R}{Z} = \dfrac{4}{20} = 0.2.$ **Current lags 78.5°.**

Ans.

(*e*) $P = EI\cos\theta = 100 \times 5 \times 0.2 = \mathbf{100\ watts.}$ **Ans.**

or $P = I^2R = 5^2 \times 4 = 100$ watts (check).

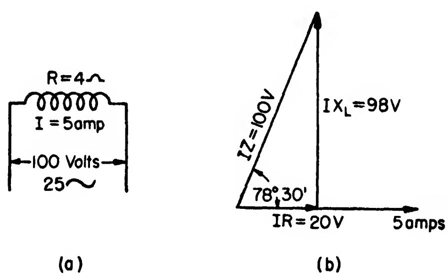


Fig. 15–4. (*a*) Coil, having resistance and inductance. (*b*) Vector diagram of voltages in (*a*).

Note that there are two voltages which must be overcome by the impressed voltage before current can flow in the coil; the IR drop, equal to 5×4 or 20 volts; and the IX_L drop, equal to 5×19.6 or 98 volts. The vector diagram is shown in Fig. 15–4(*b*).

Show a vector diagram in the solution of the following problems.

Prob. 11–4. Answer the questions in Example 4, if the frequency of the circuit were 30 cycles per second; all other data remaining the same.

Prob. 12–4. How much current will flow through a coil having 0.08 henry inductance and 20 ohms resistance, when put across a 230-volt 60-cycle circuit?

Prob. 13–4. (*a*) What is the phase relation between the voltage and current in the coil of Prob. 12–4? (*b*) What power is consumed in the circuit?

Prob. 14-4. A coil of 0.015 henry inductance and 5 ohms resistance is connected across 115-volt, 40-cycle mains. (*a*) What current flows in the coil? (*b*) What is the power factor and angle by which the voltage leads the current? (*c*) What power is consumed in the coil?

Prob. 15-4. (*a*) What current would flow in the coil of Example 5, if 120 volts at 50 cycles were put across it? (*b*) What would be the power factor and the power consumed by the coil?

Prob. 16-4. (*a*) What d-c voltage would be required to send 2 amperes through an air-cored coil of 15 ohms resistance and 0.1 henry inductance? (*b*) What a-c voltage would be required to send the same current through the coil, if the frequency were 25 cycles? 60 cycles? 120 cycles? 1000 cycles? What would be the power factor in each case?

Prob. 17-4. An air-cored coil is placed across a 60-cycle line. An ammeter, voltmeter and wattmeter connected in the circuit, read respectively, 4 amperes, 120 volts and 240 watts. What are the inductance, resistance and reactance of the coil?

Prob. 18-4. When 125 volts are impressed upon a circuit of 18 ohms impedance, the power factor is 90 per cent. (*a*) What is the resistance and the reactance of the circuit? (*b*) What power is consumed in the circuit?

Prob. 19-4. When a resistor and a coil are connected in series across a 240-volt line, 6 amperes flows in the circuit. Voltmeters, connected across the resistor and across the coil, indicate respectively 120 and 156 volts. (*a*) What power is consumed by the circuit? By the resistor? By the coil? (*b*) What is the resistance and reactance of the coil?

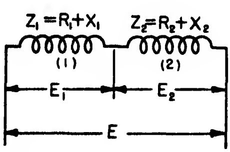


FIG. 16-4. Circuit, having two coils, or two impedances in series.

Prob. 20-4. If the inductance of the coil in Prob. 19-4 is 0.114 henry, what is the frequency of the circuit?

7-4. Impedances in Series. The resistance of a series circuit is equal to the arithmetical sum of the resistances of its parts. Similarly, the inductive reactance of a series circuit equals the arithmetical sum of the inductive reactances of its parts. But the total impedance of a series circuit is equal to the **vector** sum of the impedances of its parts.

Consider the series connection of the two coils of Fig. 16-4. Each has resistance and inductance, but the coils have different power factors. Let the resistance, reactance and impedance of coil (1) be represented as R_1, X_1, and Z_1 respectively, and of coil (2)

as R_2, X_2 and Z_2. Since the coils are in series, the same current I flows through each, and both R_1 and R_2 are in phase with it and with each other, as shown in Fig. 17–4(a). Also X_1 and X_2 are both 90° ahead of the current and in phase with each other. The impedance Z_1 of coil (1) is the vector sum of R_1 and X_1, and leads the current by θ_1°, as shown in the figure, while Z_2 of coil (2), the vector sum of R_2 and X_2, leads the current of θ_2°. Therefore, Z_1 and Z_2 differ in phase by an angle of $(\theta_1 - \theta_2)$, or θ degrees. The total impedance Z_t, is, therefore, the vector sum of the two impedances, or

$$Z_t = \sqrt{Z_1^2 + Z_2^2 + 2Z_1Z_2 \cos \theta^\circ}. \tag{19–4}$$

Since the resistances R_1 and R_2 are in phase with each other, as are the inductive reactances X_1 and X_2, the total impedance may

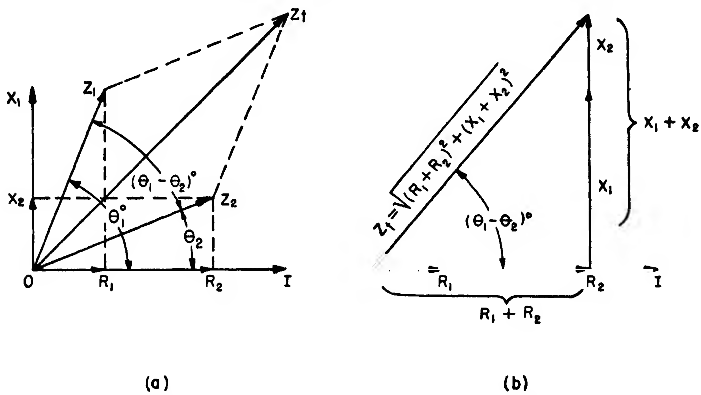


FIG. 17–4. (a) Polar vector diagram of the circuit in Fig. 16–4. The total impedance of the series combination, Z_t, is the vector sum of Z_1 and Z_2. (b) Topographic diagram equivalent to that in (a). Note that the total impedance Z_t is that of a series combination of R_1, R_2, X_1, and X_2, which are themselves merely component parts of the impedances Z_1 and Z_2 in (a).

also be computed by means of the vector diagram of Fig. 14–4(b), as

$$Z_t = \sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2} = \sqrt{R_t^2 + X_t^2} \tag{20–4}$$

or, from the relations shown in equations (15) and (16), Art. 6–4,

$$Z_t = \sqrt{(Z_1 \cos \theta_1 + Z_2 \cos \theta_2)^2 + (Z_1 \sin \theta_1 + Z_2 \sin \theta_2)^2} \tag{21–4}$$

Thus, the total impedance of a series circuit is the vector sum of the impedances of its parts, or

$$Z_t = Z_1 \oplus Z_2 \oplus Z_3 \oplus \cdots \tag{22–4}$$

Example 6. A line voltage of 120 volts at 60 cycles is impressed on the coils of Fig. 16–4. In coil (1) $R_1 = 2$ ohms, and $X_1 = 6$ ohms. In coil (2) $R_2 = 9$ ohms, and $X_2 = 3$ ohms. (*a*) What is the impedance of the circuit? (*b*) The current in the circuit? (*c*) The power factor of the circuit and lag angle of the current? (*d*) The voltage across each coil? (*e*) The power consumed in the circuit?

Solution:

(*a*) $$Z_t = \sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2} = \sqrt{(2 + 9)^2 + (6 + 3)^2}$$

$$= \sqrt{11^2 + 9^2} = \mathbf{14.21\ ohms.}\quad \mathbf{Ans.}$$

(*b*) $$I = \frac{E}{Z_t} = \frac{120}{14.21} = \mathbf{8.44\ amperes.}\quad \mathbf{Ans.}$$

(*c*) $$\text{P.F.} = \frac{R_t}{Z_t} = \frac{11}{14.21} = \mathbf{0.774,\ and\ angle\ of\ lag} = \mathbf{38.3^\circ.}\quad \mathbf{Ans.}$$

(*d*) $$Z_1 = \sqrt{R_1^2 + X_1^2} = \sqrt{2^2 + 6^2} = 6.32 \text{ ohms in coil (1).}$$

$$E_1 = IZ_1 = 8.44 \times 6.32 = \mathbf{53.3\ volts\ across\ coil\ (1).}\quad \mathbf{Ans.}$$

$$Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{9^2 + 3^2} = 9.48 \text{ ohms in coil (2).}$$

$$E_2 = IZ_2 = 8.44 \times 9.48 = \mathbf{80.1\ volts\ across\ coil\ (2).}\quad \mathbf{Ans.}$$

(*e*) $$P = EI\cos\theta = 120 \times 8.44 \times 0.774 = \mathbf{784\ watts.}\quad \mathbf{Ans.}$$

$$= I^2R = 8.44^2 \times 11 = 784 \text{ watts (check).}$$

In the solution of each of the following problems, show both the circuit and carefully drawn vector diagrams.

Prob. 21–4. A coil of 5 ohms resistance and 12 ohms reactance is joined in series with another of 8 ohms resistance and 4 ohms reactance on a 115-volt, 60-cycle line. (*a*) What is the total impedance and power factor of the circuit? (b) What current will flow?

Prob. 22–4. What would be the answer in Prob. 21–4, if the frequency of the circuit were 25 cycles? Applied voltage is 115 volts.

Prob. 23–4. Two choke coils are joined in series across a 25-cycle line.

Coil A has 12 ohms impedance and 0.03 henry inductance.
Coil B has 13 ohms impedance and 0.06 henry inductance.

(*a*) What is the total impedance of the circuit? (*b*) What is the power factor of each coil and of the circuit?

Prob. 24–4. (*a*) What voltage must be applied across the circuit in Prob. 23–4, in order that 3 amperes may flow? (*b*) What will be the voltage across each coil?

Prob. 25–4. Three impedances of 20, 16 and 10 ohms, connected in series, have power factors of 90, 65 and 40 per cent respectively, at 50

cycles. (*a*) What is the total impedance, reactance and resistance of the circuit? (*b*) What is the power factor of the circuit?

Prob. 26–4. A series circuit consists of three parts as follows:

Part *A* has 20 ohms resistance and 0.03 henry inductance.
Part *B* has 15 ohms resistance and 0.04 henry inductance.
Part *C* has 10 ohms resistance and 0.05 henry inductance.

At 60 cycles:

(*a*) What is the total impedance of the circuit? (*b*) What current will 130 volts force through the circuit? (*c*) What will be the voltage across each part? (*d*) What is the power factor of the circuit?

Prob. 27–4. Answer the questions in Prob. 26–4, if the frequency is doubled.

8–4. Resistance and Capacitance in Series. Figure 18–4 shows a series connection of a resistor *R*, and a condenser, or capacitive reactance X_c. As before, the current is the same through all parts, and the vector sum of the voltages across the parts is equal to the line voltage. Thus, in Fig. 19–4(*a*), the current *I* is laid off horizontally to

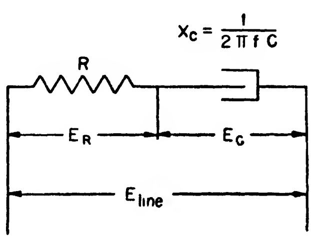


FIG. 18–4. Circuit, having resistance and capacitance in series.

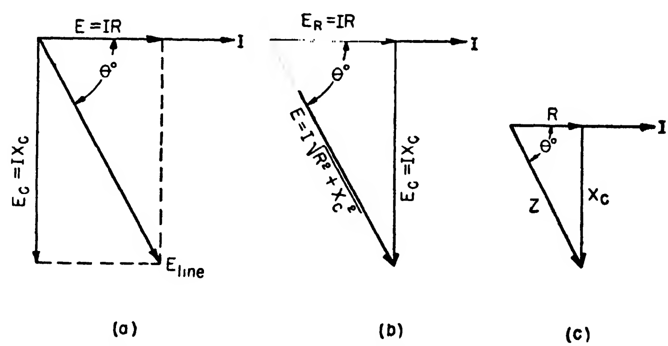


FIG. 19–4. (*a*) Polar vector diagram of voltages in the circuit of Fig. 18–4. Note that the impressed voltage, or *IZ* drop, is the vector sum of the *IR* drop, in phase with the current, and the IX_c drop, lagging the current by 90°. (*b*) Topographic diagram equivalent to (*a*). (*c*) Impedance diagram for this circuit. Note that X_c lags the current by 90°.

the right, as the reference vector, and the voltage across the resistor, or *IR* drop, in phase with the current. The IX_c drop, or voltage across the condenser, is laid off **lagging** the current by

90°, as already explained. Figure 19–4(b) shows the topographic diagram. The impressed voltage is equal to the vector sum of IR and IX at 90°; therefore, it is equal to the square root of the sum of their squares, or

$$E = \sqrt{IR^2 + IX_c^2} = I\sqrt{R^2 + X_c^2} = I\sqrt{R^2 + \left(\frac{1}{2\pi fc}\right)^2} = IZ. \qquad (23\text{–}4)$$

Transposing

$$I = \frac{E}{\sqrt{R^2 + X^2}} = \frac{E}{\sqrt{R^2 + \left(\frac{1}{2\pi fc}\right)^2}} \quad \text{or} \quad \frac{E}{Z} \qquad (24\text{–}4)$$

The expression $\sqrt{R^2 + \left(\frac{1}{2\pi fc}\right)^2}$ is the **impedance** of the circuit (where C is capacitance in farads).

The impedance diagram, showing the relation of R, X_c and Z is shown in Fig. 19–4(c).

Note that due to the presence of resistance in the circuit, the voltage lags the current by the angle θ (Fig. 19–4), which is less than 90°, and

$$\text{Tan}\,\theta = \frac{IX_c}{IR} = \frac{X_c}{R}$$

$$\text{Sin}\,\theta = \frac{IX_c}{IZ} = \frac{X_c}{Z}, \quad \text{or} \quad X_c = Z\sin\theta.$$

$$\text{Cos}\,\theta = \text{power factor} = \frac{R}{Z}, \quad \text{or} \quad R = Z\cos\theta.$$

The power consumed in the circuit equals,

$$P = EI\cos\theta = I^2R.$$

Thus, in a circuit, containing resistance and capacitance, the current leads the voltage, but by an angle less than 90°.

Example 7. The circuit of Fig. 18–4 consists of a resistor of 50 ohms and a condenser of 40 microfarads (.000040 farad) capacitance only. When 120 volts at 60 cycles are impressed: (a) What is the impedance of the circuit? (b) The current? (c) The voltage across each part? (d) The power factor and angle of current lead? (e) Power consumed by the circuit?

Solution:

$$X_c = \frac{1}{2\pi fc} = \frac{1}{2\pi 60 \times .00004} = 66.3 \text{ ohms.}$$

(*a*) $Z = \sqrt{R^2 + X^2} = \sqrt{50^2 + 66.3^2} =$ **83.1 ohms. Ans.**

(*b*) $I = \dfrac{E}{Z} = \dfrac{120}{83.1} =$ **1.445 amperes. Ans.**

(*c*) $E_R = IR = 1.45 \times 50 =$ **72.5 volts on resistor. Ans.**

$EX_c = IX_c = 1.45 \times 66.3 =$ **95.8 volts on condenser. Ans.**

$\sqrt{72.5^2 + 95.8^2} = 120$ volts on circuit (check).

(*d*) P.F. $= \cos\theta = \dfrac{R}{Z} = \dfrac{50}{83.1} = 0.6$; so that $\theta = 53.17°$. *Ans.*

(*e*) $P = EI\cos\theta = 120 \times 1.445 \times 0.6 =$ **104 watts. Ans.**

$= I^2R = 1.445^2 \times 50 = 104.4$ watts (check).

Show vector diagrams in the solution of each of the following problems.

Prob. 28–4. Solve the problem of Example 7, if the frequency were 25 cycles, all other data remaining the same.

Prob. 29–4. Solve the problem of Example 7, assuming the frequency to be 100 cycles per sec.

Prob. 30–4. A condenser of 100 microfarads and a 20 ohm resistor are connected in series. (*a*) What voltage at 60 cycles must be impressed on the circuit in order that 2.5 amperes will flow. (*b*) What will be the power factor?

Prob. 31–4. Answer the questions in Prob. 30–4, if the frequency of the supply voltage were 25 cycles.

Prob. 32–4. When 240 volts at 60 cycles is impressed on a series combination of a resistor and condenser, the current is 1.5 amperes. If the voltage across the resistor is 75 volts, what is the capacitance of the condenser?

9–4. Resistance, Inductive Reactance and Capacitive Reactance in Series. When an a-c voltage is impressed upon a circuit containing resistance, pure inductance and pure capacitance in series, as in Fig. 20–4, it is obvious that the current is the same throughout the circuit; and the line voltage is equal to the vector sum of the voltages across the parts, as before. Accordingly, in Fig. 21–4(*a*), the vector, *IR*, representing the voltage across the resistor, is drawn horizontally to the right, in phase with the current *I*; the voltage across the coil IX_L, is drawn **leading** the

current by 90°; and the voltage, across the condenser IX_c, is drawn **lagging** the current by 90°.

Thus the line voltage is equal to the vector sum of **three** voltages, of which two are at 180° and oppose each other. The relations are

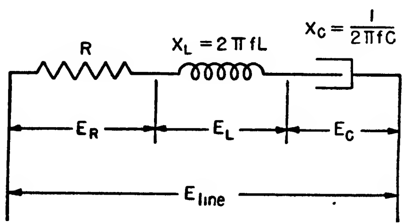


FIG. 20–4. Circuit, having resistance, inductance, and capacitance in series.

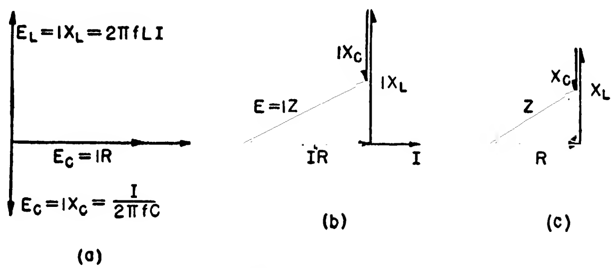


FIG. 21–4. (*a*) Polar vector diagram of voltages in the circuit of Fig. 20–4. Note that the IX_L and IX_c drops are 180° from each other. (*b*) Topographic diagram equivalent to that in (*a*). (*c*) Impedance diagram. Note also that X_L and X_c oppose each other in the circuit.

clearly shown in the topographic diagram of Fig. 21–4(*b*), and the impedance diagram of Fig. 21–4(*c*) shows that inductive and capacitive reactance also oppose, or tend to neutralize each other.

The above relations may be expressed as

$$E = \sqrt{IR^2 + (IX_L - IX_c)^2} = \sqrt{IR^2 + I(X_L - X_c)^2}$$

$$= I\sqrt{R^2 + (X_L - X_c)^2} \quad \text{or} \quad I\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fc}\right)^2} = IZ, \tag{25-4}$$

or transposing,

$$I = \frac{E}{\sqrt{R^2 + (X_L - X_c)^2}} = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fc}\right)^2}} = \frac{E}{Z}. \tag{26-4}$$

The expression $\sqrt{R^2 + (X_L - X_c)^2}$ or $\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fc}\right)^2}$ is the impedance of the circuit in ohms.

The reactance of the circuit X_t equals $(X_L - X_c)$. If X_L is greater than X_c, the resultant reactance is inductive and the current in the circuit lags the voltage. If X_c is the larger, the current leads, because the resultant reactance is capacitive.

Example 8. A coil of 25 ohms resistance and 0.1 henry inductance are joined in series with a 20 microfarad condenser across a 125-volt

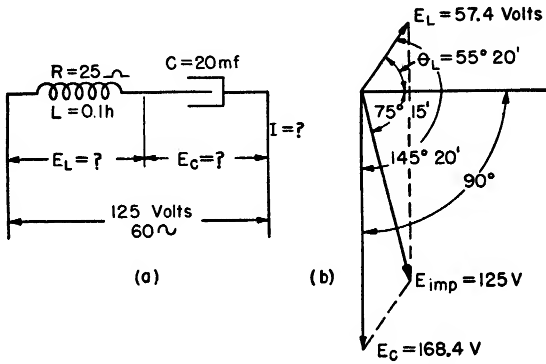


FIG. 22–4. (*a*) Series connection of a coil and a condenser. (*b*) Vector diagram of voltages for the circuit in (*a*).

60-cycle circuit, as shown in Fig. 22–4(*a*). (*a*) What is the total reactance of the circuit? (*b*) The total impedance? (*c*) The current? (*d*) The power factor and phase angle between line voltage and current? (*e*) The voltage across the coil and across the condenser?

Solution: $X_L = 2\pi 60 \times 0.1 = 37.7$ ohms.

$$X_c = \frac{1}{2\pi 60 \times .00002} = 132.6 \text{ ohms.}$$

(*a*) $X_t = (X_c - X_L) = 132.6 - 37.7 =$ **94.7 ohms. Ans.**

(*b*) $Z_t = \sqrt{R^2 + X_t^2} = \sqrt{25^2 + 94.7^2} =$ **98.1 ohms. Ans.**

(*c*) $I = \frac{E}{Z_t} = \frac{125}{98.1} =$ **1.27 amperes. Ans.**

(*d*) P.F. $= \cos\theta = \frac{R}{Z} = \frac{25}{98.1} = 0.255$; so that $\theta = 75.25°$

Since X_c is larger than X_L, the current **leads** the voltage.

(*e*) Z of coil $= Z_L = \sqrt{R^2 + X_L^2} = \sqrt{25^2 + 37.7^2} = 45.2$ ohms.

Voltage across coil $= IZ_L = 1.27 \times 45.2 =$ **57.4 volts. Ans.**

Z of condenser $= X_c$

Voltage across condenser $= IX_c = 1.27 \times 132.6 =$ **168.4 volts. Ans.**

The voltage diagram for this circuit is shown in Fig. 22-4(b).

Note, in the example above that the voltage across the condenser is greater than the line voltage. There is nothing disturbing in this, for we have already seen that the vector sum of two or more voltages may be less than any one of them, and the vector diagram makes this clear. The power factor of the coil is $\frac{R}{Z_L}$, or $\frac{25}{45.2}$, equal to 0.553, the cosine of 56.4°. Thus, the voltage across the coil **leads** the current by 56.4°, while that across the condenser **lags behind** the current by 90°. The two voltages are, therefore, 90° + 56.4° or 146.4° out of phase, as shown in the diagram; their vector sum being equal to the line voltage, or 125 volts.

Show carefully drawn vector diagrams in the solution of each of the following problems.

Prob. 33-4. Solve the problem of Example 8 above, if the frequency were 25 cycles, all other data remaining the same.

Prob. 34-4. Solve the problem of Example 8, if the frequency were 200 cycles, all other data remaining the same.

Prob. 35-4. (a) What voltage at 60 cycles would be necessary to force 1.5 amperes through a series combination of a 50 mf condenser and a coil of 0.25 henry inductance and 15 ohms resistance? (b) What will be the voltage across the coil? (c) Across the condenser? (d) Does the current lead or lag the voltage and by what angle?

10-4. Resonance in Series Circuits. When the frequency is such that the inductive and capacitive reactance in a series circuit are equal, $X_L - X_c$ equals zero, and the resistance offers the only opposition to the flow of current; that is, the impedance is equal to the resistance. With a given impressed voltage, the effective current in the circuit reaches a maximum value and the circuit is said to be in "**resonance.**" If the frequency is either increased or decreased from the resonant value, the current decreases.

Thus in the equation,

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fc}\right)^2}} = \frac{E}{Z}$$

When

$$2\pi fL - \frac{1}{2\pi fc} = 0,$$

the above equation becomes

$$I = \frac{E}{\sqrt{R^2 + 0^2}} = \frac{E}{R}.$$

The frequency at which a circuit will be in resonance, or " resonant frequency," is determined as follows:

$$2\pi fL = \frac{1}{2\pi fc}.$$

$$(2\pi)^2 f^2 LC = 1.$$

$$f^2 = \frac{1}{(2\pi)^2 LC}.$$

$$f = \frac{1}{2\pi\sqrt{LC}}. \qquad (27\text{-}4)$$

Example 9. What is the " resonant frequency " for the circuit of Example 8? $L = 0.1$ henry; $C = 20$ mf.

Solution:

$$f = \frac{1}{2\pi\sqrt{0.1 \times .000020}} = \frac{1}{2\pi \times .001414} = 112.6 \text{ cycles.}$$

When a circuit is in resonance, the power factor is one.

At resonance, the voltage across the inductive and capacitive parts of the circuit reach **extreme values, which may be dangerous** even with moderate impressed voltage. The less the resistance in the circuit, with respect to the inductive and capacitive reactances, the higher will be the voltage across the coil and across the condenser.

Example 10. (*a*) What current will flow in the circuit of Example 8 at the resonant frequency determined in Example 9? (*b*) What will be the voltage across the coil and across the condenser at this frequency? Impressed voltage = 125 volts.

Solution:

$$X_L = 2\pi fL = 6.28 \times 112.6 \times 0.1 = 70.7 \text{ ohms.}$$

$$X_c = \frac{1}{2\pi fc} = \frac{1}{6.28 \times 112.6 \times .00002} = 70.7 \text{ ohms.}$$

$$\text{Total } X \text{ of the circuit} = X_L - X_c = 70.7 - 70.7 = 0.$$

$$(a)\ I = \frac{E}{Z} = \frac{E}{R} = \frac{125}{25} = \mathbf{5 \text{ amperes.}} \quad \textbf{Ans.}$$

$$(b)\ Z(\text{coil}) = Z_L = \sqrt{R^2 + X^2} = \sqrt{25^2 + 70.7^2} = 75 \text{ ohms.}$$

$$E(\text{coil}) = E_L = IZ_L = 5 \times 75 = \mathbf{375 \text{ volts.}} \quad \textbf{Ans.}$$

$$E_{\text{condenser}} = E_c = IX_c = 5 \times 70.7 = \mathbf{353.5 \text{ volts.}} \quad \textbf{Ans.}$$

In the example above, note the increase in voltage across both parts of the circuit as compared to the values in Example 8. Also note that the current has increased from 1.27 amperes to 5 amperes, although the impressed voltage is unchanged.

If the frequency is either decreased or increased from the resonant value, the current decreases. If the frequency is decreased, $\frac{1}{2\pi fC}$ increases and $(X_c - X_L)$ is no longer zero, thereby increasing the impedance. If the frequency is increased, $2\pi fL$ increases and $(X_L - X_c)$ is again increased, increasing the impedance.

Figure 23–4 shows the general shape of the current curve as the frequency on such a circuit is changed. When the frequency is zero (corresponding to a direct current), the capacitive reactance is infinitely large and no current will flow, so the curve starts at zero. When the frequency is infinitely high, the inductive reactance is infinite in value, so the curve, theoretically, reaches zero again.

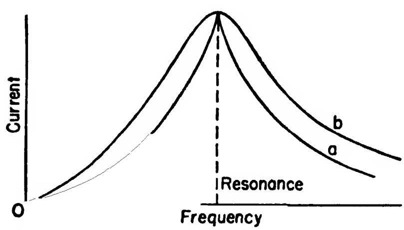


FIG. 23–4. Resonant curves for a series circuit. (*a*) A "sharply" tuned wave. (*b*) A "broadly" tuned wave.

The steeper curve (*a*) of Fig. 23–4 is called a "sharply tuned" wave, and curve (*b*) a "broadly tuned" wave. In equation (25), Art. 9–4, it is apparent that the term LC has a definite value for a given resonant frequency and may be made up of any number of combinations of L and C. The greater the value of L and the smaller the value of C, the greater is the change in the value of the term $(X_L - X_c)$; for a given change in frequency, therefore, the more sharply tuned is the current wave.

The above facts are applied in radio circuits. For instance, the aerial circuit may be "tuned" by adjusting the capacitance in a condenser, called a variometer (see Vol. I, Chapter XVI), so that the circuit is in resonance at the frequency of some particular broadcasting station. The sharper the wave, the easier it is to "tune out" interfering stations.

Of course, the lower we make the series resistance, the greater will be the current at resonance and the sharper the tuning.

Show vector diagrams in the solution of each of the following problems.

Prob. 36–4. Answer the questions in Example 8, if the resistance of the coil were 5 ohms and the capacitance of the condenser were 60 mf, all other data remaining the same.

Prob. 37–4. (*a*) What is the resonant frequency of the circuit in Prob. 36–4? (*b*) What current will flow and what will be the voltage across the coil and across the condenser?

Prob. 38–4. A coil of 0.2 henry inductance and 6 ohms resistance is connected in series with a 35.2 mf condenser across a 240-volt circuit. (*a*) Compute the current in the circuit and voltages across the parts for frequencies of 40, 50, 55, 59, 60, 61, 63, 70 and 80 cycles per second. (*b*) Plot the current wave as in Fig. 23–4.

11–4. Impedance in Parallel Circuits. It was shown in Vol. I that the total current in a parallel d-c circuit is the arithmetical sum of the currents in the branches, or

$$I_t = I_1 + I_2 + I_3 + \cdots$$

The total resistance of a parallel circuit has also been expressed as

$$R_t = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots},$$

where $\dfrac{1}{R}$ is the conductance per path, or the current per path for 1 volt impressed. **The resistance can also be computed, if any other voltage E is impressed or assumed upon the circuit,** and the equation written as

$$R_t = \frac{E}{\dfrac{E}{R_1} + \dfrac{E}{R_2} + \dfrac{E}{R_3} + \cdots} = \frac{E}{I_1 + I_2 + I_3 + \cdots} = \frac{E}{I_t}$$

It has also been shown in Chapter II that the total current in a parallel a-c circuit is equal to the vector sum of the currents in the several branches, or

$$I_t = I_1 \oplus I_2 \oplus I_3 \oplus \cdots$$

From the preceding paragraphs, we know that the current in a single circuit equals the impressed voltage divided by the impedance, or $I = \dfrac{E}{Z}\cdot$

Therefore, the total impedance in a parallel a-c circuit can be expressed as

$$Z_t = \frac{E}{\frac{E}{Z_1} \oplus \frac{E}{Z_2} \oplus \frac{E}{Z_3} + \cdots} = \frac{E}{I_1 \oplus I_2 \oplus I_3 + \cdots} = \frac{E}{I_t} \qquad (28\text{–}4)$$

where E = the voltage, **actual or assumed,** on the circuit.

Assuming 1 volt impressed equation (26) above is written

$$Z_t = \frac{1}{\frac{1}{Z_1} \oplus \frac{1}{Z_2} \oplus \frac{1}{Z_3} + \cdots},$$

where $\frac{1}{Z}$ is the current per volt in the several parallel branches. This is called the " **admittance,**" and is similar to conductance in a d-c circuit.

Thus, the impedance of a parallel a-c circuit, containing resistance and reactance, is equal to the impressed voltage (actual or assumed) divided by the vector sum of the currents in the several branches.

Note particularly, in a parallel circuit, we add **currents, not impedances,** as in a series circuit.

Also note that since the voltage across each branch of a parallel circuit is the same, the **voltage** is taken as the reference vector in constructing vector diagrams of parallel circuits. It is usually drawn horizontally to the right.

Example 11. What is the impedance and power factor of the parallel circuit of Fig. 24–4? In branch (1), R = 10 ohms; in branch (2), L = 0.05 henry, R = 6 ohms; in branch (3), C = 200 mf; frequency = 25 cycles.

Solution: For convenience, assume 100 volts impressed, drawn horizontally to the right in Fig. 25–4(a). Compute current in each branch under this assumed voltage.

Branch (1): $I + \frac{E}{R_1} = \frac{100}{10} = 10$ amps in phase with voltage as shown.

Branch (2): $Z_2 = \sqrt{R_2^2 + X_2^2} = \sqrt{6^2 + (2\pi \times 25 \times 0.05)^2}$
$= \sqrt{6^2 + 7.85^2} = 9.9$ ohms,

Branch (2): P.F. $= \cos\theta = \frac{R_2}{Z_2} = \frac{6}{9.9} = 0.61$; so $\theta = 52.3°$;
Sin $\theta = 0.792$.

$I_2 = \frac{100}{9.9} = 10.1$ amperes, lagging E by 52.3° or 50°20′.

Branch (3): $X_c = \dfrac{1}{2\pi \times 25 \times .0002} = 31.8$ ohms.

$$I_3 = \frac{E}{X_c} = \frac{100}{31.8} = 3.14 \text{ amperes, leading } E \text{ by } 90°\text{, as shown.}$$

Total current equals the vector sum of the three currents indicated in Fig. 25–4(*b*), or

$$I_t = \sqrt{(I_1 + I_2 \cos \theta_2)^2 + (I_2 \sin \theta - I_3)^2}$$
$$= \sqrt{(10 + 10.1 \times 0.61)^2 + (10.1 \times 0.792 - 3.14)^2}$$
$$I_t = \sqrt{16.16^2 + 4.87^2} = 16.87 \text{ amperes.}$$

$$\text{Total } Z \text{ of the circuit} = \frac{100}{16.87} = \mathbf{5.93\ ohms.}\quad \textbf{Ans.}$$

$$\text{Power factor} = \frac{\text{Power Comp. of } I}{\text{Total } I} = \frac{16.16}{16.87} = \mathbf{0.957.}\quad \textbf{Ans.}$$

Angle of lag = 16.8° or 16°50′.

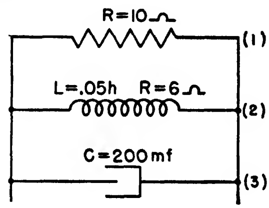


FIG. 24–4. A parallel circuit, having resistance, inductance, and capacitance.

The resistance and reactance of the combined circuit, called the "**equivalent**" R and X, in the example above can be computed as follows:

$R_t = Z_t \cos \theta = 5.93 \cos 16.8° = 5.93 \times 0.957 = 5.67$ ohms.

$X_t = Z_t \sin \theta = 5.93 \sin 16.8° = 5.93 \times 0.2896 = 1.72$ ohms.

Series-parallel a-c circuits can be computed, as in d-c, by reducing the circuit to the equivalent series circuit. After the impedance of the parallel group is com-

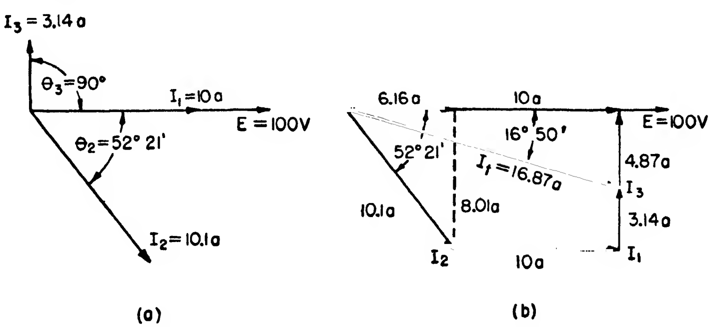


FIG. 25–4. (*a*) Polar diagram of currents in the circuit of Fig. 24–4. (*b*) Topographic diagram equivalent to that in (*a*). Note that in a parallel circuit, the voltage is taken as the reference vector.

puted, the equivalent R and X are determined, as just indicated, and the equivalent series circuit computed.

Construct a vector diagram (using E as the reference vector) in the solution of each of the following problems.

Prob. 39-4. A street lamp, containing 11 ohms resistance and 5 ohms reactance at 60 cycles, is placed in parallel with a choke coil, having 0.03 henry inductance and 6 ohms resistance across a 60-cycle circuit. What is the impedance and power factor of the combination?

Prob. 40-4. If the parallel circuit of Prob. 39-4 were placed across a 25-cycle circuit, what would be the impedance and power factor?

Prob. 41-4. An induction coil of 20 ohms impedance and 75 per cent power factor is placed in parallel with a choke coil of 18 ohms impedance and 3 per cent power factor. What is the impedance and power factor of the combination?

Prob. 42-4. Two coils and a condenser are joined in parallel. Coil A has 0.02 henry inductance and 15 ohms resistance. Coil B has 0.04 henry inductance and 4 ohms resistance; and the condenser has 150 mf capacitance. What is the impedance and power factor of the circuit at 60 cycles?

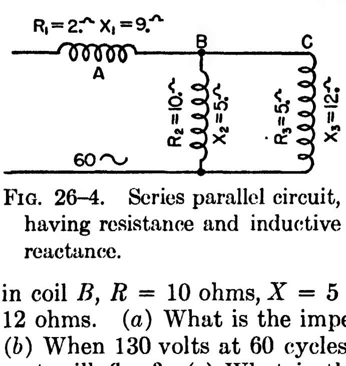


FIG. 26-4. Series parallel circuit, having resistance and inductive reactance.

Prob. 43-4. A circuit consists of a single coil, A, placed in series with a parallel combination of the two coils B and C, Fig. 26-4, on a 60-cycle circuit. In coil A, $R = 2$ ohms, $X = 9$ ohms; in coil B, $R = 10$ ohms, $X = 5$ ohms; and in coil C, $R = 5$ ohms, $X = 12$ ohms. (*a*) What is the impedance and power factor of the circuit? (*b*) When 130 volts at 60 cycles is impressed upon the circuit, what current will flow? (*c*) What is the voltage across each coil? (*d*) What current flows in each coil?

Prob. 44-4. Compute the values called for in Prob. 43-4, if coil C, in that problem, is replaced by a condenser having 20 ohms reactance at 60 cycles.

Prob. 45-4. Compute the values called for in Prob. 43-4, if coil A is replaced by a condenser having 20 ohms reactance at 60 cycles.

12-4. Resonance in Parallel Circuits. The characteristics of a resonant parallel circuit differ from those of a series circuit in that the current in the circuit is at a **minimum** value at resonant frequency and **increases** as the frequency is increased or decreased.

Pure inductance and pure capacitance, connected in parallel, would draw no current from the line at resonant frequency, as

indicated in Fig. 27–4. For at this frequency, the line current is the vector sum of two equal currents, differing 180° from each other. Therefore, the impressed voltage would simply set up a circulating, or oscillating, current in the two branches of the circuit.

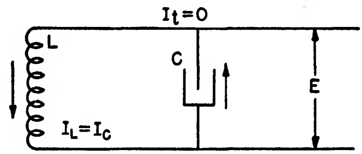


FIG. 27–4. In a parallel circuit, having inductance and capacitance only, the line current will be zero at resonant frequency.

In the actual circuit, some resistance is always present, and at resonance, the lagging reactive component of the coil current is just balanced by the leading reactive current in the condenser, leaving only the resistance or power component of the coil current to be furnished by the line. This relation is shown in Fig. 28–4(*a*) in which I_L and I_c are the currents in the coil and condenser respectively. The capacitive

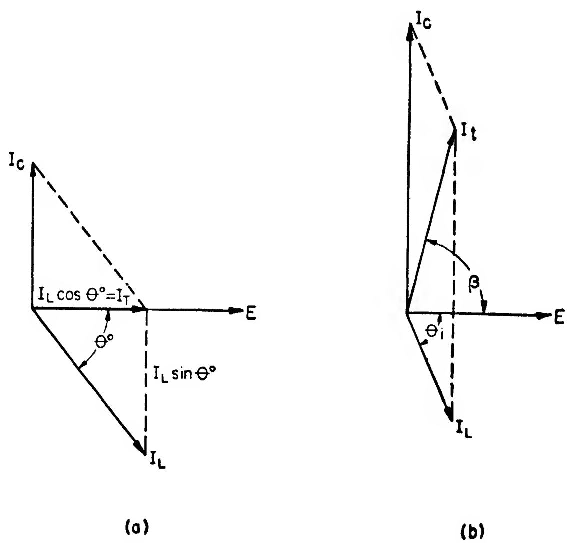


FIG. 28–4. (*a*) Vector diagram of currents in a parallel connection of a coil and a condenser at resonant frequency. The line current is at a minimum, and is equal to that component of the current in the coil, which is in phase with the voltage; thus the power factor is 1. (*b*) Vector diagram of the current in the circuit, when the frequency has been increased. Note that the condenser current has increased, the coil current has decreased, and the total has increased. The power factor is now less than one and equal to cos β.

current I_c is just equal to and 180° from the reactive current, $I_L \sin\theta$, in the coil. The resulting line current, equal to $I_L \cos\theta$ is in phase with the impressed voltage so that the power factor is **one.** This current is just necessary to supply to I^2R losses in the coil.

If the frequency is **increased,** the impedance of the coil increases, and its current and power factor **decrease;** while the condenser current increases, as indicated in Fig. 28–4(*b*). The line current is increased and now leads the voltage. If the frequency is decreased, the reverse action takes place; the line current increases and now lags the voltage.

It is instructive to consider a circuit in which both branches of a parallel arrangement contain resistance as well as reactance. In such a circuit there are two values of current at which it can be tuned to resonance, at a given frequency, by varying the capacitance. Of course, both of these values of current produce unity power factor, but one value is slightly more than minimum and the other is slightly less than maximum.

Example 12. A coil of 15 ohms and 0.2 henry inductance is in parallel with a 33.8 μf condenser, and connected across a 120-volt 60-cycle circuit. (*a*) What current flows in the coil? (*b*) In the condenser? (*c*) What are the line current and power factor of the circuit?

Solution: (*a*) $X_L = 2\pi fL = 6.28 \times 60 \times 0.2 = 75.4$ ohms.

$$Z_L = \sqrt{15^2 + 75.4^2} = 76.9 \text{ ohms.}$$

$$I_L = \frac{120}{76.9} = 1.56 \text{ amps.}$$

$$\text{P.F. (of coil)} = \frac{15}{76.9} = 0.195; \text{ so } \theta_L = 78.7°, \text{ lagging.}$$

(*b*)
$$X_c = \frac{1}{2\pi \times 60 \times .0000338} = 78.47 \text{ ohms.}$$

$$I_C = \frac{E}{X_c} = \frac{120}{78.47} = \mathbf{1.53\ amp.} \quad \textbf{Ans.}$$

(*c*)
$$\begin{aligned} I_t &= \sqrt{(I_L \cos\theta_L)^2 + (I_L \sin\theta_L - I_c)^2} \\ &= \sqrt{(1.56 \cos 78.7°)^2 + (1.56 \sin 78.7° - 1.53)^2} \\ &= \sqrt{(1.56 \times 0.195)^2 + (1.56 \times 0.981 - 1.53)^2} \\ &= \sqrt{(0.304)^2 + (1.53 - 1.53)^2} = \mathbf{0.304\ amps.} \end{aligned}$$

Ans.

$$\text{P.F.} = \frac{\text{Power Comp. of } I}{\text{Total } I} = \frac{0.304}{0.304} = 1.00. \quad \textit{Ans.}$$

Thus the circuit is in resonance at this frequency.

Prob. 46–4. Solve the problem in Example 12 at 50 cycles, all other data remaining the same. Show vector diagram.

Prob. 47–4. What value must the condenser have in Example 12 if the circuit is to be in resonance at 120 cycles? All other data remain the same. Show vector diagram.

13–4. Iron Cored Coils — Effective Resistance. In previous paragraphs, the characteristics of inductive circuits, containing air-core coils only, have been considered. The power consumed, or heat dissipated, has been computed as I^2R_o, where R_o is the **ohmic** resistance or the resistance of the conductors in the coils. This value is the "copper loss" and has been used in all computations in the construction of vector diagrams and in the power equation

$$P = EI \cos \theta = I^2R_o.$$

When iron is present in the magnetic circuit of a coil, the flux, inductance and reactance are all increased. Also the inductance is not constant but varies with the permeability. Moreover, in addition to the ohmic I^2R_o loss there are now both **hysteresis and eddy current losses** in the iron which must be supplied by the electric circuit, and which are dissipated in heat. Hysteresis and eddy current losses vary with frequency, flux density, permeability of the iron, etc. (See Vol. I, Chapters VIII and IX.)

Thus the total losses, or power dissipated, vary, not only with the current squared, but also with other quantities and

$$P = I^2R_o + W_h + W_e$$

where R_o = ohmic resistance; W_h = hysteresis loss in watts; W_e = eddy current loss in watts.

The total power dissipated in the iron cored coil is measured by a wattmeter; therefore

$$P = I^2R_o + W_h + W_e = EI \cos \theta. \tag{29–4}$$

It is evident that the value of P, as determined by the wattmeter, is greater than I^2R_o. That is,

$$P = EI \cos \theta \text{ is greater than } I^2R_o.$$

Therefore, to take account of the power used up by hysteresis and eddy currents, we may consider that the coil with iron core has an **apparent** or **effective** resistance. Since, in the air-core coil,

$$\text{Watts} = I^2R_o \quad \text{or} \quad \frac{\text{watts}}{I^2} = R_o \text{ (the ohmic resistance),}$$

we may write for the iron-core coil,

$$\frac{\text{watts}}{I^2} = R_{\text{eff}}$$

or

$$P = IE \cos \theta = I^2R_{\text{eff}},$$

in which R_{eff} is called the **effective** resistance of the coil. This is always greater than the ohmic resistance.

In many coils with air cores, the alternating flux sets up eddy currents in the conductors themselves, which apparently increases

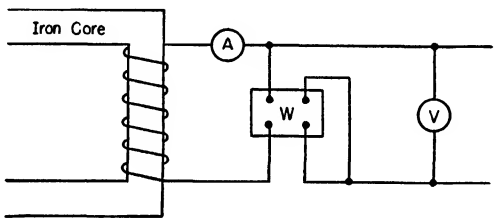


FIG. 29–4. A coil with iron in its magnetic circuit. The wattmeter measures the power dissipated in copper, hysteresis, and eddy-current losses.

their d-c resistance. Also skin effect increases the resistance above the d-c value. When these effects are appreciable, the effective resistance of the coil must be used.

Example 13. A coil with an iron core has 8 ohms resistance, as measured with direct current. When the coil is connected across 60-cycle mains, as indicated in Fig. 29–4, the wattmeter reads 50 watts; the voltmeter 115 volts; and the ammeter 1.80 amperes. (*a*) What is the effective resistance? (*b*) The reactance? (*c*) The inductance of the coil? (*d*) The power factor?

Solution:

(*a*) Effective resistance $= R_{\text{eff}} = \dfrac{W}{I^2} = \dfrac{50}{(1.80)^2} = \mathbf{15.4\ ohms.}$ **Ans.**

(*b*) $$Z = \frac{E}{I} = \frac{115}{1.80} = 63.9 \text{ ohms.}$$

$$X = \sqrt{Z^2 - R^2} = \sqrt{63.9^2 - 15.4^2} = \mathbf{62\ ohms.}\quad \textbf{Ans.}$$

(*c*) $$L = \frac{X}{2\pi f} = \frac{62}{2\pi 60} = \mathbf{0.164\ henry.}\quad \textbf{Ans.}$$

(*d*) Power factor $= \frac{W}{EI} = \frac{50}{115 \times 1.8} = \mathbf{0.2145.}$ **Ans.**

$$\frac{R_{\text{eff}}}{Z} = \frac{15.4}{63.9} = 0.2145 \text{ (check).}$$

$$\begin{aligned} E_{\text{line}} &= 115 \text{ volts, as measured by voltmeter.} \\ &= \sqrt{(IR_{\text{eff}})^2 + (IX)^2} \\ &= \sqrt{(1.80 \times 15.4)^2 + (1.80 \times 62)^2} \\ &\qquad = 115 \text{ volts (check).} \end{aligned}$$

Note the error which would occur in computing the relations in the above example, if the ohmic resistance of 8 ohms were used.

$$X = \sqrt{Z^2 - R^2} = \sqrt{63.9^2 - 8^2} = 63.4 \text{ ohms (62 ohms in Ex. 13).}$$

$$L = \frac{X}{2\pi f} = \frac{63.4}{2\pi 60} = 0.168 \text{ henry (0.164 in Ex. 13).}$$

$$\text{P.F.} = \frac{R_o}{Z} = \frac{8}{63.9} = 0.125 \text{ (0.2415 in Ex. 13).}$$

$$\text{Power} = I^2R_o = 1.8^2 \times 8 = 25.92 \text{ watts (wattmeter reads 50 watts).}$$

Prob. 48-4. A 60-cycle emf is impressed on a transformer coil, as in Fig. 29-4. The wattmeter reads 40 watts; the voltmeter, 230 volts; and the ammeter, 0.58 ampere. What are the inductance, reactance and power factor of the coil? Show vector diagram.

SUMMARY OF CHAPTER IV

When an A-C VOLTAGE OF SINE WAVE FORM is impressed on a circuit made up of any combination of constant resistances, inductances and capacitances, an ALTERNATING CURRENT OF THE SAME WAVE FORM AND FREQUENCY AS THE VOLTAGE WILL FLOW.

In a Circuit Containing Constant Resistance Only, the impressed voltage at any instant, necessary to overcome the resistance, is equal to the iR drop in the circuit at that instant; and therefore, the current is in phase with the voltage and is expressed in effective values as

$$\mathbf{E = RI.} \tag{1}$$

INDUCTANCE in a circuit sets up a counter emf which opposes any change in the value of the current. This counter voltage is proportional to the RATE OF CHANGE OF CURRENT. In an a-c circuit, the curve of counter emf is 90° behind the current curve and "chokes," or reduces, the value of the current in the circuit. Counter emf of self induction is expressed as

$$\mathbf{E_{ave} = L\frac{I_m}{\frac{1}{4f}}} \tag{2}$$

where E_{ave} = the average value of counter emf in volts.

L = inductance in henries.

I_m = maximum value of the current in amperes.

f = frequency of the circuit.

In that part of a CIRCUIT CONTAINING CONSTANT INDUCTANCE ONLY, the counter emf of self induction is equal to the impressed voltage across that part of the circuit and 180° out of phase with it. Thus the CURRENT LAGS THE IMPRESSED VOLTAGE BY 90°.

Using effective values, equation (2) may be reduced to the expression,

$$E_{impressed} = E_{counter} = (2\pi fL)I. \qquad (3)$$

The term $(2\pi fL)$ is called the Inductive Reactance, which is measured in ohms and indicated by the letter X_L,

or
$$X_L = 2\pi fL$$

Therefore
$$E_{imp} = (2\pi fL)I = X_L I. \qquad (4)$$

The term $X_L I$ or IX_L is called the INDUCTIVE REACTANCE DROP, and is ALWAYS 90° AHEAD OF THE CURRENT.

In a CIRCUIT CONTAINING CONSTANT CAPACITANCE ONLY, the CURRENT IS PROPORTIONAL TO THE RATE OF CHANGE OF VOLTAGE and leads the voltage by 90°. Expressed as an equation,

$$I_{ave} = C \frac{E_m}{\frac{1}{4f}} \qquad (5)$$

where C = capacitance in farads.

Substituting effective values equation (5) may be reduced to the expression,

$$E = \frac{1}{2\pi fc} I. \qquad (6)$$

The term $\left(\frac{1}{2\pi fc}\right)$ is called the CAPACITIVE REACTANCE. It is measured in ohms and indicated by the letter X_c.

Therefore
$$E_{imp} = \left(\frac{1}{2\pi fc}\right) I = X_c I \qquad (7)$$

The term $X_c I$, or IX_c is called CAPACITIVE REACTANCE DROP, and is always 90° behind the current.

Note similarity of equations (1), (4) and (7).

$$E = RI; \; E = X_L I; \; E = X_c I.$$

IN THE SERIES CIRCUIT

The impressed voltage is the vector sum of the voltages across the parts or

$$E_t = E_1 \oplus E_2 \oplus E_3 \oplus \cdots.$$

The current is the same in all parts. In vector diagrams of series circuits, the CURRENT is usually drawn as the REFERENCE VECTOR.

In a CIRCUIT CONTAINING RESISTANCE AND INDUCTIVE REACTANCE, impressed voltage is the vector sum of IR and IX_L at 90°, or

$$E_{imp} = \sqrt{(IR)^2 + (IX)^2} = I\sqrt{R^2 + X^2} = IZ \tag{8}$$

The term, $\sqrt{R^2 + X^2}$, or $\sqrt{R^2 + (2\pi fL)^2}$, is called the IMPEDANCE, which is measured in ohms and indicated by the letter Z. The term IZ, equal to the impressed voltage, is called the IMPEDANCE DROP. Thus it is always in phase with the voltage.

The current in the circuit equals

$$I = \frac{E}{\sqrt{R^2 + (2\pi fL)^2}} = \frac{E}{Z} \tag{9}$$

and LAGS BEHIND the voltage by an angle θ_L, which is LESS than 90°, due to the presence of resistance in the circuit.

$$\text{Power factor} = \cos\theta = \frac{R}{Z} \tag{10}$$

In a CIRCUIT CONTAINING RESISTANCE AND CAPACITIVE REACTANCE

$$E_{imp} = \sqrt{(IR)^2 + (IX_c)^2} = I\sqrt{R^2 + X^2} = IZ \tag{11}$$

The current,

$$I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{2\pi fc}\right)^2}} = \frac{E}{Z} \tag{12}$$

and LEADS the impressed voltage by an angle θ_c which is less than 90°.

The VECTOR SUM OF THE IMPEDANCES of the parts is equal to the TOTAL impedance of the circuit, or

$$Z_t = Z_1 \oplus Z_2 \oplus Z_3 + \cdots.$$

In a circuit CONTAINING RESISTANCE, INDUCTIVE REACTANCE AND CAPACITIVE REACTANCE

$$Z = \sqrt{R^2 + (X_L - X_c)^2}$$

$$\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fc}\right)^2}.$$

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fc}\right)^2}} = \frac{E}{Z}. \tag{14}$$

RESONANCE. When the frequency is such that $X_L = X_c$, the circuit is said to be in RESONANCE. The reactance of the circuit is zero and $Z = R$. The effective current reaches a maximum value, and the voltage across both the inductive and capacitive parts of the circuit may reach DANGEROUS VALUES — MUCH GREATER THAN THE IMPRESSED VOLTAGE.

The power factor of a resonant circuit = 1.

Resonant frequency may be determined by the formula,

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{15}$$

IN THE PARALLEL CIRCUIT

The voltage is the SAME across all parts and in VECTOR DIAGRAMS, the voltage is taken as the reference vector.

The total current is the vector sum of the currents in the several branches, or

$$I_t = I_1 \oplus I_2 \oplus I_3 \oplus \cdots.$$

The total Impedance is

$$Z_t = \frac{E}{\frac{E}{Z_1} \oplus \frac{E}{Z_2} \oplus \frac{E}{Z_3} \oplus \cdots} \tag{16}$$

$$= \frac{E}{I_1 \oplus I_t \oplus I_3 \oplus \cdots} = \frac{E}{I_t}$$

where E is impressed voltage, KNOWN or ASSUMED.

EQUIVALENT resistance of the circuit, $R_t = Z_t \cos\theta$,
EQUIVALENT reactance of the circuit, $X_t = Z_t \sin\theta$,

where θ = phase angle between total current and impressed voltage.

RESONANCE. When the frequency is such that the lagging reactive component of the current in the inductive branches is equal to the leading reactive component in the capacitive branches,

$$I_L \sin\theta_L - I_c \sin\theta_c = 0;$$

and the total current is the sum of the several power components of current. At this frequency, if only one branch contains resistance, the effective current in the circuit reaches a minimum value, and the circuit is said to be in RESONANCE. The power factor of the circuit at this frequency = 1.

EFFECTIVE RESISTANCE

IN AIR CORED COILS,

$$P = EI\cos\theta = I^2R_o$$

where R_o = the ohmic, or conductor, resistance of the coil.

IN IRON CORED COILS, hysteresis and eddy current losses occur in the magnetic circuit. The power consumed is measured by a wattmeter and

$$P = I^2R_o + W_h + W_e = EI \cos \theta$$

where W_h and W_e = hysteresis and eddy losses, respectively.

Therefore, $P = EI \cos \theta$ is greater than I^2R_o.

To take account of the iron losses, the coil is considered to have an APPARENT, or EFFECTIVE, resistance.

In the air cored coil,

$$\text{watts} = I^2R_o \text{ or } \frac{\text{watts}}{I^2} = R_o \text{ (ohmic resistance).}$$

In the iron cored coil, $\dfrac{\text{watts}}{I^2} = R_{\text{eff}}$,

and $P = EI \cos \theta = I^2R_{\text{eff}}$,

where R_{eff} **= THE EFFECTIVE RESISTANCE in ohms.**

This is always greater than the ohmic resistance.

PROBLEMS IN CHAPTER IV

Show a vector diagram in the solution of each of the following problems. All coils are without iron in their magnetic circuits unless otherwise stated.

Prob. 49–4. Two resistors of 23 ohms and 17 ohms are connected in series across a 115-volt, d-c circuit. (*a*) What is the current? (*b*) The voltage across each resistor? (*c*) The power consumed in each? (*d*) The impedance of the circuit?

Prob. 50–4. What would be the answers to the questions in Prob. 49–4, if the circuit had been a-c at 25 cycles? 60 cycles?

Prob. 51–4. A pressure of 120 volts at 60 cycles is applied to a coil of 0.045 henry inductance and negligible resistance. (*a*) What is the impedance? (*b*) What current will flow? (*c*) What is the power factor?

Prob. 52–4. Answer the questions in Prob. 51–4, if the frequency were 1000 cycles per second, the other data remaining the same.

Prob. 53–4. (*a*) What current will flow when 120 volts at 60 cycles is applied to a 45 mf condenser? (*b*) If the frequency were 500 cycles? (*c*) If 120 volts d-c were applied?

Prob. 54–4. What is the inductance of a coil, having 20 ohms resistance, if it takes 2.5 amperes from 125 volt 60 cycle mains?

Prob. 55–4. A circuit takes 4.5 amperes from 240 volt, 25-cycle mains at 0.6 lagging power factor. What is the inductance of the circuit?

Prob. 56–4. (*a*) How many volts at 50 cycles must be impressed upon the circuit of Prob. 55–4 in order that the same current may flow? (*b*) What is the power factor of the circuit?

Prob. 57–4. When 240 volts d-c is applied to a certain coil, the current is 16 amperes. When 240 volts at 60 cycles is applied, the current is only 3.5 amperes. What are the inductance and power factor of the circuit?

Prob. 58–4. In an inductive circuit, two impedances are in series across 230 volts at 60 cycle mains. There is the same voltage across each, but the resistance of one of them is 15 ohms, while that of the other is 7 ohms. The total power consumed in the entire circuit is 550 watts. Calculate the inductance, the power factor and the power consumed in each of these impedances. What is the voltage across each?

Prob. 59–4. One coil alone takes 10 amperes at 60 per cent power factor from 110-volt, 60-cycle mains. Another coil alone takes 10 amperes at 80 per cent power factor from the same mains. (*a*) What is the impedance, resistance, reactance and inductance of each coil? (*b*) What will be the current, power and power factor of the circuit, if the coils are connected in series across the same mains?

Prob. 60–4. In Prob. 59–4, what will be the voltage across each coil when they are in series?

Prob. 61–4. What would be the answers to Probs. 59–4 and 60–4 if the frequency were doubled?

Prob. 62–4. What would be the answers to Probs. 59–4 and 60–4 if the frequency were reduced to zero, or if d-c were used?

Prob. 63–4. If the same two coils specified in Prob. 59–4 are connected in parallel to the same mains, what will be the total amperes, watts and power factor of the combination?

Prob. 64–4. What would be the impedance, resistance and reactance of a single coil, equivalent to the parallel combination of Prob. 63–4?

Prob. 65–4. Four coils, all having the same resistance and inductance, take altogether 4 kw at 80 per cent power factor, when connected in parallel across 220-volt, 60-cycle mains. What would be the current, power and power factor, if they were connected in series across the same mains?

Prob. 66–4. A series circuit consists of two inductive parts. At 25 cycles, one part has an impedance of 12 ohms and 80 per cent power factor; the other, an impedance of 18 ohms and 25 per cent power factor. (*a*) What are the resistance, reactance and inductance of each part? (*b*) What are the impedance and power factor of the entire circuit?

Prob. 67–4. (*a*) What will be the total impedance and power factor of the entire circuit, if the two impedances of Prob. 66–4 are placed in

parallel on 25 cycles? (*b*) What will be the equivalent resistance and reactance of the circuit?

Prob. 68–4. A circuit takes 1.5 amperes at 75 per cent leading power factor from 240-volt 25-cycle mains. What is the capacitance of the circuit?

Prob. 69–4. When two condensers A and B, of pure capacitance are connected in series across 500-volt, 60-cycle mains, the current is 4 amperes, the voltage across A is 300 volts and across B is 200 volts. (*a*) What is the capacitance of the entire circuit? (*b*) The capacitance of each condenser?

Prob. 70–4. (*a*) If the two condensers in Prob. 69–4 are connected in parallel across the same mains, what total current will flow? (*b*) What will be the capacitance of the circuit?

Prob. 71–4. A coil of 0.1 henry inductance and 25 ohms resistance is connected in series with a 25.3 μf condenser across a 230-volt 60-cycle line. (*a*) What current will flow? (*b*) What is the voltage across the coil? (*c*) Across the condenser? (*d*) Power factor of the circuit?

Prob. 72–4. What is the resonant frequency in Prob. 71–4?

Prob. 73–4. A coil of 0.2 henry inductance and 12 ohms resistance is connected in series with a condenser of 50.7 μf capacitance across a 240-volt 50-cycle line. (*a*) What is the impedance of the circuit? (*b*) The current? (*c*) The voltage across each part? (*d*) The power factor of the circuit?

Prob. 74–4. Repeat Prob. 73–4, if the frequency is halved. State whether power factor is leading or lagging.

Prob. 75–4. Repeat Prob. 73–4, if the frequency is doubled. State whether the power factor is leading or lagging.

Prob. 76–4. (*a*) If the coil and condenser of Prob. 73–4 were connected in parallel across the same mains, what would be the current in each? (*b*) The total current? (*c*) The impedance and power factor of the circuit?

Prob. 77–4. Repeat Prob. 76–4, if the frequency is halved. Is the power factor leading or lagging?

Prob. 78–4. Repeat Prob. 76–4, if the frequency is doubled. Is the power factor leading or lagging?

Prob. 79–4. A series circuit consists of three appliances, A, B and C, of which A and B are inductive and C is capacitive. Impedance of A is 14 ohms with 40 per cent power factor; of B is 18 ohms with 85 per cent power factor; of C is 10 ohms with 60 per cent power factor. What is the total impedance and power factor of the circuit?

Prob. 80–4. What would be the impedance and power factor of the circuit, if the appliances in Prob. 79–4 were connected in parallel?

Prob. 81-4. A coil of 0.035 henry inductance and 10 ohms resistance, a non-inductive resistance of 15 ohms and a 40 μf condenser are connected in parallel. What is the impedance, equivalent resistance and equivalent reactance of the circuit, when the frequency is 60 cycles? State whether the equivalent reactance is inductive or capacitive.

Prob. 82-4. A coil of 3 ohms resistance and 0.0159 henry inductance is placed in series with the circuit of Prob. 81-4. (*a*) When 230 volts at 60 cycles is impressed on the combined circuit, what current flows? (*b*) What is the voltage across each part? (*c*) What current will flow in each branch of the parallel group?

Prob. 83-4. A coil with iron core takes 30 amperes from a 120-volt d-c circuit. When connected across 120 volts at 60 cycles, the power consumed is 90 watts and the current is 2.5 amperes. What is the inductance of the coil for this value of impressed voltage and frequency? What is the ohmic resistance? The effective resistance?

Prob. 84-4. A coil with iron core is in series with a 100 μf condenser across a 240-volt, 60-cycle circuit. The power consumed in the circuit is 50 watts and the current is 2 amperes. (*a*) What is the voltage across the coil and across the condenser? (*b*) What is the inductance of the coil? (*c*) What is the power factor (leading or lagging)?

CHAPTER V

POWER IN POLYPHASE CIRCUITS

The computation and measurement of power in single-phase circuits were explained in Chapter III. Methods of computing and measuring power in polyphase systems are discussed in this chapter.

1–5. Power in a Delta Connected System. Balanced Load. Consider a delta connected generator supplying power to a balanced

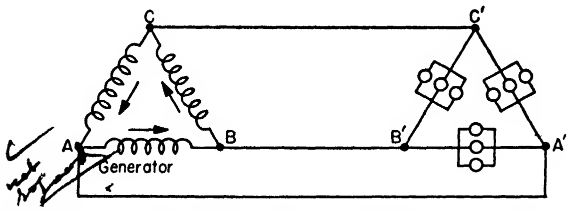


FIG. 1–5. Delta-connected generator supplying a balanced delta-connected system of incandescent lamps at unity power-factor. The total power delivered equals the sum of the power delivered to the three groups of lamps.

three-phase load of incandescent lamps, as in Fig. 1–5. Under this condition, the phase currents in the generator are all equal and in phase with the coil voltages, or phase voltages, as indicated in the vector diagram of Fig. 2–5(a).

Thus, $$I_{ab} = I_{bc} = I_{ca}.$$

and $$E_{AB} = E_{BC} = E_{CA}$$

The power in watts, delivered by the phase AB, which is equal to that delivered by each of the other phases, is

$$P_{AB} = E_{AB} \times I_{ab}. \quad \text{(power factor = 1)}$$

And the total power in watts, delivered by the generator, is equal to the sum of the power, delivered by the three phases, or

$$P_{\text{total}} = E_{AB}I_{ab} + E_{BC}I_{bc} + E_{CA}I_{ca}$$
$$= 3E_{\text{phase}}I_{\text{phase}}. \qquad (1\text{–}5)$$

lta connected system,

$$_{\text{hase}} = E_{\text{line}}$$

$$_{\text{ase}} = \frac{I_{\text{line}}}{\sqrt{3}} \text{ (Chap. II, Art. 18)}$$

Substituting these values in equation (1) above,

$$P_{\text{total}} = 3E_{\text{line}} \times \frac{I_{\text{line}}}{\sqrt{3}}$$

$$= \sqrt{3}\, E_{\text{line}} I_{\text{line}} \text{ (p.f. = 1)} \qquad (2\text{–}5)$$

Thus the power output of a delta connected generator on a **balanced load at unity power factor** is equal to $\sqrt{3}$ **multiplied by**

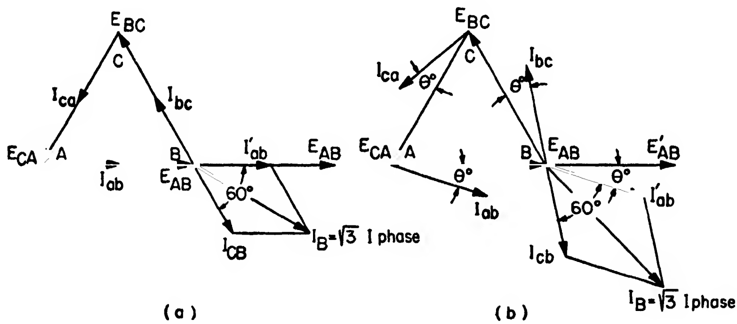


FIG. 2–5. (*a*) Vector diagram showing the relation between phase voltage phase current and line current in Fig. 1–5. The phase currents are all equal as are the phase voltages. Power per phase $= E_{AB} \times I_{ab}$. (*b*) Vector diagram showing the relation between voltage and current per phase on a balanced load when the phase currents lag behind their respective phase voltages by $\theta°$. Power per phase $= E_{AB} \times I_{ab} \cos \theta°$.

line voltage times line current. This relation is also true for the power delivered to the load.

When the balanced load of a system, similar to Fig. 1–5, is such that the current in each of the phases lags the phase voltage by an angle $\theta°$, as indicated in the diagram of Fig. 2–5(*b*), the power in watts delivered by each phase of the generator is

$$P_{\text{phase}} = E_{\text{phase}} I_{\text{phase}} \cos \theta,$$

where $\cos \theta$ is the power factor of the phase.

And the total power in watts delivered by the generator to the

line is three times this, or

$$P_{\text{total}} = 3E_{\text{phase}}I_{\text{phase}} \cos\theta;$$

$$= 3E_{\text{line}} \frac{I_{\text{line}}}{\sqrt{3}} \cos\theta;$$

$$= \sqrt{3}\, E_{\text{line}}I_{\text{line}} \cos\theta. \tag{3–5}$$

This is the standard equation for computing power in a balanced three-phase circuit and applies either to a delta or a Y connected system, as will be shown in the next article. Note that the three-phase power factor is the cosine of the angle of phase displacement between **the current per phase and the voltage per phase.** It does **not** indicate the phase difference between the current in a **line wire** and the voltage between **two line wires.**

Similarly, the reactive power in Vars may be written

$$\text{VARS} = \sqrt{3}\, E_\ell I_\ell \sin\theta, \tag{4–5}$$

Where sine θ equals the reactive factor of the load,

and
$$\text{Apparent power} = \sqrt{3}\, E_\ell I_\ell. \tag{5–5}$$

Transposing equation (3) above, we may write

$$\text{Power factor} = \frac{P_{\text{watts}}}{\sqrt{3}\, E_\ell I_\ell} \tag{6–5}$$

Also,

$$\text{Power in KW} = \frac{\sqrt{3}\, E_\ell I_\ell \cos\theta}{1000};$$

$$\text{Reactive power in KVARS} = \frac{\sqrt{3}\, E_\ell I_\ell \sin\theta}{1000}$$

$$\text{Apparent power in KVA} = \frac{\sqrt{3}\, E_\ell I_\ell}{1000}.$$

Example 1. A delta connected motor takes 100 kw at 0.8 power factor from 240-volt mains. What is the line current?

Solution:

$$P_{\text{watts}} = \sqrt{3}\, E_\ell I_\ell \cos\theta$$

$$100 \text{ kw} = 100{,}000 \text{ watts};$$

$$I_\ell = \frac{P}{\sqrt{3}\, E_\ell \cos\theta} = \frac{100{,}000}{\sqrt{3} \times 240 \times 0.8} = \textbf{301 amperes.} \quad \textbf{Ans.}$$

Alternate solution:

$$\frac{100}{0.8} = 125 \text{ kva};$$

$$\frac{125}{3} = 41.7 \text{ kva per phase};$$

$$\frac{41700}{240} = 173.75 \text{ amperes per phase};$$

$$173.75 \times \sqrt{3} = \textbf{301 amperes line current.} \quad \textbf{Ans.}$$

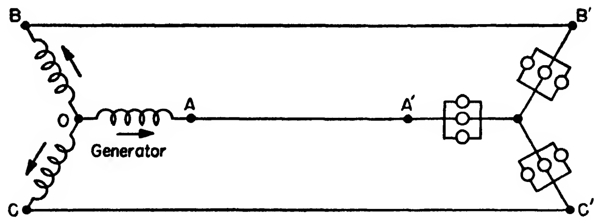


FIG. 3–5. A Y-connected generator supplying a balanced Y-connected system of incandescent lamps at unity power-factor. The total power equals the sum of the power delivered to the three groups of lamps.

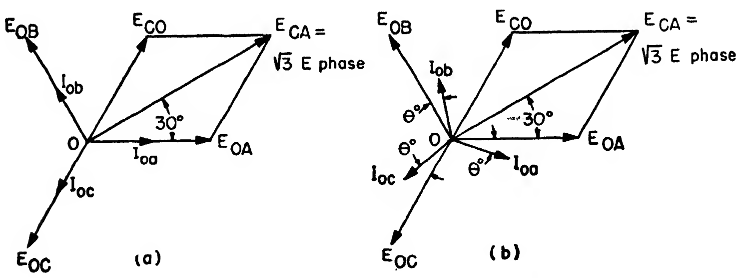


FIG. 4–5. (*a*) Vector diagram showing the relation between phase current, phase voltage and line voltage in Fig. 3–5. The phase voltages are all equal, as are the phase currents. Power per phase $= E_{OA} \times I_{oa}$. (*b*) Vector diagram showing the relation between voltage and current per phase on a balanced load when the phase currents lag $\theta°$ behind their respective phase voltages. Power per phase $= E_{OA}I_{oa} \cos \theta°$.

2–5. Power in a Y-Connected System: Balanced Load. In a balanced Y-connected system at unity power factor, as indicated in Fig. 3–5, the phase currents are all equal and in phase with the coil, or phase voltages, as shown in the diagram of Fig. 4–5(*a*).

The power in watts delivered by phase *OA* of the generator is

$$P_{OA} = E_{OA}I_{OA} \quad \text{(power factor = 1)}$$

and the total power delivered is three times this, or

$$P_{\text{total}} = 3E_{\text{phase}}I_{\text{phase}}.$$

Since, in a balanced three-phase, Y-connected system, $E_{\text{phase}} = \dfrac{E_{\text{line}}}{\sqrt{3}}$ and $I_{\text{phase}} = I_{\text{line}}$ (Chap. II, Art. 18); also indicated in Fig. 4–5(*a*), the above equation can be written as

$$P_{\text{total}} = 3\frac{E_{\text{line}}}{\sqrt{3}}I_{\text{line}}$$

$$= \sqrt{3}E_{\text{line}}I_{\text{line}} \quad (\text{p.f.} = 1.0).$$

This is exactly the same as equation (2) for the delta connection.

Also, when the load is such that the phase currents differ in phase from their voltages by an angle of $\theta°$, as shown in Fig. 4–5(*b*), the power in watts delivered by phase OA of the generator is

$$P_{OA} = E_{OA}I_{OA}\cos\theta,$$

and the total power in watts delivered by the generator is again written

$$P_{\text{total}} = 3E_{\text{phase}}I_{\text{phase}}\cos\theta$$

$$= \frac{E_{\text{line}}}{\sqrt{3}}I_{\text{line}}\cos\theta$$

$$= \sqrt{3}E_{\text{line}}I_{\text{line}}\cos\theta$$

Again, this equation is exactly similar to equation (3) in the previous article.

It is to be noted that the computations of power in terms of line current and line voltage is exactly the same for both Y and delta connected systems. And for each balanced system, the three phase power factor equals the cosine of angle of phase difference between the phase current and phase voltage.

Example 2. Assume the motor in Example 1 is reconnected in Y and operates at the same load and power factor. (*a*) On a circuit of what voltage should it operate? (*b*) What will be the line current?

Solution: Rated voltage per phase when delta connected is 240 volts. Rated line voltage when Y connected = $\sqrt{3} \times 240$, or 415 volts.

$$I_\ell = \frac{P}{\sqrt{3}E_\ell\cos\theta} = \frac{100{,}000}{\sqrt{3}\times 415\times 0.8} = \textbf{173.75 amperes.} \quad \textbf{Ans.}$$

Prob. 1–5. How much current does each lead of a 230-volt three-phase induction motor carry, if the motor requires 14 kva to operate it?

Prob. 2–5. What is the hp output of a 460-volt, three-phase induction motor, if it draws 20 amperes per lead at 70 per cent power factor and the efficiency is 85 per cent?

Prob. 3–5. What is the full-load line current of a 500-kva 2300-volt three-phase alternator?

Prob. 4–5. (*a*) What is the phase voltage and phase current, if the generator in Prob. 3–5 is delta connected? (*b*) If Y-connected?

Prob. 5–5. A 200-hp, 460-volt induction motor operates at full load with an efficiency of 92 per cent and a power factor of 90 per cent. What line current does it take?

Prob. 6–5. A three-phase, 120-volt, 5 hp induction motor takes 400 watts at no load with a power factor of 50 per cent. What line current does it take?

Prob. 7–5. If the power factor of the motor in Prob. 6–5 rises to 90 per cent at full load and the efficiency is 80 per cent, what line current does it take?

3–5. Measurement of Power in a Balanced Delta Connected System. Two-Wattmeter Method. Assume the delta connected generator of Fig. 5–5(*a*) to be delivering a balanced load at unity power factor, and it is desired to measure the power delivered by the machine. If a wattmeter could be connected to measure the power output of one phase only, the total power delivered by the generator would be three times this wattmeter reading, but since the phases are usually connected inside the frame of the machine, it becomes necessary to place the current coil of the wattmeter in the line.

Thus, if the wattmeter current coil is placed in line 1 and its pressure coil connected across to line 3, as shown in the figure, its indication will be greater than the power delivered by the phase CA. The vector relations are shown in Fig. 5–5(*b*). In the diagram, the voltage of phase CA is E_{CA}, drawn from point A. Current in phase CA is I_{ca}, or I'_{ca} drawn from point A. Therefore, the power output of this phase equals $E_{CA} \times I'_{ca}$. But the wattmeter carries the current in line 1, Fig. 5–5(*a*). This is equal to the vector sum of I'_{ca}, Fig. 5–5(*b*), and I_{ba} (I_{ab} reversed), or I_{aa}. This current, I_{aa}, is equal to the phase current times $\sqrt{3}$ and also has a phase displacement of 30° from the phase voltage E_{CA}, the

voltage on the potential coil of the wattmeter. Thus, the wattmeter reads $E_{CA}I_{aa}\cos\theta$, which is **not** the power output of one phase. Therefore, to correct the wattmeter reading, W, to obtain the power delivered by phase CA, we divide by $\sqrt{3}$ and by cos 30°, or

$$P_{(\text{phase } CA)} = \frac{W}{\sqrt{3}\cos 30^\circ},$$

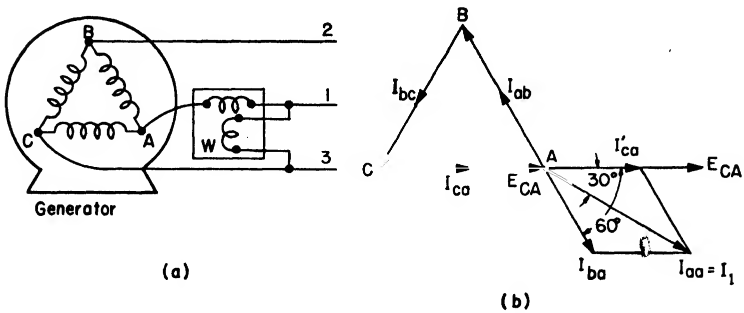


FIG. 5-5. (*a*) The wattmeter W, with current-coil in line 1 and potential-coil connected between lines 1 and 3, does NOT indicate the power output of phase CA. (*b*) The vector I_{aa} (current in current-coil of wattmeter in [*a*]) is the resultant of phase current I'_{ca} and I_{ba} and is $\sqrt{3}$ times the current in phase CA. Moreover it is 30° out of phase with I_{ca}.

and the total power delivered by the generator is three times this, or

$$P_{\text{total}} = \frac{3 \times W}{\sqrt{3}\cos 30^\circ} = \frac{3}{\sqrt{3}\cos 30^\circ} \times W$$

$$= \frac{3}{1.73 \times 0.866} \times W = \frac{3}{1.5} \times W = 2W.$$

Thus in a **balanced** three-phase load at **unity power factor,** one wattmeter with its current coil, connected in one line, and its potential coil, connected across to either of the other two lines, indicates half the power output of the generator, or half the power delivered to the load. Note the examples below.

Example 3. Assume the current per phase in the generator of Fig. 5-5(*a*) to be 10 amperes in phase with the phase voltage of 100 volts. (*a*) What will the wattmeter indicate? (*b*) What power does the generator deliver?

Solution: Current in wattmeter current-coil = line current = $\sqrt{3} \times 10 = 17.32$ amperes.

Voltage on potential coil = 100 volts.

Phase angle between current and voltage = 30°

(*a*) Wattmeter reading,

$$W = 100 \times 17.32 \times \cos 30°$$

$$= 100 \times 17.32 \times \cos 30° = \mathbf{1500\ watts.}\quad \textbf{Ans.}$$

(*b*) $2 \times W = 2 \times 1500 =$ **3000 watts. Ans.**

Also $\sqrt{3}\, E_{\text{line}} I_{\text{line}} = \sqrt{3} \times 100 \times 17.32 = 3000$ watts (check).

or $3E_{\text{phase}} I_{\text{phase}} = 3 \times 100 \times 10 = 3000$ watts (check).

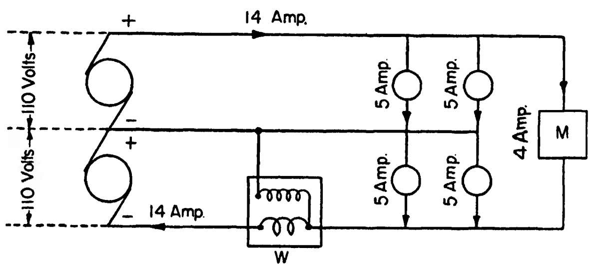


FIG. 6–5. A balanced 3-wire d-c system. The wattmeter W indicates half the total power delivered to the lamps and the motor.

The measurement of power in a balanced three-phase load at unity power factor is somewhat similar to that of the balanced three-wire d-c system of Fig. 6–5, in which the single wattmeter connected in one of the outside wires, with potential coil connected between an outside wire and the neutral, indicates half the total power delivered to the lamp and the motor.

When a three-phase load is balanced, but with a power factor less than unity (or, if the load is unbalanced), the single wattmeter, connected as in Fig. 5–5(*a*) does **not** indicate half the power delivered and, therefore, cannot be used to measure the power in the circuit.

However, the total power can be measured by two wattmeters properly connected in the line. This is called the **two wattmeter method of measuring three-phase power.**

This method is somewhat similar to the measurement of power in the unbalanced three-wire d-c system of Fig. 7–5 by the use of two wattmeters. The current coil of each wattmeter is connected

in one of the outside wires with their potential coils connected across to the neutral, as shown. The indications of the two wattmeters will not be the same, but **the sum of their readings indicates the total power delivered.**

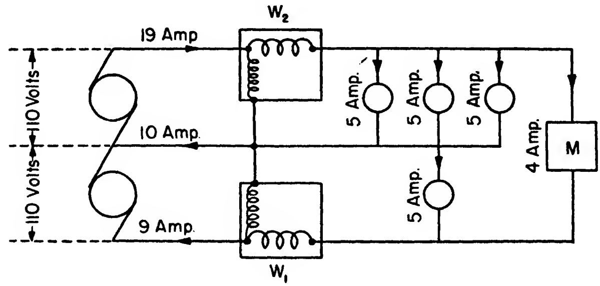


FIG. 7–5. In this unbalanced 3-wire d-c system the two wattmeters do not read alike, but the sum of their readings equals the total power delivered to the lamps and the motor.

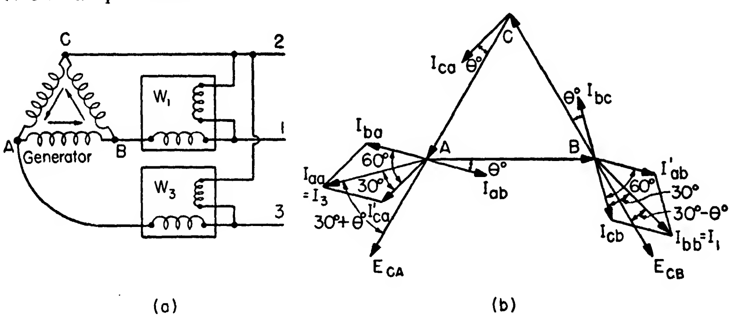


FIG. 8–5. (*a*) Two wattmeters connected to measure the power output of a delta connected generator. (*b*) Vector diagram showing the relations of the current and voltages on the wattmeters in (*a*) when the load is balanced and the phase currents lag $\theta°$ behind their respective phase voltages. Indication of wattmeter $W_1 = E_{CB}I_{bb}$ cos $(30° - \theta°)$; of wattmeter $W_3 = E_{CA}I_{aa}$ cos $(30° + \theta°)$.

In measuring three-phase power by the two wattmeter method, the current coils of the wattmeters are connected in **any two lines** with both potential coils connected across to the **third line,** as indicated in Fig. 8–5(*a*).

Let us assume the load on the generator of Fig. 8–5(*a*) is balanced, but that the phase currents lag their respective phase voltages by $\theta°$. Determine the indications of each wattmeter. The vector diagram for this load is shown in Fig. 8–5(*b*). The usual counter-clockwise sequence of phases is indicated, and phase

currents I_{ab}, I_{bc} and I_{ca} are shown lagging $\theta°$ behind their respective voltages.

The current in line 3 and in the current coil of wattmeter W_3 is the vector sum of I'_{ca} and I_{ba}, which combine at 60° in the line wire and their resultant is shown as I_{aa}, or I_3. The voltage on the potential coil of wattmeter W_3 (Fig. 8–5(a)) is that of phase CA drawn from point A, as E_{CA}. From an inspection of the diagram, the phase angle between E_{CA} and $I_{aa}(I_3)$ is seen to be $30° + \theta°$. Therefore, the indication of wattmeter W_3 is

$$W_3 = E_{CA} I_3 \cos (30° + \theta°).$$

The current in line 1 and in the current coil of wattmeter W_1 is equal to the vector sum of I'_{ab} and I_{cb}, drawn from point B (Fig. 8–5(b)), which also combine at 60°. Their resultant is I_{bb}, or I_1. The pressure on the potential coil of this wattmeter is the voltage of the phase BC, but the positive direction of this voltage is reversed with respect to I_{CB} and I'_{AB}, and therefore, with their resultant I_{bb}, or I_1. Thus, this voltage is reversed and drawn from point B as E_{CB}, as shown. An inspection of the diagram shows the phase angle between the voltage E_{CB} and current I_{bb}, or I_1, to be $30° - \theta°$. Therefore, the indication of wattmeter W_1 in line 1 is

$$W_1 = E_{CB} I_1 \cos (30° - \theta°).$$

We may therefore write

$$W_1 = E_p I_p \cos (30° - \theta°); \tag{7–5}$$

$$W_3 = E_p I_p \cos (30° + \theta°). \tag{8–5}$$

This is further illustrated in the examples below.

Example 4. Assume the voltage of the delta connected generator of Fig. 8–5(a) to be 100 volts, the current per phase 10 amperes, and the angle θ, 20° lagging; (a) What is the total power output of the generator? (b) What will wattmeter W_1 indicate? (c) Wattmeter W_3?

Solution: Since the phase currents combine at 60° in the lines, the line currents, or current in the wattmeters $= \sqrt{3} \times 10 = 17.32$ amperes. Phase voltage = line voltage = voltage on potential coils of the wattmeters = 100 volts.

(a) Indication of wattmeter:

$$\begin{aligned} W_1 &= 100 \times 17.32 \cos (30° - 20°) \\ &= 100 \times 17.32 \cos 10° \\ &= 100 \times 17.32 \times 0.9848 = \textbf{1705 watts.} \quad \textbf{Ans.} \end{aligned}$$

(b) $W_2 = 100 \times 17.32 \cos (30° + 20°)$

$= 100 \times 17.32 \times \cos 50°$

$= 100 \times 17.32 \times 0.0643 =$ **1114 watts. Ans.**

(c) Total power output $= 1705 + 1114 =$ **2819 watts. Ans.**

Also total power $= \sqrt{3}\, E_\rho I_\rho \cos \theta°$

$= \sqrt{3} \times 100 \times 17.32 \times \cos 20°$

$= \sqrt{3} \times 100 \times 17.32 \times 0.9397 = 2819$ watts (check).

Example 5. Assuming for the generator of Fig. 8–5(a) the same voltage and current per phase as in Example 4, but with a load power

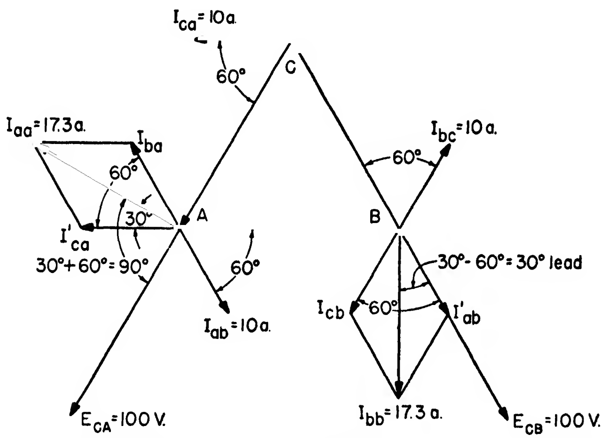


FIG. 9–5. Vector diagram when the phase currents of the generator in Fig. 8–5(a) lag 60° behind their respective phase voltages. Indication of wattmeter $W_1 = E_{CB} I_{bb} \cos [(30° - 60°)$ or $30°]$; of wattmeter $W_3 = E_{CA} I_{aa} \cos [(30° + 60°)$ or $90°]$.

factor of 50 per cent, compute the readings of each wattmeter, and the total power delivered.

Solution: Construct the vector diagram of Fig. 9–5, but note that the phase currents I_{ab}, I_{bc} and I_{ca} of 10 amperes each, lag 60° behind their respective phase voltages. These currents again combine at 60° in the lines, so that each of the line currents $= \sqrt{3} \times 10 = 17.32$ amperes.

From an inspection of the diagram, note the current I_{bb} in line 1 flows in the current of coil of wattmeter W_1 and lags 30° behind the voltage E_{CB} on its potential coil. Also note the current I_{aa} in line 3 flows in the current coil of wattmeter W_3, and lags 90° behind the voltage E_{CA} on its potential coil.

Therefore, the indications of the wattmeters are,

$$\begin{aligned} W_1 &= E_{CB} I_{bb} \cos 30^\circ \\ &= 100 \times 17.32 \cos 30^\circ \\ &= 100 \times 17.32 \times 0.866 = \textbf{1500 watts.} \quad \textbf{Ans.} \end{aligned}$$

$$\begin{aligned} W_1 &= E_{CA} I_{aa} \cos 90^\circ \\ &= 100 \times 17.32 \times 0 \quad = \textbf{0 watts.} \quad \textbf{Ans.} \end{aligned}$$

$$\text{Total power delivered} \quad = 1500 \text{ watts.} \quad \textit{Ans.}$$

Also, Total power $= \sqrt{3} \times 100 \times 17.32 \cos 60^\circ = 1500$ watts (check).

The examples above show that when power is measured in a balanced three-phase system by two wattmeters, the indications of the two instruments are the same, if the power factor is unity, or 1.00. As the power factor decreases, the indications of the wattmeters differ in greater degree until, at a power factor of 0.5, corresponding to a phase difference of 60° between the current and voltage of the phases, one of the wattmeters reads zero, and the other indicates the total power delivered.

In the following problems in this chapter, the fact will be brought out that when the power factor is less than 0.5, that is, when the voltage and the current of the phases differ more than 60°, the angle $(30^\circ + \theta^\circ)$ is greater than 90°. Since the cosine of an angle greater than 90° is negative, the indication of one of the wattmeters will be negative and must be subtracted from that of the other to obtain the total power, or

$$P_{\text{total}} = W_1 - W_3.$$

Therefore, care must be used to be certain a negative reading is indicated. It is always the lower reading instrument which is suspected. This can be checked by removing the potential lead of this instrument from the common wire (2 in Fig. 8–5(a)) and touching it to the line containing the other wattmeter (line 1 in Fig. 8–5(a)). If now **the pointer of the low reading wattmeter reverses,** the original reading is **negative.** If the pointer does **not** reverse, the original reading is positive. In either case the potential lead must be returned to its former connection, and its reading properly subtracted or added in accordance with the above test.

To reverse the deflection of the pointer of a wattmeter, **either** the current or the potential coil, preferably the former, must be reversed in the circuit.

4-5. Measurement of Power in a Y-Connected System — Two-Wattmeter Method. Power in a balanced three-phase, three-wire, Y-connected system, also may be measured by two wattmeters, and their indications will differ in exactly the same way as in the delta connection. So that at unity power factor, the two instruments read alike; at 50 per cent power factor, one wattmeter indicates zero and the other indicates the total power; while at other power factors, the sum (or difference) of the two instrument readings indicates the power delivered. This is illustrated in the following example.

Example 6. Assume the generator of Example 4 and Fig. 8-5(*a*) to be reconnected in Y, 100 volts per phase and to be delivering the same

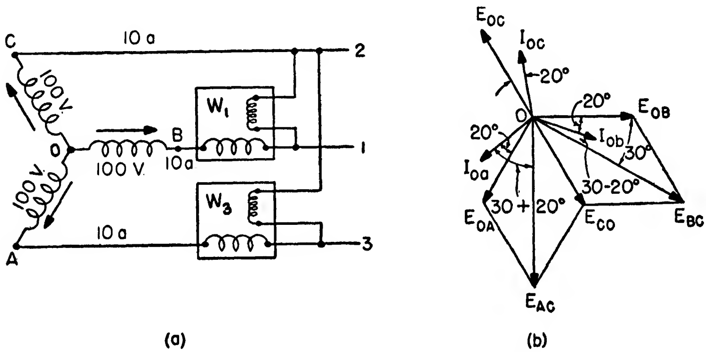


Fig. 10-5. (*a*) Two wattmeters connected to measure the output of a Y-connected generator. (*b*) Vector diagram showing the relation of currents and voltages on the wattmeters in (*a*) when the load is balanced and the phase currents lag 20° behind their respective phase voltages. Indication of wattmeter $W_1 = E_{BC}I_{ob}\cos(30° - 20°)$; of wattmeter $W_3 = E_{AC}I_{oa}\cos(30° + 20°)$.

current of 10 amperes per phase, lagging 20° behind the phase voltage, as in that example. Thus the machine is delivering the same power. The wattmeters are connected, as in Fig. 10-5(*a*), with their current coils in lines 1 and 3, similar to the arrangement in Fig. 8-5(*a*). What will each wattmeter indicate?

From the vector diagram of Fig. 10-5(*b*), the following relations are shown:

In wattmeter, W_1

$$\text{Voltage on potential} = E_{BC}\ (E_{OB} \oplus E_{OC}\ \text{reversed})$$
$$= 100 \times \sqrt{3} = 173.2 \text{ volts.}$$

Current in current coil $= I_{OB} = 10$ amperes.

Phase angle between E_{BC} and $I_{OB} = (30° - \theta°) = (30° - 20°)$;

$$\text{Wattmeter reading} = 173.2 \times 10 \times \cos 10^\circ$$
$$= 173.2 \times 10 \times 0.9848 = \mathbf{1705\ watts.} \quad \textbf{Ans.}$$

In Wattmeter W_3,

$$\text{Voltage on potential coil} = E_{AC}\ (E_{OA} \oplus E_{OC}\ \text{reversed})$$
$$= 100 \times \sqrt{3} = 173.2\ \text{volts.}$$

$$\text{Current in current coil} = I_{OA} = 10\ \text{amperes;}$$
$$\text{Phase angle between } E_{AC} \text{ and } I_{OA} = (30^\circ + \theta^\circ) = (30^\circ + 20^\circ);$$
$$\text{Wattmeter reading} = EI \cos 50^\circ$$
$$= 173.2 \times 10 \times 0.6428 = \mathbf{1114\ watts.} \quad \textbf{Ans.}$$

$$\text{Total power delivered} = 1705 + 1114 = 2819\ \text{watts;}$$

or $$\text{Total power} = \sqrt{3} \times 173.2 \times \cos 20^\circ = 2819\ \text{watts (check).}$$

Note that the wattmeter readings are exactly the same for the same power delivered at the same power factor for both a delta and a Y connected system.

In the solution of the following problems, use counter-clockwise sequence of phases in all vector diagrams.

Prob. 8–5. A 240-volt three-phase delta connected generator supplies 50 amperes per phase at 70.7 per cent power factor. Power output is to be measured by two wattmeters, connected in lines 1 and 3 (see Fig. 8–5(a)). Show a diagram of circuit connections and compute the readings of each wattmeter by aid of vector diagram.

Prob. 9–5. Compute the readings of each wattmeter, if the generator in Prob. 8–5 were Y connected, same phase, voltage and current, if the power factor is 0.8 lagging. Meters are connected in lines 1 and 3. (See Fig. 10–5(a).) Show diagram of connections and vector diagram.

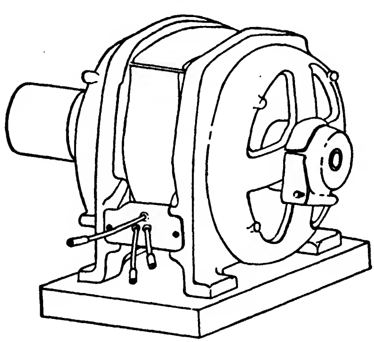


FIG. 11–5. Three-phase induction motor.

Prob. 10–5. Three single phase loads, each drawing 60 amperes at 0.6 power factor lagging, are connected in delta on a 230-volt line. Power is measured by two wattmeters, connected in lines 1 and 2. Show a diagram of connections and compute the reading of each wattmeter by aid of vector diagram.

Prob. 11–5. A 125-volt delta-connected induction motor, Fig. 11–5, takes 12 kw at 0.8 lagging power factor. If power input is measured by two wattmeters, connected in lines 1 and 3, what will each instrument indicate? Show diagram of connections and vector diagram.

Prob. 12–5. What would be the readings of the two wattmeters if a synchronous motor, taking the same power, were used in Prob. 11–5, but with a power factor of 0.8 leading?

Prob. 13–5. What would each wattmeter in Prob. 11–5 read, if the motor had a lagging power factor of 50 per cent, same kw input?

Prob. 14–5. What would each meter in Prob. 11–5 read, if the power factor were 35 per cent lagging, same kw input?

5–5. Power Factor from Wattmeter Readings — Two Wattmeter Method. It was shown in Art. 3–5 that when power in a balanced three-phase circuit is measured by two wattmeters, their indications can be expressed as

$$W_1 = E_{\text{line}} I_{\text{line}} \cos (30° - \theta°);$$

$$W_2 = E_{\text{line}} I_{\text{line}} \cos (30° + \theta°).$$

Or, from the trigonometric formula (see Appendix A)

$$W_1 = EI(\cos 30° \cos \theta° + \sin 30° \sin \theta°); \tag{9–5}$$

$$W_2 = EI(\cos 30° \cos \theta° - \sin 30° \sin \theta°). \tag{10–5}$$

Subtracting equation (10) from equation (9),

$$\begin{aligned} W_1 - W_2 &= 2EI \sin 30° \sin \theta° \\ &= 2EI \times 0.5 \sin \theta° = EI \sin \theta°. \end{aligned} \tag{11–5}$$

Adding equations (9) and (10).

$$\begin{aligned} W_1 + W_2 &= 2EI \cos 30° \cos \theta° \\ &= 2EI \times 0.866 \cos \theta = 1.732EI \cos \theta°. \end{aligned} \tag{12–5}$$

Dividing equation (11) by equation (12),

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{EI \sin \theta°}{\sqrt{3}EI \cos \theta°};$$

or

$$\sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \frac{\sin \theta°}{\cos \theta°} = \tan \theta°. \tag{13–5}$$

Since the tangent of the angle of phase difference between phase voltage and current can be computed from the wattmeter readings, the power factor, or cos θ, may be determined from a table of tangents and cosines.

Example 7. From the wattmeter readings in the system of Example 4, determine the power factor of the circuit.

Solution: From Example 4,

W_1 indicates 1705 watts;
W_3 indicates 1114 watts.

$$\sqrt{3}\,\frac{W_1 - W_3}{W_1 + W_3} = \tan\theta;$$

$$\sqrt{3}\,\frac{1705 - 1114}{1705 + 1114} = \sqrt{3}\,\frac{591}{2819} = \sqrt{3} \times 0.2096 = 0.363;$$

$$0.363 = \tan 20°$$

Thus $\theta = 20°$

Power factor $= \cos 20° = 0.939$. *Ans.*

This value checks with the power factor originally assumed in that example.

Example 8. In measuring the power delivered to a certain balanced three-phase system by the two-wattmeter method, one wattmeter reads 1224 watts, while the other indicates a negative value of 448 watts. What is the power factor of the system?

Solution:

$$\text{Tan}\,\theta = \sqrt{3}\,\frac{1224 - (-448)}{1224 + (-448)} = \sqrt{3}\,\frac{1672}{776} = 3.73$$

$$3.73 = \text{tangent } 75°$$

$$\text{Power factor} = \cos 75° = \mathbf{0.2588.}\quad \textbf{Ans.}$$

Prob. 15-5. Power in a balanced three-phase, three-wire system is measured by two wattmeters which indicate 5000 and 2500 watts respectively. What is the power factor and phase angle between the current and voltage per phase?

Prob. 16-5. What would be the power factor and phase angle, if the wattmeter readings in Prob. 15-5 had been 5000 and −2500 watts respectively?

6-5. Unbalanced Three-Phase System — Two-Wattmeter Method. In three-phase distribution of power, the load is generally balanced. This is desirable in any polyphase system, since both the generators and the line operate more efficiently under this condition. Three-phase motors and transformers, etc., under normal conditions comprise a balanced load. When the load consists of a number of single-phase motors, transformers and lighting circuits, these may be so distributed between the three lines that the load is only slightly unbalanced. However, unbalanced loads do occur, as has been explained in Chapter II, Art. 16. Power in such unbalanced three-phase three-wire circuits can be measured by the two-wattmeter method, as illustrated in the example below.

Example 9. A 125-volt three-phase line supplies three single-phase loads, as follows: Load I, a 4 kw lighting circuit at unity power factor; load II, a 5 hp motor at full load, 82 per cent efficiency and 80 per cent lagging power factor; load III, a 3 kw induction furnace with 60 per cent lagging power factor. These loads are connected across the three lines as shown in Fig. 12–5(*a*). The power taken by the combined load is measured by two wattmeters connected in lines 2 and 3, as shown. What will each wattmeter indicate?

Solution: Since the power taken by the 3-phase system is the sum (or difference) of the readings of the two wattmeters, we have only to

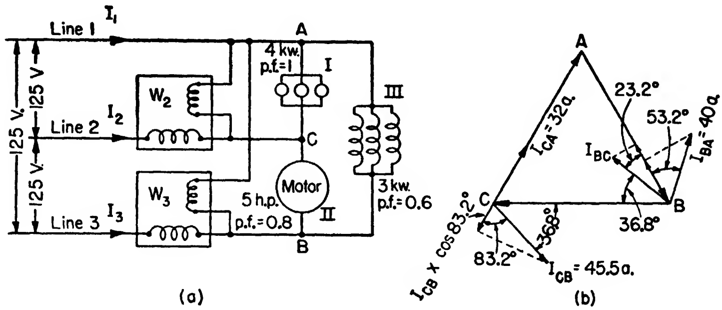


FIG. 12–5. (*a*) Two wattmeters connected to measure the power taken by the unbalanced delta-connected load of Example 9. (*b*) Vector diagram of the currents and voltages in (*a*). Indication of wattmeter W_2 = 125 (32 − 45.5 cos 83.2°); of wattmeter W_3 = 125 (45.5 cos 23.2° + 40 cos 50.2°).

determine the current through each wattmeter and the phase angle which each current makes with the voltage across the respective wattmeter.

The current through W_2 is I_2 and equals the vector sum of I_{CA} and I_{CB}.

The voltage across W_2 is E_{CA}.

In finding the reading of wattmeters W_2 it is much easier to find those **components** of the currents I_{CA} and I_{CB} which are in phase with the voltage E_{CA} and multiply their **algebraic** sum by E_{CA}.

$$I_{CA} = \frac{4000}{125} = 32 \text{ amp in phase with } E_{CA}$$

$$I_{CB} \doteq \frac{5 \times 746}{0.82 \times 125 \times 0.80} = 45.5 \text{ amp lagging } E_{CB} \text{ by } 36.8°$$

In order to find the algebraic sum of those components of I_{CA} and I_{CB} which are in phase with E_{CA}, construct Fig. 12–5(*b*), drawing the voltage triangle first.

Draw I_{CA} in phase with E_{CA}.

Draw I_{CB} lagging E_{CB} by 36.8°, by first sketching in $\mathbf{I}_{BC}$ **lagging** $\mathbf{E}_{BC}$, and then reversing the direction of I_{BC} in order to obtain I_{CB}.

We now draw I_{CB} from the point C on our voltage diagram because we want to find the currents leaving the point C on our circuit diagrams, Fig 12–5(a).

Note particularly that I_{CB} also lags E_{CB} by 36.8° but that the product $I_{CB} \times$ cos (angle between I_{CB} and E_{CA}), is negative and has the value $-45.5 \cos 83.2°$.

Thus W_2 reads,

$$125\ (32 - 45.5 \cos 83.2°)$$
$$= 125\ (32 - 5.4)$$

or

$$W_2 = 125 \times 26.6 = 3325 \text{ watts}$$

Wattmeter W_3 carries the current I_3 which is made up of the vector sum of I_{BC} and I_{BA} flowing from the point B. The voltage across the wattmeter is E_{BA} which is the negative of E_{AB}. In Fig. 12–5(b) we have already drawn I_{BC} **lagging** E_{BC} by 36.8° and **leading** E_{BA} by (60° − 36.8°) or 23.2°.

We have seen that I_{AB} is 40 amp and lags E_{AB} by 53.2°. Therefore I_{BA} must equal 40 amp and lag E_{BA} by 53.2°, as drawn from point B in Fig. 12–5(b). The components of I_{BC} and I_{BA} which are in phase with the wattmeter voltage must be, $I_{BC} \cos 23.2°$ and $I_{BA} \cos 53.2°$, and have the values

$$I_{BC} \cos 23.2° = 45.5 \times 0.919 = 41.8$$
$$I_{BA} \cos 53.2° = 40 \times 0.60 = \underline{24}$$
$$I_{BC} \cos 23.2° + I_{BA} \cos 53.2° = 65.8 \text{ amps.}$$

Wattmeter W_3 therefore reads $125 \times 65.8 = 8225$ watts.

The sum of the readings $(W_2 + W_3) = 3325 + 8225 = 11{,}550$ watts.

Total power delivered $= 8225 + 3325 =$ **11,550 watts. Ans.**

Check,

$$\text{Power delivered to load I} = 4000 \text{ watts;}$$
$$\text{Power delivered to load II} = \frac{5 + 746}{0.82} = 4550 \text{ watts;}$$
$$\text{Power delivered to load III} = \underline{3000 \text{ watts;}}$$
$$\text{Total power delivered} = 11{,}550 \text{ watts, check.}$$

Since power in any balanced or unbalanced three-phase three-wire circuit can be measured by the two-wattmeter method, this is the common, or standard, method of power measurement in such systems.

Prob. 17–5. Figure 13–5 represents three groups of lamps connected to a three-phase system. Each lamp takes 4 amperes in phase with the voltage. (a) What will each wattmeter indicate? (b) What total power is supplied to the lamps?

Prob. 18–5. What current flows in each of the line wires in Prob. 17–5?

Prob. 19–5. Each lamp in Fig. 14–5 takes 3 amperes in phase with the voltage. The output of the motor is 5 hp with a lagging power

factor of 0.80 and an efficiency of 75 per cent. (*a*) What will wattmeter W_1 indicate? (*b*) Wattmeter W_2?

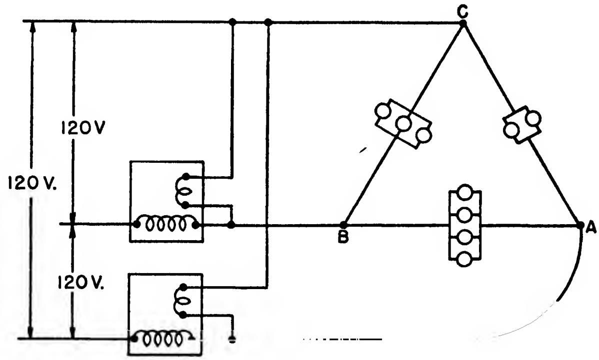


FIG. 13–5. Two wattmeters connected to measure the power taken by an unbalanced delta-connected lamp load at unity power-factor.

Prob. 20–5. What current flows in each line wire in Prob. 19–5?

Prob. 21–5. Three single-phase loads are connected to a three-phase line as indicated in Fig. 15–5, and the power delivered to the system is

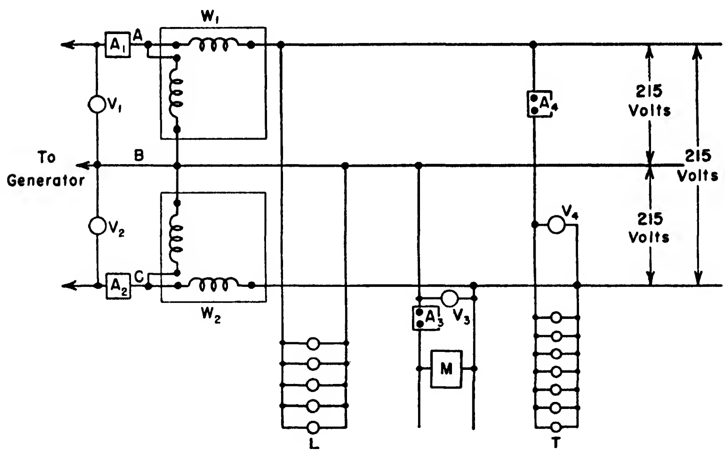


FIG. 14–5. An unbalanced delta-connected three-phase circuit.

measured by wattmeters W_1 and W_2, connected in lines A and B respectively, as shown. (*a*) What does wattmeter W_1 indicate? (*b*) Wattmeter W_2? (*c*) What is the total power delivered? Compute by two methods.

Prob. 22–5. What will each wattmeter indicate in Fig. 16–5?

Prob. 23–5. Find current in each line in Fig. 17–5.

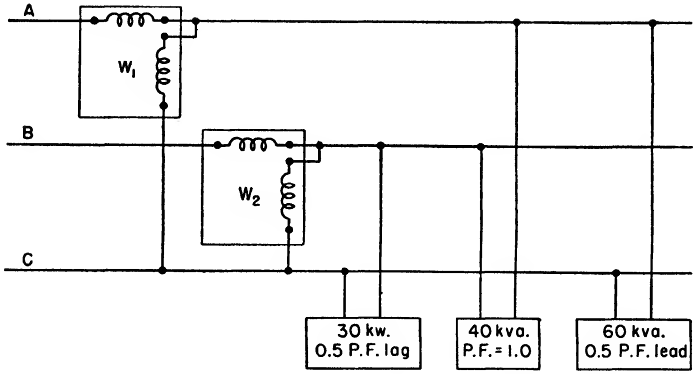


FIG. 15–5. Power to an unbalanced three-phase delta-connected circuit, measured by the two-wattmeter method.

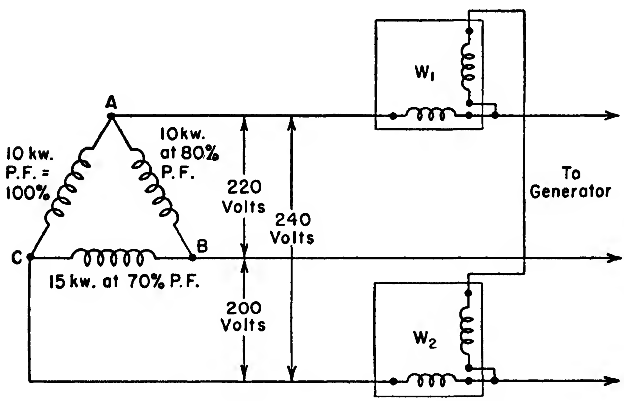


FIG. 16–5. An unbalanced delta-connected three-phase load in which the voltages also are unbalanced.

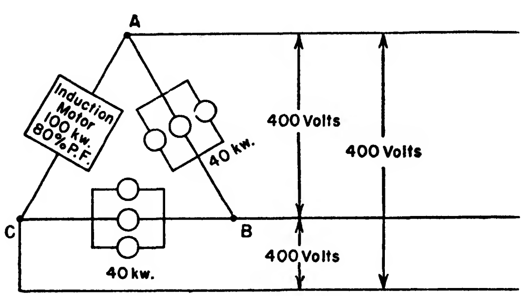


FIG. 17–5. An unbalanced load on a three-phase three-wire system.

7–5. Power Factor in Unbalanced Three-Phase Systems. In a balanced polyphase system, the power factor, as we have seen, is the cosine of phase difference between the voltage and current per phase. This is actually the ratio of the watts to the volt-amperes in each phase and is the same for all phases.

Thus, in a balanced three-phase system, in which W_1, E_1 and I_1 are the watts, volts and amperes in one and the same phase of a generator,

$$\frac{W_1}{E_1 I_1} = \frac{W_2}{E_2 I_2} = \frac{W_3}{E_3 I_3} = \text{power factor} \tag{14–5}$$

where $(W_1 = W_2 = W_3)$, $(E_1 = E_2 = E_3)$ and $(I_1 = I_2 = I_3)$

Also, from equation (14), we may write,

$$\frac{W_1 + W_2 + W_3}{E_1 I_1 + E_2 I_2 + E_3 I_3} = \text{power factor} \tag{15–5}$$

Therefore, the power factor of the balanced system is the ratio of the total power in all the phases of the system to the total apparent power, or volt amperes, in the system, and is equal to the power factor per phase.

But if the system is **unbalanced** and the voltages of the phases are unequal, or the amperages unequal, or the wattages unequal, the power factor of the several phases is not the same. The system, or three-phase, power factor, as computed from equation (15) above, is practically meaningless, because it is a theoretical or fictitious quantity, which does not actually refer to any of the circuits in the generator, or in the load.

If the three-phase system is not greatly unbalanced, it is common practice to compute the **approximate value** of the power factor by dividing the total watts in the circuit by ($\sqrt{3}$ times the product of the average volts between lines and the average amperes per line wire), thus:

$$\text{Power factor} = \frac{\text{total watts}}{\sqrt{3}\,\dfrac{E_1 + E_2 + E_3}{3} \times \dfrac{I_1 + I_2 + I_3}{3}}. \tag{16–5}$$

8–5. Three-Wattmeter Method of Measuring Power in Balanced or Unbalanced Three-Phase Systems. In any **three-phase three-wire** system on balanced, or unbalanced load, if the neutral is available, as in either a Y-connected generator, or load, three

wattmeters may be used to measure the power. In Fig. 18–5, the current coils of the wattmeters W_1, W_2 and W_3 are connected respectively in the three lines and their potential coils are each connected from line to the neutral point, as shown.

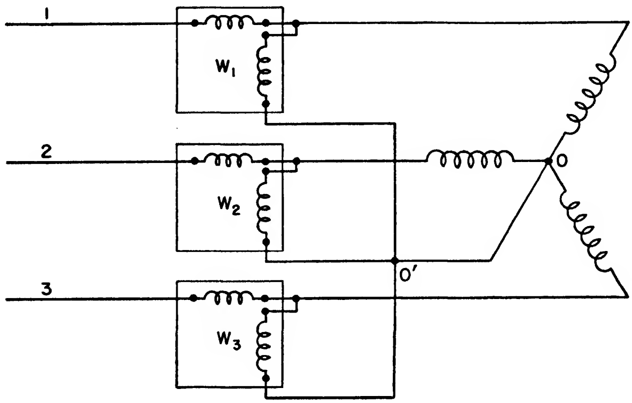


FIG. 18–5. Three-wattmeter method of measuring three-phase power when the neutral point of the system is available.

Also in a Y-connected three-phase **four-wire** system, three wattmeters **must** be used if the load is **unbalanced**, since in this case the neutral also carries current. The three wattmeters are

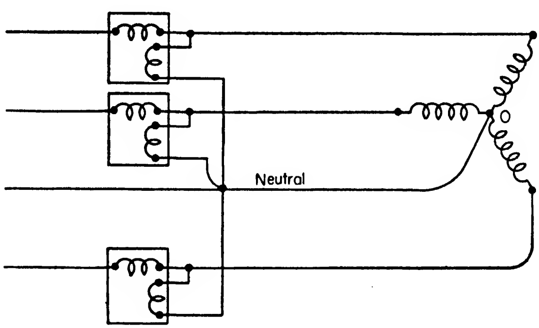


FIG. 19–5. Three-wattmeter method of measuring power in a three-phase four-wire system.

connected in the lines as before, with the potential coils all connected to the neutral line, as shown in Fig. 19–5.

In either case above, each wattmeter indicates the power in one phase, regardless of the power factor, or degree of unbalance of the system.

The total power is the sum of the three wattmeter readings, or

$$P_{\text{total}} = W_1 + W_2 + W_3.$$

If the load is balanced, the three wattmeters read alike, or

$$W_1 = W_2 = W_3.$$

When a neutral point is not available, three wattmeters may also be used to measure the power in a three-phase system. In Fig. 18–5, if the potential coils of the wattmeters have the same resistance, the currents in these potential circuits are the same and form a symmetrical system displaced 120°. Therefore, no current

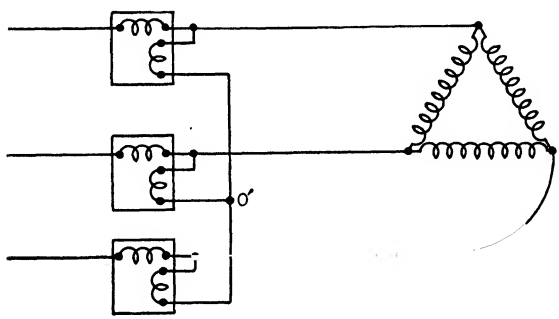


FIG. 20–5. Three-wattmeter method of power measurement in a three-phase system when the neutral is *not* available. If the potential circuits of the wattmeters are identical, they form a symmetrical Y-system displaced 120° with O′, the neutral point.

flows between 0 to 0′; these points are at the same potential and the connection between 0 and 0′ can be broken. Thus point 1 becomes an artificial neutral and each wattmeter, if the load is balanced, reads one third the total power, regardless of the power factor.

Thus the three wattmeters may be connected, as in Fig. 20–5, in a delta connected system. If the resistances of the potential coils are not the same, the potential of point 0′ is shifted and the indications of the three wattmeters differ from each other, but it can be shown that the sum of their readings is equal to the total power. The three wattmeters, connected in this manner, indicate the total power, regardless of the power factor or unbalance of the system.

The measurement of power by the three-wattmeter method, while it takes an extra instrument, has the advantage that, when the power factor of the load varies over a wide range, the indica-

tions of all three instruments are always positive. Thus it is unnecessary to reverse any of their connections, as in the two-wattmeter method.

9-5. One-Wattmeter Method. Balanced Three-Phase Three-Wire Systems. When the load on a three-phase three-wire system, either Y or delta connected, is **balanced** and **steady,** one wattmeter may be used to determine the power. Figure 21-5(a) shows the connections in a motor circuit. The current coil of the wattmeter is connected in any one of the three lines, No. 2 in the figure. The pressure coil is connected through a single-pole double-throw switch S, first to line 1 and the meter read: then the switch is quickly thrown to the other position, transferring the

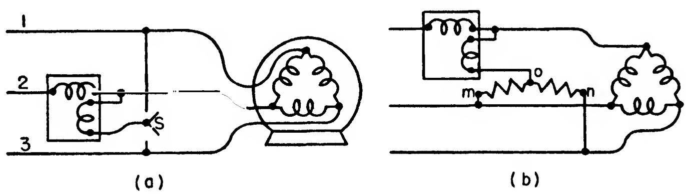


FIG. 21-5. (a) One wattmeter may be used in a balanced circuit to measure power by the two-wattmeter method. (b) Measurement of power in a balanced three-phase circuit by means of one wattmeter and a " Y box." The resistances *om* and *on* must be the same as that of the potential circuit of the wattmeter. The instrument measures one third the total power.

potential coil to line 3 and a second reading taken. The sum of the two readings is the total power delivered to the motor. Thus, when only one wattmeter is available, the power can be measured by the two-wattmeter method. However, this method is applicable to balanced loads only.

Power in a three-phase circuit may also be measured by one wattmeter by means of an arrangement called a Y box. In Fig. 21-5(b), two non-inductive resistances, **om** and **on**, equal in value to that of the potential circuit of the wattmeter, are connected in Y with this circuit across the three lines. Thus an artificial neutral is obtained at 0 and the wattmeter indicates one third the power in the circuit. Here also the measurement applies to a balanced load only, but is correct for any power factor.

Problem 24-5. If the motor of Fig. 21-5(a) takes a line current of 34.6 amperes at 240 volts and 0.939 power factor, compute, by the aid of a vector diagram, the reading of the wattmeter: (a) When the potential circuit of the wattmeter is connected to line 1. (b) When the switch is thrown, connecting the terminal of this circuit to line 3. (c)

What is the total power delivered to the motor? (*d*) Check the value determined in (*c*) with the power computed by the standard equation for power in a three-phase circuit.

10–5. Measurement of Power in Two-Phase Systems — Balanced and Unbalanced Load. When the two phases of a two-phase machine are not connected, as indicated in Fig. 22–5, two wattmeters must be used, if the load is unbalanced; one connected in each separate phase. On a balanced load, regardless of the power factor, one wattmeter reads half the total power.

Figure 23–5 shows the arrangement of two wattmeters for measuring power in a balanced, or unbalanced, three-wire two-

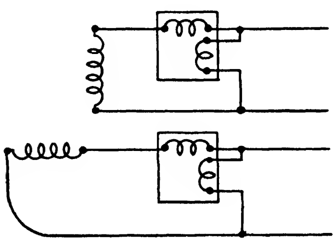


FIG. 22–5. Arrangement of wattmeters to measure power in a two-phase four-wire circuit in which the two phases of the generator and the load are entirely separate.

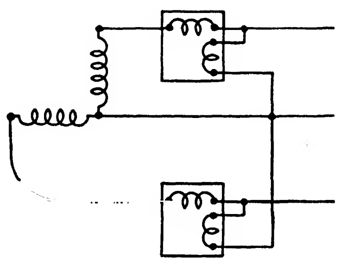


FIG. 23–5. Two wattmeters, with current coils in the outside lines, measure the power in any two-phase three-wire system.

phase system. Note that the current coils of the wattmeters are connected in the outside lines, not in the common line. On a balanced load, regardless of the power factor, the two wattmeters read alike, as each wattmeter indicates half the total power.

If the phases are interconnected and the machine supplies a four-wire system, as indicated in Fig. 24–5(*a*), three wattmeters measure the total power, regardless of the degree of unbalancing or the power factor. When a neutral is brought out to form a five-wire two-phase system, as in Fig. 24–5(*b*), four wattmeters must be used, if the load is unbalanced. On a balanced load one wattmeter measures one quarter of the total power.

Figure 25–5 shows a "mesh" connection of a two-phase alternator. The armature consists of four equal windings displaced 90°. This is similar to the three-phase closed-delta connection; no current will flow in the closed circuit since the sum of

four equal emfs at 90° is zero. A generator so connected is sometimes called a "quarter-phase" or "four-phase" machine. Three wattmeters must be used to measure the power in such a four-wire circuit.

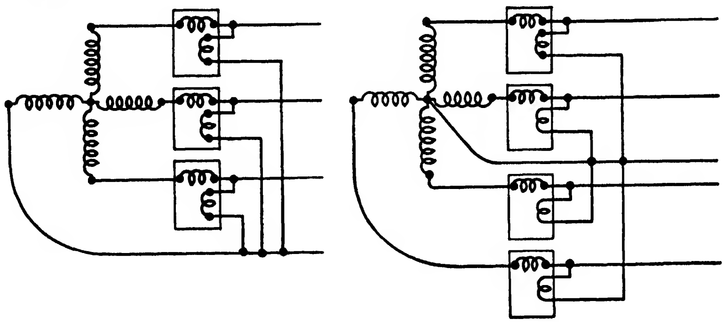


FIG. 24–5. (*a*) In a two-phase four-wire interconnected system, three wattmeters are used to measure the power. (*b*) Four wattmeters must be used to measure the power in two-phase five-wire system if the load is unbalanced.

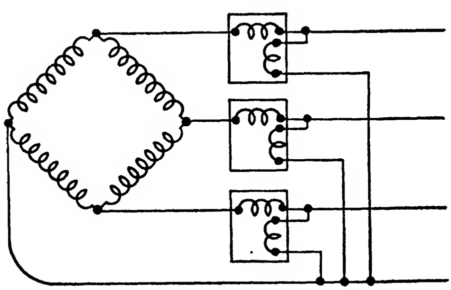


FIG. 25–5. In a two-phase or quarter-phase "mesh"-connected system three wattmeters must be used to measure the power.

In any interconnected polyphase circuit, the number of the wattmeters required may be said to be **one less than the number of line wires.**

11–5. Line Current and Power Factor in Parallel-Connected Three-Phase Loads. When two or more three-phase appliances are connected in parallel to the same line, the power taken by all of them is, of course, the sum of the power taken by each. But in computing the current in the line and the power factor of the system, it is convenient to consider but one phase of each machine. Since the machines are joined in parallel, corresponding phases are in parallel, and the combined current of the several machines **per**

phase is easily determined together with its phase angle. Note the example below.

Example 10. Assume a three-phase Δ-connected induction motor M in Fig. 26–5, taking 100 kw at 0.8 lagging power factor, is operating on the same 230-volt line with a three-phase Δ-connected synchronous motor S, taking 100 kw at 0.9 leading power factor. Find the line current and the power factor of the combined load.

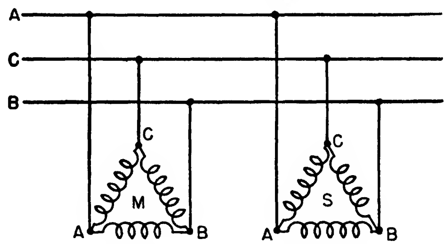


FIG. 26–5. The 100-kw synchronous motor S, at 0.9 leading power factor, is operating in parallel with the 100-kw induction motor M at 0.8 lagging power factor (Example 10).

Solution: Current taken per terminal by the induction motor,

$$I_t = \frac{100{,}000}{\sqrt{3} \times 230 \times 0.8} = 314 \text{ amperes.}$$

$$\text{Current per phase} = \frac{314}{\sqrt{3}} = 181 \text{ amperes.}$$

This current lags 36.8° behind its corresponding phase voltage.
Current taken per terminal by the synchronous motor,

$$I_s = \frac{100{,}000}{\sqrt{3} \times 230 \times 0.9} = 279 \text{ amperes.}$$

$$\text{Current per phase} = \frac{279}{\sqrt{3}} = 161 \text{ amperes.}$$

This current leads its corresponding phase voltage by 25.8°.

Thus the vector sum of the two phase currents above is the total current per phase for both motors. The current per phase for each of these motors is shown in Figs. 27–5(a) and 27–5(b).

In the vector diagram of Fig. 27–5(c), if E_{AB} is the voltage across the phase AB of both motors, the total current per phase for the system is computed as follows:

$$I_{\text{phase}} =$$

$$\sqrt{(181 \cos 36.8° + 161 \cos 25.8°)^2 + (181 \sin 36.8° - 161 \sin 25.8°)^2}$$

$$\sqrt{(181 \times 0.8 + 161 \times 0.9)^2 + (181 \times 0.6 - 161 \times 0.436)^2}$$

$$= \sqrt{289.7^2 + 38.4^2} = 292.2 \text{ amperes.}$$

Total current per line per system $= \sqrt{3} \times 292.2 =$ **506 amperes. Ans.**

$$\text{Power factor of system} = \frac{289.7}{292.2} = \mathbf{0.99\ lagging.}\quad \textbf{Ans.}$$

Total Kva $= \sqrt{3} \times 230 \times 506 = 201.7.$

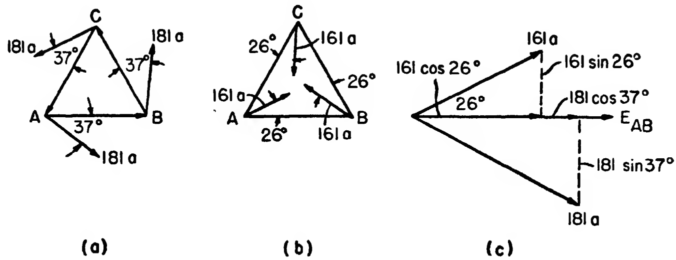


FIG. 27–5. (*a*) Relation of phase currents and voltages in the induction motor *M* of Fig. 26–5. (*b*) Relation of phase currents and voltages in the synchronous motor *S* of Fig. 26–5. (*c*) Vector diagram to determine the total current per phase taken by both motors of Fig. 26–5.

If only the total Kva and power factor of the system are desired, these may be determined by the " power method " as described in Chapter III, Art. 9, which applies either to single-phase or to polyphase loads. The solution of the system of Example 10, by this method, is shown in Example 11 below.

Example 11. Kva taken by induction motor of Example 10 is

$$\frac{100}{0.8} = 125 \text{ Kva, lagging } 36.8°.$$

$$\text{Kva taken by synchronous motor} = \frac{100}{0.9} = 111 \text{ Kva leading } 25.8°.$$

From Fig. 28–5, total Kva of the system

$$= \sqrt{(125 \cos 36.8° + \cos 25.8°)^2 + (125 \sin 36.8° - 111 \sin 25.8°)^2}$$

$$= \sqrt{(100 + 100)^2 + (75 - 48.4)^2} = 201.7 \text{ Kva.}$$

This checks closely with the result in Example 10.

$$\text{Power factor} = \frac{200}{201.7} = 0.99 \text{ lagging.}$$

Prob. 25–5. At a certain woolen mill a 600-kva synchronous motor is used to improve the power factor of a 1050-kw induction motor load which has a power factor of 64 per cent. At what power factor does the synchronous motor operate, if it raises the total power factor to 85 per cent when loaded to take 200 kw? Motors are three-phase at 550 volts.

Prob. 26-5. A 500-Kva three-phase 440-volt synchronous motor is operated at rated Kva at 84 per cent leading power factor on the same line with 440-volt three-phase induction motors, totaling 1000 Kva at 80 per cent power factor. What is the power factor of the line and the total current per line?

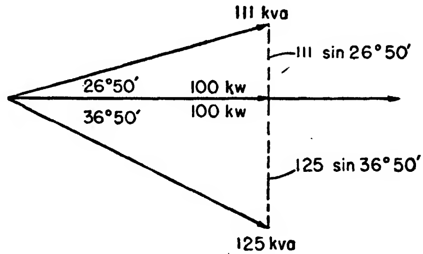


FIG. 28-5. Vector diagram to determine the power factor of the line and total kva supplied to the two motors of Fig. 26-5, by the " power " method (Example 11).

12-5. Polyphase Wattmeter. When power is to be measured by the two-wattmeter method, a polyphase wattmeter may be used. This instrument is the type generally seen on switchboards, and is essentially two single wattmeters in one, as the entire

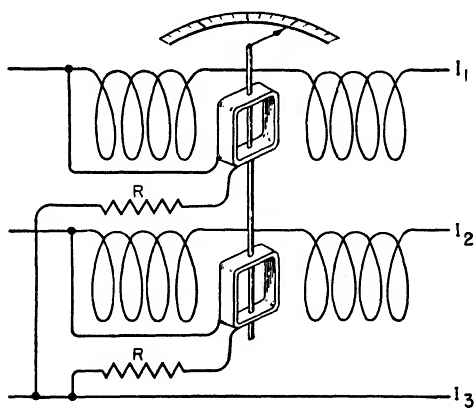


FIG. 29-5. Connection of a polyphase wattmeter in a three-phase line.

mechanism is assembled in a single case. The dynamometer type is the more common. It consists of two sets of current coils and two potential coils. The potential coils are both mounted on the same spindle to form one moving element. A pointer is attached to the spindle which moves over a single scale, as indicated in Fig. 29-5.

Torque is exerted between each potential coil and one set of current coils exactly as in a single phase wattmeter. Also the arrangement of the coil connections in a three-phase circuit is similar to that of two single wattmeters: that is, one set of current coils is connected in each of two line wires with one end of each potential coil joined to the third line, as shown. The torque on the moving element is the net torque on the two potential coils and the deflection of the pointer is proportional to the total power in the circuit.

Polyphase wattmeters of the portable type are also manufactured.

13–5. Power-Factor Meter. The usual type of power-factor meter, particularly for switchboards, is based on the fact that when

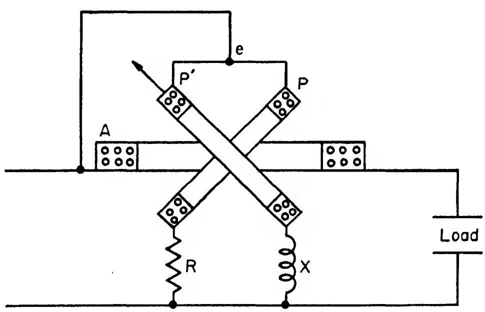


Fig. 30–5. Single-phase power-factor meter — dynamometer type.

a coil, carrying a current and free to turn, is placed in a magnetic field, it assumes a position such that the plane of the coil is perpendicular to that field.

Single-phase meter. In Fig. 30–5, A is a fixed coil connected in the line. P and P' are flat coils, wound of fine wire and fastened rigidly together at approximately 90 space degrees on a spindle which is free to turn; that is, there is no control, such as a spring, connected to the moving element. To the spindle is attached a pointer which moves over a scale, graduated, either in degrees or in values of the cosines (power factor). One terminal of each of these moving coils is connected at e, as shown in the figure, and to the same side of the circuit as A. The other end of coil P is connected through a non-inductive resistance, R, to the other side of the circuit; while a high inductive reactance X, is connected in series with coil P'. The currents in the two coils thus differ in phase by approximately 90 electrical, or time, degrees.

If it be assumed that the currents in coil P and P' differ in phase by exactly 90°, then the current in P is in phase with the voltage, while that in P' lags 90° behind the voltage.

On a load of unity power factor, the current in P' and, therefore, its flux lags 90° in time behind the flux of coil A. The fluxes of P' and A are thus always 90° out of phase, and the net torque is zero. The current and flux of P are in phase with that of A, and torque is exerted causing P to turn into the plane of coil A.

On a load of zero power factor, the current and flux of P is displaced 90° in time from the flux of A and no torque is developed; while the current in P' and its flux is now in phase with that of A and a torque is exerted, causing P' to turn into the plane of A.

Thus the coils P and P' move through a space angle of 90° between unity and zero power factor, and assume an intermediate

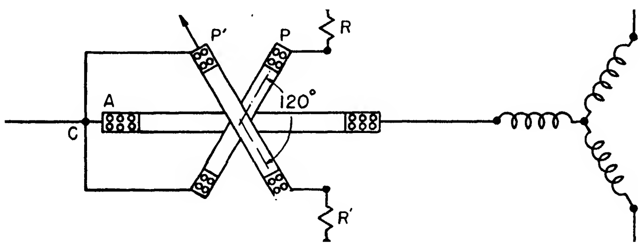


FIG. 31-5. Three-phase power-factor meter — dynamometer type.

position for power factors between these values. If the power factor is leading, the torque is such that the coils move in the opposite direction. Thus, at unity power factor, the pointer stands at the midpoint on the scale, and moves in one direction for lagging currents and in the opposite direction for leading currents.

Since no circuit can be composed of inductive reactance only, the currents in coils P and P' cannot differ in phase by exactly 90°. However, the meter reads correctly, when the space angle between the two moving coils is made equal to the difference in the phase angle between their currents.

The meter also may be constructed with two fixed coils connected in series and placed on the same axis. The moving element in this case is mounted between them. A condenser may also be substituted for the inductive reactance in series with one coil.

The current is lead into the moving coils through flexible strips of silver foil, which offer no appreciable opposition to their free movement.

Three-phase meter. If the coils P and P' are fastened to the spindle at a space angle of 120°, the instrument can be used to measure three-phase power factor, **if the load is balanced, or nearly so.** The fixed coil (or coils) is connected in one of the three lines, as shown in Fig. 31–5; and the common terminals of P and P' are connected to the same line at C. The other terminals are connected respectively in series with non-inductive R and R' to the other two lines.

14–5. Frequency Meter. The same principle, employed in the power-factor meter described above, is also used in the construction of an instrument to indicate frequency.

In the frequency meter, the fixed coil (or coils) is made of fine wire and connected in series with the movable coil P and P', mounted rigidly on the spindle at 90 space degrees, which in turn are joined in parallel through the resistance R and reactance X respectively, as shown in Fig. 32–5. The entire arrangement is connected across the two wires of a two-wire circuit or across any two wires of a three-phase circuit.

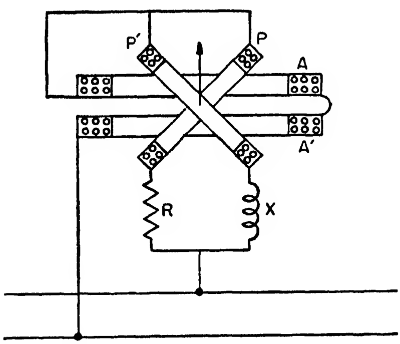


FIG. 32–5. A frequency meter — dynamometer type.

Since the instrument has no control, when no voltage is impressed upon it, the pointer may stand at any position on the scale.

When potential is applied, the current in P' is such that torque is exerted, tending to move the coil and the pointer to the left, while that in P tends to move the pointer to the right. Thus the torques on the two coils are in opposite directions. If the frequency is such that the current and, therefore, the torques exerted on the two coils are equal, the pointer stands at the center, or mid-point of the scale. An increase in frequency increases the reactance, X, and decreases the current and the torque of P'. The torque of coil P now predominates and the pointer moves to the right. A decrease in frequency decreases X, and increases the current in P' and its torque, and the pointer now moves to the left.

In an instrument designed for a 60-cycle circuit, the circuits of P and P' are so proportioned that at the frequency the pointer stands

approximately at the midpoint of the scale which is marked 60 cycles. The scale is graduated from this point, higher frequencies to the right and lower to the left.

15–5. Reactive Factor Meter. A single wattmeter may be used in a balanced three-phase circuit to indicate the "**reactive factor,**" or a sine of the angle θ between phase voltage and current.

The current coil of the wattmeter is connected in any one of the three lines, while the terminals of the potential coil are joined to the other two lines, as in Fig. 33–5(*a*).

At unity power factor, the phase currents I_{ab}, I_{bc} and I_{ca} of Fig. 33–5(*b*) are in phase with their respective voltages. The current in line 2 and in the current coil of the wattmeter is I_2.

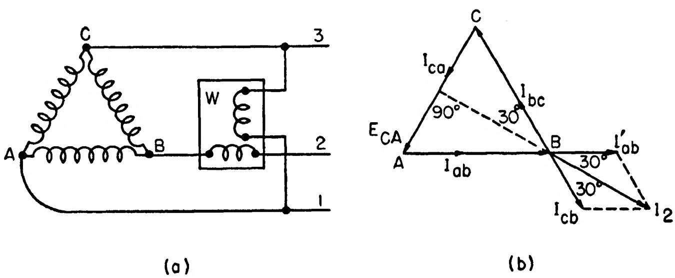


FIG. 33–5. (*a*) A single-phase wattmeter connected in a three-phase circuit to indicate the "reactive factor" of the load. (*b*) At unity power-factor the line current I_2, in the current coil of the wattmeter in (*a*), is displaced 90° from the voltage *CA*, across the potential coil of the instrument. The wattmeter, therefore, reads zero at unity power-factor load.

This current is seen to be 90 electrical, or time degrees from the voltage *CA* on the potential coil. Thus, the instrument reads zero ($\sin \theta = 0$).

At zero power factor, the currents in the two coils of the wattmeter are in time phase with each other, as can be seen from the diagram, and the instrument indication is a maximum.

Therefore, the readings of the meter are proportional to the sine of the angle of phase difference, or reactive factor of the load. The pointer can be set to indicate a zero value at the midpoint of the scale; and the scale calibrated to read reactive factor directly, leading values in one direction and lagging values in the other; or a special inverse scale may be used to indicate the power factor rather than the reactive factor.

In circuits operating close to unity power factor, the reactive

factor, or $\sin\theta$, changes more with a given change in power factor near 100 per cent than does the cosine; and hence, the indications of the above instrument are more precise as a power-factor meter than are those of the usual type of power-factor meter. This instrument is used principally in systems supplying synchronous converters, where close adjustment of power factor is desirable.

SUMMARY OF CHAPTER V

POWER in a BALANCED 3-PHASE SYSTEM — EITHER Y OR DELTA CONNECTED is computed in terms of VOLTS BETWEEN LINES and LINE CURRENT, and is expressed as

$$P = \sqrt{3}\, E_L I_L \cos\theta.$$

APPARENT POWER is Volt-amperes in a balanced 3-phase circuit is computed as

$$VA = \sqrt{3}\, E_L I_L.$$

REACTIVE POWER or VARS is computed as

$$VARS = \sqrt{3}\, E_L I_L \sin\theta.$$

In above equations

$$\begin{aligned} E_L &= \text{voltage between lines;} \\ I_L &= \text{current per line;} \\ \cos\theta &= \text{the power factor;} \\ \sin\theta &= \text{the reactive factor.} \end{aligned}$$

These equations apply either to the output of the generator or the input to the load.

The POWER FACTOR of a balanced 3-phase circuit is the cosine of the angle between the volts per phase and the current per phase.

Power Factor in unbalanced circuits has little meaning.

MEASUREMENT OF THREE-PHASE POWER.

TWO WATTMETER METHOD.

POWER in BALANCED or UNBALANCED three-phase systems (Y or delta connected) can be measured by two wattmeters. The current coils of the instruments are inserted, respectively, in two of the three lines; and the potential coils connected, respectively, between each of these lines and the third. The algebraic sum of the readings of the two instruments is equal to the power in the circuit.

ON A BALANCED LCAD.

AT UNITY POWER FACTOR, both instruments read alike.

AT POWER FACTORS LESS THAN 1.00 AND GREATER THAN 0.5, the indications of the instruments are not the same.

AT POWER FACTOR OF 0.5, one instrument reads zero and the other measures the total power in the circuit.

AT POWER FACTORS LESS THAN 0.5, the indication of one instrument is NEGATIVE, and the DIFFERENCE of the two readings equals the total power in the circuit.

POWER FACTOR FROM WATTMETER READINGS. When power in a balanced load is measured by two wattmeters,

$$\tan\theta = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} \text{ where } \theta = \text{angle of displacement}$$

between current and voltage per phase. $\cos\theta$ = power factor.

THREE-WATTMETER METHOD. Three wattmeters may be used to measure the power in either a balanced or unbalanced three phase system. The current coils of the three instruments are connected, respectively, in one of the three lines; while the potential coils are connected, respectively, between that line and the neutral, when it is available, as in a Y-connected circuit. When the neutral is not available, one terminal of all the potential coils are joined together. The sum of the readings of the three instruments indicate the total power.

IN A THREE-PHASE FOUR-WIRE SYSTEM, THREE WATTMETERS MUST BE USED, IF THE LOAD IS UNBALANCED.

ONE-WATTMETER METHOD — BALANCED LOAD ONLY.

FIRST METHOD: The current coil of the wattmeter is connected in one of the three lines, and potential coil is connected between this line and the second through a double throw switch, and a reading taken; the switch is quickly thrown, connecting potential coil to the third line, and a second reading taken. The sum of the two readings is the total power, if the load is steady.

SECOND METHOD: Another method is that of connecting the potential coil to a "Y box," consisting of two resistances, having the same resistance as that of the potential circuit of the meter. These resistances are connected to the other two lines, and an artificial neutral point is formed at the terminal of the potential coil, and the instrument measures one third the power in the circuit.

MEASUREMENT OF TWO-PHASE POWER — UNBALANCED LOAD.

IN A 3-WIRE SYSTEM, TWO WATTMETERS ARE USED. Current coils are connected respectively in the outside lines with potential coils connected, respectively, between these lines and the common line.

IN A 4-WIRE SYSTEM in which the phases are **NOT INTERCONNECTED**, power is measured by **TWO WATTMETERS** — one instrument connected in each phase. When the phases **ARE INTERCONNECTED**, three wattmeters must be used.

In a 5-wire system, four wattmeters must be used.

In general. **IN ANY INTERCONNECTED POLYPHASE SYSTEM,** the number of wattmeters necessary to measure the power must be one less than the number of line wires.

THE POLYPHASE WATTMETER consists of two sets of current coils connected, respectively, in two lines of the three-phase circuit; and two potential coils mounted on the same spindle and connected to the third line. The indication of the instrument is proportional to the net torque on the spindle, and is proportional to the total power.

The pointer is attached to the spindle and moves over a single scale. All coils are enclosed in a single case. This instrument measures power by the two-wattmeter method and is the type generally seen on switchboards.

POWER FACTOR, FREQUENCY and REACTIVE FACTOR meters of the dynamometer type are generally used on switchboards.

PROBLEMS ON CHAPTER V

Prob. 27–5. A certain 2300-volt Y-connected generator is rated at 2000 Kva. (*a*) What is its full load current rating per terminal? (*b*) What is the rated phase current and phase voltage?

Prob. 28–5. If the machine in Prob. 27–5 is reconnected in delta, what is its voltage and current rating?

Prob. 29–5. (*a*) If the generator in Prob. 27–5 supplies a load of 2000 Kw at 2300 volts and 0.8 power factor, what is the current per terminal in per cent of full load? (*b*) What is the load in Kva? (*c*) What KVARS, or reactive Kva, does it supply?

Prob. 30–5. Three non-inductive resistors of 20 ohms each are connected in Y on a 230-volt, 3-phase 3-wire line. (*a*) What is the voltage across each resistor? (*b*) The current in each line? (*c*) The power taken by the combination? (Solve by two methods.)

Prob. 31–5. Answer (*a*), (*b*), and (*c*) of Prob. 30–5, if the resistors in that problem are connected in delta on the same three-phase line.

Prob. 32–5. In Prob. 30–5, if a neutral wire is brought out from the junction point of the three resistors to the neutral of the generator, what would be the current in each of the four line wires? Show vector diagram.

Prob. 33–5. What is the ratio of the power taken by the two circuits in Probs. 30–5 and 31–5 above?

Prob. 34–5. A 200-hp, 2300-volt delta-connected induction motor at full load has an efficiency of 92 per cent and a power factor of 0.88. Compute (*a*) the line current; (*b*) the phase current; (*c*) the kw input to the motor; (*d*) the kva input; (*e*) the kvar input.

Prob. 35–5. If the motor of Prob. 34–5 is supplied by a Y-connected generator and is the only connected load, what will be the phase voltage and current in the generator? Neglect losses in the line between motor and generator.

Prob. 36–5. A 5000-kva, 6900-volt, 3-phase, Y-connected alternator is supplying 3500 kw at 6900 volts and 400 amperes. (*a*) What is its power factor? (*b*) The kva delivered? (*c*) The kvars?

Prob. 37–5. A 100-hp 2300-volt Y-connected induction motor has an efficiency of 0.92 and a power factor of 0.85 at full load. (*a*) What

is the rated current per terminal and the kva input? (*b*) What is the phase voltage and current?

Prob. 38–5. (*a*) If the motor in Prob. 37–5 is reconnected in delta, what will be its new voltage rating? (*b*) What is the current per terminal at full load? (*c*) The phase current?

Prob. 39–5. A 575-volt, 3-phase synchronous motor takes 150 amperes and 100 kw from the line. (*a*) What is its power factor? (*b*) What kva does it take?

Prob. 40–5. Three impedances, each consisting of 12 ohms resistance and 16 ohms inductive reactance, are connected in Y across a 460-volt 3-phase line. (*a*) What is the line current and the total power consumed in the impedances? (*b*) What is the power factor of the circuit?

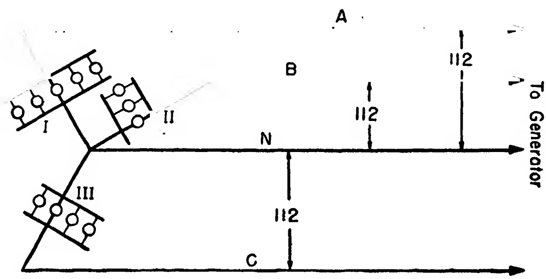


FIG. 34–5. The three banks of lamps are Y-connected to a four-wire three-phase system.

Prob. 41–5. In Fig. 34–5, each lamp takes 5 amperes at unity power factor. What current flows in each of the four lines? Show vector diagrams.

Prob. 42–5. A 460-volt, 3-phase generator supplies 600 kw to a load at 0.8 power factor. The power is measured by the two-wattmeter method. What will each wattmeter indicate? Show diagram of connections and solve by means of a vector diagram.

Prob. 43–5. Three single-phase loads, each taking 20 kw at 0.707 power factor lagging, are connected in delta to a 230-volt 3-phase line. If the power input is measured by two wattmeters, compute the reading of each wattmeter. Show diagram of connections and vector diagram.

Prob. 44–5. If the power input to the motor of Prob. 37–5 is measured by two wattmeters, what will each instrument indicate?

Prob. 45–5. The a-c power input to a 500-volt delta-connected synchronous motor is 12 kw with a leading power factor of 0.9. If power is measured by two wattmeters, what will each read?

Prob. 46–5. The power input to a 3-phase induction motor is measured by two wattmeters. When the three line voltages are each

600 volts, one wattmeter reads 7.35 kw and the other, 3.85 kw. (*a*) What power does the motor take? (*b*) What is the power factor?

Prob. 47–5. Power in a balanced three-phase circuit is measured by two wattmeters. When the voltage between each pair of line wires is 500 volts, one wattmeter reads +60 kw and the other −25 kw. (*a*) What is the power factor? (*b*) The line current?

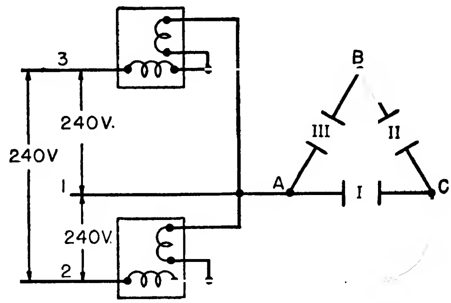


FIG. 35–5. Two wattmeters connected to measure the power in an unbalanced three-phase system.

Prob. 48–5. The power input to an unbalanced 240-volt 3-phase load is measured by two wattmeters, connected as shown in Fig. 35–5. Load I is 7.2 kva at 0.6 power factor lagging; load II, 12 kva at 0.8 power factor lagging; load III, 6 kva at 0.5 power factor lagging. What is the reading of each wattmeter?

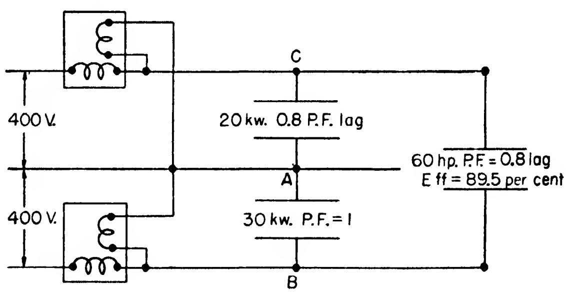


FIG. 36–5. An unbalanced delta-connected three-phase system, in which the three-phase voltages are equal.

Prob. 49–5. What would each wattmeter in Prob. 48–5 read, if the power factor of load II were leading? All other data the same.

Prob. 50–5. Three single-phase loads are connected in delta across a 400-volt 3-phase line, as indicated in Fig. 36–5. (*a*) What will each wattmeter indicate when connected in each line as shown? (*a*) What is the current in each line?

Prob. 51–5. Power supplied to the 440-volt 3-phase circuit of Fig. 37–5 is measured by the two wattmeters connected as shown. (*a*) What will each instrument indicate? (*b*) What is the total power supplied? (Check by two methods.)

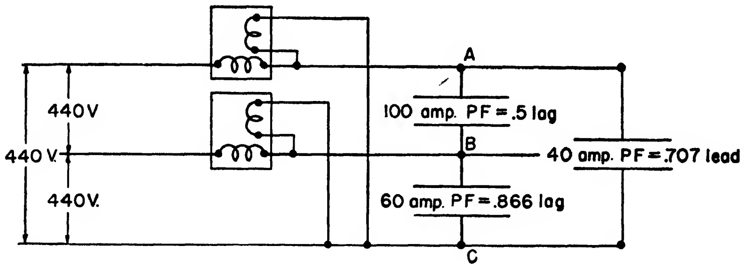


FIG. 37–5. To determine the indications of each wattmeter on an unbalanced three-phase load.

Prob. 52–5. The voltage between lines in Fig. 38–5 is 230 volts. Load *L* takes 40 amperes at unity power factor; load *T*, 60 amperes at unity power factor; and the motor, *M*, 80 amperes at 0.8 lagging power factor. Potential circuits of wattmeters are alike. (*a*) What is the voltage across the pressure coil of each wattmeter? (*b*) What current

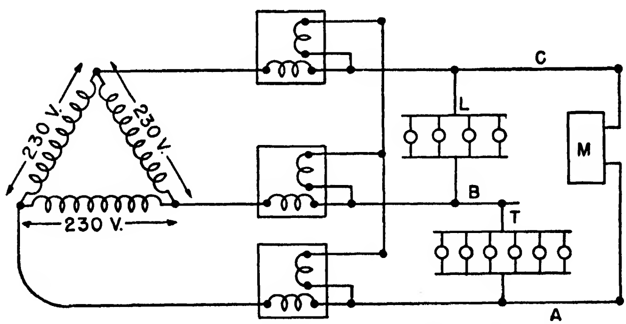


FIG. 38–5. Power to an unbalanced three-phase delta-connected load is measured by three wattmeters.

flows in each line wire? (*c*) How much power is taken altogether by the three loads? (*d*) What will each wattmeter indicate? (*e*) Compare the total power, as indicated by the three wattmeters, with that computed in (*c*).

Prob. 53–5. A 240-volt 4-wire 3-phase generator supplies an unbalanced load, as indicated in Fig. 39–5. The phase voltages are all equal. Power is measured by three identical wattmeters, connected as shown. (*a*) What will each wattmeter indicate? (*b*) What current flows in the neutral wire?

Prob. 54–5. In a 230-volt 2-phase 4-wire motor, having a power factor of 85 per cent, the two phases are balanced and entirely separate. At full load, the motor takes 15 amperes per terminal. (*a*) What power does it take at this load? (*b*) Show by diagram how you would connect the wattmeters to measure the load. (*c*) What would each wattmeter read?

Prob. 55–5. (*a*) If the motor in Prob. 54–5 were operated at correct voltage on a 3-wire 2-phase system, what current would each line carry? (*b*) What should be the voltages between line wires? (*c*) Show by diagram how you would connect wattmeters to measure the power input. (*d*) What would each wattmeter indicate?

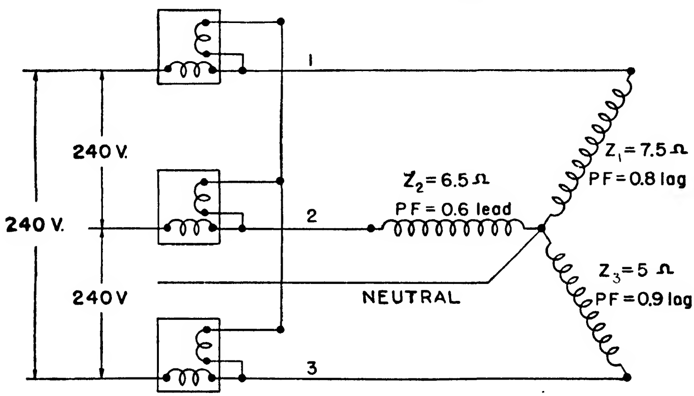


FIG. 39–5. Power to an unbalanced three-phase four-wire system is measured by three wattmeters.

Prob. 56–5. A 1500-kva, 2300-volt, two-phase generator has two windings. (*a*) What is the rated current per phase? (*b*) If the mid-points of the phases are joined and the machine supplies a five-wire system, show by diagram the connection of the wattmeters necessary to measure the output under all conditions of load. (*c*) What should be the voltage between the neutral and each of the other lines?

Prob. 57–5. A generator supplies two induction motors in parallel which take 7 kva each at 230 volts. Each has a power factor of 80 per cent lagging. (*a*) What is the total kva output of the generator? (*b*) Total watts output? (*c*) Power factor of the line? (*d*) (1) What is the current in each line wire, if the system is single-phase? (2) If the system is two-phase four wire? (3) If the system is three-phase three wire?

Prob. 58–5. In order to improve the power factor of the line in Prob. 57–5, one of the induction motors is exchanged for a synchronous motor, which carries the same load but is adjusted to take a leading current at 0.9 power factor. (*a*) What apparent load in kva does the synchronous motor take? (*b*) What is the total kva output of the

generator and compare with (*a*) in Prob. 57–5? (*c*) What is the power factor of the line? (*d*) What is the current in each line wire, if the system is single-phase? Two-phase four-wire? Three-phase three-wire?

Prob. 59–5. An induction motor, taking a lagging current of 20 amperes with a power factor of 0.75, is connected in parallel with a synchronous motor, taking a leading current of 35 amperes with a power factor of 0.85. What is the line current of the 230-volt 3-phase system which supplies the two motors?

Prob. 60–5. What is the power factor of the system in Prob. 59–5?

Prob. 61–5. If the power to the combined load in Prob. 59-5 is measured by the two-wattmeter method, what would each instrument indicate?

CHAPTER VI

THE ALTERNATOR-CONSTRUCTION AND ARMATURE WINDINGS

The alternating-current generator, or alternator, does not differ fundamentally from the direct-current generator. Both machines have a field structure and a field winding excited by direct current. And both machines have an armature in which the emf is generated. The two machines, however, differ in several respects.

First, the d-c generator is usually self-excited, while the alternator — except in rare cases — is separately excited from an outside source.

Second, the armature of the d-c machine is always the rotating member, while in the alternator, the armature generally is stationary and the field rotates. Small low-voltage a-c generators often have a rotating armature and a stationary field, but all large alternators are of the rotating-field type.

The generation of an emf in an armature conductor depends solely upon the **relative** motion of the conductor and the field flux; so **either** the armature or the field may be the rotating member. In the d-c generator, due to the commutator, the armature must rotate; for, if the armature is stationary, the brushes must rotate with the field. Since the alternator has no commutator, either the field or the armature may be the rotating member.

In all a-c machines the **rotating member** is generally called the **rotor**, and the **stationary member** the **stator.**

1–6. Advantage of the Rotating-Field-Type Alternator. In a polyphase alternator with rotating armature, the terminals of the armature winding must be brought out to **three or more** collector rings for carrying the current from the armature to the external circuit, as shown in Fig. 1–6. These collector rings are exposed and, except in low-voltage alternators, are difficult to insulate from the frame of the machine and from one another. This is particularly true when the armature is wound for such voltages as 2300, 6900, or 13,800 volts — the potential at which many alternators are operated. At these voltages the collector rings become a source of trouble, due to arc-overs, short circuits, etc., besides

offering an additional hazard to the operator. Furthermore, an armature wound for such voltages must be very carefully insulated. It is more difficult to insulate such a winding when it is subjected to stresses due to centrifugal forces and the vibration caused by rotation.

On the other hand, a stationary armature, such as shown in Fig. 2–6, can be more easily insulated. It is not subjected to

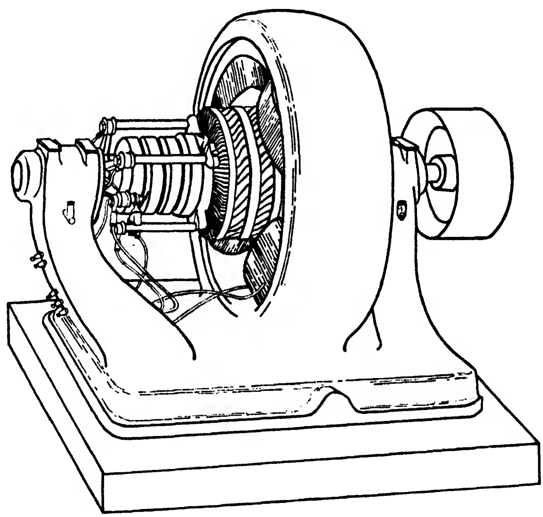


FIG. 1–6. An alternating-current generator having a revolving armature.

centrifugal forces and requires no a-c collector rings. The terminals can be brought out through the frame of the machine, and the leads continuously insulated from the winding to the external circuit.

In a revolving field alternator, the exciting current is supplied to the field coils through two collector rings only, as indicated in Fig. 3–6, which is the field structure for the armature of Fig. 2–6. This current is supplied at low-voltage d-c, from 110 to 250 volts, and therefore these rings are not a particular source of trouble.

The stator of a large slow-speed water-wheel alternator is shown in Fig. 4–6.

In high-voltage low-speed alternators and in high-speed turbo-alternators it is often difficult to find space on the armature for the necessary conductors. This can only be obtained by increasing the depth of the slots in the armature core. This **decreases** the

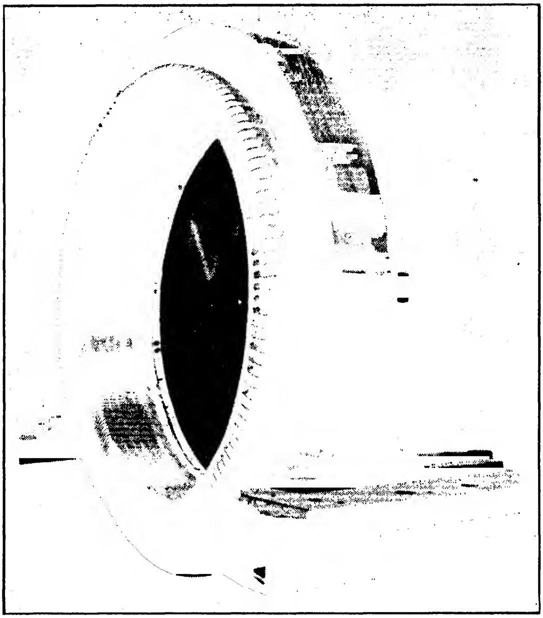


FIG. 2–6. The stationary armature of an a-c generator with a revolving field. (*General Electric Co.*)

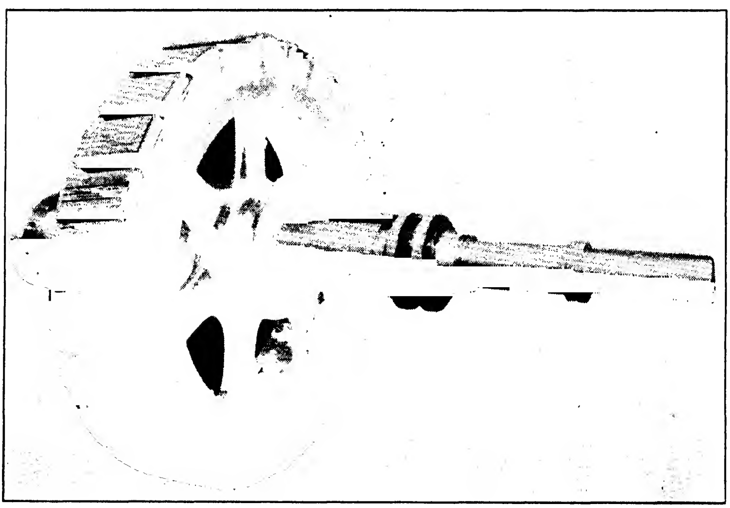


FIG. 3–6. The revolving field of an a-c generator showing the collector rings by means of which direct current is supplied to the field coils. (*General Electric Co.*)

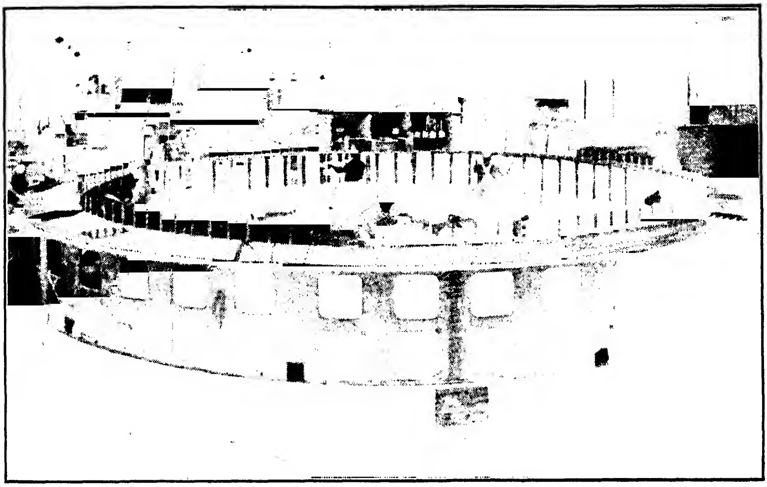
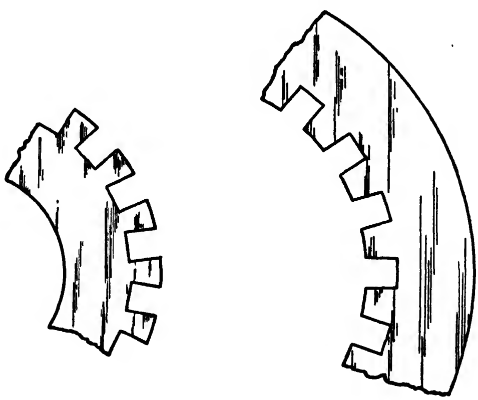


FIG. 4-6. Stator of 38,889-KVA Fort Peck vertical a-c generator. (*Allis-Chalmers Manufacturing Co.*)



(a) (b)

FIG. 5-6. (*a*) Shows the decreased width at the bottom of the teeth in a revolving armature core. (*b*) Shows the increased width at the bottom of the teeth in a stationary armature core.

cross-section area of the " neck " of the teeth at the bottom of the slots in a rotating armature, as shown in Fig. 5-6(*a*), and increases the reluctance of the magnetic path; while in the stationary armature of the revolving field machine, increasing the depth of the slots increases the area at the neck of the teeth, as shown in Fig. 5-6(*b*).

2–6. Definite-, or " Salient "-Pole, Alternator. The field structure of small alternators with rotating armature does not differ greatly from that of the d-c generator, as seen in Fig. 1–6. Figures 3–6 and 4–6 show the construction of the field structure for low-speed rotating-field alternators, driven by reciprocating engines or water wheels. These field structures consist of laminated cores or pole pieces bolted or keyed to a steel spider which in turn is keyed to the shaft. The field coils are wound on these laminated pole pieces. Machines of this type are called " definite-pole " or " salient-pole " alternators.

3–6. Field Structure of the Turbo-Alternator Wound Rotor. Most large alternators are driven by steam turbines, which are high-speed machines. An alternator with a rotor, such as that in Figs. 3–6 or 4–6, is not suitable for high speed. Such a rotor is difficult to balance at high speed and vibration would be excessive; the projecting poles would make the machine exceedingly noisy; the windage losses would be high; and the centrifugal forces set would exceed safe limits.

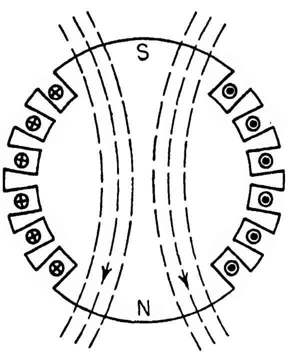


Fig. 6–6. Cross-sectional diagram showing how the field winding sets up magnetic poles on the surface of a round rotor. This is a two-pole field.

Therefore the field structure for the high-speed turbine-driven alternator is cylindrical in form and of small diameter relative to its length. It is manufactured as a solid steel cylinder, or of thick disks keyed to the shaft. Slots are cut in the surface of this cylinder parallel to the shaft, and the field winding — usually of copper strap properly insulated — is wound in these slots. Magnetic poles are thus set up on the surface of the cylinder, as shown diagrammatically in Fig. 6–6 for a two-pole rotor. Since there are no projecting poles in such a field structure, the machine is called a " round-rotor " alternator. Figure 7–6 shows the field winding in process of assembly on the rotor for a turbo-alternator. Figure 8–6 shows the laminations being stacked for the stator of a large turbo-alternator and Fig. 9–6 shows the coils being placed in the slots of the stator. Note the great length of this machine compared to its diameter.

FIG. 7–6. The field winding in the process of assembly on the rotor of a turbo-alternator. (*General Electric Co.*)

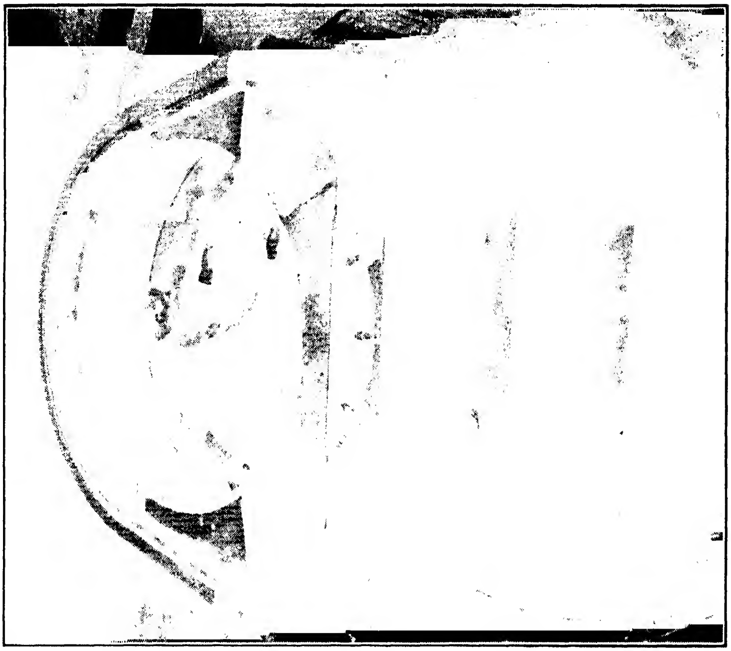


FIG. 8–6. Stacking the laminations of the stationary armature of the turbo-alternator of Fig. 7–6. (*General Electric Co.*)

FIG. 9-6. Coils being placed in the slots of armature of Fig. 8-6. (*General Electric Co.*)

4-6. Armature Windings. There is no fundamental difference between the armature of an a-c generator and that of the d-c machine. It has been shown in Vol. I, Chapter X, that the emf induced in the individual coils of a d-c generator is alternating, and the function of the commutator is merely to periodically reverse the connection of the armature coils to the external circuit. Therefore, if a d-c winding is properly tapped and leads are brought out, not to a commutator, but to collector rings mounted on the shaft, an a-c emf will be induced between these rings, and an alternating current may be taken from the armature. Such a winding is called a " closed-circuit " winding.

Armature windings for a-c generators may be either of the " closed-circuit " or the " open-circuit " type. A delta-connected armature is a closed-circuit winding, while a Y connection is an

open-circuit winding. In the latter case, one end of each of the three separate sets of coils is brought out either to three collector rings, if the armature rotates; or directly through the frame of the machine to stationary insulated terminals, if the field rotates.

Armature windings of commercial alternators are of the " drum " type, as shown in the preceding figures. But, since the circuits in

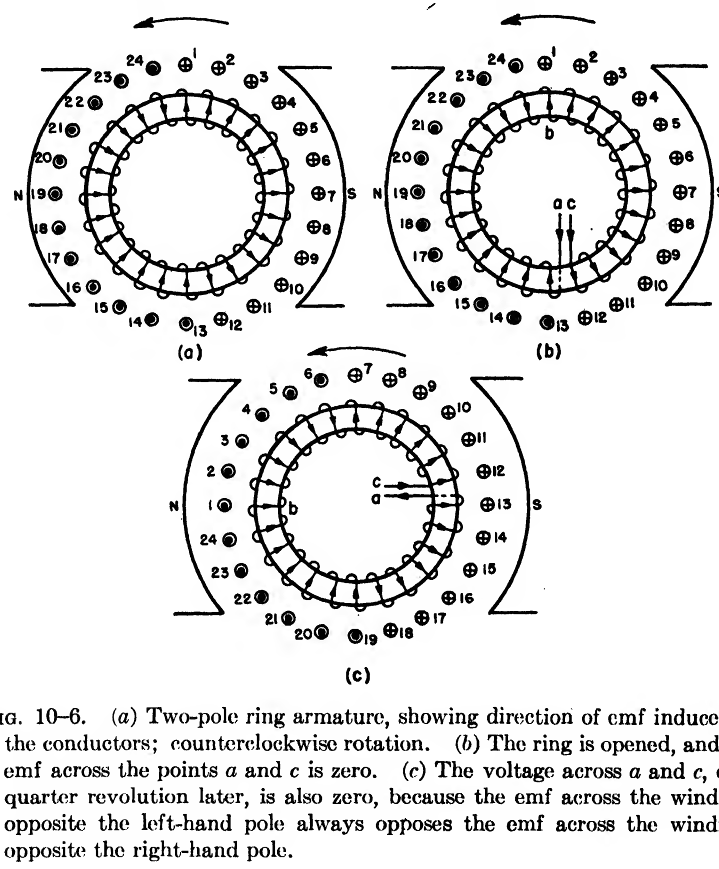


FIG. 10–6. (*a*) Two-pole ring armature, showing direction of emf induced in the conductors; counterclockwise rotation. (*b*) The ring is opened, and the emf across the points *a* and *c* is zero. (*c*) The voltage across *a* and *c*, one-quarter revolution later, is also zero, because the emf across the windings opposite the left-hand pole always opposes the emf across the windings opposite the right-hand pole.

rotating " ring " armatures are more easily shown in simple diagrams, the latter type will be first used in the following discussion. Simple windings of 24 conductors are shown in most of the following figures. The drum type of winding will be considered later in this chapter.

5–6. Summation of the EMF's in a Closed-Circuit Winding. Figure 10–6(*a*) is a diagram of a simple two-pole, 24-conductor ring

armature without taps or commutator. The conductors are equally spaced on the surface of the armature and symmetrically arranged with respect to the poles.

Assuming counter-clockwise rotation, the direction of the emf's induced in the conductors numbered from 1 to 12, under the south pole, is **in**, and that induced in the conductors numbered from 13 to 24, under the north pole, is **out**, as shown.

If the ring be opened and leads brought out at *a* and *c*, as shown in Fig. 10–6(*b*), the emf's induced at this instant, in the series conductors 1 to 12, are in additive series and in a direction through the winding from *c* to *b*; while the emf's induced in the series conductors 13 to 24 are also in additive series but in a direction through the winding from *a* to *b*. Since an equal number of conductors in each half cut the same flux at the same speed, the induced emf in each half is the same, but in the opposite direction in the series circuit; so it is apparent that the emf across the terminals *ac* is zero. When the armature turns one-quarter revolution, or 90 electrical degrees (electrical degrees = space degrees in a two-pole machine), as shown in Fig. 10–6(*c*), the emf induced in one-half the conductors is again equal and opposite to that induced in the other half; and the emf across *ac* in this position, also, is seen to be zero. This same relation holds for any position during the rotation of the armature; thus the total induced emf in the closed ring is zero.

6–6. Single-Phase Closed-Circuit Winding. In Fig. 11–6*a*, the same two-pole armature is tapped at two diametrical points, and leads *a* and *b* are brought out to two collector rings, as shown. These taps divide the armature into **two parallel** paths, **bra** and **bsa,** each generating equal emf's, but in opposite directions in the closed ring; that is, an emf is set up in each path in the armature, tending to send a current through the winding from *b* to *a*. At the instant shown, the emf across the collector rings is at its maximum value with ring *a* positive.

When the armature has turned (counter-clockwise) through 90 electrical degrees, as indicated in Fig. 11–6(*b*), each conductor is generating an emf; but half the conductors in path **bsa** lie under a north pole, and half under a south pole, so the net voltage in this path is zero. The same holds true for the conductors in path **bra.** Thus the voltage across the collector rings is zero.

When the armature turns through another 90 degrees, as indicated in Fig. 11–6(*c*), the emf induced in all the conductors in each

path is in the same direction; and the voltage across the collector rings is again a maximum. However, the polarity of the rings and the direction of the emf in the individual conductors is reversed. In any other position of the armature, the instantaneous emf, generated by some of the conductors in either path, will be in opposition to the emf generated in the remaining conductors in that

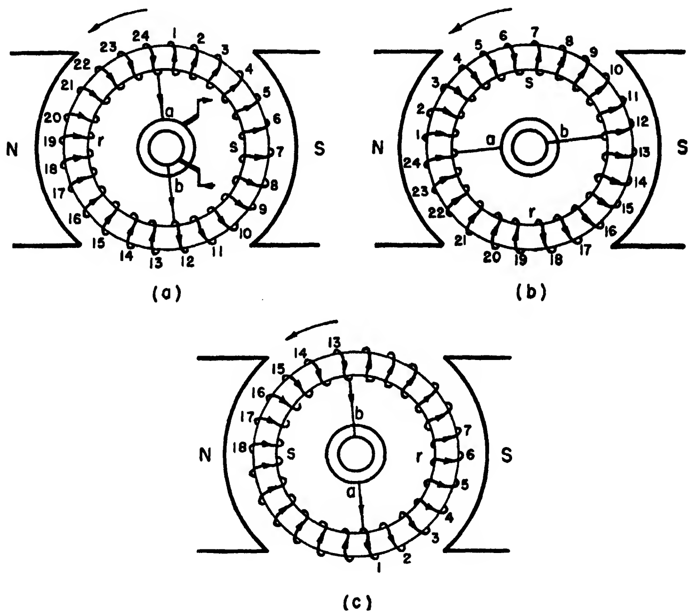


FIG. 11–6. (*a*) The winding is tapped at two diametral points and brought out to collector rings. The voltage between rings *a* and *b* is a maximum, at this instant, with ring *a* positive. (*b*) The voltage between *a* and *b*, one-quarter revolution later, is now zero. (*c*) The voltage between *a* and *b*, one-quarter revolution later than (*b*), is a maximum again with ring *b* positive.

path; and the voltage across the collector rings will be less than the maximum value.

Thus, as the armature rotates, an alternating emf is generated between the rings, and an alternating current will flow, if the rings are connected through brushes to an external circuit. This current, at any instant, will divide equally between the two identical parallel paths in the winding. It will be shown later that, if the flux is properly distributed in the air gaps, this emf and current will be of sine-wave form.

7–6. Single-Phase Open-Circuit Winding. If instead of tapping this armature, as in Fig. 11–6, it is cut at diametrically opposite points, as shown in Fig. 12–6(*a*), the winding is separated into two equal coils. When these coils are joined in series at x in Fig. 12–6(*b*), so that their respective emf's add in the same direction in the circuit, an "open-circuit" winding is formed. The terminals a and b are connected to collector rings, not shown in the

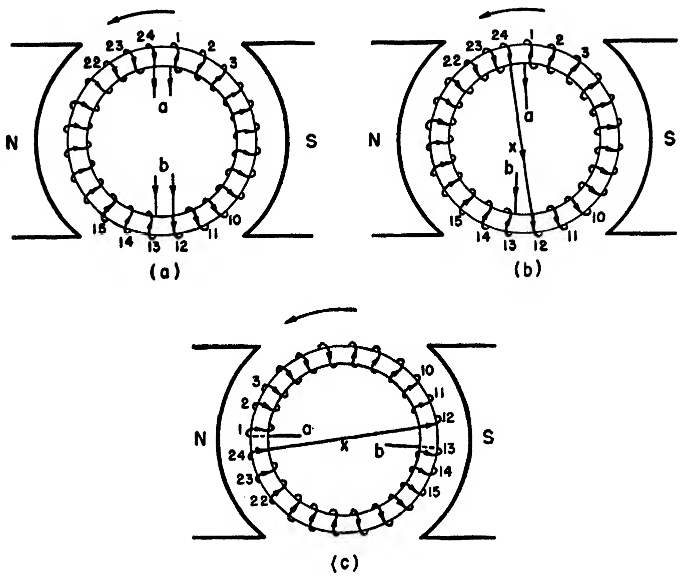


Fig. 12–6. (*a*) The winding is opened at two diametrical points dividing the winding into two identical coils. (*b*) The two coils are joined in series so that their emfs add together. The voltage between a and b, or between collector rings, is a maximum, and twice that in Fig. 11–6. (*c*) The voltage across a and b, one-quarter revolution later, is zero.

diagram. At this instant the emf across the rings is a maximum in a direction through the winding from b to a. When the armature reaches the position of Fig. 12–6(*c*), 90 electrical degrees later, the voltage across the rings is zero, since the emf induced in half the conductors in each coil is equal, and opposite to that induced in the other half.

The voltage generated by the open-circuit connection of this armature is just twice that of the closed-circuit arrangement of Fig. 11–6, since the emf's in the two groups, or coils, are in phase with each other. However, the armature now will safely deliver

only half the current, since there is only **one path** through the winding, and the external current is the same as the current per conductor. Thus the capacity of the armature in volt-amperes is the same, either with the open-circuit or with the closed-circuit connection.

Prob. 1–6. (*a*) If the armature of Fig. 11–6 generates a sine wave of voltage and is rated at 120 volts and 100 amperes line-current at full load, what effective current flows in each armature conductor, and what is the volt-ampere rating of the machine? (*b*) If the armature is connected as an open-circuit winding, as in Fig. 12–6, what is the rated voltage and line current and the volt-ampere rating of the machine? (*c*) If the armature of Fig. 11–6 is delivering rated load at unity power factor, how many amperes are flowing in each conductor at the instant it is moving through the position shown in Fig. 11–6(*a*)? (*d*) How many amperes would the armature be delivering when it is moving through the position shown in Fig. 11–6(*b*), and what would be the current in each conductor?

Prob. 2–6. (*a*) If the armature of Prob. 1–6 and Fig. 11–6 is delivering full load current at rated voltage, but the current is lagging one-quarter period, or 90 electrical degrees, behind the voltage, what is the value and direction of the current in each numbered conductor at the instant the armature is passing through the position, shown in Fig. 11–6(*b*), when the emf is zero? (*b*) Which conductors are opposing the motion of the armature (carrying current in the same direction as the induced emf and acting as generators), and what conductors are aiding the motion of the armature (carrying current in a direction opposite to the induced emf and acting as motors)? (*c*) How do the net total generator action and net motor action compare at the instant shown in Fig. 11–6(*b*)?

Prob. 3–6. (*a*) For the same effective values of voltage and current and the same phase lag of current behind emf (zero power factor), what is the value and direction of current in each numbered conductor when the armature is passing through the position shown in Fig. 11–6(*a*)? (*b*) From comparison of results of Probs. 2–6 and 3–6 and similar ones for other positions of the armature, what can you say concerning the torque and power required to drive a generator on a purely inductive load with zero power factor? The armature resistance is neglected here.

8–6. Instantaneous EMF Induced in an Armature Conductor. We have learned that when a conductor cuts 100,000,000, or 10^8, lines of magnetic flux in one second, the average emf induced is one volt. Therefore if a conductor cuts 50,000,000, or 5×10^7, lines in $\frac{1}{2}$ second, or 100,000 lines in $\frac{1}{1000}$ second, it is **cutting at the rate of 10^8 lines per second,** and the average emf induced is one volt.

We may consider a period of time so short that the average value is practically an instantaneous value. Thus, if the rate of cutting is 10^8 lines per second at any instant, the induced voltage at that instant is one volt; the instantaneous voltage, thus, is equal to the rate of cutting at that instant divided by 10^8.

A conductor, even when rotating at uniform speed, is constantly changing its direction of motion relative to the flux, and is cutting through a field in which the lines are not uniformly distributed; that is, the field intensity, B, relative to the motion of the conductor changes with the change in position of the conductor. Therefore the value of the emf induced in the conductor varies from instant to instant.

If we know the relative speed of the conductor to the flux where the conductor is at any instant, and the cutting length of the conductor, we can compute the rate of cutting and the emf which is being generated at just that instant. Expressed as an equation,

$$e = \frac{B\ell V}{10^8} \tag{1-6}$$

where e = instantaneous emf in volts,

B = lines per square inch (or per square centimeter) in the field where the conductor is moving at the given instant. The area is to be measured in the plane in which the conductor moves.

ℓ = cutting length of the conductor in inches (or centimeters).

V = velocity of conductor relative to the field in inches per second (or centimeters per second).

Care must be used to measure units of length, either all in inches or all in centimeters. Thus if B is measured in lines per square inch, ℓ must be in inches and V in inches per second.

Example 1. In a rotating field alternator, similar to Fig. 3–6, the diameter of the frame at the depth of the winding = 8 ft; cutting length of conductor = 6 in.; speed = 12 rpm.

What would be the instantaneous emf induced in a conductor as that part of a pole face was passing it where the flux density became 6300 lines per square centimeter?

Solution:

$$e = \frac{B\ell V}{10^8}$$

$B = 6300 \times 2.54^2 = 40{,}600$ lines per square inch.

$\ell = 6$ inches.

$$V = \frac{8 \times 12 \times 3.1416 \times 1200}{60} = 1005 \text{ inches per second.}$$

$$e = \frac{40{,}600 \times 6 \times 1005}{10^8} = 2.45 \text{ volts.} \quad \textit{Ans.}$$

Prob. 4–6. At an instant later than that in Example 1, the portion of the pole face nearer the trailing tip was passing the same conductor, and the flux at this point was 6500 lines per square centimeter. What emf was induced in the conductor at this instant?

Prob. 5–6. Assume the greatest flux density under the poles of the generator in Example 1 was 7000 lines per square centimeter. At what speed would the rotor have to turn in order to induce a maximum voltage of 1.28 volts in a single conductor?

Prob. 6–6. If the greatest allowable peripheral speed of the rotor of Prob. 5–6 is a mile per minute, what is the greatest instantaneous emf which can safely be set up in a single inductor? Assume the outside diameter of the rotor is 95 inches.

9–6. Instantaneous EMF at the Terminals of a Closed-Circuit Winding. In the armature of Fig. 11–6, the emf across the collector rings, at any instant, is equal to the arithmetical sum of the instantaneous emfs in the series conductors in one half the armature at that instant, as each half the armature generates the same voltage at any instant, and the two halves are in parallel between the rings.

Since the instantaneous emf in each conductor is determined by the flux density of the field at the position where the conductor is at that instant, the emf across the collector rings at succeeding instants depends upon the distribution of the flux in the air gaps.

First assume in Fig. 13–6, which is the same armature shown in Fig. 11–6, that the flux is distributed uniformly under the face of each pole and that no flux enters the armature beyond the edges of the poles, as indicated. In the position shown in the figure, conductors 1, 2, 11 and 12 are cutting no flux and generating no emf. Thus the voltage across the collector rings is that induced in conductors 3 to 10 inclusive, which are all cutting flux at the **same rate**. As the armature rotates (counter-clockwise), conductor 11 enters the magnetic field and begins to generate an emf, while conductor 3 leaves the field and ceases to generate a voltage; so the emf across the collector rings is unchanged. As the armature continues to turn, conductor 1 enters the magnetic field under the other pole, and begins to generate an emf in opposition to that of the other series conductors in the path of the armature, and the

voltage across the rings decreases. This process continues in steps with the rotation, the voltage decreasing to zero and rising again to a maximum in the opposite direction, etc.

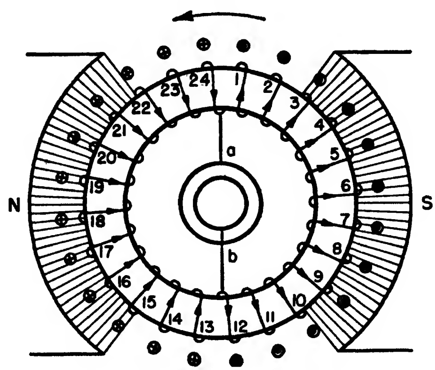


FIG. 13–6. The winding of Fig. 11–6 in a magnetic field which is uniform at every point in the air gap under the poles.

A generator, with a distribution of flux in the air gaps similar to that in Fig. 13–6, is said to have a rectangular "flux wave" (see Vol. I, page 296).

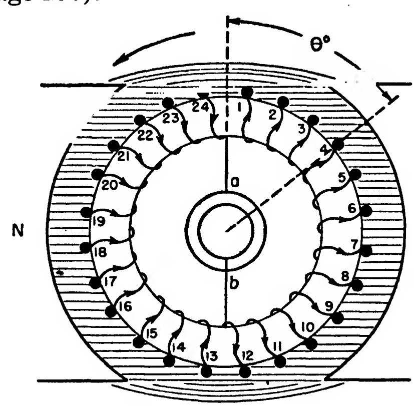


FIG. 14–6. The winding of Fig. 11–6 in a magnetic field, the density of which varies from point to point in a manner proportional to $B \sin \theta$.

Now assume that the flux in the generator of Fig. 11–6 fringes at the pole tips and crosses the air gap in the manner shown in Fig. 14–6. The flux density now varies from place to place around the entire circumference of the armature; and at any point θ electrical degrees from the neutral point, the flux density of useful flux

crossing the air gap can be expressed as $B \sin \theta$, where B is the maximum flux density at the center of a pole face, or 90 electrical degrees from the neutral point. It is to be noted that when the flux takes the path indicated in Fig. 14–6, **all** the conductors are cutting flux at various rates except when they happen to be at the exact neutral point. In the position shown, the induced emf across the rings is at a maximum. In this case, the instantaneous emf, induced in any conductor θ electrical degrees from the neutral point, is expressed as,

$$e = \frac{B \sin \theta l V}{10^8} \tag{2-6}$$

When the flux is distributed in the manner shown in Fig. 14–6, the flux distribution, or the " flux wave," is said to be sinusoidal.

In the position shown in Fig. 14–6, conductor 1 is 7.5 electrical degrees from the neutral point, and the flux density is $B \sin 7.5°$. When the armature moves 15 electrical degrees (counter-clockwise), conductor 1 is in a field of the same flux density, but the direction of induced emf in this conductor is reversed, thereby reducing the voltage across the collector rings.

The solution of the following problems will show that the shape of the curve of voltage is determined by the distribution of the flux in the air gaps.

Prob. 7–6. In the generator of Fig. 13–6, assume the flux density to be 50,000 lines per square inch at every point in the air gap between the armature and pole faces with no fringing; diameter of the armature, 6 inches; frequency 60 cycles; poles covering 66.7 per cent of the armature circumference; cutting length of conductor 8 inches. (*a*) Compute the emf in each numbered conductor. (*b*) Compute the total emf between collector rings in the position shown. (*c*) Compute the total emf in the closed mesh of the winding.

Prob. 8–6. (*a*) Compute the total emf between rings, as in Prob. 7–6, for successive positions of the armature of Fig. 13–6, $\frac{1}{24}$ revolution apart, or successive instants, 15 electrical degrees apart, for one complete cycle. (*b*) Draw a curve between electrical degrees as abscissa, and instantaneous emf between rings as ordinates, for a complete cycle. (*c*) Compute the effective value of the curve. (Note that it is **not** a sine curve.)

Prob. 9–6. Assume the flux density in the generator of Fig. 14–6 to be 50,000 lines per square inch in the air gaps at the centers of the poles, other data as in Prob. 7–6. (*a*) Compute the induced emf in each numbered conductor at the instant shown in Fig. 14–6. Note that conductors 1 and 12 (also 13 and 24) lie 7.5 electrical degrees

from the neutral point. (*b*) Compute the total emf between collector rings in the position shown. (*c*) Compute the total emf in the closed mesh of the winding.

Prob. 10–6. (*a*) Compute the emf between collector rings, as in Prob. 8–6, for successive positions of the armature, $\frac{1}{24}$ revolution (15 electrical degrees) apart, for one complete cycle. (*b*) Draw the curve of emf as in (*b*) of Prob. 8–6. (*c*) Compute the effective value of this curve.

10–6. Tapping Points on a Closed-Circuit Winding. In Fig. 11–6, the two-pole winding is tapped at two diametrical points and the number of conductors in each path is the same. Also the

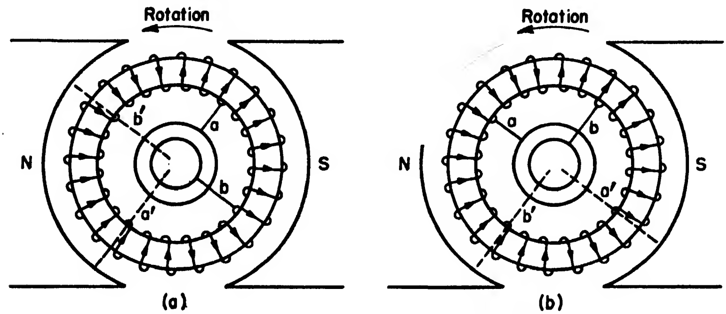


FIG. 15–6. (*a*) The winding tapped at two points, 90 electrical degrees apart, instead of 180° as in Fig. 11–6. The voltage between rings *a* and *b* is a maximum at this instant. Note that the emfs in the coils between *a* and *b′* are opposed and supply no emf to the rings. The same is true of the coils between *a′* and *b*. The emf between rings is less than in Fig. 11–6. (*b*) One-quarter revolution later, the emf between rings is zero.

voltage induced in each path at any instant is the same, and opposite in direction through the winding.

If the taps on the two collector rings be made at some other points on the bipolar armature of Fig. 11–6, such as *a* and *b* in Fig. 15–6(*a*), the emf between rings, or taps, is still alternating, but has a smaller maximum value. The sum of the emfs in path *ba* is, at every instant, equal to the sum of the emfs in the path *ba′b′a*, although the number of conductors in the latter path is greater than in path *ba*. To show this, extend the line of *a* through the center to *a′* and extend *b* to *b′*. The sum of the emfs in that part of the armature from *b* to *a′* is zero, since the emfs in one half the conductors in this path are equal and opposite to those in the other half. This is also true for the sum of the emfs in the conductors in path *b′a*.

The sum of the emfs between a' and b', is always equal to the sum of the emfs between b and a at the same instant. Both emfs are in a direction toward the same collector ring, and each is equal to the total emf across the rings. The position of the armature in Fig. 15–6(*a*) is such that both these voltages and that across the rings is at a maximum, which is seen to be less than the maximum for the armature when tapped as in Fig. 11–5.

Figure 15–6(*b*) shows the position of the armature when the emf between rings is zero.

Prob. 11–6. (*a*) At rated full load, a certain two-pole single-phase ring-wound armature, like that in Fig. 11–6, delivers from its rings a current of sine wave form and 100 amperes, effective value; what should be the effective value of amperes flowing in each path, or in each conductor? (*b*) If the two rings are tapped, as in Fig. 15–6, and assuming the impedance of either path to be proportional to the number of conductors in the path, compute the current in each path in the armature when 100 amperes is taken from the rings. (*c*) When delivering the same amperes output, in what ratio are the watts used up in heating each path in Fig. 15–6, greater than they would be if the two rings were properly tapped, as in Fig. 11–6?

Prob. 12–6. In Prob. 11–6, by what percentage are the total watts, lost in heating in the whole armature, greater or less, when tapped, as in Fig. 15–6, than when properly tapped, as in Fig. 11–6, assuming the same amperes output in both cases, and the same winding?

Prob. 13–6. If 100 amperes can safely be taken from the rings when the armature is tapped, as in Fig. 11–6, what current can be taken from the rings when tapped, as in Fig. 15–6, without increasing the amperes and the heating in any conductor above its rated value?

11–6. Armature Taps for a Synchronous Converter. If the closed-circuit armature of a d-c generator is properly tapped and equipped with both collector rings and a commutator, as shown in Fig. 16–6, both direct and alternating current can be taken from the winding at the same time. It is then called a **double-current generator.**

Instead of tapping separately from the winding to the commutator bars and to the collector rings, it is possible to tap from the rings to the proper commutator bars. Thus single-phase alternating current may be had, from any d-c generator, simply by mounting upon the shaft two insulated rings and connecting them to two commutator bars, separated by the proper distance.

The machine may also be operated from an a-c source receiving power at the collector rings to drive it as a **synchronous motor and**

to deliver direct current to the commutator. It is then called a **rotary,** or **synchronous converter;** in this case, a single-phase converter. It is important to note that, neglecting the voltage drop in the armature, the d-c voltage at the commutator is equal to the maximum a-c voltage at the collector rings.

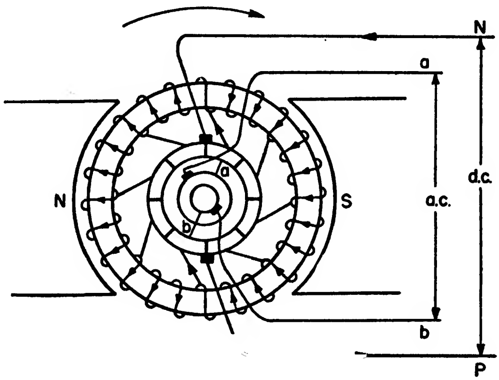


FIG. 16–6. The armature of Fig. 11–6 tapped and connected to both commutator and collector rings. It can now be used as a "double-current" generator to deliver both alternating and direct current; or as a "synchronous converter," receiving alternating current at the collector rings as a synchronous motor, and delivering d-c from the commutator.

Prob. 14–6. When the speed and field excitation of a two-ring, double-current generator, like that in Fig. 16–6, are such as to produce 230 volts at the terminals N and P, the a-c voltage between the rings a and b, if of sine wave form, is about 163 volts. If the total load of 10 kw were equally divided between the a-c and the d-c circuits, how many amperes must be delivered to each circuit? The a-c circuit is non-inductive.

Prob. 15–6. Suppose the dynamo, shown in Fig. 16–6, were operated as an " inverted " converter (converting direct emf to alternating emf), taking d-c from 230-volt mains and delivering a-c at 163 volts. When delivering 10 kw to the single-phase circuit at unity power factor, it operates at an efficiency of 88 per cent. What are the a-c amperes output and the d-c amperes input?

Prob. 16–6. If the converter of Prob. 15–6 were delivering its 10 kw to an inductive load having a power factor of 0.8, what would be the amperes and kva on the a-c side, and the amperes and kw on the d-c side? The efficiency in this case is 86 per cent.

Prob. 17–6. (a) On the basis of a sine wave of emf between collector rings of the machine in Fig. 16–6, what can you say of the ratio of a-c voltage to d-c voltage? (b) Assuming no losses, or 100 per cent effi-

ciency, for this machine when used as a converter, what is the ratio between the a-c current input and the d-c current output?

12–6. Multipolar Armatures. When we consider a **four-pole** alternator with a closed winding, we find the method of tapping differs from that which is proper for a two-pole machine. Figure 17–6 shows a closed ring winding, without collector rings attached, placed in a four pole field. It is seen that the conductors are now

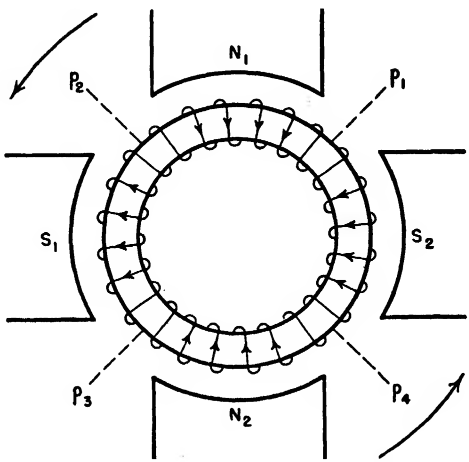


FIG. 17–6. A ring winding in a four-pole field. Note that the winding divides into four similar groups with the emfs in the conductors in each group in the same direction, and that there are four neutral points.

naturally divided into four groups, or coils, giving four neutral points p instead of two. The emf induced in the conductors in any one group is in the same direction. However, the direction of emf in adjacent groups is in opposition through the winding, and the total emf induced around the ring again is zero.

When the same two-pole armature tapped at two diametrical points, as in preceding figures, is placed in a four-pole field, as shown in Fig. 18–6(a), it is apparent that the emf across the collector rings is zero. And when the armature turns through 90 electrical degrees (one-eighth revolution in a four-pole field), as shown in Fig. 18–6(b), the emf across the rings is again observed to be zero. Also in any other position of the armature, the emf between rings is zero. Therefore, a closed-circuit armature, tapped properly for two poles, will develop no voltage between collector rings, when placed in a four-pole field. This is also true

when a two-pole armature is placed in a field of any other number of poles.

Figure 19–6(*a*) shows the same closed winding, properly tapped for a four-pole field. Note that each collector ring is tapped to

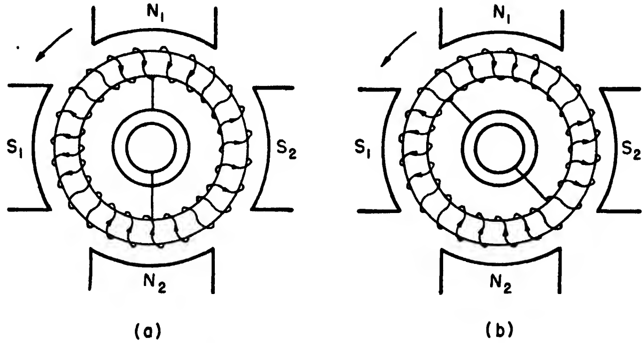


FIG. 18–6. (*a*) The winding tapped at two diametrical points, as in Fig. 11–6, in a four-pole field. Note that emf across the collector rings is zero. (*b*) The armature in (*a*) 90 electrical degrees (45 space degrees) later. Again the emf across the collector rings is zero.

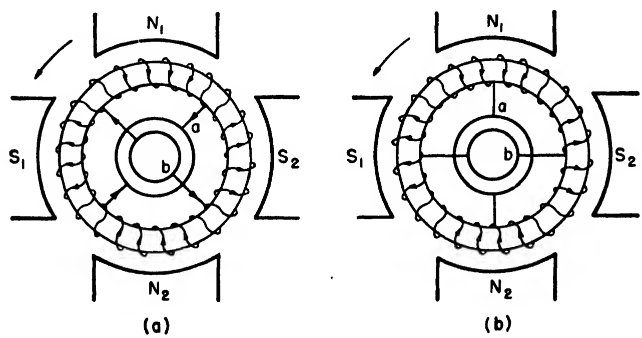


FIG. 19–6. (*a*) The winding of Fig. 17–6, properly tapped for a four-pole field. The emf across the rings is a maximum with ring *a* positive. (*b*) The armature of (*a*) is now 90 electrical degrees, or one-eighth revolution, later. The emf across the rings is zero.

two points on the winding, which are electrically similar. At the instant shown, the armature is in the position of maximum emf between rings. In Fig. 19–6(*b*), the armature has turned through 90 electrical degrees from Fig. 19–6(*a*) and the emf between rings is zero. It is to be noted that there are four parallel paths in this four-pole armature.

In a **six-pole** winding, each collector ring must be tapped to **three** electrically similar points on the armature and there are six paths in parallel; in an eight-pole armature, each ring is tapped to four points and there are eight paths, etc.

It is also important to note that the number of conductors in each coil, or group, on the armature must be the same and similarly placed with respect to the poles, if the current delivered by the machine is to divide equally among the several paths.

Prob. 18–6. A single-phase six-pole closed-circuit armature delivers 120 amperes, effective value, to the external circuit. What current flows in each armature conductor?

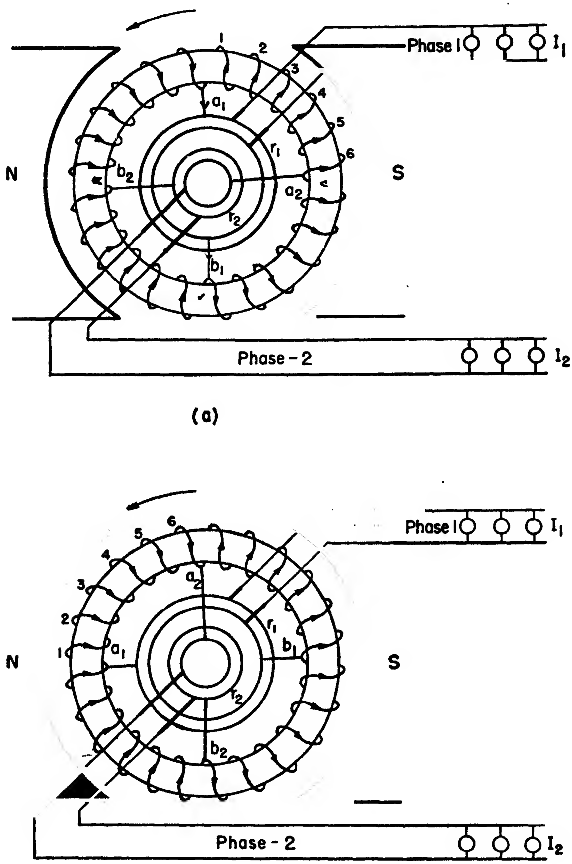
13–6. Two-Phase Closed-Circuit Armature. If the taps are brought out to collector rings from proper points of a closed circuit armature, the winding can be used to deliver polyphase current to the external circuit.

Consider the same ring-wound bipolar armature, shown in the previous figures, but with four collector rings mounted upon the shaft, instead of two, as indicated in Fig. 20–6 (*a*). Let one pair of these rings, r_1, be tapped to diametrically opposite points on the winding, as a_1 and b_1. The other pair of rings, r_2, will be tapped also to diametrically opposite points on the winding, as a_2 and b_2, midway between the points a_1 and b_1. From the figure, it will be seen that the emf b_1 to a_1 has its maximum instantaneous value at the same instant at which the emf b_2 to a_2 has its zero value. The conditions, one quarter period, or 90 electrical degrees later, are shown in Fig. 20–6 (*b*), and it is apparent that the emf b_1 to a_1 has decreased to its zero value, while that from b_2 to a_2 has increased to its maximum value. Thus, the voltage between each of the two pairs of collector rings have the same maximum and effective values, but there is a phase difference of 90° between them. Therefore, a single phase circuit can be taken from each pair of rings, as indicated, and the machine used to supply a two-phase four-wire system.

It should be noted here that a closed circuit armature cannot be used to supply a two-phase three-wire system, for two collector rings would necessarily be connected to the common wire, and it is apparent that this would short-circuit part of the winding.

The armature in the figures above can be indicated diagrammatically, as in Fig. 21–6. This is called a " mesh " connection. The winding in this case is actually a four-phase, or " quarter-

phase," winding, as the taps divide it into four distinct groups or coils, b_1b_2, b_2a_1, a_1a_2, etc.; and a separate single-phase circuit can be taken from any two taps, or collector rings. The emfs induced in



(b)

FIG. 20-6. (*a*) The ring armature in a two-pole field, tapped at four points for two phase. The voltage across a_1–b_1 is at a maximum at this instant, while that across taps a_2–b_2 is zero. Therefore, there is a phase difference of 90° between these two voltages. (*b*) One-quarter revolution later, the voltage across a_1–b_1 is zero, while that across a_2–b_2 is at a maximum.

any two adjacent coils, or groups, on the armature also are displaced 90 electrical degrees. That is, assuming counter-clockwise rotation, when the middle of coil b_2a_1, for instance, is opposite the center of the North pole, its emf is a maximum; while the emf in

coil a_1a_2 will not reach a maximum until the armature has turned through one quarter revolution or 90 electrical degrees.

If we consider a positive direction of emf in the coils of Fig. 20–6 or 21–6 to be in **clockwise** direction through the winding (that is, from b_1 to b_2, b_2 to a_1, a_1 to a_2, etc.), the emf in coil b_1b_2 leads that in coil b_2a_1 by 90 degrees. And their vector sum, b_1a_1, is the emf across the collector rings r_1 in a direction through the winding from b_1 to a_1. The emf of coil b_2a_1 leads that of coil a_1a_2 by 90 degrees, and their vector sum b_2a_2 is the emf across the rings r_2 in a direction through the winding from b_2 to a_2. Also the emf of coil a_1a_2 leads that of a_2b_2 by 90 degrees; and their vector sum a_1b_1 is the emf across the rings r_1, but in a direction through the winding from a_1 to b_1.

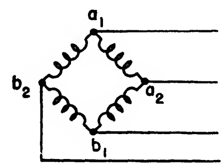


Fig. 21–6. Diagrammatic representation of the circuits in Fig. 20–6, showing that in a "mesh" connection, there are really *four phases* at 90° to each other.

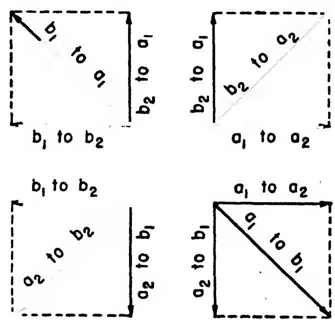


Fig. 22–6. Vector diagrams of the voltages across the different parts of the armature in Fig. 20–6 and Fig. 21–6. These show the phase relations and relative values of these emfs with respect to one another.

Note that the positive direction of emf in path $a_1a_2b_1$ is 180 degrees from the positive direction of emf in path $b_1b_2a_1$. And finally, the emf of coil a_2b_1 leads that of coil b_1b_2 by 90 degrees. Their vector sum a_2b_2 is the emf across the rings r_2 in a direction through the winding from a_2 to b_2. Note again the positive direction of emf in path $a_2b_1b_2$ is 180 degrees from that in path $b_2a_1a_2$. The complete vector diagram of these emfs is drawn in Fig. 22–6, and shows that the emf across rings r_1 leads that across rings r_2 by 90 electrical degrees.

Since, in this closed-circuit armature, the emf across each phase of the two-phase circuit is the vector sum of the emfs of two coils at 90 degrees, the voltage across each pair of collector rings is 1.41, or $\sqrt{2}$, times the voltage of each coil. That is, the emf across

rings r_1 is $\sqrt{2}$ times the emf of either coil b_1a_2 or coil a_2a_1, not twice the emf of one coil.

14–6. Current in the Coils of a Two-Phase Closed-Circuit Armature. Consider the generator of Fig. 20–6 to be delivering a balanced load to the two-phase circuit. Assume a current of I_1 amperes is supplied from the terminals a_1b_1 to Phase 1, and an equal current of I_2 amperes — at the same power factor, is supplied from the terminals a_2b_2 to Phase 2.

The current I_1 divides equally in the two parallel paths, $b_1b_2a_1$ and $b_1a_2a_1$, so that $I_1/2$ amperes flows in each path, as indicated

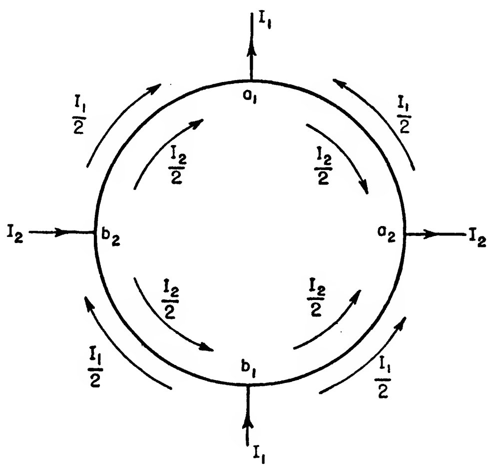


FIG. 23–6. Diagram showing how the line currents, I_1 and I_2 combine in the coils of a closed-circuit, or mesh-connected, two-phase armature.

in Fig. 23–6. Similarly, the current I_2 divides equally in the two parallel paths, $b_2a_1a_2$ and $b_2b_1a_2$; so that $I_2/2$ amperes flows in each path. Thus each coil in the winding carries one half the line current for each phase.

Since the voltages across the terminals a_1b_1 and a_2b_2, or across each pair of collector rings, differ in phase by 90 degrees, and the line currents of the phases have the same power factor on a balanced load, the currents $I_1/2$ and $I_2/2$ combine at 90 degrees in each coil, as shown in Fig. 24–6. So the current per armature coil is

$$I_c = \sqrt{\left(\frac{I_1}{2}\right)^2 + \left(\frac{I_2}{2}\right)^2} = \sqrt{\frac{I_1^2}{4} + \frac{I_2^2}{4}} = 1/2\sqrt{I_1^2 + I_2^2};$$

but $\sqrt{I_1^2 + I_2^2} = 1.41I;$

therefore $I_c = \dfrac{1.41I}{2} = 0.707I,$

where I_c = current per coil, or per conductor.

I = line current per phase.

Example 2. The generator of Fig. 20–6 delivers a line current of 100 amperes per phase to the external circuit at unity power factor. What is the current in each armature coil?

Solution: Current delivered by each parallel path to Phase 1 = $\frac{100}{2}$ or 50 amperes. Current delivered by each parallel path to Phase 2 = $\frac{100}{2}$ or 50 amperes. Total current per coil = $\sqrt{50^2 + 50^2}$ = 70.7 amperes. *Ans.*

Note that the current per coil on balanced load is 70.7 per cent of the line current per phase.

If the generator in Example 2 were delivering an unbalanced load of 160 amperes to one phase, and 80 amperes, at the same

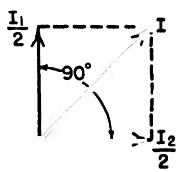


FIG. 24–6. Vector diagram of the currents in each path of the armature represented in Fig. 23–6. Balanced two-phase load.

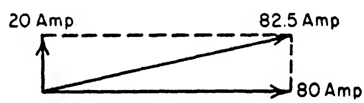


FIG. 25–6. Vector diagram of the current per path in Fig. 23–6 for an unbalanced load at unity power factor.

power factor, to the other, the total current output of the machine would not be changed. However, the current in each armature path to one load now equals 160/2 or 80 amperes; and in each path to the other equals 40/2 or 20 amperes. These currents combine at 90 degrees and the current per coil is now $\sqrt{80^2 + 20^2}$ or 82.5 amperes, as shown in Fig. 25–6. In this case the machine is delivering the same output in volt-amperes as in the example above, provided the terminal voltages remain constant; yet the heating has increased in the ratio $\dfrac{82.5^2}{70.7^2}$, or $\dfrac{1.36}{1.00}$. Thus an unbalanced load decreases the capacity of the generator.

From the diagram of Fig. 23–6, it is seen that the current in coils

b_2a_1 and b_1a_2 is the sum of two currents, $\frac{I_1}{2} + \frac{I_2}{2}$, while in b_1b_2 and a_1a_2, it is the **difference** of these same two currents. On balanced loads of any power factor, and on unbalanced loads at unity power factor, the currents in each armature coil combine at 90 degrees. Since the **sum of two currents at 90° has the same value as their difference,** the currents in all the armature coils are of the same value.

However, when the power factors of the two phases are not the same, these currents do not combine at 90° in the coils, so their **difference** in one coil is **not equal** to their **sum** in another coil. Thus while the currents in half the coils may be decreased that in the other half will be increased, thereby decreasing the capacity of the machine.

15–6. Two-Phase, Open-Circuit Winding. If the two-phase armature of Fig. 20–6 is cut open at the taps b_1, a_2, a_1 and b_2, and leads are brought out, the winding is separated into four equal coils, as indicated in Fig. 26–6(a), all generating equal emfs. When the middle of coil 1, Fig. 26–6(b), is under the center of the North pole, the middle of coil 3 is under the center of the South pole, so the emfs in these two coils reach a maximum at the same instant, but their emfs are in opposite directions, or at 180 electrical degrees, in the closed ring. At this same instant, the middle of coils 2 and 4 are at the neutral point and their emfs are zero.

Again, assuming the positive direction of emf in the coils to be in clockwise direction around the rings (either direction may be taken as positive) and starting with coil 1, we label the terminals of the coil S (start) and F (finish), as we follow the positive direction of emf progressively around the ring. That is, S_1, S_2, S_3 and S_4 are similar terminals of all the coils, and F_1, F_2, F_3 and F_4 are the other similar terminals.

If we connect F_1 to F_3, as in Fig. 26–6(b), coils 1 and 3 are now joined in series with coil 3 reversed in the circuit, and the emfs of the two coils combine in phase with each other. S_1 and S_3 are the terminals of one phase and are brought out to collector rings. Similarly, coils 2 and 4 are connected in series, F_2 to F_4 with coil 4 reversed, and their emfs combine in phase. S_2 and S_4 are the terminals of the other phase, also brought out to another pair of collector rings. In Fig. 26–6(b), the emf across S_1S_3 is at its maximum value, while that across S_1S_4 is zero and lagging 90 electrical degrees. Figure 26–6(c) shows the position of the coils

one-quarter revolution, or 90 electrical degrees, later in the cycle. The emf across S_1S_3 has dropped to zero while that across S_2S_4 is now a maximum.

The emf across the phase terminals, S_1S_3 and S_2S_4 is twice that of one coil. Since the emf of coil b_1b_2 (Fig. 20–6(*a*)) is 0.707 of the

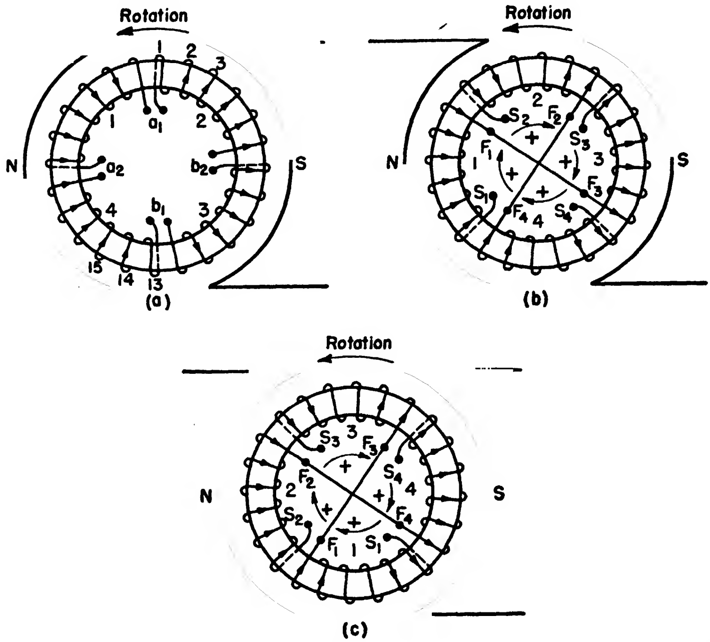


FIG. 26–6. (*a*) The two-phase armature of Fig. 20–6 is opened at the tapping points, dividing the winding into four equal coils. (*b*) Coils 2 and 4 are joined in series so that their emfs combine in phase, forming one phase of a two-phase open-circuit winding. Coils 1 and 3 are similarly joined, forming the other phase. The winding now has one coil per pole per phase. At this instant, the voltage between terminals S_1S_3 is a maximum, and between S_2S_4 is zero. (*c*) One-quarter revolution later, the voltage between S_1S_3 is zero, and between S_2S_4 is a maximum.

emf of the two coils b_1 to a_1 in series, the emf across the rings of this open-circuit connection is twice this or 2×0.707 or 1.41 times the emf of the closed-circuit arrangement of the same armature. However, the armature will now safely deliver only 0.707 as much current per phase, so the load capacity of the machine is unchanged.

Each phase in this two-pole armature consists of two coils and, therefore, the winding is said to have **one coil per pole per phase.**

In this respect it is similar to the windings of commercial machines which are wound with a definite number of coils, or **coil groups per pole per phase.**

Prob. 19–6. A 230-volt two-phase alternator, with closed winding similar to Fig. 20–6, delivers, at rated load, 100 amperes to each phase of the external circuit. (*a*) If the external loads are both of unity power factor, what is the total kw output of the alternator? (*b*) If the load is inductive with 80 per cent power factor in both phases, what is the kw output? (*c*) What is the total kva rating of the alternator? (*d*) What is the rated full load current in each conductor on the armature?

Prob. 20–6. If the alternator of Prob. 19–6 were loaded as a four-phase generator, what is the largest value of current which could be taken by each of the four phases, without exceeding the rated full load current per conductor, as determined in Prob. 19–6? The four phases are balanced (same amperes in each) and the loads are non-inductive.

Prob. 21–6. When the generator of Prob. 19–6 is operating at unity power factor, but the phases have become unbalanced, so that one of them carries 140 amperes, instead of 100 as rated, what is the greatest current that may be taken by the other phase without exceeding the rated full load current in any armature conductor?

Prob. 22–6. Under the unbalanced load conditions of Prob. 21–6, assuming the terminal voltages remain constant, what would be the allowable kw output of the machine? By what percentage is the kw output increased or decreased from the rated value on balanced load?

Prob. 23–6. (*a*) If only one of the two phases of Prob. 19–6 were loaded at unity power factor and the output of the other phase were zero, how many amperes could be delivered to the loaded phase without developing a greater I^2R loss in the whole winding than is permitted under rated load conditions? (*b*) What is the allowable kw output under the conditions in (*a*)?

Prob. 24–6. (*a*) If the armature of Prob. 19–6 is re-connected as an open-circuit winding, similar to Fig. 26–6, what will be the rated voltage between collector rings? (*b*) What will be the rated, or full load, current delivered to each phase? (*c*) At unity power factor load what will be its rated kw output?

Prob. 25–6. What changes of connections in Fig. 20–6 would make the armature suitable as a single-phase generator in a four-pole field? Show sketch.

Prob. 26–6. (*a*) If the taps on the armature of Prob. 19–6 are changed so that it can be operated as a single-phase generator in a four-pole field, how many amperes can it deliver to the outside circuit without exceeding the rated load current per conductor? (*b*) At the

same speed and flux per pole, what would be the rated voltage between collector rings? (*c*) What would be the kva rating of the machine?

16–6. Three-Phase Closed-Circuit Ring Winding — Delta Connection. Figure 27–6 shows the same ring armature used in previous figures tapped at three equidistant points on the winding to supply a three-phase circuit. This divides the armature into three equal coils with the same number of series conductors.

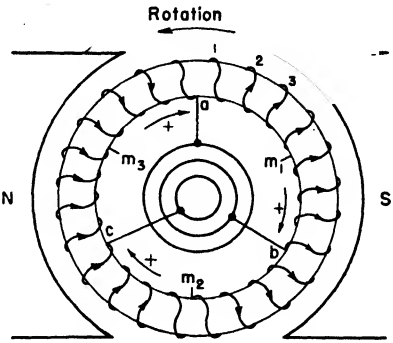


FIG. 27–6. The armature of previous figures tapped at three equidistant points on the winding now forms a delta-connected three-phase armature.

The armature rotates counterclockwise and the direction of the emf induced in the individual conductors in the position shown is indicated in the figure. At this instant, the total emf induced in coil *bc* is zero; while coils *ca* and *ab* are generating emfs which are equal and opposite to each other in the closed ring. However, the maximum and effective emfs in all three coils are the same.

If we again assume the positive direction of emf in each coil to be in clockwise direction through the closed ring, the emf in coil *ca* at the instant the middle of this coil, m_3, passes the center of the North pole. Also, the emf in coil *ab* reaches its maximum positive value in a direction from *a* to *b* at the instant its middle, m_1, passes the center of the North pole. This occurs just one third period or 120 electrical degrees after the maximum from *c* to *a*. And the emf in *bc* reaches its maximum value in the same direction from *b* to *c*, when its middle, m_2, passes the center of the same pole. This is the two-thirds period, or 240° after the maximum emf, *c* to *a* in coil *ca*, or 120° after the maximum from *a* to *b* in coil *ab*. Therefore, the coils *ab*, *bc* and *ca* differ in phase from one another, successively, by 120°, and the sum of these three emfs around the closed ring again is seen to be zero. This is shown by the polar and topographic diagrams of Fig. 28–6, which also represent the voltages between collector rings.

As the armature coils of Fig. 27–6 are connected together and tapped, the winding represents a **mesh,** or **delta,** connection of coils.

However, the different phases of the windings of commercial alternators with drum wound armatures are usually connected in Y or "star." This is an open-circuit winding and, as has been

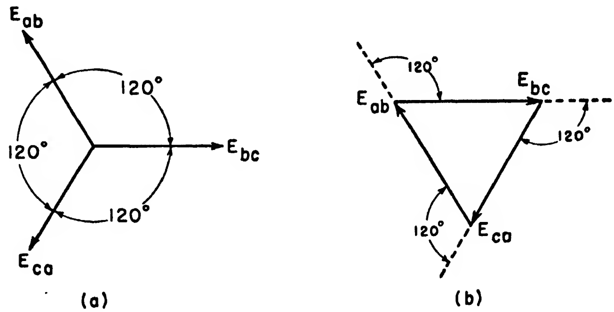


FIG. 28–6. (*a*) Polar vector diagram of the three emfs between the points of tapping of the armature in Fig. 27–6. (*b*) Topographic vector diagram of the three emfs.

shown in previous chapters, gives a greater terminal voltage for a given voltage per coil. This is particularly desirable in generators for high tension circuits.

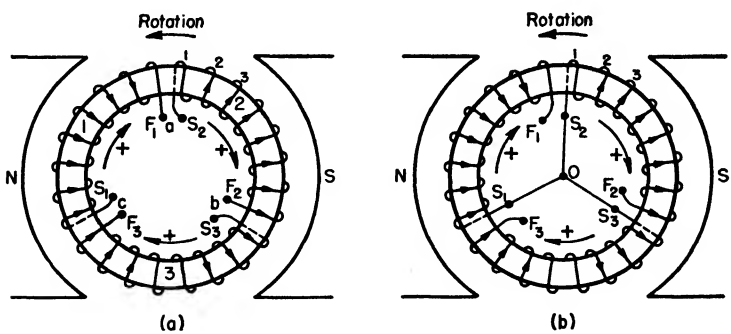


FIG. 29–6. (*a*) The armature of Fig. 27–6 opened at the points of tapping is divided into three separate coils with emfs at 120°. Terminals S_1, S_2, and S_3 are the corresponding emfs of the coil. (*b*) When S_1, S_2, and S_3 are joined together at O the winding becomes a Y-connected armature. Terminals F_1, F_2, and F_3 are brought out to collector rings.

17–6. Three-Phase Open-Circuit Winding: Y, or Star, Connection. If we cut open the winding of Fig. 27–6 at the three equidistant points (*a*, *b*, and *c*), as shown in Fig. 29–6(*a*), three separate groups or phases are formed from the winding, all generating emfs of the same value. These emfs differ in phase suc-

cessively from one another by 120 electrical degrees; that is, coil 2 lags 120° behind coil 1, and coil 3 lags 120° behind coil 2. The terminals of these coils are again labeled S to F, as we follow the positive direction of emf progressively around the ring, as shown by the arrows. Therefore S_1, S_2, and S_3 are similar ends of the three phases and F_1, F_2 and F_3 are the other similar ends.

If now, we connect any three similar ends as S_1, S_2 and S_3 together at O, as shown in Fig. 29–6(*b*), we have between the three

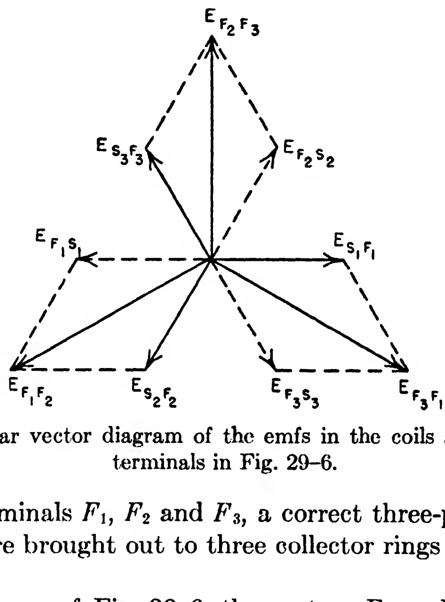


FIG. 30–6. Polar vector diagram of the emfs in the coils and between the terminals in Fig. 29–6.

remaining terminals F_1, F_2 and F_3, a correct three-phase emf and these latter are brought out to three collector rings (not shown in the figure).

In the diagram of Fig. 30–6, the vectors $E_{S_1F_1}$, $E_{S_2F_2}$ and $E_{S_3F_3}$ represent the emfs of the three coils. With respect to the positive direction of emf through the coils, the voltage across the terminals F_1 to F_2 is the vector sum of the emfs of coil 1 (S_1F_1) reversed and coil 2 (S_2F_2) direct; voltage across terminals F_2 to F_3 is that of coil 2 (S_2F_2) reversed and coil 3 (S_3F_3) direct; and voltage across terminals F_3 to F_1, that of coil 3 (S_3F_3) reversed and coil 1 (S_1F_1) direct. (Check these relations in the circuits of Fig. 29–6(*b*).)

It has been shown in Chapter II that the voltage across the terminals of the Y-connected armature is $\sqrt{3}$ times that of the same armature delta connected. However, the line current is only $\frac{1}{\sqrt{3}}$ that of the delta connection; therefore, the capacity of the machine in volt-amperes remains the same.

Prob. 27-6. A delta connected three-phase armature is rated to deliver 100 amperes from each terminal at full load, with 230 volts between each pair of terminals. What would be its rated full load amperes per line and volts between terminals, if the phases were reconnected in Y?

Prob. 28-6. What is the maximum steady load in kilowatts which can safely be obtained from the machine in Prob. 27-6: (*a*) Connected in delta? (*b*) Connected in Y?

Prob. 29-6. In general, if the internal connections are changed from Y to delta, what difference will be produced in the rating of a generator as to: (*a*) Amperes per terminal (or per line wire)? (*b*) Volts between terminals (or line wires)? (*c*) Kilovolt-ampere rating (or kw at unity power factor)?

Prob. 30-6. A three-phase delta-connected armature is rated at 100 kva, 2300 volts. (*a*) What is the current in each part of the armature and line current at rated full load output? (*b*) What would be the amperes, volts and kva rating of this machine, if the phases were reconnected in Y?

Prob. 31-6. A three-phase delta-connected alternator is rated 200 kva at 6600 volts. (*a*) If this machine were used to deliver its full rated load (kva) to a single-phase circuit from one pair of terminals, what would be the amperes in each section of the armature winding, assuming the impedance in each armature path is directly proportional to number of turns, or conductors? Draw a sketch to illustrate your solution. (*b*) By what percentage would the rate of heat developed in each armature coil (or phase) exceed that permitted under normal rated load?

Prob. 32-6. In the generator of Prob. 31-6, what are the greatest amperes and kva that can be delivered at rated voltage from one pair of terminals without exceeding the normal full load current in any one phase or armature path?

Prob. 33-6. What is the ratio of the rated three-phase load to the maximum steady single-phase load which may safely be taken from the alternator of Prob. 31-6?

18-6. Relative Capacity of an Armature, Tapped for Single Phase, Two Phase, and Three Phase. Consider the same closed armature used in previous figures, tapped for single phase, two phase, and three phase, as shown in Fig. 31-6.

Single Phase. Assume the winding will safely deliver 100 amperes, single phase, at 100 volts from terminals a_1b_1. The greatest allowable current in any single coil or conductor, thus, is $\frac{1}{2}$ of 100 or 50 amperes. The armature operated as a single-phase

machine, therefore, will deliver 100 × 100 or 10,000 volt-amperes; or, at unity power factor, 10,000 watts or 10 kilowatts.

Two-Phase. Now, consider the same winding tapped for two phase at points a_1 and b_1 and at a_2 and b_2. It has been shown, in Art. 14, Chap. VI, that the current in each coil of the two-phase closed-circuit armature, on balanced load, is 0.707 times the line current per phase. Therefore, the line current which can be taken from the terminals a_1b_1 is $\dfrac{50}{0.707}$ amperes. The same current can also be taken from terminals a_2b_2. The voltage across each pair of terminals still is 100 volts.*

Therefore, this armature, when tapped for two phase, can deliver 2 (100 × 70.7) or 14,140 volt-amperes; or at unity power factor, 14,140 watts or 14.4 kilowatts. Thus, with the same weight of copper and iron this armature, connected for two phase, will safely deliver 1.414 or $\sqrt{2}$ times the output of the single-phase connection. This is a very definite advantage in favor of the two-phase alternator.

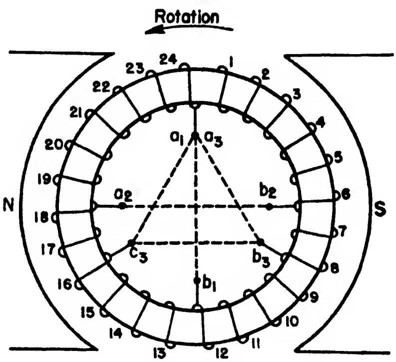


FIG. 31-6 The armature of the previous figures tapped for single-phase, two-phase, and three-phase.

Three Phase. Before attempting to determine the capacity of this winding, when tapped at points, a_3, b_3 and c_3 (Fig. 31-6) for three phase, let us first consider the phase relations of the emfs in the several conductors on the armature. In Fig. 31-6 and previous diagrams, an armature having 24 conductors equally spaced upon its surface, has been shown. Since the field is bipolar, one revolution completes one cycle in every conductor As we follow the winding in clockwise direction around the ring, the emf in each successive conductor lags one-twenty-fourth of a cycle, or 15 electrical or time degrees, behind that in the preceding conductor, as shown in Fig. 32-6. The vectors, representing the emfs in the individual conductors, lie on the circumference of a circle, as shown.

* The effects of armature reaction in altering the flux and the induced voltage are here neglected. They will be considered in the next chapter.

Thus, the emf b_1 to a_1 of Fig. 31–6 is equal to the diameter of the circle, or the line b_1a_1 in Fig. 32–6, and represents 100 volts. The taps a_3, b_3 and c_3 in Fig. 31–6 divide the armature into three coils of eight conductors each (1 – 8), (9 – 16) and (17 – 24). In Fig. 32–6, draw vectors a_3b_3, b_3c_3 and c_3a_3, each spanning the eight vectors representing the emfs in these conductors. These lines

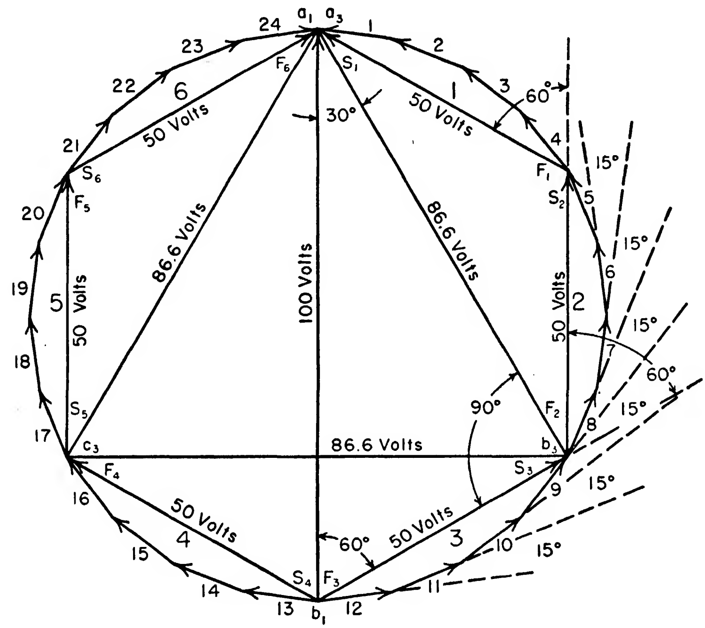


FIG. 32–6. Topographic vector diagram of the emfs in the individual conductors of the 24-conductor armature, shown in previous figures. The voltage across any combination of conductors may be determined from this diagram.

represent the three-phase voltages. Also draw the line b_1b_3, and triangle $a_3b_3b_1$ is a right triangle inscribed in a semicircle. Therefore, the length of vectors b_3a_3 equals b_1a_1 sin 60°, and represents 100×0.866, or 86.6 volts. This is the voltage of the armature when tapped for three phase, as in Fig. 31–6.

Since the safe allowable current per coil is 50 amperes, the line current for the armature, tapped for three-phase delta, is $\sqrt{3} \times 50$ or 86.6 amperes. And the output $(\sqrt{3}\,E_LI_L)$, therefore, is $\sqrt{3} \times 86.6 \times 86.6$, or 13,000 volt-amperes, or 13 kw at unity power factor.

This would indicate that more power can be taken from the armature tapped for two phase than tapped for three phase. However, **this is not true in practice** because the three-phase winding can be tapped to better advantage than shown at $a_3b_3c_3$ in Fig. 31–6. The winding, in this case, is divided into three large groups of coils a_3b_3, b_3c_3 and c_3a_3 per **pair** of poles.

If the winding is so arranged that each phase has **one group of coils for each pole,** as noted in Art. 15, Chap. VI, so that each phase is symmetrically distributed with regard to every single pole (not

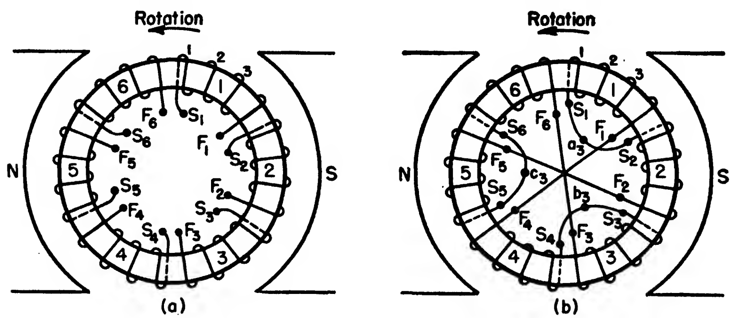


FIG. 33–6. (*a*) The winding of Fig. 27–6 cut up into six equal coils. At the instant shown, the emfs in coils 2 and 5 are at a maximum. Emfs in coils 3 and 6 will reach a maximum at the same time; the same is true for emfs in coils 1 and 4. The positive direction of emf (not the instantaneous emf) is in a direction from S and F in each coil. (*b*) Coils 1 and 4 are connected in series to form one phase; 3 and 6 another phase; and 5 and 2 the third phase; and the phases are connected in delta. The winding now has *one coil per pole per phase.*

with regard to every pair of poles, as in Figs. 27–6 and 29–6), the voltage of each phase, with the same speed and flux per pole, can be increased.

Therefore, let us cut the winding into six equal coils, as in Fig. 33–6(a), each consisting of four series conductors, and again assume the positive direction of emf to be from S to F through each coil. In Fig. 33–6(b), coils 1 and 4 are joined in series and comprise one phase of the winding; coils 3 and 6, another phase; coils 5 and 2, the third phase; and the winding is delta connected. Note that each phase now consists of two equal groups, or coils, **per pair of poles,** or one group, or coil, per pole. Thus the winding has **one coil per pole per phase.**

It is apparent, from the figure, that the emfs in each pair of coils, comprising one phase, are in time phase with each other. That is,

the emfs in coils 2 and 5, for instance, reach a maximum at the instant shown, but 180 electrical degrees from each other in the closed ring. Thus they are joined in series with coil 2, reversed with respect to its positive direction of emf; and these two emfs combine in phase.

To clarify the relations of the coils in Fig. 33–6, they are concentrated at six equidistant points on the ring, as shown in Fig. 34–6. It is readily seen that, with a positive direction of emf from S to F through all the coils, the emf of coil 2 lags 60 electrical, or time degrees behind that of coil 1; emf of coil 3 lags 60° behind that of coil 2, etc., as we proceed in clockwise direction around the ring.

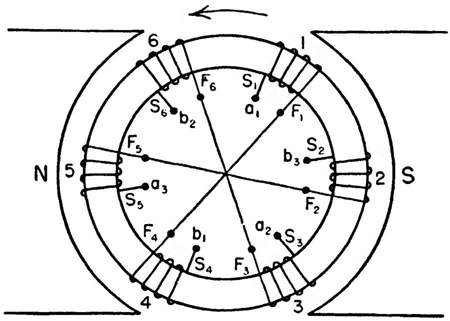


FIG. 34–6. The coils of Fig. 33–6(a) are concentrated at six equidistant points on the ring, and connected in series in pairs to form three separate phases at 120°.

Therefore, the emf of phase a_2b_2, consisting of a series combination of coils 3 direct and 6 reversed, lags 120° behind that of phase a_1b_1, consisting of a series combination of coils 1 direct and 4 reversed. And phase a_3b_3, consisting of coils 5 direct and 2 reversed, lags 120° behind that of phase a_2b_2. Thus a correct three-phase voltage may be obtained from these several pairs of terminals.

Now, in our original assumption, the armature of Fig. 31–6 tapped at a_1b_1 for single phase, developed 100 volts, shown as b_1a_1 in Fig. 32–6. From this latter figure, it is seen that the vector, representing the resultant emf of four conductors in series, or one coil, in Fig. 34–6, is one side of the hexagon inscribed in the circle. This is always equal to the radius of the circle, and so represents 50 volts in the diagram, since the diameter represents 100 volts.

It has already been shown that the resultant emf of two adjacent coils in series (Fig. 32–6) is the vector b_3a_3, or 86.6 volts. However, if coils 2 and 5, etc., are properly joined in series, their emfs

combine in phase and produce 50 + 50, or 100 volts per phase. Thus, when the phases of Fig. 34–6 are connected in delta, b_1 to a_2, b_2 to a_3 and b_3 to a_1, as in Fig. 35–6, the emf across each pair of the terminals, T_1T_2, and T_3, is 100 volts.

Since the allowable current per coil in this armature is 50 amperes, the full load line current per terminal is $50 \times \sqrt{3}$ or 86.6 amperes, as before. And the allowable output $(\sqrt{3}\ E_L I_L)$ is $\sqrt{3} \times 100 \times 86.6$, or 15,000 volt-amperes; or 15 kw at unity power factor.

Also, if the phases are connected in Y as in Fig. 36–6(*a*), the emf between terminals, or collector rings, is $\sqrt{3} \times 100$, or 1732 volts.

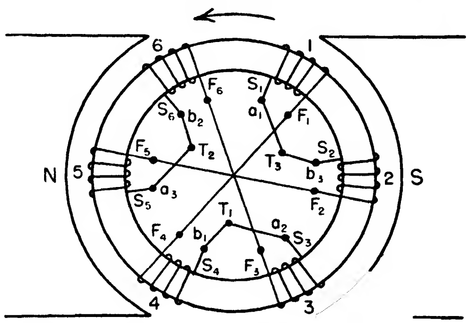


FIG. 35–6. The three phases of Fig. 34–6, connected in delta, similar to Fig. 33–6(*b*).

Current per terminal is 50 amperes, and the allowable output is $\sqrt{3} \times 173.2 \times 50$, or 15,000 volt-amperes, or 15 kw, as before.

Figure 36–6(*b*) is another arrangement of the coils which produces a Y connection with **longer end connections** to the common neutral. Note that coils 5 and 2 are both reversed with respect to each other, thereby giving an emf for this phase reversed 180° from the connection of Fig. 36–6(*a*). But their connection to the neutral point is also reversed, so that their combined emf is in the same rotation to that of the other two phases, as before. The capacity of the armature, so connected, is unchanged.

The foregoing discussion shows the capacity of the same armature when tapped for three phase is $\dfrac{15}{14.14}$ of its capacity when tapped for two phase and $\frac{15}{10}$ when tapped for single phase.*

* These relations are general and apply to any type of commercial winding, regardless of the number of coils or the number of poles, provided only that all coils are exactly similar and the total number of coils per pole is divisible by the numbers of phases.

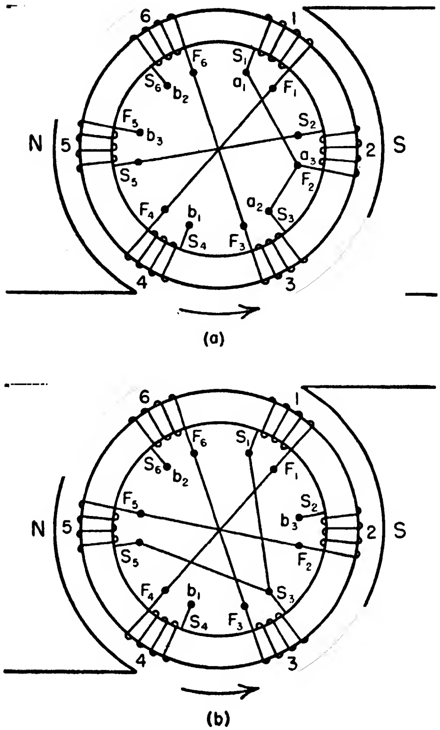


FIG. 36–6. (*a*) The winding of Fig. 34–6, connected in Y. (*b*) Another arrangement of the coil in Y with shortened end-connections to the common neutral point.

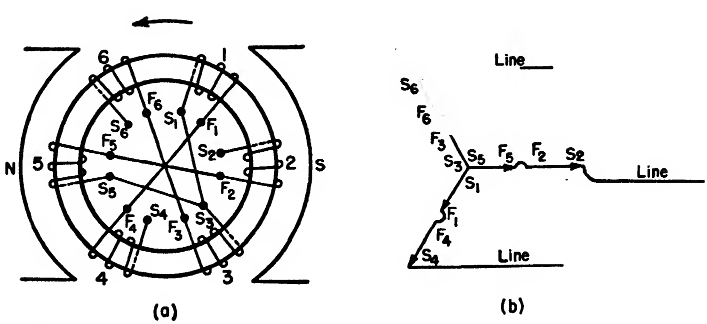


FIG. 37–6. (*a*) The best Y connection for this armature, similar to Fig. 36–6(*b*). (*b*) The corresponding vector diagram.

Thus for the same weight of material, there is a very definite advantage in favor of the three-phase alternator, which fact explains why most a-c machines of any considerable size are three-phase.

Example 3. The two-pole three-phase alternator of Fig. 37–6(*a*) (similar to Fig. 36–6(*a*)), shows the connections of a ring armature equivalent to a drum winding having **one group of coils per pole per phase.** All groups in each phase are in phase with each other, thus producing at the terminals the greatest full-load voltage possible to obtain from this armature at normal speed and flux. Assume this machine so connected that it can safely deliver 200 kva at 2300 volts

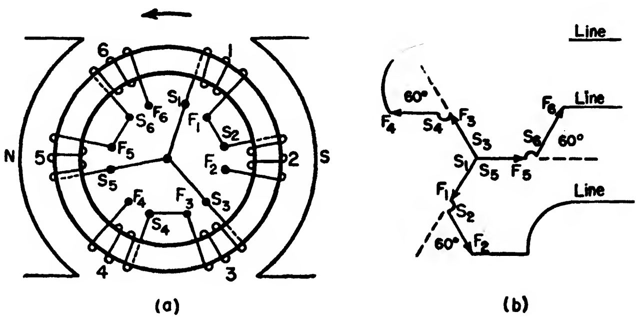


FIG. 38–6. (*a*) A different arrangement of the coils in Y. Note that the emfs in the two coils S_1F_1 and S_2F_2 are not in phase. (*b*) The corresponding vector diagram.

between terminals on a balanced load. (*a*) What is the full load current per coil? (*b*) What is the normal voltage per coil?

Solution:

$$(a)\ 200{,}000 \text{ volt-amperes} = \sqrt{3} \times 2300 \times I_{\text{line}}.$$

$$I_{\text{line}} = \frac{200{,}000}{\sqrt{3} \times 2300} = 50.2 \text{ amperes} = \text{phase current. } \textit{Ans.}$$

$$(b)\ E_{\text{phase}} = \frac{2300}{\sqrt{3}} = 1328 \text{ volts.}$$

Since the emfs of the two series connected coils per phase are in phase with each other, as shown by the vector diagram in Fig. 37–6(*b*),

$$E_{\text{coil}} = \frac{1328}{2} = 664 \text{ volts. } \textit{Ans.}$$

Prob. 34–6. If the winding of Fig. 37–6(*a*) is reconnected, as in Fig. 38–6(*a*), and generates the same voltage per coil as in Example 3, what will be the rated current per terminal, voltage between terminals

and kva capacity? The corresponding vector diagram is shown in Fig. 38–6(*b*).

Prob. 35–6. Solve Prob. 34–6 on the basis of the connections shown in Fig. 39–6(*a*). Note this is another Y connection with F_1S_4 joined

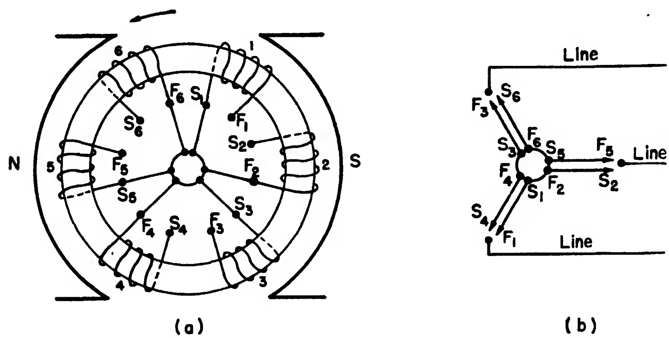


FIG. 39–6. (*a*) Another Y connection for this armature. Note that coil ends F_1 and S_4 are connected together to form one terminal; F_3 and S_6 to form the second; and F_5S_2 to form the third. (*b*) The corresponding vector diagram.

together to form one terminal winding; F_3S_6 another terminal; and F_5S_2 the third. Thus the two coils in each phase are connected in parallel, as shown by the corresponding vector diagram of Fig. 39–6(*b*).

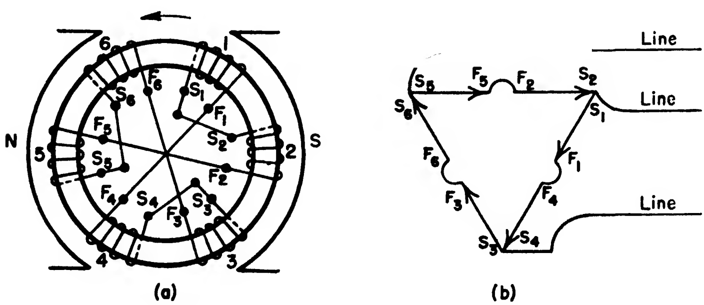


FIG. 40–6. (*a*) The best delta connection for this armature, similar to Fig. 35–6. (*b*) The corresponding topographic vector diagram.

Prob. 36–6. Solve Prob. 34–6 on the basis of the connections shown in Fig. 40–6(*a*). Note that this is a delta, so arranged as to give the greatest voltage per phase. Figure 40–6(*b*) is the corresponding vector diagram.

Prob. 37–6. Solve Prob. 34–6 on the basis of the connections shown in Fig. 41–6(*a*) — another delta connection. Figure 41–6(*b*) is the corresponding vector diagram.

Prob. 38–6. Solve Prob. 34–6 on the basis of the connections shown in Fig. 42–6(*a*) — another parallel connection of the coils. Figure 42–6(*b*) is the corresponding vector diagram.

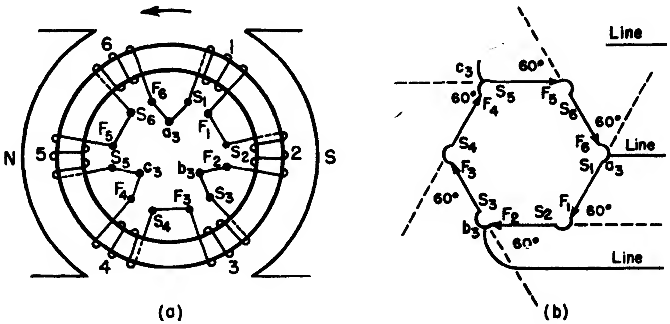


FIG. 41–6. (*a*) Another arrangement of the coils in a delta, or mesh, connection. (*b*) The corresponding topographic vector diagram.

Prob. 39–6. From the solutions of the five preceding problems and Example 3, explain which methods of connection should be used to obtain each of the following results:

(*a*) Obtain the greatest possible three-phase line current from the given coils.

(*b*) Obtain the greatest possible three-phase terminal voltage from the given coils.

(*c*) Obtain the greatest possible kva from the given coils.

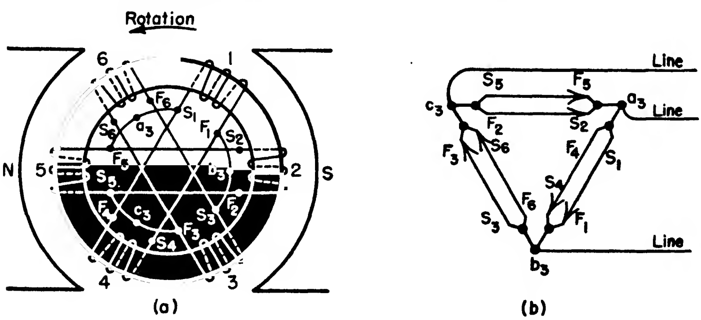


FIG. 42–6. (*a*) Another arrangement of the coils in delta. (*b*) The corresponding vector diagram.

Prob. 40–6. Explain, by means of vector diagrams, what results would be obtained by making the following connections in Fig. 34–6. (*a*) a_1 to a_2, b_2 to b_3, a_3 to b_1, " junction " points being armature terminals. (*b*) b_1 to b_2, a_2 to a_3, b_3 to b_1, junction points being armature terminals. (*c*) b_1 to b_3, a_3 to b_2, a_2 to a_1, junction points being armature terminals. (*d*) a_1, b_2 and b_3 together, others as terminals of the armature. (*e*) b_1, b_2 and b_3 together, others as terminals of the armature.

DRUM ARMATURE WINDINGS

19–6. General Principles — Definitions. The general principles which govern direct-current windings also hold for alternating-current drum windings.

Commercial drum windings are made up of coils, usually form-wound, both sides of which lie in slots in the surface of the armature core. The span of each coil must be such that the two sides lie under adjacent poles. All coils must be so connected that their emfs **add** together in the series circuit.

When the coil spans exactly the distance between the centers of adjacent poles, called the **pole pitch,** the coil and the winding is said to be **full pitch.** One pole pitch is equivalent to 180 electrical space-degrees on the periphery of the armature and may be measured in inches, or winding spaces, but is generally expressed in terms of slots. If the span of each coil, called the **coil pitch,** differs from the pole pitch, the coil and the winding is said to be **fractional pitch.** If the span of the coil, or coil pitch, is less than the pole pitch, it is said to be a **short-pitch,** or **short-chord,** coil or winding and is measured in percentage of full pitch. For instance, if the pole pitch, or full pitch, is equal to 10 slots on the armature core, and the coil pitch is 8 slots, the winding is called an $\frac{8}{10}$ pitch winding. This is equivalent to $\frac{8}{10}$ of 180°, or 144 electrical degrees. The use of fractional pitch windings generally has the effect of smoothing out the emf wave, making it approach more closely to a sine wave.

A concentrated winding is one which occupies only **one slot per pole.** A partially distributed winding occupies more than one slot per pole, but only part of the available surface of the armature. A fully distributed winding occupies all the slots, which are uniformly spaced over the entire periphery of the armature core.

A-c windings, like d-c windings, may be single layer or double layer. Regardless of the number of turns per coil, a single-layer winding has only one coil side per slot, while a two-layer winding has two coil sides, or two winding spaces per slot. In a two-layer winding, the coil pitch in winding spaces is odd.

20–6. Elementary Drum Windings. Diagrammatic Representation. Figure 43–6(a) represents the simplest four-pole rotating armature with four conductors only, and one slot per pole — a concentrated winding. The emf induced in the conductors under the South pole is **in,** while that in the conductors under the North

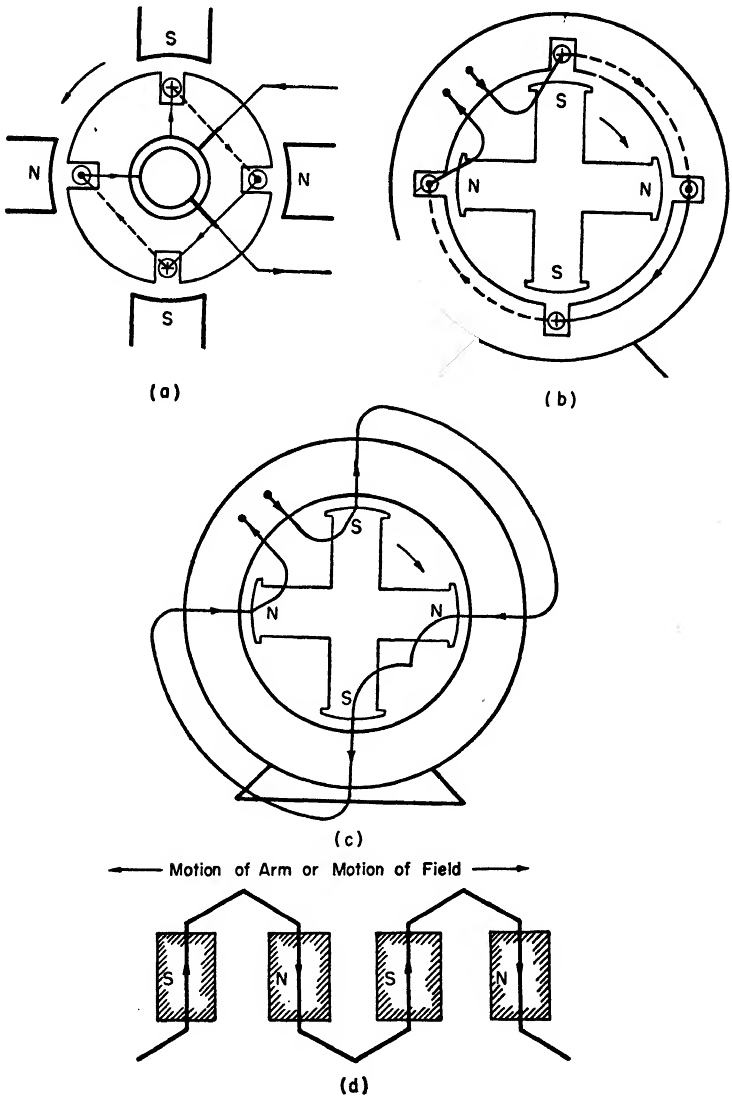


FIG. 43-6. (*a*) Simplest four-pole drum winding for a rotating armature, one slot and one conductor per pole. (*b*) The same simple winding for a rotating field alternator. (*c*) Another way of representing the winding in (*b*). Conductors are indicated by radial lines over the poles. Arrows on conductors indicate direction of induced emf; arrow pointing away from center indicates an emf of *in*. (*d*) A developed diagram of the winding in (*a*) and (*b*).

pole is **out.** At the instant shown, the emf in each conductor is at a maximum and the conductors are so connected that their emfs add together in the series circuit. The end connections are shown as chords of the circle; end connections at the rear, or pulley, end of the armature being shown as broken lines. Note that this is a **full-pitch** winding. So the emfs in the two sides of each coil are displaced 180 electrical degrees, but are so connected that their emfs combine in phase. Thus, the emf at the collector rings is the arithmetical sum of the emfs induced in the four conductors.

Figure 43–6(*b*) is a diagram of the same simple winding for a rotating field generator. It is full pitch and the emfs in all four conductors combine in phase. Note that counter-clockwise rotation of the armature in (*a*) and clockwise rotation of the field in (*b*) induce emfs in the same direction in the individual conductors.

Another diagram of the same winding with rotating field is shown in Fig. 43–6(*c*). The short radial lines over the poles represent the conductors perpendicular to the plane of the paper, and the arrows on these lines show the direction of induced emfs for clockwise rotation at the instant shown. An arrow pointing away from the center of the figure represents a voltage **in,** and in the opposite direction, a voltage **out.**

Figure 43–6(*d*) is a development of this winding and the arrows indicate the direction of induced emf in the various conductors, either for rotating fields or rotating armature.

21–6. Lap, Wave and Spiral Windings. Alternator armature windings may be grouped in three general classes in which, as in d-c windings, there are many variations. An armature may be **lap wound, wave wound,** or **spiral wound.**

Figure 44–6 is that of a six-pole partially distributed **lap** winding with four slots per pole. One turn per coil is shown in the figure. Figure 45–6 shows the same armature, connected as a wave winding.

In a d-c armature, for a given total number of armature conductors, the wave winding has a **greater number of series conductors** (fewer parallel paths) and **generates a higher voltage** than the lap winding, if the other conditions remain the same. In the alternator, the wave winding, for a given total number of armature conductors, has the **same number** of series conductors and generates the **same** voltage as the lap winding, if the other conditions remain the same. A comparison of Figs. 44–6 and 45–6 shows this. Thus, in the generation of a high voltage, the wave winding has no

advantage over the lap wound armature. Furthermore, the length of the end connections between coils is, in general, shorter in the lap-wound armature.

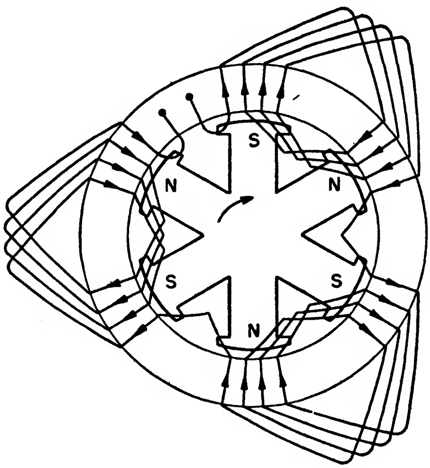


FIG. 44–6. A single-phase partially distributed *Lap* winding.

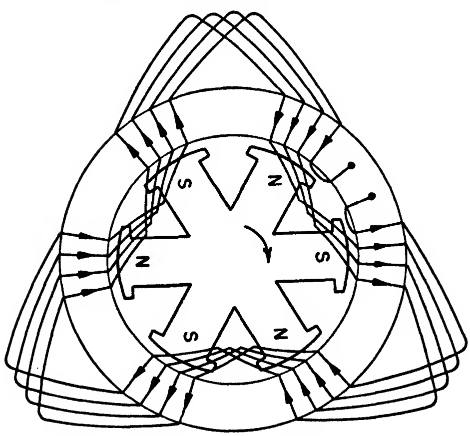


FIG. 45–6. Single-phase partially distributed *Wave* winding.

From the general appearance of the coils in each coil group of the lap and wave windings above, they are called **barrel** windings.

Figure 46–6 shows the same armature connected as a **spiral,** or **concentric** winding with the same number of conductors and slots per pole. Note that each coil group is connected in the form of a spiral and uses (in this case) two different shapes of coils with

different pitches. A winding with six slots per pole would require coils of three different shapes; with eight slots per pole, four different shapes, etc. This is a distinct disadvantage when additional coils must be kept in reserve for repairs. However, in the spiral windings, there are fewer crossings of the end connection, which reduces insulation problems. The spiral wound armature is always a single layer, while the lap and wave windings are usually two layer.

The lap and wave windings above are full pitch, since the coils each span just 180 electrical degrees. In the Fig. 46–6, the coils

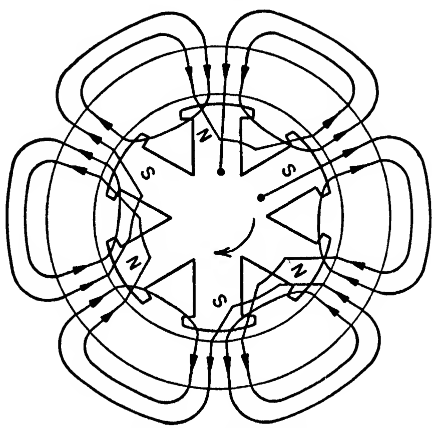


Fig. 46–6. Single-phase partially distributed *Spiral* winding.

are not full pitch. However, the voltage induced in any two adjacent coil groups is the same, under the same conditions, as that induced in any one coil group, in Figs. 44–6 or 45–6. Therefore, this spiral winding is equivalent to a full pitch winding.

The wave and spiral windings are in little use today in the United States, the lap winding being the more common type.

Prob. 41–6. Draw a single-phase four-pole lap winding, similar to that in Fig. 44–6, in which there are three slots per pole.

Prob. 42–6. Repeat Prob. 41–6 for a wave winding, similar to that in Fig. 45–6. How do the number of series conductors compare with that in Prob. 41–6?

Prob. 43–6. Draw a four-pole spiral winding similar to that in Fig. 46–6 with six slots per pole.

22–6. Single Phase Lap Windings. It was shown in Art. 18, Chap. VI, that, for the same weight of material, the output of a

two-phase armature is 1.4 times greater than a single-phase armature; for a three-phase armature 1.5 times greater. Because of this fact, single-phase windings are seldom used except in the stators of small motors. However, since the polyphase armatures consist merely of two or three separate single-phase windings on the

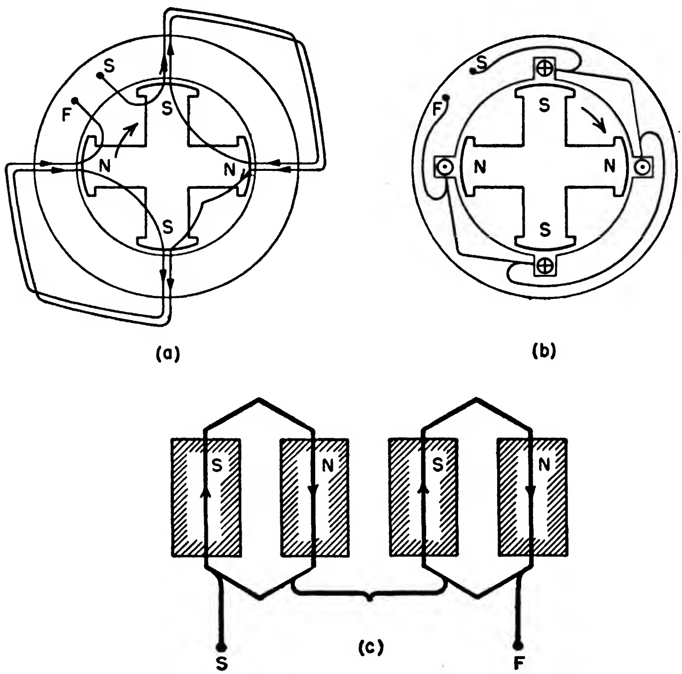


FIG. 47–6. Single-layer *half-coiled* lap winding — one slot per pole. (*a*) Winding has two turns per coil. (*b*) Appearance of the end connections. (*c*) Development of the winding.

same core, it is necessary to first understand the single-phase winding.

Figure 47–6 represents a simple four-pole lap winding, one slot per pole. Two turns per coil are indicated in (*a*). The loops around the conductors in this figure indicate they both lie in the same slot. The arrangement of the end connections of the coils with several turns is illustrated in (*b*); and (*c*) shows the development of the winding. The terminals of the winding are labeled *S* and *F* as shown. This is a single-layer full-pitch winding. It is also a " **half-coiled** " winding. Note that there are **two** coils in this **four-pole** winding or one half as many coils as poles. When a

winding has **one half** as many coils or coil groups **per phase** as there are poles, it is called a **half-coiled** winding. When a winding has the **same** number of coils or coil groups as poles, it is called a **whole-coiled** winding. Figures 44–6 and 45–6 represent half-coiled windings, while Fig. 46–6 is a whole-coiled winding.

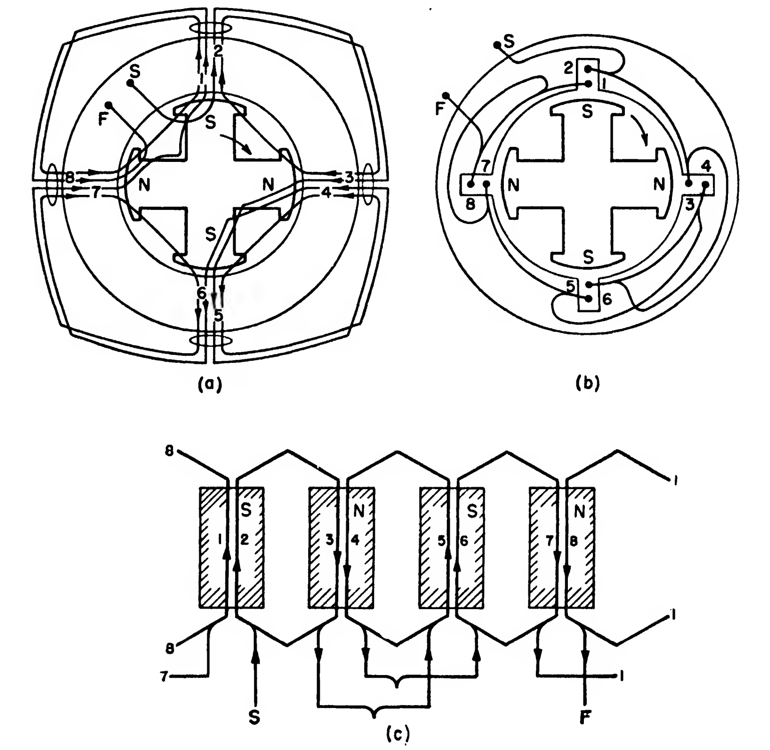


FIG. 48–6. Two-layer *whole-coiled* lap winding — one slot per pole. (*a*) Winding has two turns per coil, and two coil-sides per slot. (*b*) Appearance of the end connections. (*c*) Development of the winding.

Figure 48–6 is a four-pole full-pitch concentrated lap winding with one slot per pole, but with four coils having two coil sides per slot. It is, therefore, a two-layer winding. Since there are the same number of coils as poles, it is also a whole-coiled winding. Figure (*a*) shows two turns per coil and four conductors per slot. The arrangement of the end connections of the coil is shown in (*b*); and (*c*) shows the development of the winding.

Concentrated windings are seldom, if ever, used in practice;

since both a poorly shaped voltage wave is generated, and the surface of the armature core is not used to advantage.

Figure 49–6 is a partially distributed four-pole two-layer lap winding, having two slots per pole. One turn per coil is indicated and the winding is whole coiled. Starting at one terminal of the winding, S, we go out on conductor 1 and back on 6, which completes one coil, then pick up conductors 3 and 8 for the second coil in the first coil group. We then proceed through a long end connection and pick up conductors 12 and 7; then 10 and 5, the two coils in the second coil group. Then through a second end connection, we pick up conductors 9 and 14, 11 and 16, for the third coil group. By means of a third end connection, we pick up,

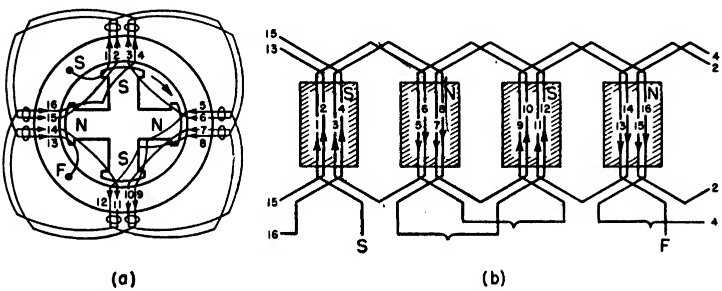


Fig. 49–6. Single-phase two-layer whole-coiled lap winding — two slots per pole. (*a*) Winding has one turn per coil and two coil-sides per slot. (*b*) Development of the winding.

for the fourth coil group, conductors 4 and 15, 2 and 13, from which we bring out the other terminal of the winding F. This is a full pitch progressive winding and the emfs in the two sides of each coil combine in phase. The emfs in the two coils of each coil group, however, are out of phase with each other, but the emfs of the four coil groups are in phase. Figure 49–6 shows more clearly than do Figs. 47–6 and 48–6 the general appearance of a lap winding.

Figure 50–6 is the development of a four-pole single-phase lap winding on an armature core having 24 slots, or 6 slots per pole. This winding occupies four slots per pole and, for clearness, is shown with one turn per coil. Starting at one terminal of the winding, S, we go out on conductor 1 and back on conductor 10, comprising one coil, and the **coil pitch in slots** is six. Since there are 24 slots in this four-pole winding, the slots are $\frac{720}{24}$, or 30 electrical degrees apart; and a coil span of six slots equals 6×30, or 180°. The winding is thus full pitch. From conductor 10, we pick up 3, 12,

5, 14, 7 and 16, completing one coil group A (four coils); then pass through a **short** end connection and pick up conductors 17, 26, 19, 28, 21, 30, 23 and 32 to complete the second coil group C. We than pass through a **long** end connection and pick up conductors 8, 31, 6, 29, 4, 27, 2 and 25, completing the third coil group D. Then through another **short** end connection, we pick up conductors 24, 15, 22, 13, 20, 11, 18 and 9, for the fourth coil group B, from which we bring out the other terminal F. All conductors are so connected that their emfs add together in the series circuit. The arrows on the conductors indicate the proper direction of induced emf when the poles move to the right. Poles are assumed to lie underneath the conductors in the figure. In the position shown,

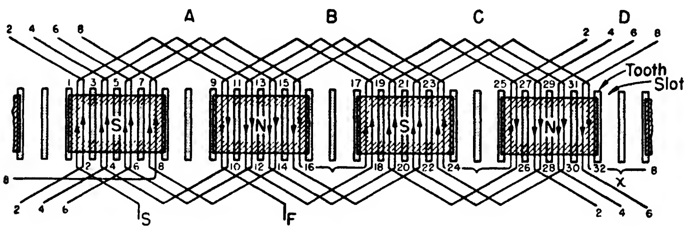


FIG. 50–6. Winding uses only two-thirds of the slots in the armature core. Coil groups are connected in the order A, C, B, D

the induced voltage is at a maximum. Note that the span of each coil in winding spaces is odd, just as in the d-c armature.

Figure 50–6 is a whole-coiled winding, since there are the same number of coil groups (A, B, C and D) as poles. The emfs in all the several coil groups are in phase and the order in which they may be joined in series may be varied. In this figure, the coil groups are connected in the order, A, C, B, D, a progressive winding. Note, that after picking up conductor 32, we may pass forward, by means of an end connection spanning six slots to conductor 8, as shown; or, by means of another end connection, spanning six slots, we may pass back from conductor 32 to conductor 24, picking up conductors 15, 22, 13, 20, 11, 18 and 9; then by a short end connection to conductor 8, picking up 31, 6, 29, 4, 27, 2 and 25, from which the second terminal is brought out. In this latter arrangement the coil groups would be connected in the order A, C, B, D, a retrogressive winding. Note that in both these arrangements we have **one long** and **two short** end connections. The generated emf would be the same in both cases.

Figure 51–6 shows the same winding, as in Fig. 50–6, but with the coil groups connected in a manner similar to Fig. 49–6. Instead of passing from conductor 16 (Fig. 50–6) through a short end connection to conductor 17 in coil group *C*, we pass through a long end connection and pick up conductor 24 in coil Group *B*. The coil groups then are picked up in the order *A*, *B*, *C*, *D*, another progressive winding. This arrangement gives the same emf at the terminals. While the diagram of Fig. 51–6 is easier to trace, it has the disadvantage of three **long end** connections.

A careful inspection of Figs. 50–6 and 51–6 will show that, if the winding is opened at its midpoint, *x*, we have two coil groups in each half, generating equal emfs in phase with each other. These

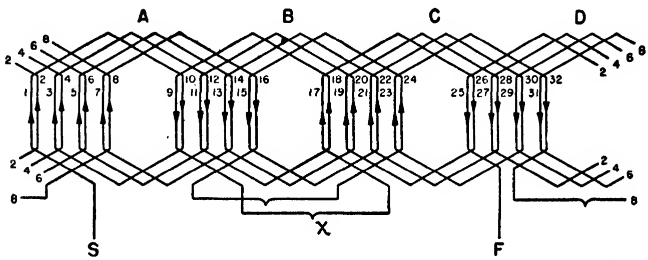


FIG. 51–6. The same winding as Fig. 50–6, except that the coil groups are connected in the order *A*, *B*, *C*, *D*.

two sections may be connected in parallel so that their emfs are in the same direction, and the normal voltage of the winding is reduced one half. This is a common arrangement, particularly in small motors, so they may be operated either from 120 or 240-volt circuits.

The windings of Figs. 50–6 and 51–6 occupy only four of the six slots per pole. If the other two slots were used, approximately fifty per cent more wire would be required which would add but little to the induced voltage. Since the slots are displaced 30 electrical degrees in space, the emf in coil one (conductors 1 and 10) leads that induced in coil two (conductors 3 and 12) by 30°; thus in each coil group, the emf in each succeeding coil lags 30° behind that of the preceding coil. Were two additional coils per group used to fill the remaining slots their emfs would combine at 120° and 150° respectively from that of the first coil.

Prob. 44–6. Draw a single-phase six-pole single-layer full-pitch half-coiled lap winding, having two slots per pole, which differ in position by 20 electrical space degrees. (See Fig. 47–6(*c*).)

Prob. 45–6. Draw a single-phase six-pole two-layer full-pitch whole-coiled lap winding for the armature core of Prob. 44–6.

Prob. 46–6. Draw a single-phase full-pitch two-layer whole-coiled lap winding for a four-pole armature in which there are 20 slots. Winding occupies three slots per pole.

Prob. 47–6. Draw a single-phase two-pole full-pitch whole-coiled lap winding for which there are 30 slots. Winding occupies 20 slots.

23–6. Alternator EMF: Differential Belt Factor. In both the alternator and the d-c machine, the generated emf depends upon the **rate of cutting** of the fixed flux. In Vol. I, Chapter X, the average voltage generated in a d-c generator is given as,

$$E_{\text{ave}} = \frac{\Phi PZN}{10^8 \times 60 \times a}$$

where Φ = flux per pole;
P = number of poles;
$\dfrac{Z}{a}$ = number of series conductors; $\dfrac{N}{60}$ = rev per second.

The average emf, induced per conductor in either an a-c, or a d-c generator, can therefore be expressed as

$$E_{\text{ave}} = \frac{\Phi PN}{10^8 \times 60} \tag{3–6}$$

In the a-c generator, this relation may be expressed in terms of the frequency. Since frequency (f) = $rps \times$ pairs of poles;

$$f = \frac{P}{2} \times \frac{N}{60}, \quad \text{or} \quad 2f = \frac{PN}{60}.$$

Substituting in Eq. (3) above, we have,

$$E_{\text{ave}} = \frac{\Phi 2f}{10^8} \tag{4–6}$$

And if the distribution of the flux in the air gap is such that a sine wave of emf is generated, the expression for the effective voltage per conductor becomes

$$E_{\text{eff}} = \frac{0.707}{0.636} \times \frac{\Phi 2f}{10^8} = \frac{1.11 \times \Phi 2f}{10^8} = \frac{2.22\Phi f}{10^8} \tag{5–6}$$

In a concentrated winding of one slot per pole and Z conductors in series per phase, the emfs in all conductors are in time phase,

and the fundamental equation for effective voltage per phase is written:

$$E_{\text{eff}} = \frac{2.22\Phi Zf}{10^8} \qquad (6\text{–}6)$$

However, the commercial winding occupies several slots per pole per phase, and the emfs in all the coils in any one coil group are not in time phase with each other. The terminal emf, therefore, is the vector sum of the emfs in the series conductors and equation 6 above must be corrected by a "factor," K_b, called the "**differential belt factor.**" This factor is the **ratio of the vector**

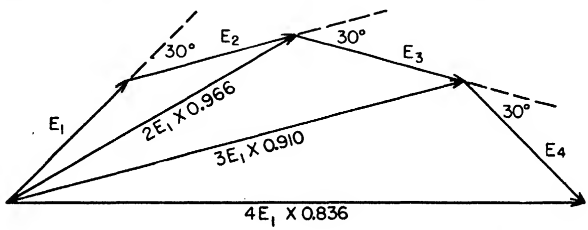


FIG. 52–6. Vector diagram for determination of "belt factor."

sum of all the emfs in any one coil group to their arithmetic sum, and varies with the number of slots per pole per phase and the space angle in electrical degrees between slots.

Equation (6), therefore, is written,

$$E_{\text{eff}} = \frac{2.22K_b\Phi Zf}{10^8} \qquad (7\text{–}6)$$

This equation gives the effective value of the emf for a full-pitch winding, generating a sine wave of voltage.

The differential belt factor for the winding of Figs. 50–6 or 51–6 may be determined by means of the vector diagram of Fig. 52–6. The vectors E_1, E_2, E_3, etc., represent the emfs of the four coils in a coil group, and are drawn at 30° to conform to the phase angle between the coils. From the diagram, the vector sum of these emfs is readily found to be 0.836 of their arithmetic sum.*

* It can be shown that the differential belt factor may be computed from the expression $\dfrac{\sin n\frac{\beta}{2}}{n \sin\frac{\beta}{2}}$, where n = the number of slots per pole per phase; β = electrical degrees between adjacent slots.

The differential belt factor is computed from the diagram as:

$$K_b = \frac{4E_1 \times 0.836}{4E_1} = 0.836$$

Also from the diagram, the belt factor for two slots per pole is 0.966; and for three slots per pole, 0.91.

Example 4. What effective emf will be generated by the winding of Fig. 50–6, if it has 5 turns per coil; the flux per pole is 2×10^6 and the frequency is 60 cycles?

Solution: This winding, as shown, has 16 coils, or 32 conductors. With 5 turns per coil, $Z = 5 \times 32$ or 160 conductors. $K_b = 0.836$. Substituting in Eq. (7) above,

$$E_{\text{eff}} = \frac{2.22 \times 0.836 \times 2 \times 10^6 \times 160 \times 60}{10^8} = 356 \text{ volts.} \quad \textit{Ans.}$$

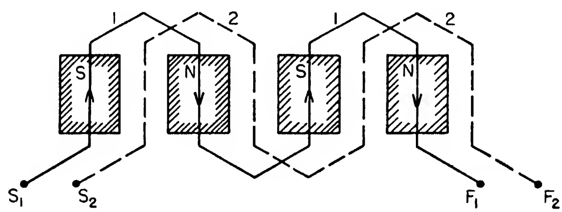


FIG. 53–6. Development of a very simple two-phase winding, one slot per pole per phase.

Prob. 48–6. What is the effective emf induced per conductor in the winding of Example 4?

Prob. 49–6. What is the differential belt factor for the winding in Prob. 46–6?

24–6. Two-Phase Lap Winding. If a second winding is placed on the armature core of Fig. 43–6, so that the coil sides of this winding lie in slots, midway between those of the first, as shown in Fig. 53–6, this second winding will be displaced 90 electrical degrees in space from the first, and a very elementary two phase winding is obtained. S_1F_1 are the terminals of phase 1 and S_2F_2 the terminals of phase 2. This is a single-layer concentrated winding of one slot per pole per phase. In the position shown, the instantaneous emf of phase 1 is at a maximum, while that in phase 2 is zero.

Figure 54–6 is a two-phase four-pole armature wound on the same 24-slot core, as in Figs. 50–6 and 51–6. This is a fully distributed whole-coiled lap winding. Each phase is separate and

wound in a different set of slots from the other. S_1F_1 are the terminals of one phase and S_2F_2 the terminals of thé other. One turn per coil is indicated in the figure. The pitch of each coil is six slots or 180 electrical degrees, so this is a full-pitch winding. The winding is two layer and the two coil sides in any one slot are in the same phase. There are three coils in each coil group, instead of four, and the winding has **three slots per pole per phase.** The order of connecting the coil groups in each phase is the same as in Fig. 51–6, namely, *A*, *B*, *C*, and *D*, and the emfs in the four coil groups are all in phase. An inspection of Fig. 54–6 shows that the emfs in the two phases are displaced 90 electrical degrees and the emf in phase 1 is at a maximum. A winding table is also shown in

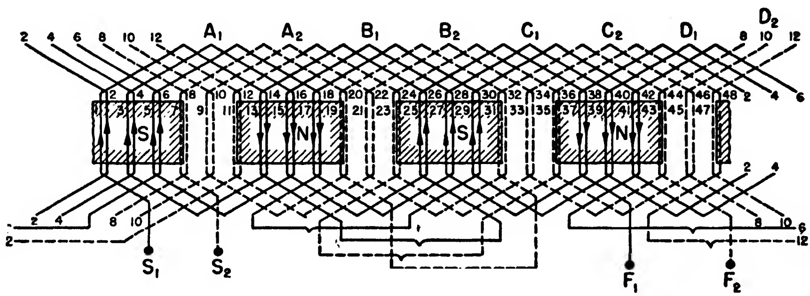


FIG. 54–6. A two-phase full-pitch lap winding on the same 24-slot armature core used in Figs. 50–6 and 51–6. There are three slots per pole per phase. A winding table is also shown.

the figure. Since this winding has three coils per coil group and adjacent coils in each group are displaced 30°, the belt factor is 0.91, as shown in Fig. 52–6.

In a symmetrical polyphase winding, the number of slots should be divisible by the number of phases and also by the number of poles.

$$\textbf{The number of slots per pole per phase} = \frac{\text{Total slots}}{\text{Poles} \times \text{No. of phases}}.$$

Thus in the winding of Fig. 54–6, the number of slots per pole per phase equals $\dfrac{24}{4 \times 2}$, or 3.

Prob. 50–6. If the emf wave of the armature of Fig. 54–6 is of sine wave form, what voltage is induced per phase when, as in Example 4, the frequency is 60 cycles, the flux per pole is 2×10^6 and there are 5 turns per coil?

Prob. 51–6. Draw a fully distributed two-phase four-pole whole-coiled two-layer full-pitch lap winding for an armature core in which there are 32 slots. Show how you compute the number of slots per pole per phase.

Prob. 52–6. The slots in the armature core of Prob. 51–6 differ in position by how many electrical space degrees? What is the differential belt factor?

Prob. 53–6. What voltage is induced per phase in the winding of Prob. 51–6, if the frequency is 25 cycles, the flux per pole is 3×10^6, and there are 4 turns per coil?

Prob. 54–6. Draw a 6-pole 36-slot full-pitch whole-coiled two-phase lap winding.

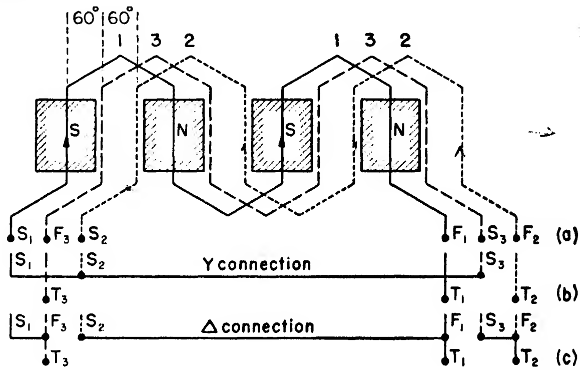


FIG. 55–6. (*a*) Development of a very simple three-phase winding, one slot per pole per phase. (*b*) Arrangement of coil ends for a Y-connection. (*c*) For a delta connection.

25–6. Three-Phase Lap Winding. Figure 55–6(*a*) is a developed diagram for the simplest three-phase four-pole drum winding. There are twelve equally spaced slots in the armature core which are, therefore, displaced $\frac{720}{12}$ or 60 electrical degrees. Each of the three windings occupies four slots, or one slot per pole per phase. Phase 1 lies in four slots directly under the four poles, and we mark the terminals S_1F_1, as shown. Phase 3 lies in the four slots, displaced 60 electrical degrees from phase 1. Phase 2 lies in the remaining slots, 60° from phase 3 or 120° from phase 1; and we mark the terminals S_2F_2. Now, if we consider the positive direction of emf in phase 3 to be reversed, its emf is displaced 120° from both that of phases 1 and 2; and we mark the terminals in reverse order S_2F_2 as shown. The terminals S_1, S_2 and S_3 are, therefore, the similar or corresponding ends of three windings at 120° with each other. Figure 55–6(*b*) shows how these ends may

be joined for a Y connection of the winding, the ends F_1, F_2 and F_3 being brought out as the three-phase terminals T_1, T_2 and T_3. Figure 55–6(*c*) indicates the arrangement for a delta connection.

In Figure 56–6, the same 24 slot armature core, as shown in Figs. 50–6, 51–6 and 54–6, is used for a three-phase, fully-distributed two-layer lap winding. This is a full pitch (six slots per coil) whole-coiled winding, exactly similar in the order of connecting the coil groups, to Figs. 51–6 and 54–6, except that that there are now **two** slots per pole per phase $\left(\dfrac{24}{4 \times 3} = 2\right)$. A winding table for each phase is also shown. Note that the terminals S_3F_3 of phase 3 are reversed in sequence with respect to

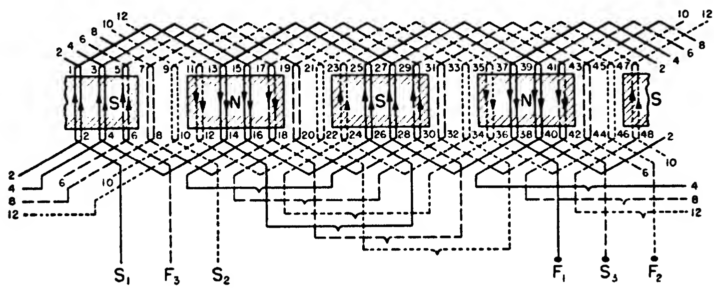


FIG. 56–6. Three-phase full-pitch lap winding on the same 24-slot armature core previously used. Winding has two slots per pole per phase. A winding table is shown.

the other two phases, exactly as in the simple winding of Fig. 55–6, so that three emfs, all differing in phase by 120°, are obtained. Also the emfs in all four coil groups in each phase are in phase with each other.

Since adjacent coils in each coil group are also displaced 30 electrical degrees and there are two coils per group, or two slots per pole per phase, the belt factor, as shown in Fig. 52–6, is 0.966.

Prob. 55–6. (*a*) If the emf wave of the armature in Fig. 56–6 is of sine wave form, what voltage is induced per phase when, as in Example 4, the frequency is 60 cycles, the flux per pole is 2×10^6 and there are 5 turns per coil? (*b*) What will be the emf of the machine when Y connected? (*c*) What is the voltage induced per conductor?

Prob. 56–6. Draw a three-phase fully-distributed full-pitch four-pole lap winding for an armature core having 36 slots.

Prob. 57–6. What is the angle in electrical space degrees between adjacent slots in Prob. 56–6? What is the belt factor?

Prob. 58–6. Assuming sine wave of emf, what voltage is induced per phase in the armature of Prob. 56–6, if the flux per pole is 3.5×10^6 lines, turns per coil is 4 and the speed is 750 rpm?

Prob. 59–6. Draw a three-phase six-pole lap-winding for an armature having 54 slots.

26–6. Fractional Pitch Windings: Coil Pitch Differential Factor. The windings previously shown are all full pitch. Armatures in most commercial alternators are wound with **fractional pitch** windings. This improves the shape of the emf wave, as stated in Art. 19 and requires somewhat less copper, due to reduced span of the coils.

Figure 57–6 is a three-phase four-pole fractional-pitch lap winding wound on the same 24-slot armature core previously used.

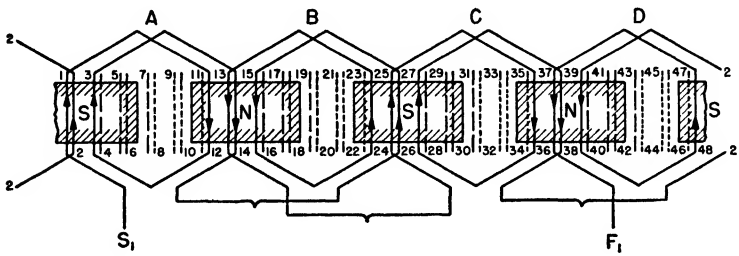


FIG. 57–6. Three-phase five-sixths-pitch lap winding. The circuit for one phase only is shown. This winding, except for the pitches, is identical to that in Fig. 56–6.

For clearness in the diagram, only one phase is shown, the other two being identical. This winding is exactly similar to that in Fig. 56–6 in the number of coils per coil group, number of groups per phase and the order of connecting these groups in series, except that each coil spans only five slots, instead of six. Full pitch, as we have seen, is six slots; therefore, this is a $\frac{5}{6}$ pitch winding.

Since the slots in this four pole winding are displaced 30 electrical degrees in space (in this case) the two sides of a coil, such as conductors 1 and 12, are 5×30 or 150° apart, and are so connected that their emfs combine at 30° (instead of in phase with each other, as in the full pitch winding). The emfs of the several coils in each group still combine at 30°, as before, and a careful inspection of Fig. 57–6 shows that the emfs of the several coil groups also combine in phase with each other.

Note that, except for one slot per pole in each phase, no two coil sides of the same phase lie in a single slot. This arrangement

somewhat reduces the inductive effect and so reduces the reactance of the winding.

Because each coil is less than full pitch, and the emfs in the two coil sides do not combine in phase, the emf per coil and per phase is reduced. Therefore, another correction, known as the "**coil pitch differential factor**" (K_p), commonly called the "**pitch factor,**" must be applied to the formula for effective voltage per phase (Equation 7–6).

The corrected equation is

$$E_{\text{eff}} = \frac{2.22K_pK_b\phi Zf}{10^8} \tag{8–6}$$

In Fig. 52–6, Chap. VI, vectors E_1, E_2, E_3, etc., represent the arithmetic sum of the emfs per coil in each coil group of a full-

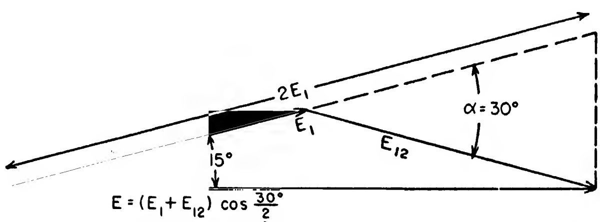


FIG. 58–6. Vector diagram for determination of "pitch factor."

pitch winding. In the fractional-pitch winding E_1, E_2, etc., each is the vector sum of the emfs in the two sides of a coil. In Fig. 58–6, E_1 and E_{12} represent the emfs in conductors 1 and 12 of Fig. 57–6, and the diagram shows that the emf per coil (E_c) is

$$E_c = E_1 + E_{12} \cos \frac{30°}{2} = 2E_1 \cos 15° = 2E_1 \times 0.966.$$

Coil pitch factor in this case is 0.966.

Expressed as a general equation

$$E_c = (E_a + E_b) \cos \frac{\alpha}{2} = 2E_a \cos \frac{\alpha}{2}, \tag{9–6}$$

where $E_a = E_b =$ the emfs per coil side;
$\alpha =$ phase displacement of the two coil sides.

The constant 0.966 is the coil pitch factor only for a winding in which the emfs in the two coil sides differ by 30 electrical degrees.

Prob. 60–6. If the winding of Fig. 57–6 generates a sine wave of emf, what voltage is induced per phase, when, as in Example 4, the frequency is 60 cycles, the flux per pole is 2×10^6 and there are 5 turns

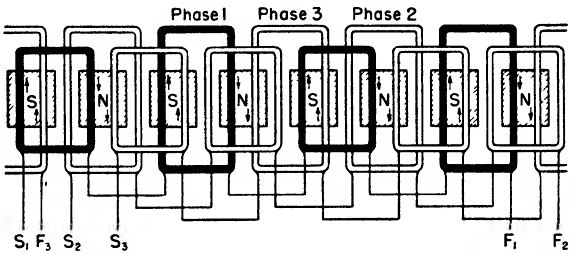


FIG. 59–6. Three-phase chain winding, one slot per pole per phase.

per coil? Compare this voltage with that induced per phase in the same full-pitch winding of Prob. 55–6.

Prob. 61 6. Draw a seven-eighth-pitch two-phase four-pole two-layer lap winding. There are to be 32 slots in the armature core.

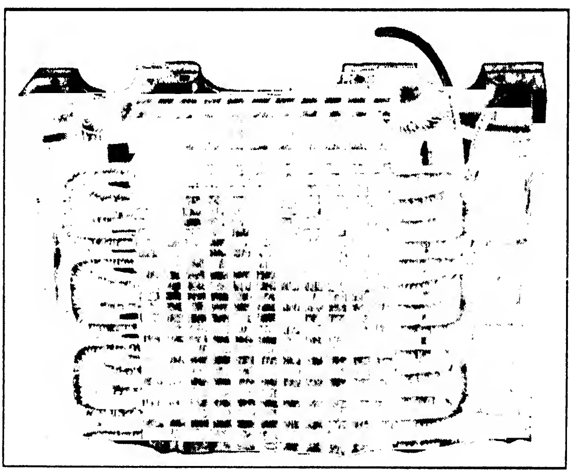


FIG. 60–6. Section of completed chain winding, one slot per pole per phase.

Prob. 62–6. What is the pitch factor of the winding in Prob. 61–6?

Prob. 63 6. Draw an eight-ninth-pitch three-phase four-pole two-layer lap winding for which there are 36 slots.

27–6. Chain and Basket Windings. A **chain** winding is a spiral winding for a polyphase armature, having **one slot per pole per phase.** It has half as many coils as slots and, therefore, is a single

layer winding. The chain winding is so named from the appearance of the assembled coils. Figure 59–6 shows a developed diagram of an eight-pole three-phase chain winding, having several turns per coil. Note that there are half as many coils per phase as poles, and the winding is full pitch. In each phase, the

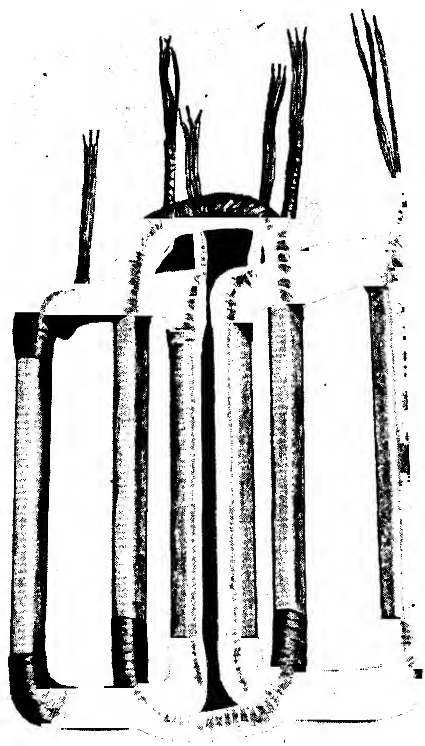


Fig. 61–6. Appearance of the coils in a chain winding similar to that in Fig. 60–6. Note that they consist of two different forms.

emfs in the four coil groups are all in phase with each other. In the figure, the coils in phase 2 are in the position of zero emf.

The appearance of a section of a completed winding is illustrated in Fig. 60–6. Since the end connections of the coils lie in a single plane, perpendicular to the generator shaft, the winding is called "single range." The appearance of the coils is shown in Fig. 61–6. Note that they consist of two different forms to avoid crossing of the end connections.

Figure 62–6 is the development of a three-phase four-pole winding on the same 24 slot armature previously shown. This winding has **more** than one slot per pole per phase — two, in this case —

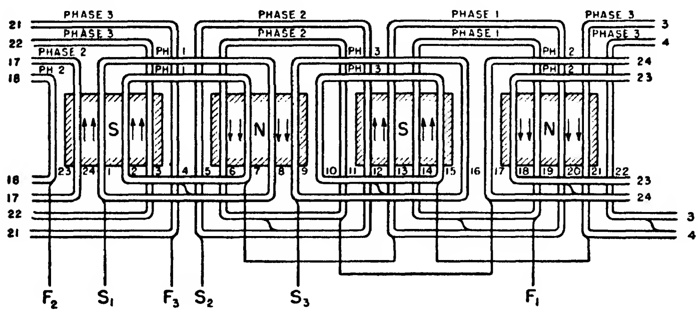


FIG. 62–6. Three-phase basket winding, two slots per pole per phase.

and is, therefore, called a " **basket winding.**" Note that each coil group consists of two concentric wound coils, and the coil ends lie

FIG. 63–6. Appearance of a section of a three-phase basket winding, three slots per pole per phase.

in two vertical planes perpendicular to the shaft, therefore, the winding is called " two-range." The two coils per group have different pitches, but with the same number of series conductors per coil, as in the lap winding of Fig. 56–6, will generate the same

voltage per phase. It is, therefore, equivalent to a full-pitch winding. The appearance of a section of a basket winding with **three** slots per pole per phase, is shown in Fig. 63–6. This winding is three range.

When the number of pairs of poles is odd, the number of coil groups per phase is also odd, and coils having one long side and one short side, as shown in Fig. 64–6, must be used to complete the winding. This occurs in windings of 6, 10, and 14 poles, etc. The solution of Prob. 65–6 illustrates this.

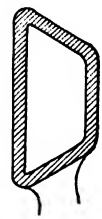


FIG. 64–6. A coil with one long and one short side must be used in a chain winding for which the number of pairs of poles is odd.

The chain winding is particularly adapted to slow speed alternators of relatively large capacity. Since most large generators are driven by high speed turbines, this type of winding is not in general use today.

Prob. 64–6. A four-pole alternator has 20 equally spaced slots in the armature core. Draw a single-phase spiral winding which occupies but three slots per pole.

Prob. 65–6. Draw a three-phase six-pole chain winding in which there is one slot per pole per phase.

Prob. 66–6. Draw a two-phase four-pole basket winding with two slots per pole per phase.

Prob. 67–6. Draw a three-phase four-pole basket winding for an armature in which there are 36 slots.

28–6. Shape of the EMF Wave. The instantaneous value of the induced emf per conductor has been given in Eq. (1), Chap. VI, as $e = \frac{B\ell v}{10^8}$. When the speed is constant, V is constant, and the emf is proportional to B. If the flux distribution, or the flux wave, is uniform under the poles without fringing, as in Fig. 13–6, Chap. VI, a flat emf wave per conductor, similar to the solid line in Fig. 65–6, results. (See Vol. I, Fig. 44–10(a), page 296.) If the distribution of the flux, or the flux wave, varies from point to point around the surface of the armature in a manner proportional to $B \sin \theta$, as in Fig. 14–6, the desired sine wave of emf per conductor results. However, neither of these conditions ordinarily exists in the actual machine, for the flux wave fringes at the pole

tips, as indicated by the dotted line in Fig. 65–6; and it is extremely difficult to obtain a flux distribution similar to that in Fig. 14–6.

To obtain approximately sinusoidal distribution of flux in the air gap of the definite pole machine, the pole tips are beveled, as

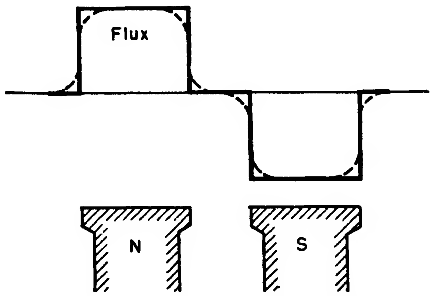


FIG. 65–6. Uniform distribution of flux under the pole faces.

indicated in Fig. 66–6, to give the greatest flux density at the centers of the poles. In the wound rotor alternator, the field coils are distributed in several slots. In Fig. 67–6(a), the field coil occupies six slots, a, b, c, d, e and f. In Fig. 67–6(b), the magnetic lines, or the flux, set up by the ampere turns in slots cd above is shown in (1); that set up by ampere turns in slots be alone, is shown in (2); and that by af alone, in (3). The combined magnetic lines, or flux, set up by the entire field coil, is indicated in Fig. 67–6(c). This gives a flux wave form in "steps." However, fringing smoothes out the wave and it approaches more nearly to that of a sine curve, as shown.

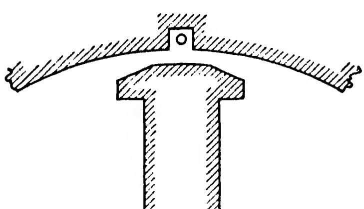


FIG. 66–6. Pole face is beveled to obtain highest flux density at the center of the pole.

In a concentrated winding, one slot per pole, the emfs in all the conductors, in any one phase, are in phase. Thus the emf wave at the terminals is exactly the same as that for one conductor, and any irregularities in the flux wave are reproduced in the emf wave.

In a distributed winding, occupying several slots per pole per phase, the emfs in the separate coils in a coil group are not in

phase, so the emf per coil group is the sum of several emf waves displaced from each other. This also aids in smoothing out the voltage wave at the terminals of the winding.

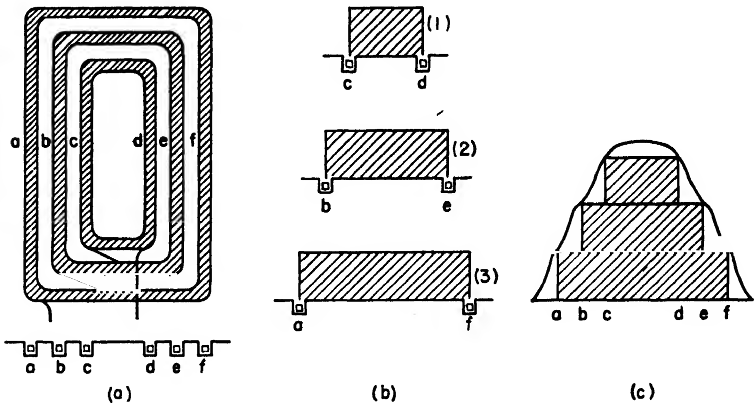


FIG. 67–6. (*a*) Distributed field winding in wound-rotor alternator. (*b*) Mmf, or flux, set up by each of the three coils in (*a*). (*c*) Combined mmf, or flux, set up by the three field coils.

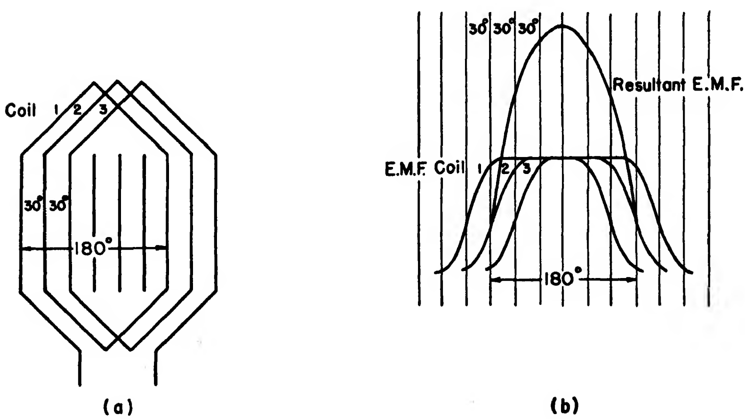


FIG. 68–6. (*a*) One coil group in an armature winding. (*b*) Form of emf wave induced in each coil in (*a*), and resultant emf of the entire group.

Figure 68–6(*a*) represents the coil sides in one coil group of a full pitch winding having three slots per pole per phase. Figure 68–6(*b*) shows the emf waves, generated by each of the three coils in the group. The emf of coil 2 lags 30° behind that of coil 1, and the emf of coil 3 lags behind that of coil 2 by the same angle. The

combined emf wave of the group is the sum of these three curves and approaches closely to sine wave form. Since the emfs in all the coil groups in the winding are in phase, the wave shape of the coil group is the same as that of the entire winding.

In the case of a fractional pitch winding, the emfs in the two sides of each coil would still have the same wave shape; but since the two coil sides are not in phase, the emf of the coil group would be (in this case) the sum of six emf waves, displaced from each other. This further improves the shape of the emf wave, both for coil group and at the armature terminals, as stated in Art. 19.

SUMMARY OF CHAPTER VI

There is no fundamental difference between a d-c generator and an alternator. Both have an armature in which the emf is generated. Both have a field winding, excited by direct current. The field in D-C GENERATOR is SELF EXCITED. The field in an A-C GENERATOR is SEPARATELY EXCITED.

AN ELECTROMOTIVE FORCE is generated whenever there is RELATIVE MOTION between a CONDUCTOR and a MAGNETIC FIELD. In a D-C GENERATOR, it is always the CONDUCTORS (FORMING THE ARMATURE) WHICH ROTATE. In an ALTERNATOR, EITHER THE FIELD OR THE ARMATURE MAY ROTATE. Most alternators have rotating fields. Small low voltage alternators sometimes have a rotating armature and a stationary field.

In all A-C GENERATORS and MOTORS, the ROTATING MEMBER is generally called the ROTOR, and the stationary member the STATOR.

ADVANTAGE OF ROTATING FIELD. Alternators are wound to generate high voltages, 2300 to 13,000 volts, and must be carefully insulated. Polyphase ROTATING ARMATURES must have three more collector rings which are difficult to insulate for these voltages and cause trouble due to arc waves. It is more difficult to insulate windings when they are subject to vibration, etc., due to rotation.

STATIONARY ARMATURES require no collector rings as the winding is brought out through the frame of the machine, and is not subjected to vibration or centrifugal forces. Only TWO collector rings are needed to supply low voltage direct current to the rotating field. Also the cross section area at the bottom of the teeth is relatively greater in stationary armatures, thereby reducing the reluctance of the magnetic circuit.

IN SLOW SPEED ALTERNATORS, the field poles are bolted or keyed to a spider mounted on the shaft, and the field coils are wound on these pole pieces. Machines so constructed are called " DEFINITE " or " SALIENT POLE " alternators.

IN HIGH SPEED TURBINE DRIVEN ALTERNATORS, the field structure consists of a solid cylinder, keyed or bolted to the shaft.

The field winding is set in slots in the surface of this cylinder and there are no projecting poles. Machines so constructed are called ROUND ROTOR ALTERNATORS.

A CLOSED ARMATURE WINDING for an alternator does not differ fundamentally from a d-c winding since an alternating emf is induced in both. The COMMUTATOR serves merely as a reversing switch, and rectifies the emf delivered to the terminals.

THE SUMMATION OF ALL THE EMFS IN A CLOSED-CIRCUIT WINDING IS ZERO. If TWO COLLECTOR RINGS are substituted for the commutator, a single phase alternating current can be taken from the winding. Taps are brought out to the rings in such a manner that the greatest possible emf exists between them; otherwise unbalanced currents will flow in the winding and the output will be reduced.

THE INSTANTANEOUS EMF induced in any conductor is found from the equation

$$e = \frac{B \ell V}{10^8}$$

where e = emf at any instant; B = flux density in which the conductor lies at that instant; ℓ = cutting length of conductor; V = relative speed of field and conductor at that instant.

(B, ℓ and V must all have the same length units.)

INSTANTANEOUS EMF BETWEEN TAPS in a closed circuit armature is equal to the arithmetical sum of the instantaneous emfs in all the series conductors between those taps.

BOTH COMMUTATOR AND COLLECTOR RINGS may be attached to the same armature winding. The same machine may then supply both alternating and direct current at the same time, as a DOUBLE-CURRENT generator; or it may receive a-c power at the collector rings, and deliver d-c from the commutator as a "synchronous converter."

TWO PHASE ARMATURE. When the closed winding is tapped at four equally spaced points on the winding per pair of poles and these taps are properly joined to four collector rings, the armature may be used to deliver two-phase alternating current to a four-wire circuit.

A FOUR-PHASE SYSTEM may be taken from a closed circuit winding, tapped for two phase. In this case, the emf of each phase of the two phases equals $\sqrt{2}$ or 1.41 times the emf of each of the four phases.

THE CURRENT IN EACH COIL of a two-phase generator with closed winding, on balanced load, equals 0.707 of the current in each line wire.

AN UNBALANCED LOAD on a two-phase closed-circuit winding, due either to unequal currents or power factors of the phases, causes unequal currents and unequal heating in the various coils in the armature. To avoid over-heating, the output of the armature must be reduced below the rated value.

OPENING A CLOSED WINDING at the tapping points, if tapped for single phase, forms two separate coils per pair of poles. These coils may be connected in series so that their emfs combine in phase to form a single-phase open-circuit winding of twice the generated emf at the terminals. However, the rated current is reduced one half and the output is unchanged.

If the closed winding, TAPPED FOR TWO PHASE, is opened at the tapping points, four separate coils per pair of poles results. They may be connected in series in two groups, so that the emfs in the coils in each group combine in phase to form a two-phase open-circuit winding. The emf per phase is now 1.41 its closed circuit value, and the rated current per phase is only 0.707 of its former value and the capacity of the machine is unchanged.

THREE PHASE ARMATURE. When a closed winding is tapped at three equally spaced points per pair of poles, and these taps are properly joined to three collector rings, three equal Δ-connected emfs are obtained. The armature may be used to supply a three-phase three-wire circuit.

If the armature is opened at the points of tapping, three separate groups of coils, per pair of poles, result. The emfs per group are displaced 120° and they may be connected in Y to supply a three-wire three-phase system, or used to supply a three-phase four-wire system.

For the same weight of copper and iron, the same speed and flux pole, THE RELATIVE CAPACITY of an armature, tapped for SINGLE, TWO, and THREE PHASE, is in the ratio of 1 : 1.41 : 1.5, respectively.

DRUM ARMATURE WINDINGS

COMMERCIAL DRUM WINDINGS generally consist of form-wound coils placed in slots on the surface of the armature core. The SPAN OF EACH COIL must be such that the two sides lie under adjacent poles. The POLE PITCH is the distance between the centers of adjacent poles, equal to 180 electrical space degrees. In a FULL PITCH winding, the span of the coil equals the POLE PITCH. The emfs in the two sides of a full pitch coil combine in phase with each other. In a FRACTIONAL PITCH WINDING, the span of the coil is less than the pole pitch and is generally measured in slots. If full pitch is 8 slots and coils span 7 slots the winding is called a $\frac{7}{8}$ pitch winding.

A CONCENTRATED WINDING occupies only one slot per pole.

Alternator windings may be SINGLE LAYER, or TWO LAYER; a single-layer winding has one coil side per slot; a two-layer winding two coil sides per slot. A WHOLE COILED winding has as many coil groups as poles; a HALF COILED winding has HALF as many coil groups as poles.

A-C WINDINGS may be LAP wound, WAVE wound or SPIRAL wound. Lap wound armatures are the more common type. Lap and Wave windings are generally two-layer, while the Spiral winding is always single layer.

A POLYPHASE SPIRAL WINDING of one slot per pole per phase is called a **CHAIN** winding; if the winding has two or more slots per pole per phase, it is called a **BASKET** winding.

When the Voltage Wave is of SINE WAVE FORM, the Effective EMF, induced in a CONCENTRATED winding, is found from the equation:

$$E = \frac{2.22\Phi Z f}{10^8}$$

where Φ = Flux per pole; f = frequency;
Z = Number of conductors per phase.

When a FULL PITCH winding occupies two or more slots per pole per phase, the equation above must be corrected by a factor (K_b), called the "DIFFERENTIAL BELT FACTOR." The "belt factor" is the RATIO of the VECTOR sum of the emfs in each coil group to their ARITHMETICAL sum.

When the winding is FRACTIONAL PITCH, the emfs in the two sides of the respective coils do not combine in phase, and the equation above must also be corrected by another factor (K_p), called the "COIL PITCH DIFFERENTIAL FACTOR." The coil pitch factor is the RATIO of the VECTOR sum of the emfs in the two sides of each coil to their ARITHMETICAL sum.

The effective emf of any winding, having a sine wave of emf, is found from the equation

$$E = \frac{2.22 K_p K_b \Phi Z f}{10^8}$$

In a full pitch winding $K_p = 1$.

In a symmetrical polyphase winding,

$$\text{Number of slots per pole per phase} = \frac{\text{Total number of slots}}{\text{Number of Poles} \times \text{Number of Phases}}.$$

In the commercial alternator, AN EMF WAVE CLOSELY APPROXIMATING A SINE WAVE is obtained as follows:

(1) The pole faces, in a Definite Pole Machine, are beveled at the pole tips to obtain a greater flux density at the centers of the poles.

(2) In a Round Rotor Machine, the fixed winding is wound in several slots per pole to obtain the effect in (1).

(3) By distributing the armature winding in several slots per pole per phase, the combined emf waves of the several coils per group, which are out of phase with each other, gives a resulting wave per group, approaching that of a sine curve.

(4) By using a fractional pitch winding, the effect in (3) is increased.

PROBLEMS IN CHAPTER VI

Prob. 68–6. A single-phase four-pole 60-cycle alternator has a concentrated winding, similar to Fig. 48–6, Chap. VI. There are 9 turns per coil and 2.5×10^6 magnetic lines per pole. Assuming sine wave of emf, determine the generated voltage.

Prob. 69–6. A three-phase 8-pole 25-cycle alternator has one slot per pole per phase and 24 conductors per slot. The flux per pole is 3×10^6 lines and the emf wave is a sine curve. What voltage does the machine generate?

Prob. 70–6. Draw a single-phase 6-pole full-pitch two-layer lap winding for an armature of 42 slots. Winding occupies only four slots per pole.

Prob. 71–6. Draw a two-phase 6-pole 25-cycle full-pitch two-layer lap winding in which there are 48 slots.

Prob. 72–6. Draw the winding of Prob. 71–6 with seven-eighth pitch.

Prob. 73–6. (*a*) What is the belt factor for the winding of Prob. 71–6? (*b*) For Prob. 72–6?

Prob. 74–6. What is the coil-pitch factor for the winding of Prob. 72–6?

Prob. 75–6. What voltage is generated per phase by the armature of Prob. 72–6, if the emf is a sine wave, the flux per pole is 4×10^6 lines and there are 10 turns per coil?

Prob. 76–6. Draw a three-phase, four-pole 60-cycle full-pitch two-layer lap winding in which there are 48 slots.

Prob. 77–6. Draw a five-sixth-pitch two-layer lap winding for the armature in Prob. 76–6.

Prob. 78–6. What is the belt factor for the windings of Probs. 76–6 and 77–6?

Prob. 79–6. What is the coil-pitch factor for the winding of Prob. 77–6?

Prob. 80–6. Assuming a sine wave of emf, what voltage is generated per phase by a three-phase four-pole 60-cycle eight-tenths-pitch two-layer lap winding in which there are 5 slots per pole per phase, 9 turns per coil and 2.6×10^6 lines per pole?

Prob. 81–6. Draw a three-phase 6-pole full-pitch two-layer lap winding in which there are 72 slots.

Prob. 82–6. Draw a three-phase 6-pole basket winding in which there are two slots per pole per phase.

Prob. 83–6. (*a*) If the alternator specified in Prob. 31–6, Chap. VI, were reconnected in Y for a three-wire system, what would be the rated

voltage and current output? (*b*) If full load current were taken from one pair of terminals, at rated voltage, what percentage of its rated full load KVA does it supply? (*c*) The average watts I^2R loss in the whole armature under these conditions is what percentage of normal full load value? Load is non-inductive.

Prob. 84–6. A certain two-phase six-pole 1200 rpm generator with two-layer full-pitch lap winding is rated at 240 volts, 30 KVA and has 72 slots in its armature core. (*a*) How many slots per pole per phase? (*b*) How many coils per pole per phase? (*c*) What is the voltage across each coil group? (*d*) What is the phase difference between adjacent coils in each coil group? (*e*) Assuming sine wave of emf, what is the voltage per coil? (*f*) What is the rated full-load current per coil?

Prob. 85–6. Assuming the same speed and flux per pole, answer the questions in Prob. 84–6 for the same machine when rewound for three phase and delta connected. (*a*) What would be the rated voltage and KVA capacity of the machine in this case? (*b*) The rated current per terminal? (*c*) What would be the rated voltage, KVA capacity and current per terminal, if the machine were Y connected?

Prob. 86–6. A three-phase 60-cycle 900-rpm alternator has 96 slots and a full-pitch two-layer lap winding. It is rated at 100 KVA, 2300 volts and is Y connected. (*a*) How many slots per pole per phase? (*b*) What is the phase difference of emfs in adjacent coils of each coil group? (*c*) Assuming sine wave of emf, what is the voltage per coil? (*d*) What is the rated full load current per coil?

CHAPTER VII

THE ALTERNATOR
PERFORMANCE AND OPERATION

In the operation of the alternator we are particularly concerned in its performance with respect to the following features:

First: How does it behave when it is delivering power? That is, what conditions affect the terminal voltage, or the regulation, of the machine?

Second: How does the alternator interact with other machines on the same system. In other words, what conditions affect its satisfactory operation in parallel with other generators?

Third: What load can it deliver and what is its efficiency?

1–7. Regulation. Since the field winding of an alternator is excited from a separate d-c source, the machine, in this respect, is similar to the separately excited direct-current generator. The voltage regulation of a d-c generator is the percentage rise in voltage between full-load and no load under the conditions of rated terminal voltage at full load, constant field-current and rated speed. Similarly, the regulation for an alternator is defined as the percentage **change** in terminal voltage from rated value under the same conditions, when the rated load in Kva at a given power factor is removed.

It has been shown in Vol. I, Chapter X, that the terminal voltage of a separately excited d-c generator decreases as the load on it is increased. This is due to two causes — the IR drop in the armature and armature reaction. Both IR drop and armature reaction occur also in the alternator, and ordinarily cause a decrease in terminal voltage as the load is applied. In the alternator, there is still another factor which causes a loss of voltage in the armature as the load is increased. This is the **reactance** of the armature, due to the leakage flux and consequent inductive effect of the alternating current in the armature coils. The regulation of the alternator is poor compared to that of the d-c generator, due to the **reactance drop** in the a-c machine, which is not present in the d-c generator,* and to the greater effect of armature reaction in the

* Reactance does not occur to any appreciable extent in a d-c armature, since all coils carry a steady current except during commutation. Inductance shows its presence only in the coils as they are short circuited by the brushes. In the alternator armature, the current reverses at the same instant in all the coils in any one phase.

alternator. Furthermore, the regulation of the alternator depends upon the power factor of its load. The regulation may be very satisfactory at unity power factor, but very poor for the same current output at low lagging power factors. Also at low leading power factors the regulation may be **negative.** That is, the terminal voltage may actually **rise** as the load on the alternator is increased.

The percentage regulation of an alternator in itself is usually not important, especially for large machines, as these alternators are generally equipped with voltage regulators (see Art. 15, Chap. VII) which automatically hold the terminal voltage constant. Information about the regulation of the alternator, however, is important in determining many necessary facts regarding its installation and operation. For instance, the regulation should be known in order to determine the size of the d-c generator, called the exciter, used to supply direct current for the field of the alternator. To keep the terminal voltage of the separately excited d-c generator constant we have learned that the field current must be increased as the load on the machine is applied. This is also true for the alternator. An alternator with poor voltage regulation will require a greater increase in field current, and a greater output of its exciter, to keep the terminal voltage constant as load is applied, than one with good regulation. Again the regulation of an alternator determines to large extent the value of the armature currents which will flow on short circuit. The forces set up by this current in large machines are enormous and dangerous to the safety of the machine. The value of these currents is used in determining the size of current limiting reactances placed in the armature loads, the size of circuit breakers and other switching equipment. Also, the satisfactory operation of an alternator in parallel with other generators is very definitely related to its regulation.

2–7. Armature Reactance. When current flows in the coils of an alternator armature, an alternating flux is set up interlinking the turns of the coils. This is indicated in Fig. 1–7, in which the two sides of a single coil of a two layer winding are shown in cross section. The flux, so set up, is independent of the field flux and is called a leakage flux. This flux gives **inductance** to the conductors, and therefore to the winding. This inductance, when multiplied by 2π and the frequency, gives **reactance** to the winding. Thus, when current flows in the winding, there is a loss of voltage due to

the **reactance drop.** That is, a part, or component, of the generated emf must be used to overcome the counter emf of self-inductance in the coils. This is the armature IX_L drop.

The end connections and all parts of the coils have reactance but the conductors imbedded in the slots in the armature core con-

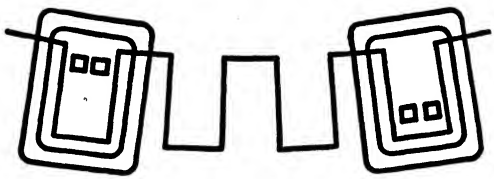


FIG. 1–7. Leakage flux linking the conductors in a simple coil of an armature. This flux produces reactance in the armature.

tribute by far the greater part. In Fig. 2–7(*a*) the conductors lie in a deep narrow slot, while in Fig. 2–7(*b*) they lie in a wide shallow slot. The reluctance in the magnetic path of the leakage flux around the conductors is almost entirely in the slot, as that in the iron part of the path is negligible. Thus, the reluctance of the

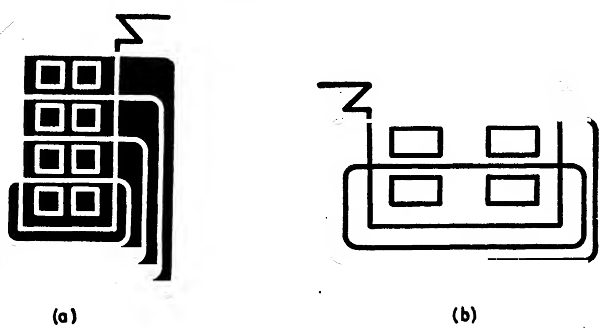


FIG. 2–7. Leakage flux: (*a*) In a deep narrow slot, (*b*) in a shallow wide slot.

wider slot is greater and flux set up by a given current per conductor is less, so the inductance is also less. Therefore, an armature wound in wide shallow slots will have a smaller reactance than one wound in deep narrow slots.

3–7. Armature Resistance. It has been shown in Chapter IV, Art. 11, that when an alternating current flows in a coil with an iron core, hysteresis and eddy current losses occur in the magnetic circuit, in addition to the ohmic I^2R loss in the conductors, all of which must be supplied by the current. The coil thus acts as

though it had a greater resistance than that determined by direct-current measurement. This is called the **effective** resistance.

Similarly, in an alternator armature, the coils embedded in the slots have virtually an iron core. When current flows in these coils, the alternating leakage flux set up (Fig. 2–7) produces hysteresis and eddy current losses in the teeth and core. This flux is approximately proportional to the current. Since hysteresis loss varies with $B^{1.6}$ and eddy currents with B^2, the losses due to this flux vary approximately with the **square** of the current. Also, when the cross section of the armature conductors is large, this flux sets up appreciable eddy current losses in the conductors themselves, which also vary with the square of the current. Thus, the losses due to the flow of current in the armature, or the effective resistance, is greater for alternating than for direct current. The effective resistance, then, depends largely upon the dimensions of the slots and teeth and the size of the conductors. It varies roughly from 115 to 175 per cent, and more, of the d-c resistance. In commercial alternators the voltage drop due to resistance is small compared to that due to reactance, so that considerable variation in the value of effective resistances does not greatly affect the terminal voltage. In problems in this chapter the a-c, or effective armature resistance, is taken as 150 per cent or 1.5 times its d-c value.

4–7. Armature Impedance Drop. In the direct-current generator we **add numerically** the armature IR drop to the terminal voltage to obtain the induced voltage. While in the alternator, we have both the effective IR_e drop and the IX_L drop (or IZ_L drop) which must be **added vectorially** to the terminal voltage to obtain the induced voltage.

Current in phase with terminal voltage. Figure 3–7(*a*) shows a polar vector diagram of conditions when the current I is in phase with the terminal voltage, E_t, and the power factor of the load is unity. The effective resistance drop, IR_e, is laid off in phase with the current I, and the IX_L drop leading it by 90°, their vector sum being the IZ_L drop. The induced emf, E_i, is now the vector sum of E_t and IZ_L which form adjacent sides of the parallelogram.

The corresponding topographic diagram is shown in Fig. 3–7(*b*), in which the IR_e drop is laid off at the end of vector E_t and in phase with the current, while the IX_L drop is laid off at the end of IR_e and leading it by 90°. The induced voltage, E_i, is now the hypothenuse of a right triangle of which $(E_t + IR_e)$ is one side

and IX_L is the other, or

$$E_i = \sqrt{(E_t + IR_e)^2 + (IX_L)^2} \qquad (1\text{–}7)$$

Note that the induced voltage, E_i, in the diagram is out of phase with the terminal voltage by the small angle α.

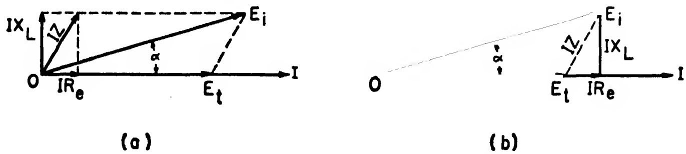


FIG. 3–7. Alternator polar and topographic diagrams for unity power factor. The induced voltage under load, E_i, is the vector sum of the terminal voltage. E_t, the effective resistance drop, IR_e, and the leakage reactance drop, IX_L.

Example 1. The armature of a 30-kva, 240-volt, single-phase, alternator has an effective resistance of 0.04 ohm and an inductive reactance of 0.2 ohm. What is the induced voltage when the machine is delivering full load current at rated terminal voltage to an external load having unity power factor?

Solution: Current $I = \dfrac{30{,}000}{240} = 125$ amperes

$$IR_e \text{ drop} = 125 \times 0.04 = 5 \text{ volts.}$$

$$IX_L \text{ drop} = 125 \times 0.2 = 25 \text{ volts}$$

$$E_i = \sqrt{(240 + 5)^2 + 25^2} = 246 \text{ volts.} \quad \textit{Ans.}$$

(240 +5)² + 5²

Current lagging the terminal voltage. The same method is used to compute the induced voltage when the current lags the terminal

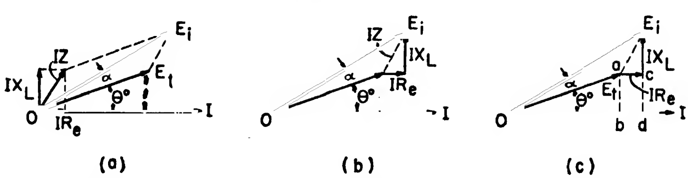


FIG. 4–7. Alternator polar and vector diagrams for lagging current, at power factor, cos θ.

voltage by any angle $\theta°$, as indicated in Fig. 4–7. In the polar diagram of Fig. 4–7(*a*), the terminal voltage E_t is laid off from O leading the current by $\theta°$, the IR_e drop in phase with the current, and the IX_L drop leading it by 90°, as before. The IZ_L drop (vector sum of IR_e and IX_L) and the terminal voltage, E_t, now

form two sides of a parallelogram of which the induced voltage, E_i, is the diagonal.

Figure 4–7(*b*) shows the topographic diagram in which the IR_e drop is drawn from the end of vector, E_t, in phase, or parallel, to the current, I, and the IX_L drop is drawn from the end of IR_e and leading it by 90°. The sum of these three vectors, or E_i, is the induced voltage. This diagram is easily solved by the 90° component method explained in Chapter II, Art. 11, method (*d*). Thus in Fig. 4–7(*c*), we draw the construction lines *ab* and *cd* to the current vector and perpendicular to it. The induced voltage, E_i, is now the hypothenuse of the right triangle odE_i. By this

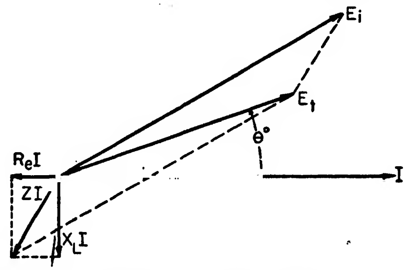


Fig. 5–7. The terminal voltage of the alternator, E_t, is actually the vecto sum of the induced voltage under load, E_i, the resistance drop, R_eI, and th reactance drop, X_LI, due to the leakage flux.

construction the terminal voltage, E_t, is thrown into two 90° components, *ob* equal to $E_t \cos\theta$ and *ab* (= *cd*) equal to $E_t \sin\theta$. *bd* is equal to IR_e and

$$E_i = \sqrt{[(E_t \cos\theta) + IR_e]^2 + [(E_t \sin\theta) + IX_L]^2} \qquad (2\text{–}7)$$

In reality the terminal voltage, E_t, can be considered in Fig. 5–7 as consisting of three components, the induced emf, E_i, the resistance reaction, or R_eI, necessary to overcome the effective resistance of the armature, 180° to the current, and the counter emf of self-induction due to the leakage flux, or X_LI, which lags 90° behind the current (see Ch. IV, Arts. 1 and 2). The total reaction opposing the flow of current in the armature is $\sqrt{R_eI^2 + X_LI^2}$, or Z_LI. The terminal voltage is thus the vector sum of E_i and Z_LI, as shown.

Example 2. Compute the induced voltage in the alternator of Example 1, when the machine is delivering full load current at rated terminal voltage to an external load having a power factor of 0.6 lagging.

Solution: As before, Current $I = \dfrac{30{,}000}{240} = 125$ amperes.

$$IR_e \text{ drop} = 125 \times 0.04 = 5 \text{ volts.}$$

$$IX_L \text{ drop} = 125 \times 0.2 = 25 \text{ volts.}$$

$$\cos\theta = 0.6;\ \theta = 53^\circ\ 10';\ \sin\theta = 0.8.$$

$$E_i = \sqrt{[(240 \times 0.6) + (5)]^2 + [(240 \times 0.8) + (25)]^2} = 263 \text{ volts.} \quad \textit{Ans.}$$

Note that with the same current and impedance drop in Examples 1 and 2, a lagging current causes a greater numerical difference in volts between **induced** and **terminal** voltages. This is because the IX_L drop, with lagging current, is thrown more nearly in phase with the terminal voltage. At unity power factor, the IR_e drop has the greater effect on the terminal voltage.

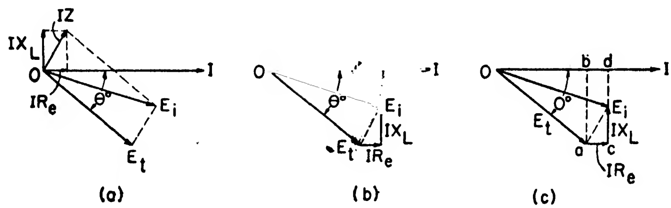


FIG. 6–7. Alternator polar and topographic diagrams for leading current, at power factor, $\cos\theta$.

On lagging load the IX_L drop produces the greater effect between induced and terminal voltages.

Current leading the terminal voltage. When the load on the alternator takes a current leading the terminal voltage by an angle θ°, the conditions are as indicated in the polar and topographic diagrams of Fig. 6–7. As before, the IR_e and IX_L drops are drawn in phase and 90° leading the current respectively. In Fig. 6–7(*c*) we drop the perpendicular *ab* and *cd* to the current vector I, as before, and *ob* equals $E_t \cos\theta$; *ab* (= *cd*) equals $E_t \sin\theta$; and $bd = IR_e$. The induced voltage E_i is the hypothenuse of the right triangle odE_i, or

$$E_i = \sqrt{[(E_t \cos\theta) + IR_e]^2 + [(E_t \sin\theta) - IX_L]^2} \qquad (3\text{–}7)$$

(Compare with equation (2) and note the change in sign.)

Example 3. Compute the induced voltage in the alternator of Example 1, when the machine is delivering full load current at rated terminal voltage to an external load having a power factor of 0.6 leading.

Solution: As before, Current $I = \dfrac{30{,}000}{240} = 125$ amperes.

$$IR_e \text{ drop} = 125 \times 0.04 = 5 \text{ volts.}$$

$$IX_L \text{ drop} = 125 \times 0.2 = 25 \text{ volts.}$$

$$\cos\theta = 0.6;\ \theta = 53^\circ\ 10';\ \sin\theta = 0.8.$$

$$E_i = \sqrt{[(240 \times 0.6) + 5]^2 + [(240 \times 0.8 - 25)]^2} = 223.8 \text{ volts.}$$

Note in this case that, when the current leads the terminal voltage by a sufficiently large angle, the induced voltage is numerically **less** than the terminal voltage. This is caused by the angular position of the IX_L drop with respect to the terminal voltage.

It should be clearly understood that the induced emf, E_i, in the diagrams and examples above is **not** the zero load voltage, but is the induced voltage per phase when the machine is delivering a current I, to a load. In the commercial machine it is practically impossible to determine accurately the value of E_i. Before the no-load, or open-circuit, voltage can be computed, the effect of armature reaction must be considered.

Prob. 1–7. A 50-kva, 460-volt, single-phase alternator has an effective armature resistance of 0.15 ohm and a reactance of 0.6 ohm. What is the induced emf when the machine delivers rated current at rated terminal voltage and at unity power factor?

Prob. 2–7. (*a*) Repeat Prob. 1–7 for a lagging load of 0.75 power factor. (*b*) For a leading load of 0.75 power factor.

Prob. 3–7. A 100-kva, 2300-volt, single-phase alternator has an effective armature resistance of 0.6 ohm and a reactance of 0.85 ohm. What is its induced emf when it delivers rated current at unity power factor and at rated terminal voltage?

Prob. 4–7. (*a*) Repeat Prob. 3–7 for lagging load at 0.8 power factor. (*b*) For leading load at 0.8 power factor.

Prob. 5–7. A 200-kva, 2300-volt, three-phase, delta-connected alternator has an effective armature resistance per phase of 0.8 ohm and a reactance per phase of 2.5 ohms. When the machine delivers rated current at unity power factor and at rated terminal voltage, what is the induced voltage per phase?

Prob. 6–7. (*a*) Repeat Prob. 5–7 for lagging load at 0.8 power factor. (*b*) For leading load at 0.6 power factor.

5–7. Armature Reaction. It will be remembered that, in the separately excited d-c generator, the induced emf decreases with increase in armature current, due to armature reaction. The

terminal voltage is also further reduced by the armature IR drop. (See Vol. I, Fig. 47–10.)

Similarly, in an alternator with unity power factor or lagging load, the ordinates of curve A, Fig. 7–7, represent the emf induced by the field ampere-turns alone, which are constant and independent of the load current. The ordinates of curve B represent the induced emf at various load currents, which decreases with increase of armature current, due to armature reaction. And finally, the ordinates of curve C represent the terminal voltage, which is reduced by the armature impedance drop, and which also increases with the value of the current.

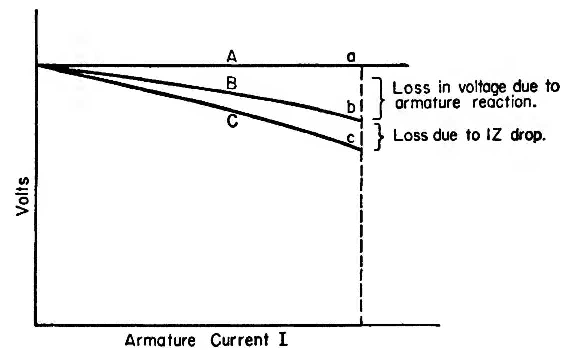


FIG. 7–7. The ordinate of point a represents the induced voltage due to field ampere-turns, or open-circuit emf; ab represents the voltage lost due to armature reaction, and bc that caused by armature IZ drop. (Note that bc numerically is *not* equal to the IZ drop.) The ordinate of point c is the terminal voltage at an armature current I.

Thus, for an armature current, I, in Fig. 7–7, a is the emf induced by the field flux; ab is the loss in emf because of armature reaction; b is the induced emf at this armature current and corresponds to E_t in Figs. 3–7 to 6–7; bc is the voltage loss due to IZ_L drop; and the ordinate of point c represents the terminal voltage at this load current. Therefore, when the alternator delivers a current of I amperes, the open-circuit voltage, represented by the ordinate of point a, is reduced, because of armature reaction and armature IZ drop, to a value represented by the ordinate of point e.

The leakage reactance of an armature is difficult to measure or calculate accurately and its effect is generally combined with that of armature reaction, as will be shown later.

6–7. Single-Phase Armature Reaction. Armature reaction in the alternator, as in the d-c generator, is the effect of the magnetic

action, or the magnetomotive-forces, set up by the armature current, upon the field flux.

When the brushes of a d-c generator are set on the mechanical neutral point, the armature current sets up cross ampere-turns, or an mmf, 90 electrical degrees to the field flux, or field mmf (see Vol. I, Fig. 43–10). And when the brushes are given a forward lead, the armature current sets up both a cross magnetizing and a demagnetizing action, which reduces the effective flux in the air gap. The greater this angle of brush shift, the greater is the demagnetizing action for a given armature current. Since the alternator has no commutator, the shift of the brushes does not affect the air gap flux. However, the power factor of the load on an alternator has a marked effect on the useful flux in the air gaps and upon the terminal voltage of the machine.

Current in phase with induced voltage. Figure 8–7(*a*) represents a revolving field alternator and shows one phase of a two-layer armature having one slot per pole per phase. The poles move from left to right and the direction of the field flux, *m*, is indicated. The induced voltage at this instant is a maximum. If the external circuit is closed, the armature will deliver current which, at unity power factor, is practically in phase with the induced emf (except for the small angle x_1, Fig. 8–7). Figure 8–7(*b*) indicates only the direction of the current in the armature conductors and the flux, or mmf, set up by this current at the same instant shown in (*a*). Note that the armature mmf is practically at 90 electrical space degrees from the field flux, shown in (*a*). This would be the actual case in a wound rotor alternator in which the air gap is uniform around the entire periphery of the field structure. In the definite-pole alternator, due to the high reluctance of the space between the poles, the armature mmf acts through the iron path of the field poles. Figure 8–7(*c*) shows the combined effect of the armature and field mmfs for the same instant shown in (*a*) and (*b*). The armature leakage flux is not indicated. Note particularly in this figure that the field flux, or mmf, *m*, and the armature mmf, *a*, act in the **same** direction in the **trailing** half of the poles and **oppose** each other in the **leading** half. This causes cross magnetization, changes the distribution of the flux in the air gaps and distorts the shape of the emf wave. This, in addition to the resistance and reactance drop in the armature, also reduces the terminal voltage.

Thus, at a power factor approximating unity, the net effect of the armature current is to shift the flux under the poles. This

action is similar to that in the d-c generator when the brushes are set on the mechanical neutral.

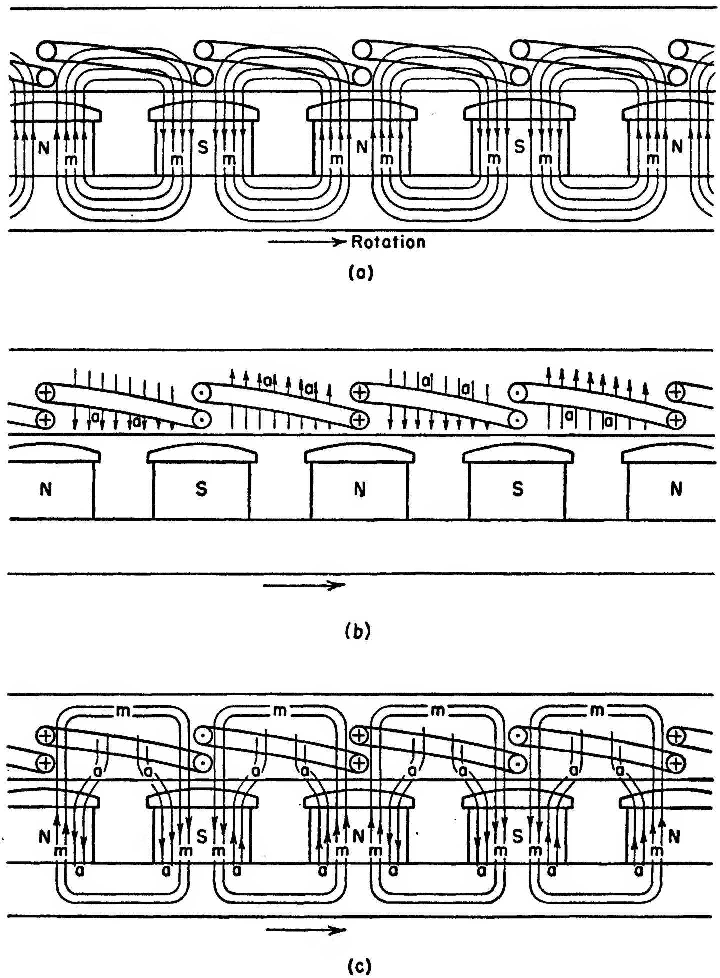


FIG. 8–7. Armature reaction in a definite pole alternator at unity power factor. (*a*) Flux distribution, due the field ampere-turns only. (*b*) Mmf, or flux, set up by the armature current, or ampere-turns, only. (*c*) Combined effect of armature and field ampere-turns.

Current lagging 90° behind induced voltage. When the armature current lags behind the induced voltage by 90°, the current does not reach a maximum in the direction indicated in Fig. 8–7 until the poles have moved through 90 electrical space degrees

(to the right) and the induced voltage is zero. Figure 9–7 shows the condition at this instant. Note that the armature flux, or mmf, *a*, is now in **opposition** to the field flux, *m*, at every point along the magnetic path. This reduces the total flux across the air gap and decreases the induced voltage.

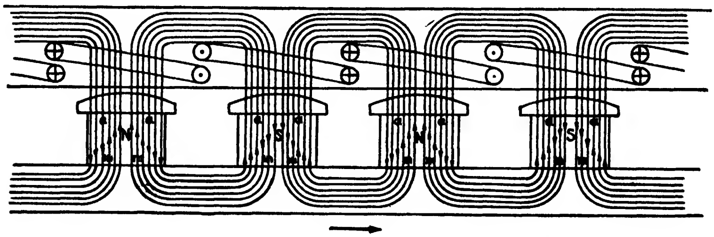


FIG. 9–7. Combined effect of armature and field ampere-turns. Current lagging 90°.

Thus, armature reaction in an alternator on lagging load of practically zero power factor results in a **demagnetizing** effect upon the main field and reduces the air gap flux. It is somewhat similar to the effect of armature reaction in a d-c generator if the brushes

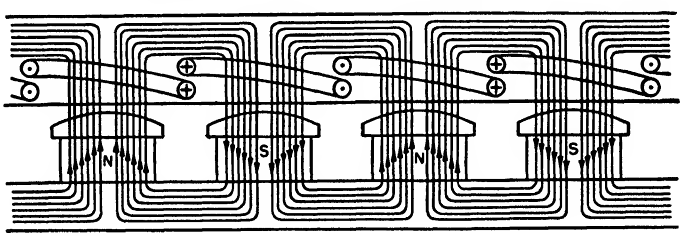


FIG. 10–7. Combined effect of armature and field ampere-turns. Current leading 90°.

were to be given a forward lead of 90 electrical degrees. In this case also, armature reaction would consist entirely of back ampere-turns.

Current leading 90° ahead of induced voltage. Figure 10–7 shows the relative position of armature conductors and poles at the instant when the current in the conductors is a maximum. Note that the direction of the current is reversed, or at 180°, with respect to that in Fig. 9–7. The armature flux, or mmf, *a*, now is in the **same** direction as the field flux, *m*, at every point along the magnetic path. Therefore, on a leading load of practically zero power

factor, armature reaction has a **magnetizing** effect on the main field and tends to **increase** the value of the induced voltage.

Thus, in general, armature reaction in an alternator,

(1) distorts, or shifts, the field flux on unity power factor load;

(2) demagnetizes, or weakens, the field flux on lagging power factor loads;

(3) magnetizes, or strengthens, the field flux on leading power factor loads.

In the figures above, single-phase armature reaction is shown only at the instant when the current is a maximum. As the field poles rotate, their position with respect to the armature conductors changes and the value of the current also changes from a maximum to zero four times in a cycle, so that total armature reaction is not constant, but varies in intensity during the cycle. In fact, even when the current is out of phase by 90°, cross magnetization takes place at certain instants in the cycle. This is explained in the following section.

7–7. Pulsating Armature Reaction—Single Phase. Figures 11–7 to 13–7 represent a simple bi-polar alternator with rotating armature. For clearness, the armature is represented as a single turn coil and the collector rings and brushes are not shown. The marks ⊙ and ⊕ indicate direction of current, **not** emf, in the conductors.

(1) Current in phase with induced voltage. Figures 11–7 (*a* to *e*) show the conditions at various instants in the cycle when the current is in phase with the induced voltage. In these figures the length and direction of the vector n represent the strength (ampere-turns) and direction of the armature reaction; that is, of the magnetic action of the armature current. During one cycle of induced emf the end of vector armature-reaction n moves twice around the dotted circle.

In Fig. 11–7(a) the current in conductors A and B is a maximum and the armature reaction, n_1, is also a maximum and 90° to the field mmf (as in Fig. 8–7). Armature reaction is all **cross magnetization** at this instant. In Fig. 11–7(b), 45° later in the cycle, the current and also the armature reaction, n_2, has decreased in value. n_2 now consists of two components, a **cross magnetizing** action, c, and a **demagnetizing,** or weakening, action, w. In (c) 45° later than (b), the current has dropped to zero and armature reaction is zero. In (d), the direction of current in the conductors A and B has reversed and is increasing in value. The

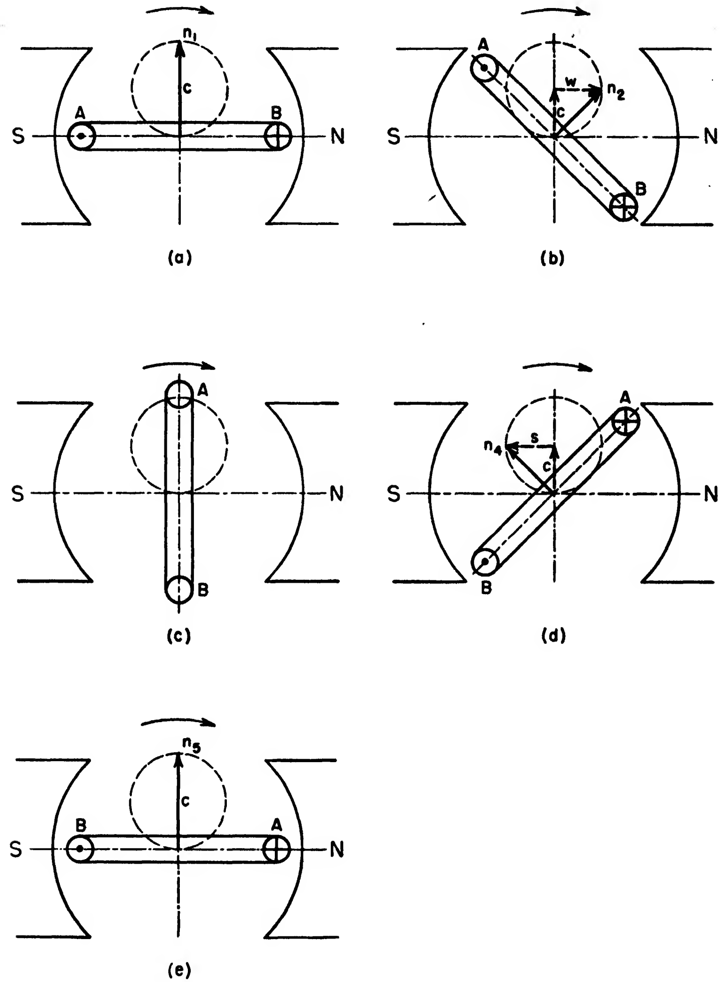


FIG. 11–7. Armature reaction in a single-coil armature at various instants in the cycle. Current in phase with induced voltage.

armature reaction, n_4, now consists of a **cross** magnetizing action, c, and a **magnetizing** action, s, tending to strengthen the main field. In (e), after completing one half cycle, the current and armature reaction, n_5 is a maximum again and all **cross** magnetizing. The same sequence of events follows during the remainder of the cycle. Thus, when the current is in phase with the induced voltage, armature reaction:

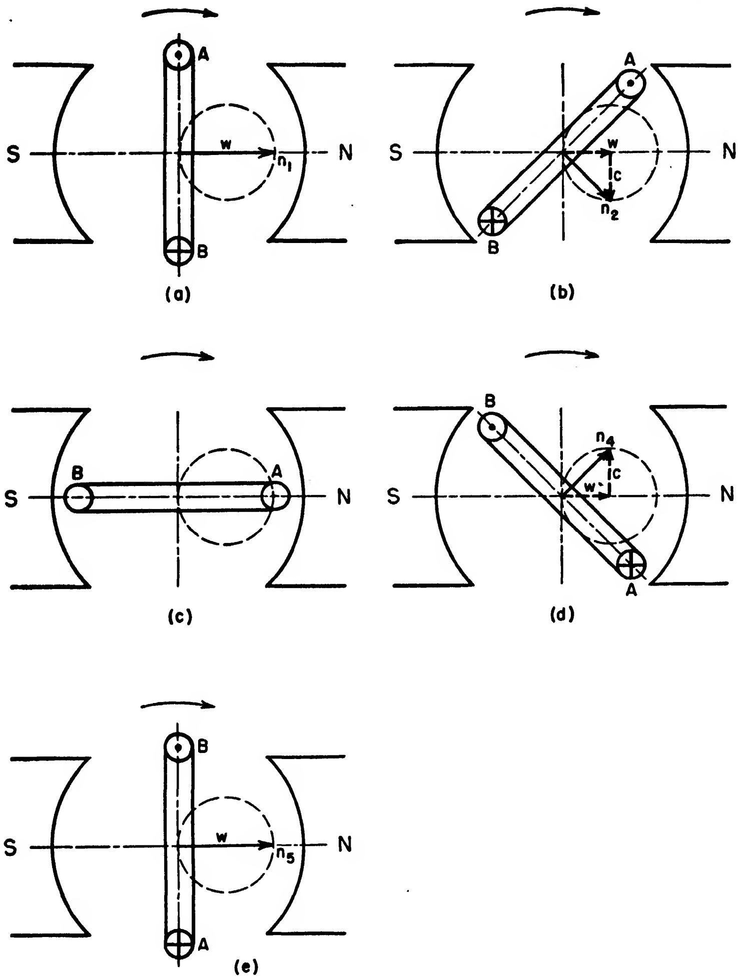


Fig. 12–7. Armature reaction in single-coil armature current lagging 90°.

In Fig. 11–7(*a*) is all cross magnetizing;
in (*b*) is cross magnetizing and weakening;
in (*c*) is zero;
in (*d*) is cross magnetizing and strengthening;
in (*e*) is all cross magnetizing.

(2) **When the current lags 90° behind the induced voltage,** the varying effect of armature reaction is shown in Figs. 12–7(*a* to *e*).

In (*a*) the current in the conductors and armature reaction, n_1, is a maximum. Note the current in conductors A and B is in the same direction as in Fig. 11–7(*a*), but the armature has turned 90° further in its revolution before the current reaches this maximum. Now n_1 directly opposes and weakens the field mmf. In (*b*), 45° later, n_2 has both a cross magnetizing component, *c*, and a demagnetizing, or weakening, component *w*. In (*c*), 45° later than (*b*), the current and armature reaction are zero. In (*d*) the current in the conductors has reversed and is increasing. n_4 now has both a cross magnetizing component, *c*, and a demagnetizing, or weakening, component, *w*. In (*e*), at the end of the half cycle, the current and n_5 are again a maximum, and n_5 is all demagnetizing, or weakening. The same sequence follows during the second half cycle. Thus when the current lags 90°, armature reaction:

In (*a*) is all demagnetizing, or weakening;
in (*b*) is cross magnetizing and demagnetizing;
in (*c*) is zero;
in (*d*) is cross magnetizing and demagnetizing;
in (*e*) is all demagnetizing, or weakening.

(3) **When the current leads the induced voltage by 90°,** the conditions are shown in Figs. 13–7(*a* to *e*). In (*a*) the current in conductors A and B is a maximum in the direction shown. This is 90° **before** the coil reaches the position of maximum induced voltage. n_1 is now in the same direction as the field mmf, therefore, acts only to magnetize it. In (*b*), n_2 is reduced in value and consists of both a cross magnetizing component, *c*, and a strengthening component, *s*. In (*c*), both current and armature reaction are zero. In (*d*), the current has reversed and is increasing, and n_4 has both a cross magnetizing component, *c*, and a strengthening component, *s*. And in (*e*), the current is again a maximum, and n_5 acts directly to magnetize, or strengthen, the field mmf. The sequence during the remainder of the cycle is the same. Thus, when the current leads 90°, armature reaction:

In (*a*) is all magnetizing, or strengthening;
in (*b*) is cross magnetizing and strengthening;
in (*c*) is zero;
in (*d*) is cross magnetizing and strengthening;
in (*e*) is all magnetizing, or strengthening.

The three sets of diagrams above show that:

When the current is in phase with the induced emf, the magnetizing and demagnetizing actions neutralize each other over the cycle and the average effect of armature reaction is **cross magnetizing.**

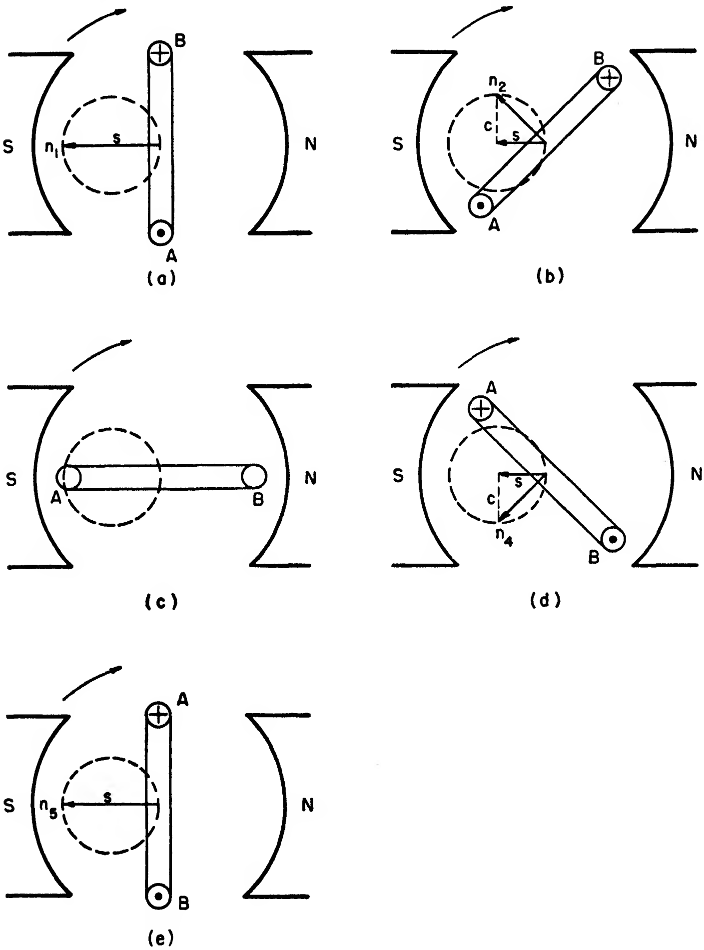


FIG. 13-7. Armature reaction in single-coil armature current leading 90°.

When the current lags 90°, the average effect is **demagnetizing:**

When the current leads 90° the average effect is **magnetizing.**

On lagging loads with power factors greater than zero, similar sets of diagrams will show both magnetizing and demagnetizing action over the cycle. However, the demagnetizing action will

be the greater. On leading loads with corresponding power factors, the magnetizing action will be the greater.

This changing, or pulsating, armature reaction in single-phase machines causes the resultant flux to oscillate across the pole faces. This increases the hysteresis and eddy current losses and sets up an alternating emf in the field winding.

8–7. Polyphase Armature Reaction. When a polyphase alternator supplies a balanced load, that is, equal currents at the same power factor in all the phases, the armature reaction is constant in value for any given current and fixed in direction for any given power factor.

If the current per phase is **in phase** with its induced voltage, the effect of armature reaction is a steady distorting, or cross magnetizing, action which pushes the field flux toward the trailing pole tip. This is shown in Figs. 14–7(*a* to *e*) for a simple three-phase alternator. In (*a*), n_1 represents the armature mmf, or armature reaction, for phase 1 at the instant of maximum current. In (*b*) and (*c*), n_2 and n_3 represent the mmfs of phases 2 and 3 respectively, at the same instant. In (*d*) the corresponding mmfs n_1, n_2 and n_3 are added vectorially, giving the total three-phase armature reaction, n, both as to value and direction. This value is constant throughout the cycle. It can be shown that n is 1.5 times the maximum value for a single phase. Figure 14–7(*e*) shows the condition 30° later in the cycle. The current in phase 3 has decreased to zero and n_3 is zero, while n_1 has decreased, and n_2 has increased. The total armature reaction, n, is now the vector sum of n_1 and n_2 and equal to the same value as before.

When the same current per phase **lags** the induced voltage by an angle, θ, as in Fig. 15–7(*a*), the total armature reaction has the same value as before, but takes the position **on**. Note that **on** now has both a **cross-magnetizing** and a **demagnetizing** component. And if the current **leads** by an angle, θ, as in Fig. 15–7(*b*), **on** takes a corresponding position. It now has both a **cross-magnetizing** and a **magnetizing** component. In both these cases n has a constant value for a given current, with a constant cross-magnetizing and demagnetizing (or magnetizing) action throughout the cycle.

Since armature reaction in the three-phase (and two-phase) alternator is constant, or steady, it has no effect on the field current. This is another advantage of the polyphase alternator over the single-phase machine.

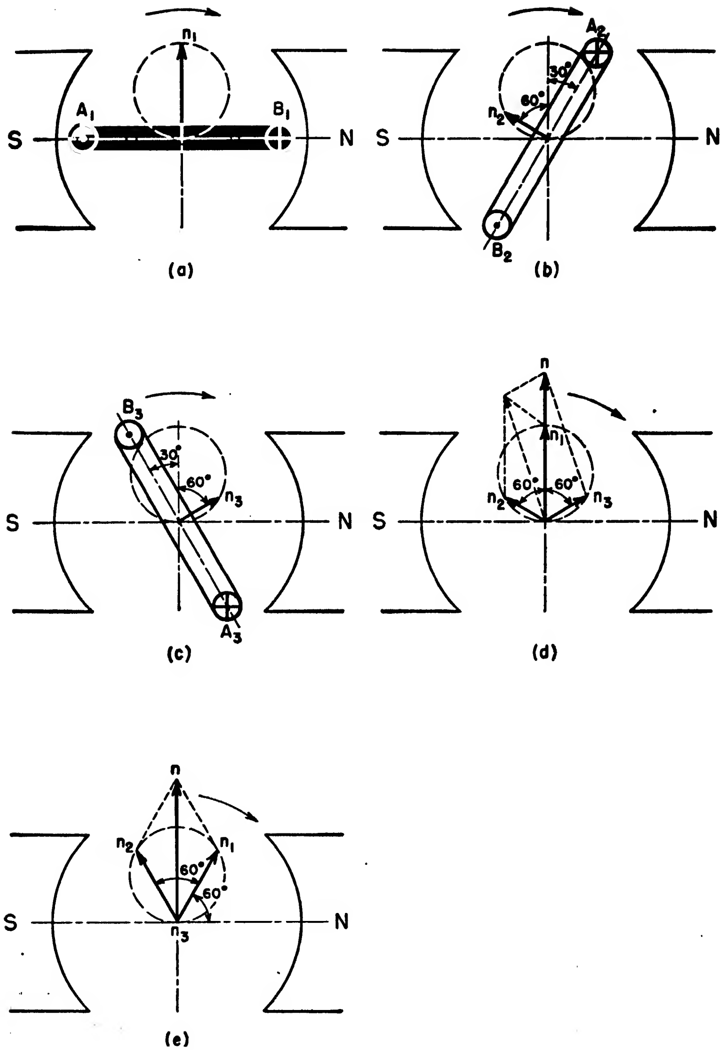


FIG. 14-7. (*a*, *b*, and *c*) Armature reaction in each of the coils of a simple three-phase alternator at the same instant. (*d*) Combined effect of armature reaction in the three coils at the instant shown in (*a*, *b*, and *c*). (*e*) Combined effect of three-phase armature reaction 30° later in the cycle when the current in coil A_3B_3 in (*c*) has dropped to zero.

9-7. Alternator Regulation — Synchronous Impedance. The obvious method of obtaining the voltage regulation of an alternator is to put an external load on it, the power factor of which can be controlled. This is not difficult with machines of small kva capacity. However, in large alternators in common use today, it is very difficult, and generally impossible, to obtain a balanced load at the desired power factor. Besides, such a test uses a large amount of power and the expense is prohibitive. For such alternators, several methods have been devised to compute the regulation, either from the design data before the machine is built, or from no-load tests consuming but little power. All of these

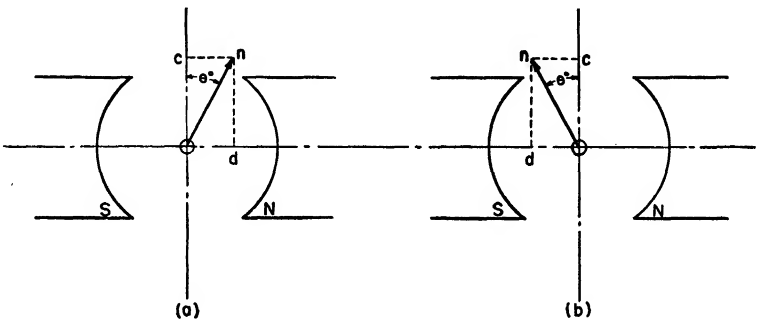


FIG. 15-7. (*a*) Three-phase armature reaction when the current lags $\theta°$ behind the induced voltage. (*b*) When the current leads by $\theta°$.

methods give results which are more or less accurate approximations of the actual performance of the machine.

The simplest method of computing alternator regulation is the so-called **EMF, or Synchronous Impedance Method.** The effect of armature reaction is treated as a voltage drop by this method, and the reasoning is as follows.

The mmf induced by the current in the armature conductors has two effects. It sets up the leakage flux (Figs. 1-7 and 2-7) which is proportional to the current. It also sets up the flux that causes armature reaction and which distorts, demagnetizes, or magnetizes the main field.

We have seen that the leakage flux sets up an induced voltage ($X_L I$ in Fig. 15-7) proportional to and lagging 90° behind the current. This causes a loss of voltage, and is equivalent to a reactance drop; shown as IX_L in Fig. 4-7. Similarly, if we consider separately the flux which causes armature reaction, we can

assume this flux also sets up a voltage proportional to and lagging 90° behind the current. Since armature reaction usually acts to reduce the terminal voltage, we may consider this induced voltage as equivalent to a reactance drop in phase with the leakage reactance drop. This assumption is not exact but may be taken as a fairly close approximation to actual conditions.

Thus, in Fig. 16–7(a), the heavy lines are identical to the diagram of Fig. 4–7. The vector X_aI, 90° behind the current, represents the induced voltage due to the armature reaction flux. This is equivalent to the reactance drop, IX_a, laid off from the end of vector IX_L and in phase with it. In the diagram, $IX_L + IX_a =$

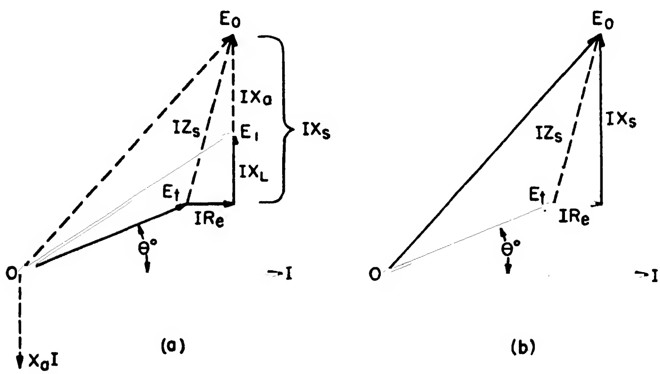


FIG. 16–7. Alternator vector diagrams in which the effect of armature reaction is combined, as a reactance drop, with the leakage reactance drop to produce the synchronous reactance, or the IX_S drop.

IX_s. IX_s is in reality a fictitious reactance drop in which X_s is called the **synchronous reactance.** And $IZ_s = \sqrt{(IR_e)^2 + (IX_s)^2}$, where Z_s is the **synchronous impedance.** Figure 16–7(a) is simplified and redrawn in standard form in (b). This shows clearly that the no-load, or open-circuit voltage, E_o, is the vector sum of the terminal voltage, E_t, the effective resistance drop IR_e, and the synchronous reactance drop, IX_s. Therefore, the induced voltage, E_t, under load conditions (shown in Fig. 16–7(a) and discussed in Art. 4) need not be known.

If E_t and I, in Fig. 16–7, are the **rated** terminal voltage and current respectively, at any power factor cos θ, the regulation can be computed as:

$$\text{Regulation in per cent} = \frac{E_o - E_t}{E_t} \times 100 \qquad (4\text{–}7)$$

Note that all alternator vector diagrams apply to one phase only in a polyphase machine with balanced load, in which E_t and I are the rated terminal voltage and current per phase; IR_e and IX_s the effective resistance and reactance drop per phase. The voltage regulation of one phase in an alternator, either Y- or delta-connected on a balanced load, is the same as that across the machine terminals.

The synchronous impedance methods of computing alternator regulation gives results **poorer** than the actual value. For this reason it is often called the **pessimistic** method.

Prob. 7–7. A 1000-Kva, 6600-volt, Y-connected alternator has an effective armature resistance of 0.5 ohm and a synchronous reactance of 10 ohms per phase. Compute:

(*a*) The rated full load terminal voltage per phase.
(*b*) The rated full load current per phase.
(*c*) The effective IR drop per phase at full load.
(*d*) The synchronous IX drop per phase at full load.
(*e*) The induced voltage per phase on open circuit when the load is removed.
(*f*) The voltage regulation in per cent, when the machine is supplying rated current at rated terminal voltage to a unity power factor load.

Illustrate solution by vector diagram.

Prob. 8–7. (*a*) Repeat Prob. 7–7 when the alternator is supplying an 0.8 power factor lagging load. (*b*) An 0.8 power factor leading load.

Prob. 9–7. Repeat Prob. 7–7 on the assumption that the alternator is delta connected at the same phase voltage with unity power factor.

Prob. 10–7. (*a*) Repeat Prob. 9–7 when the alternator is supplying an 0.8 power factor lagging load. (*b*) An 0.8 power factor leading load.

10–7. Determination of Synchronous Reactance and Regulation — Single-Phase Alternator. The synchronous reactance of an alternator is determined in the following manner.

First, the saturation, or magnetization, curve is obtained at no load and rated frequency, or speed, exactly in the same manner as for a d-c generator. This curve is plotted with induced emf, E, as ordinates and field current, I_f, as abscissae, as in Fig. 17–7.

Second, with reduced field current, the armature is short circuited through an ammeter, as indicated in Fig. 18–7. While the machine is driven at rated frequency, readings of field current, I_f, and short-circuit current, I_a, are taken, as I_f is gradually increased, until I_a approaches 40 to 50 per cent above rated full load value. A short-circuit curve is now plotted on the same

graph with the saturation curve, plotting I_f as abscissae to the same scale and I_a as ordinates to any convenient scale, as in Fig. 17–7.

Third, the resistance of the armature is measured with direct current.

Consider any value of field current, I'_f, in Fig. 17–7. The voltage induced on open circuit by this field current is E'. On

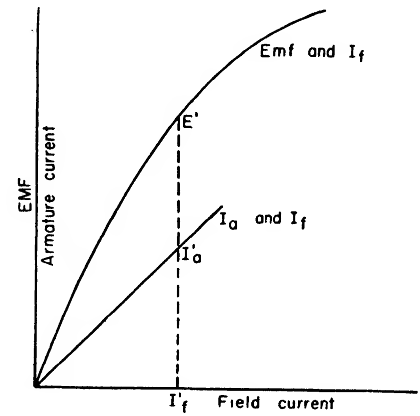


FIG. 17–7. Open-circuit and short-circuit curves of an alternator.

short circuit, the same field current, I'_f, produces the armature current, I'_a. The terminal voltage on short circuit, of course, is

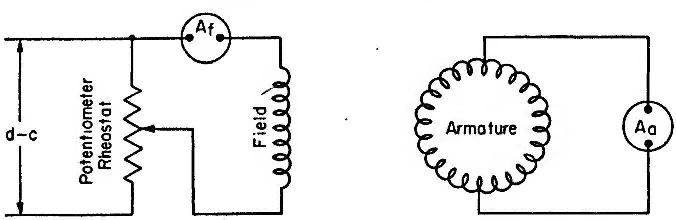


FIG. 18–7. Connections for the short-circuit test of a single-phase alternator.

practically zero. In Fig. 18–7 it is the drop across the ammeter. Therefore the induced voltage E' is considered as entirely consumed in forcing the current, I_a, through the impedance of the armature, and is thus the IZ drop in the armature. This, of course, includes the loss in voltage due to armature reaction.

Thus, $$E' = I'_a Z_s, \quad \text{or} \quad Z_s = \frac{E'}{I'_a}. \tag{5-7}$$

where, E' and I'_a are the induced voltage and the short-circuit current, respectively, at the field current, I'_f, and Z_s is the **synchronous impedance** of the armature.

Since $Z_s = \sqrt{R_e^2 + X_s^2}$, where R_e is effective resistance and X_s is synchronous reactance, $X_s = \sqrt{Z_s^2 - R_e^2}$.

The short-circuit curve is obtained at relatively low flux density of the magnetic circuit. The mmf due to armature reaction has a relatively greater effect on the field flux at low flux density. Furthermore, on short circuit, since the resistance is small compared to the reactance, the current lags nearly 90° and the effect of armature reaction is greater than under normal operating conditions. Therefore the synchronous impedance calculated by this method is correspondingly **high** and the regulation is **worse** than the actual value, as already stated. This error, from the standpoint of the purchaser, is on the "right side," however, for he knows the performance of the machine is better than the results of the test indicate.

Example 4. A 240-volt, 60 Kva, single-phase alternator at normal frequency has an open-circuit voltage of 175 volts when the field current is 10 amperes. On short circuit, when the field current is 10 amperes, the armature current is 376 amperes. The resistance of the armature, as measured with direct current, is 0.05 ohm. Determine the per cent regulation on a lagging load of 0.8 power factor.

Solution: Assuming effective resistance is 1.5 times the d-c resistance, effective armature resistance = 15 × 0.05 = 0.075 ohm.

$$Z_s = \frac{175}{376} = 0.455 \text{ ohm}$$

$$X_s = \sqrt{0.455^2 - 0.075^2} = \sqrt{0.206 - 0.007} = 0.448 \text{ ohm.}$$

$$\text{Full load current} = \frac{60{,}000}{240} = 250 \text{ amperes}$$

$$\cos\theta = 0.8;\ \theta = 36^\circ\ 50';\ \sin\theta = 0.6$$

$$\begin{aligned} E_{\text{no load}} &= \sqrt{[(E_t\cos\theta)+IR]^2+[(E_t\sin\theta)+IX]^2} \\ &= \sqrt{[(240\times0.8)+(250\times0.075)]^2+[(240\times0.6)+(250\times0.448)]^2} \\ &= \sqrt{(192+18.75)^2+(144+112)^2}=332 \text{ volts} \end{aligned}$$

$$\text{Per cent regulation} = \frac{332 - 240}{240} \times 100 = 38.3. \quad \textit{Ans.}$$

Note that there is slight difference between Z_s and X_s in the example above, so that very little error will result if the impedance is used instead of the reactance.

11–7. Determination of Synchronous Reactance and Regulation — Delta-Connected Alternator. It was stated in Art. 9 that the alternator diagram, and also the regulation of an alternator, always refers to the change in voltage in a single phase, or winding. Therefore, in a polyphase machine, the resistance and synchronous reactance **per phase** must be obtained from the test data.

Figure 19–7 (*a*) shows the connections for obtaining the open-circuit, or magnetization curve of a delta-connected alternator. The voltmeter reading, E, indicates the induced voltage per phase in this case. Figure 19–7 (*b*) shows the connections for obtaining the short-circuit data. Three exactly similar ammeters, and short-

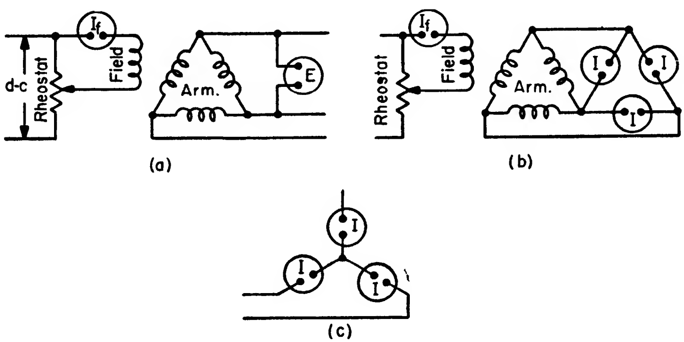


FIG. 19–7. (*a*) Connections for the open-circuit test of a delta-connected alternator. (*b*) Connections for the short-circuit test; ammeters connected in delta. (*c*) Ammeters connected in Y.

circuit connections of **equal resistance** must be used, or the currents in the three phases will be unbalanced. The ammeters are here connected in delta and each instrument measures the current per phase. **Synchronous impedance per phase** is, therefore, computed for a given field current, directly from the curves, or $Z_s = \dfrac{E'}{I'_a}$, exactly as in Fig. 17–7. When the ammeters are connected in Y, as in Fig. 19–7 (*c*), each instrument indicates the **line current** on short circuit. Synchronous impedance per phase for a given field current is now computed as:

$$Z_s = \frac{E'}{\dfrac{I'_a}{\sqrt{3}}} \tag{6-7}$$

The d-c resistance of the delta-connected alternator is measured across any two terminals, as indicated in Fig. 20–7. The resistance computed from the instrument readings is $\frac{2}{3}$ the resistance of one phase. Thus, if R is the resistance per phase,

$$\text{Computed resistance} = \frac{1}{\frac{1}{R} + \frac{1}{2R}} = \frac{1}{\frac{3}{2R}} = \frac{2}{3}R \qquad (7\text{–}7)$$

The d-c resistance per phase is, therefore, $\frac{3}{2}$ times the value computed from the instrument readings.

Example 5. The open-circuit, short-circuit curves obtained by test from a 1200-Kva, 2300-volt, 60-cycle, delta-connected alternator are shown in Fig. 21–7. These curves are plotted with **terminal** emf and **line** amperes as ordinates and field current as abscissa, as previously explained. Note that the scale of field current is the same for both curves. The d-c armature-resistance, as measured between terminals, is 0.125 ohm. Determine: (*a*) the effective resistance per phase; (*b*) the synchronous reactance per phase; (*c*) the regulation on a lagging load of 0.8 power factor.

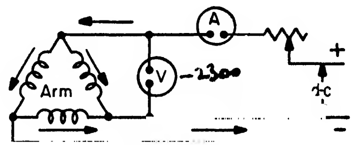


Fig. 20–7. Connections for resistance measurement of a delta-connected alternator.

Solution: (*a*) D-c armature resistance per phase $= 0.125 \times \frac{3}{2} =$ 0.1875 ohm. (See Fig. 20–7 and equation (7).) Effective resistance per phase $= 1.5 \times 0.1875 = 0.281$ ohm. (*b*) Maximum value of line current on short circuit, from Fig. 21–7, = 550 amperes. This requires 40 amperes field current.

Open-circuit terminal voltage at 40 amps field current = 1600 volts. This is the induced emf per phase.

$$Z_s = \frac{E}{\frac{I}{\sqrt{3}}} = \frac{1600}{\frac{550}{\sqrt{3}}} = \frac{1600}{318} = 5.04 \text{ ohms.}$$

$$X_s = \sqrt{Z_s^2 - R_e^2} = \sqrt{5.04^2 - 0.281^2} = 5.03 \text{ ohms}$$

$$(c) \text{ Rated line current} = \frac{1{,}200{,}000}{\sqrt{3} \times 2300} = 302 \text{ amps.}$$

$$\text{Rated current per phase} = \frac{302}{\sqrt{3}} = \underline{174.5 \text{ amps.}}$$

Rated terminal voltage per phase = 2300 volts.

Full load IR drop per phase $= 174.5 \times 0.281 = 49$ volts.

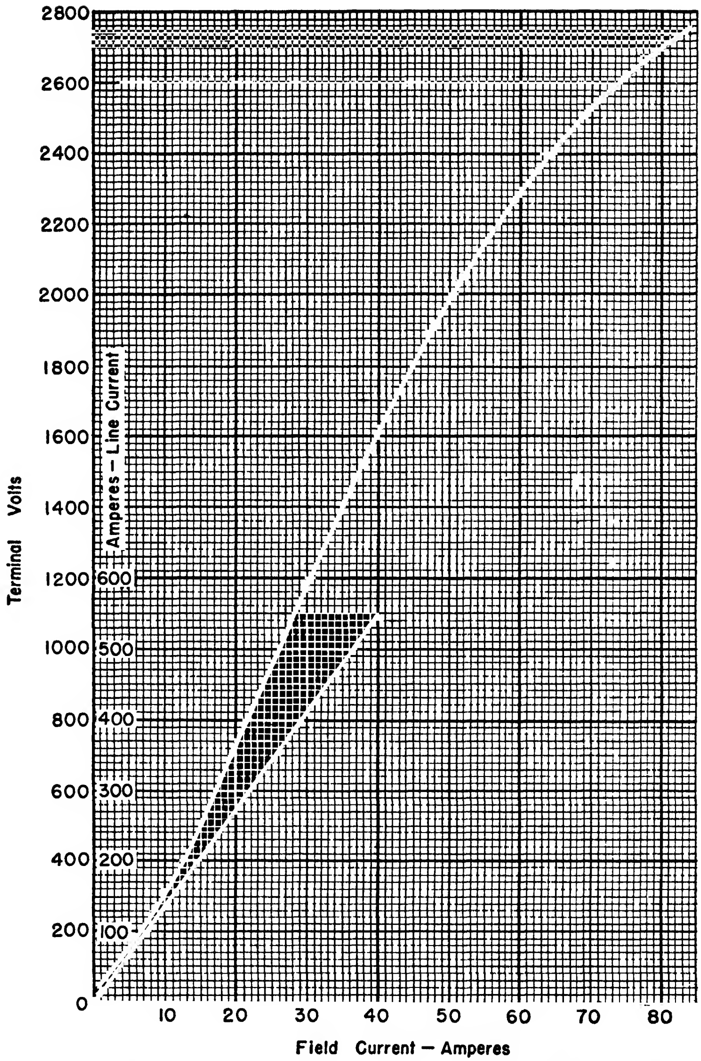


FIG. 21–7. Open-circuit and short-circuit curves for a 1200-Kva, 2300-volt, 60-cycle, three-phase alternator.

Full load IX drop per phase $= 174.5 \times 5.03 = 878$ volts. Z_s

$\cos\theta = 0.8$; $\theta = 36° \, 50'$; $\sin\theta = 0.6$.

From Fig. 22–7,

$$E_{\text{no load}} = \sqrt{[(2300 \times 0.8) + 49]^2 + [(2300 \times 0.6) + 878]^2} = 2900 \text{ volts}$$

$$\text{Per cent regulation} = \frac{2900 - 2300}{2900} \times 100 = 27.8. \quad \textit{Ans.}$$

Prob. 11–7. Compute the regulation of the alternator in Example 5: (*a*) On unity power factor load. (*b*) On leading load at 0.7 power factor.

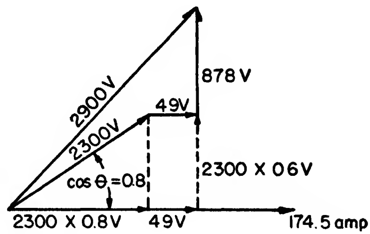


FIG. 22–7. Vector diagram for the alternator in Example 5.

12–7. Synchronous Reactance and Regulation, Y-Connected Alternator. Figure 23–7 shows the connections for the open-circuit, short-circuit tests of a Y-connected alternator. Note that the voltmeter on open circuit indicates a value equal to $\sqrt{3}$ times the induced emf per phase, while the ammeter on short circuit indicates armature current per phase. Thus, for any

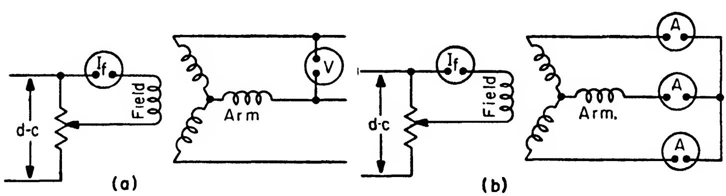


FIG. 23–7. (*a*) Connections for the open-circuit test of a Y-connected alternator. (*b*) Connections for the short-circuit test.

value of field current, as I_f' in Fig. 17–7, the induced emf per phase on open circuit is $\dfrac{E'}{\sqrt{3}}$; and on short circuit the armature current per phase is I_a'. Therefore, the synchronous impedance per phase is,

$$Z_s = \frac{\left(\dfrac{E'}{\sqrt{3}}\right)}{I_a'} \tag{8–7}$$

ns for obtaining the d-c resistance of the armature Fig. 24–7. The instruments measure the resistance of two phases in series; therefore, the resistance per phase is $\frac{1}{2}$ the measured value.

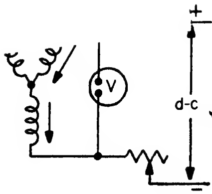


FIG. 24–7. Connections for resistance measurement of a Y-connected alternator.

Example 6. Assume the 1200-Kva, 2300-volt alternator of Example 5 is Y-connected and the curves of Fig. 21–7 represent the results of the open-circuit short-circuit tests on the machine, connected as in Fig. 23–7. As before, these curves are plotted with terminal volts and **line** amperes. Also, the d-c armature resistance measured between terminals, as in Fig. 24–7, is 0.125 ohm. Determine: (*a*) Effective armature resistance per phase. (*b*) Synchronous reactance per phase. (*c*) Regulation on a lagging load on 0.8 power factor.

Solution:

(*a*) D-c resistance per phase $= \dfrac{0.125}{2} = 0.0625$ ohm.

Effective resistance per phase $= 1.5 \times 0.0625 = 0.094$ ohm

(This is just $\frac{1}{3}$ the resistance for the delta connection in Example 5.)

(*b*) At 40 amperes field current (Fig. 21-7), open-circuit terminal emf = 1600 volts, as before.

For the same field current, short-circuit line current = 550 amperes, which is also the phase current.

$$\text{Synchronous impedance per phase, } Z_s = \frac{\left(\dfrac{1600}{\sqrt{3}}\right)}{550} = 1.68 \text{ ohms.}$$

$$X_s = \sqrt{1.68^2 - 0.094^2} = 1.677 \text{ ohms.}$$

(This is also $\frac{1}{3}$ the reactance determined in Example 5.)

(*c*) Rated full load terminal voltage per phase $= \dfrac{2300}{\sqrt{3}} = 1330$ volts.

Rated line current (as in Example 5) = phase current = 302 amperes.

IR drop at full load current $= 302 \times 0.094 = 28.4$ volts.

IX drop at full load current $= 302 \times 1.677 = 506$ volts.

$E_{\text{no load}} = \sqrt{[(1330 \times .08) + 28.4]^2 + [(1330 \times 0.6) + 506]^2} =$ 1700 volts.

Regulation in per cent $= \dfrac{1700 - 1330}{1330} \times 100 = 27.8.$ *Ans.*

Again note that $X_s = Z_s$, nearly. For most computations Z_s can be used for X_s without appreciable error.

It will also be noted that, from the same test data, the regulation of the machine computed in Example 6 as a Y-connected armature is exactly the same as in Example 5 in which the armature is assumed to be delta connected. This is of great convenience, for it is often difficult, or impossible, to inspect the windings in the commercial machine to determine the manner in which the phases are connected, and this information is not always given on the name plate.

Prob. 12–7. Compute the regulation of the alternator in Example 6: (*a*) On unity power factor load. (*b*) On leading load at 0.7 power factor.

Prob. 13–7. The results of open-circuit, short-circuit tests at rated speed on a 2140-Kva, 6300-volt, 50-cycle delta-connected alternator are given below.

Open-Circuit Test

Volts between Terminals	Field Amperes
0 0	0.0
1200	13 2
2400	26 7
3600	40 5
4800	55 5
6000	72.5
6300	77 1
6930	90 0
7800	118.0
8550	155 0

Short-Circuit Test

Short-Circuit Amperes per Terminal	Field Amperes
0.0	0.0
99.	15.3
195.	30.0
294.	46.
390.	60.5

D-c Armature Resistance between terminals = 0.078 ohm.

From these data draw curves with field current as abscissas as already explained. (*a*) Compute the effective armature resistance per phase. (*b*) Compute the synchronous impedance per phase corresponding to each value of armature current observed in the short-circuit test. (*c*) Does the synchronous impedance appear to be a constant quantity within the range of the test data? (*d*) From the average value of the synchronous impedance in (*b*), compute the synchronous reactance per phase.

Prob. 14–7. Compute the voltage regulation of the alternator in Prob. 13–7 when delivering rated full load at rated voltage and at unity power factor. Draw complete vector diagram to illustrate your solution.

Prob. 15–7. (*a*) Repeat Prob. 14–7 for a lagging load at 0.8 power factor. (*b*) For a leading load at 0.8 power factor. Show vector diagrams.

Prob. 16–7. What would be the voltage regulation in Prob. 15–7, if the resistance drop were neglected and the synchronous impedance were used in place of the synchronous reactance?

Prob. 17–7. By means of vector diagrams compute the terminal voltage of the alternator in Prob. 13–7 when the current output is reduced to one half of full load value while keeping the excitation constant at full load value. Power factor of the load remains constant at 0.8 power factor.

Prob. 18–7. Repeat Prob. 13–7 on the assumption the alternator is Y-connected and the test data are as given in that problem.

Prob. 19–7. Repeat Probs. 14–7 and 15–7 using the results obtained in Prob. 18–7.

13–7. Percentage Resistance and Reactance.

The IR and IX drops in an alternator at **full load current** per phase are often given as a percentage of the rated terminal voltage per phase. This is called the **per cent resistance and the per cent reactance.** Thus, from Example 6,

$$IR \text{ drop at full load current} = 302 \times 0.094 = 28.4 \text{ volts}$$

$$\text{Effective resistance} = \frac{28.4}{1330} \times 100 = 2.1 \text{ per cent}$$

$$IX \text{ drop at full load current} = 302 \times 1.677 = 506 \text{ volts}$$

$$\text{Synchronous reactance} = \frac{506}{1330} \times 100 = 38 \text{ per cent}$$

14–7. Excitation for Alternators.

It has been shown that the inherent change in terminal voltage of an alternator, with change of load, or power factor, is so great that some external means of adjustment, or control, must be used to keep this voltage constant. This is accomplished either by means of a rheostat in the alternator field circuit, or by control of the exciter voltage, or by both, as indicated in Fig. 25–7. In the latter case the alternator field rheostat is set to give approximately the desired terminal voltage, closer adjustments being made by the rheostat in the shunt field of the exciter. Exciters in capacities of 100 kw and over are generally compound wound and flat compounded, while smaller sizes are generally shunt wound. The maximum capacity of exciters for large turbo-alternators is generally about 0.5 per cent of the

alternator Kva rating, while that for small and slow-speed engine-driven machines may be as high as 40 per cent. Rated exciter-voltages range from 125 volts for small and medium sized machines to 250 volts for large alternators. Normal field current for large alternators may be close to 1000 amperes.

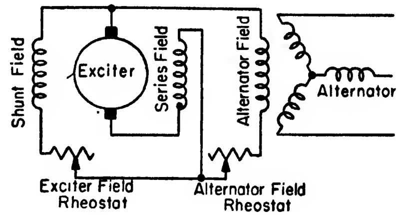


FIG. 25–7. The voltage of an alternator is controlled by a rheostat in the alternator field circuit and, also, by a rheostat in the shunt-field circuit of the exciter.

Since an alternator cannot generate a voltage nor deliver energy unless supplied with direct current for its field, a reliable source of power to drive the exciter must be available at all times. In generating plants having two or more alternators, there are three general methods for supplying excitation to the several machines.

(1) In some plants each alternator may have its own direct-connected exciter. Often, each exciter may have sufficient capacity to supply excitation to two alternators in case of failure of any one of the exciter units.

(2) In other plants, the exciters are not direct-connected to their alternators, but are driven by a separate power source and connected in parallel to a common pair of conductors, of d-c "bus bars," to which the field circuits of all the alternators are connected. The exciters, in this case, are generally driven by induction motors supplied from the a-c output of the station. However, one or more exciters in the station must be steam driven, if the station is a steam plant; or driven by a separate water wheel, if in an hydraulic plant. This is necessary, since power to drive at least one exciter must always be available before the plant can be put into operation, or in case of a general shutdown due to a major short circuit on the system, etc.

(3) A combination of the two schemes above may be employed. The direct-connected exciters may be used only to excite their own machines, or switched to the d-c bus. The bus may be "sectionalized," or divided, so that one group of exciters may supply only part of the alternators in the station.

Schemes (1) and (2) above each have advantages and disadvantages and there is no common agreement among operators as to the better method.

Storage batteries are also often installed as a reserve in case of failure of the exciter units.

15–7. Automatic Voltage Control—The Diactor Regulator. The regulation of the alternator is poor, as has been shown, and commercial loads fluctuate widely. Yet the terminal voltage must be held constant by adjustment of the field current. Thus if the load

FIG. 26–7. The Diactor Voltage Regulator. (*General Electric Co.*)

increases, the terminal voltage of the alternator **falls** and resistance in the field rheostat must be **decreased;** if the load **decreases,** the terminal voltage **rises** and the resistance in the rheostat must be **increased.** Hand adjustment requires the constant attendance of an operator and is unsatisfactory, particularly with rapid fluctuations of load. Therefore, several automatic devices have been developed for control of voltage.

The Diactor Regulator is one of the most successful devices for maintaining the terminal voltage of a-c generators constant at a

set value. It operates by regulating the current in the d-c field of the a-c generator. It does this indirectly by automatically increasing or decreasing a resistance connected in series with the field coils of the d-c generator, thereby raising or lowering the d-c voltage across the field of the a-c generator. The resistor consists, primarily, of stacks of carbon plates, small changes in the resistance of which are made by changing the area of contact

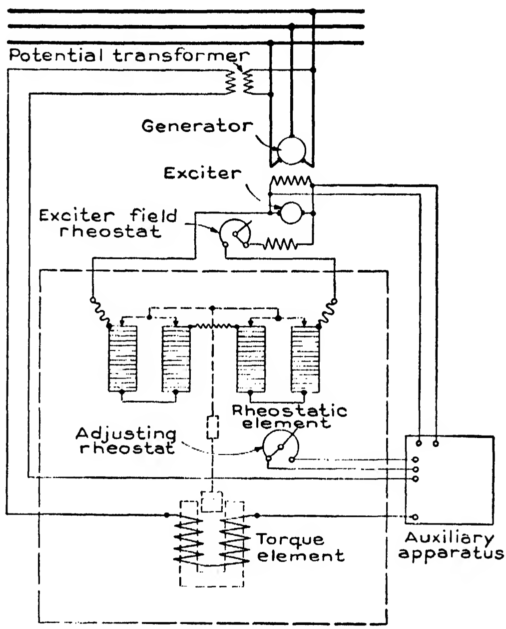


FIG. 27–7. Simplified diagram of circuits in a Diactor Regulator for automatic voltage control of an alternator. (*General Electric Co.*)

between the plates, and not by the usual method of changing the pressure between them. The larger changes in resistance are made by opening and closing the contacts between silver buttons which are inserted in the edge of the carbon plates. Thus the resistance of the regulator can rapidly and smoothly be changed by infinitesimal steps from practically zero to the maximum resistance of the carbon stacks.

Figure 26–7 is a front view of a Diactor Voltage Regulator without the enclosing case, and Fig. 27–7 is a simplified diagram of a regulator controlling the voltage of a three-phase generator.

By means of the knob shown in Fig. 26–7, the regulator can be

set for any particular a-c voltage within the range of the apparatus. As long as the generator maintains this voltage there is no motion of any part of the regulator. It has no vibrating contactors.

The box labelled Auxiliary Apparatus in Fig. 27–7 is attached to the rear of the panel of Fig. 26–7 and contains a full-wave copper-oxide rectifier and a stabilizer which prevents the elements of the regulator from over-shooting or chattering; every correction in the a-c voltage being completed in a small fraction of a second.*

Another type of voltage regulator for very large alternators consists of a motor operated rheostat in the alternator field,

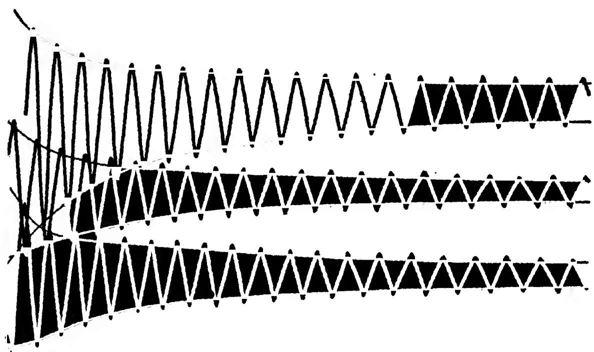


FIG. 28–7. Oscillogram of short-circuit currents in the three phases of an alternator. The maximum current is about 1400 amp. In the one-third of a second shown in the oscillogram, it had dropped to about 200 amp, maximum. (*Allis-Chalmers Co.*)

controlled by a high speed relay. If the alternator voltage falls, the motor drives the rheostat arm in a direction to reduce resistance in the rheostat; if the voltage rises, the rheostat arm is driven in the opposite direction.

Some recent types of voltage regulators make use of the electronic tube.

16–7. Short Circuits on Alternators — Current Limiting Reactors. When an alternator, operating under normal conditions of voltage and speed, is suddenly short circuited, the current instantly rises to a momentary value many times the rated full load current, and then gradually drops to a steady, or " sustained " short-circuit current, usually from about 1.5 to 4 or 5 times rated

* For complete details of construction and operation see General Electric Bulletin G E A 2022F.

full load current, as indicated in Fig. 28–7. At the instant of short circuit, if the short is near the alternator, only the resistance and the true reactance of the armature, as a coil, limit the current. This short-circuit current is of low power-factor and it takes an appreciable period of time for the armature reaction to weaken the field (that is, for the synchronous reactance to take effect) and reduce the current to the steady, or sustained, short-circuit value.

This first rush of current, which lasts only a few cycles, as shown in the figure, is particularly dangerous to the machine, as the

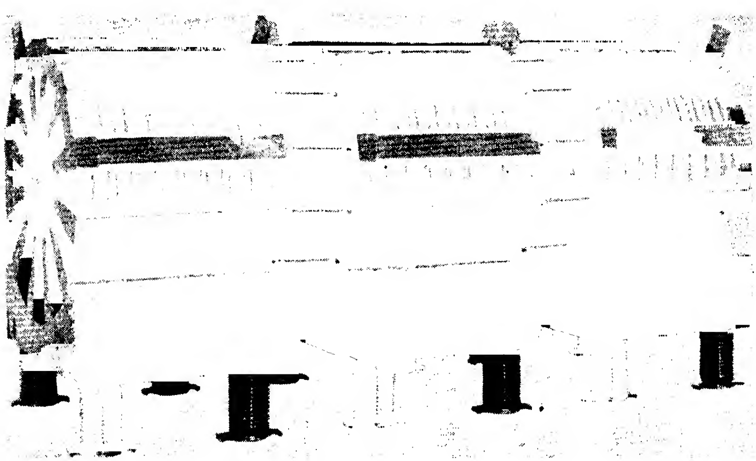


Fig. 29–7. "Current Limiting Reactors" to limit the short-circuit current in a three-phase alternator. (*General Electric Co.*)

magnetic forces set up vary with the square of the current. These forces are enormous and may twist the coils out of shape, or tear the winding apart, even when braced as shown in Fig. 9–6.

To protect the alternator and its circuit breaker, particularly if it is of large capacity, a reactance coil, or **current limiting reactor,** is placed in each alternator lead between the machine and the breaker. These reactors, illustrated in Fig. 29–7, consist of coils of copper strap, or cable, usually wound on reinforced concrete frames, and set on insulators. They have the characteristics of air-cored coils. These coils may produce a reactance voltage drop, at full-load current, as high as 15 or 20 per cent of the rated alternator voltage, and very materially reduce the instantaneous, or momentary short-circuit current.

The alternator is also protected by an oil switch, or circuit breaker, one type of which is illustrated in Fig. 30–7. The breaker contacts are immersed in oil in an enclosed container. If these contacts open during the first surge of current, great forces are generated at the switch break, often wrecking it. Oil switches have been known actually to explode under such abnormal conditions. Therefore, the oil switch is usually equipped with automatic relays, generally of the over-load-inverse-time-limit type.* Figure 31–7 shows one of these relays.

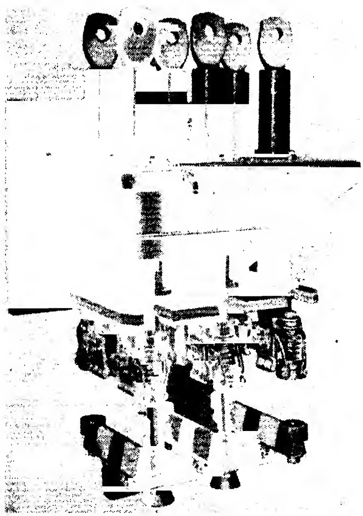


Fig. 30–7. Three-phase oil switch, or circuit breaker, for a medium size alternator. (*General Electric Co.*)

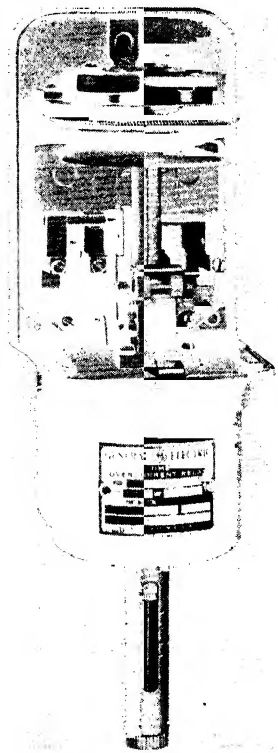


Fig. 31–7. A time-delay over-load relay. (*General Electric Co.*)

Relays may be set to open a circuit at a definite interval of time, from a small fraction of a second to several seconds in duration.

* Relays are in general use today for protection of apparatus and for numerous switching operations. There are many forms and their functions are varied. The art of relaying is almost a separate field in itself and beyond the scope of this text.

The interval of time required for the operation of an overload-inverse-time-limit relay varies inversely with the value of the current. That is, the relay may be set to open the circuit in a given time under the action of a given current. If the current reaches a higher value, the relay operates in a **shorter** period of time.

To relieve a short circuit, the relays are generally set to open the oil switch at a definite instant **after** the current has dropped to its sustained short-circuit value. Since continuity of service is of prime importance in any plant, the action of the relay is delayed in the hope the short circuit will clear itself, and so avoid disconnecting the alternator from the line.

Prob. 20–7. A 720-Kva, 11,000-volt, 60-cycle Y-connected alternator has an inherent reactance per phase of 8 per cent (see Art. 13, Chap. VII). (*a*) What is the reactance in ohms? (*b*) If this machine is short circuited when operating at normal terminal voltage and frequency, what momentary short-circuit current will flow before armature reaction reduces it to the sustained or steady value? Neglect resistance of armature and leads. (*c*) What percentage of full load value is the momentary short-circuit current?

Prob. 21–7. What would be the momentary short-circuit current in the alternator of Prob. 20–7 if current limiting reactors, each having an inductance of 0.075 henry, were placed in each armature load?

17–7. " Hot Spot " Temperature. Alternators ordinarily are not protected against overload. The safe Kva load is determined by the temperature rise in the windings. In large alternators, thermocouples (see Vol. I, Ch. XVII) are usually embedded in the bottom of several slots in the armature. This location is generally considered the **hottest spot** in the winding. Each of these thermocouples can be connected in turn to a milliammeter on the switchboard, so that the operator may easily check the temperature of the machine. In armature windings insulated with mica, a temperature rise of 70° to 75° C is allowed. Where varnished cambric, or impregnated cotton, is used, a temperature rise of 50° to 55° C is allowed.

PARALLEL OPERATION OF ALTERNATORS

18–7. Alternators in Parallel. Commercial loads on alternating-power plants vary over a 24 hour period in much the same way as on direct-current plants. Therefore, the reasons for operating direct-current generators in parallel (see Vol. I, Ch. XII) apply

also to alternators. The number of alternators operating in a plant depends upon the capacity of the individual machines and upon the total load on the plant. The loads on large power stations in heavily populated areas may be several hundred-thousand kilovolt-amperes. No single alternator, so far constructed, can carry such a load, although a very few alternators of 200,000 Kva rating are in operation today. There are, also, in operation several machines of 100,000 Kva rating, but most alternators have ratings considerably below this figure (see Introduction to Ch. I). It is therefore evident that the generation and distribution of electrical energy at constant voltage necessitates operating alternators in parallel.

19–7. Parallel Operation of Alternators vs. Direct-Current Parallel Operation. Similarities and Differences Compared. The conditions required for the paralleling of two alternators, or for paralleling an alternator with the common station bus, are practically the same as for paralleling two sets of storage batteries, or two direct-current generators. The points of similarity and difference are briefly compared in this section.

(*a*) When two d-c generators are to be paralleled, the emf of the incoming machine must first be adjusted to the same value as the terminal voltage of the other, or to that of the bus bars.

The same adjustment should be made before paralleling two alternators.

(*b*) The polarity of two d-c generators must be such that their voltage, or emfs, are in opposition to each other in the local circuit between the machines. That is, the positive (+) terminals must be connected together and negative (−) terminals so connected, as in Fig. 32–7. The polarity of a d-c machine is determined by the voltmeter across its terminal.

The same condition must be fulfilled when paralleling two alternators. However, the direction of the emf in an alternator reverses several times a second and the voltmeter across the terminals does **not** show the polarity of the machine, since the voltmeter indicates effective values. Therefore, in paralleling two alternators, they must be **synchronized.** This is explained in the following article.

(*c*) After two d-c generators are properly paralleled with equal terminal voltages, these voltages are in opposition and balance each other in the local circuit between the machines. And no current will flow in the armature of the incoming generator.

Similarly, after two alternators are properly **synchronized,** their voltages are in opposition, or 180 electrical degrees from each other, in the load circuit. And, if their emfs have the same effective value and wave form, no currents will flow in the incoming machine.

(*d*) Direct-current generators will operate satisfactorily in parallel regardless of the speed at which they are driven, provided only that their characteristic curves are approximately the same.

Alternators, on the other hand, must be driven at the **same electrical** speed; that is, at the **same frequency.** For instance, a four-pole alternator, driven at 1800 rpm, may be operated in

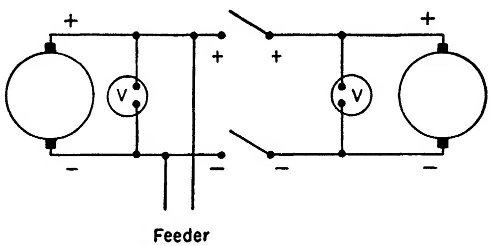


FIG. 32–7. The emfs of parallel-connected direct-current generators must be opposed to each other in the local circuit between the machines. That is, positive terminals must be connected together, also negative terminals connected.

parallel with a six-pole alternator driven at 1200 rpm. Each generates a 60-cycle emf and, therefore, is driven at the same electrical speed.

(*e*) The terminal voltages of all parallel connected generators must be the same, since their terminals are all respectively connected together.

If the emf of the incoming d-c generator is raised by field rheostat adjustment, the machine will supply a current to the line of such value, that the armature *IR* drop is just equal to the difference between its generated emf and the bus-bar voltage. Thus the load on the incoming machine is increased.

After synchronizing, if the emf of the incoming alternator is raised by field adjustment, this alternator will also supply a current of such value, that the armature *IZ* drop is just equal to the difference between its emf and the bus-bar voltage. However, this current is almost wholly reactive, that is, lags almost 90° behind the alternator emf. As the power factor of the external

circuit is determined by the power factor of the load, this reactive current cannot flow in the external circuit, but must circulate in the circuit between the machines. Thus the load on the incoming machine is practically unchanged. This is explained in a following article.

(*f*) In order for direct-current generators to operate satisfactorily in parallel they must have **drooping voltage characteristics,** that is, the voltage must fall as the load is increased. (The exception is in the parallel operation of overcompounded generators with an equalizer.)

In order for alternators to operate satisfactorily in parallel their **prime movers** must have a drooping speed characteristic. This is also explained later.

20–7. Synchronizing. The conditions for paralleling, or synchronizing, two alternators, as stated above, are:

(1) Their terminal voltages must be equal, or approximately so.

(2) Their polarities must be reversed, or opposed to each other in the local series circuit between the machines.

(3) Their frequencies must be the same.

The first condition can be determined by voltmeters across the machine terminals, but some means must be provided for determining the relative polarity and frequency of the machines. This is determined by the use of "synchronizing lamps."

Figure 33–7 shows a simple diagram of connections for synchronizing two single-phase alternators. An incandescent lamp is connected across each blade of the switches S_1 and S_2, as shown. Assume that alternator, A_1, is operating normally and supplying energy to the feeders through the bus bars. Switch S_1 is closed. In order to synchronize alternator, A_2, it is brought up in speed until its frequency is nearly that of A_1. The voltage of A_2 is now adjusted, by field rheostat, until its emf is equal to the terminal voltage of A_1. Although the switch S_2 is open, the machines are connected together through the lamps, L_2L_2, across S_2 as shown.

If the lamps now become alternately bright and dark, it means the frequencies of A_1 and A_2 are different. The number of light beats per second shows the **difference** between the frequencies of the two machines. The lamps do not show which alternator is fast or slow, so the speed of the incoming machine A_2 is raised or lowered until the bright and dark periods are long and follow each other very slowly.

While the lamps are dark, they show that the resultant voltage

of the two machines in series is **low** and that the machine voltages are **approximately in opposition** in the local circuit. Since a lamp filament ceases to glow even when there is considerable voltage across it, the lamps in Fig. 33–7 do not indicate the instant when the voltages of the two machines are in exact opposition to each other. This occurs at the **middle of a dark period.** At this in-

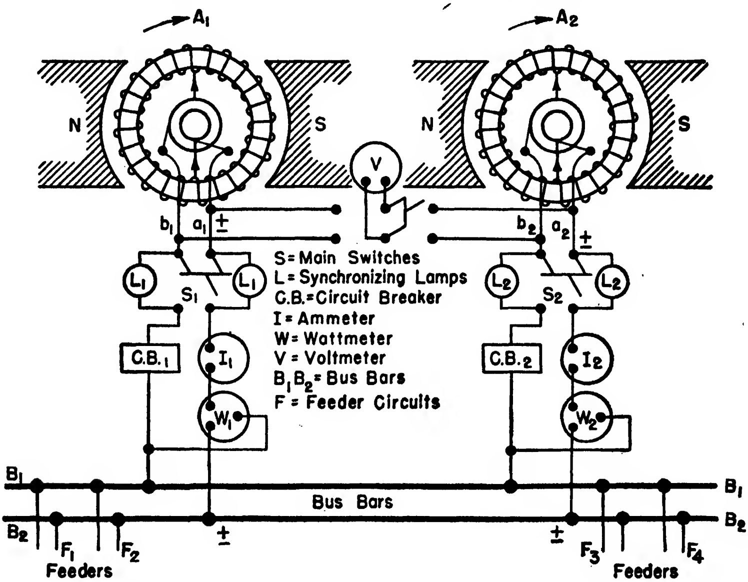


FIG. 33–7. Simple diagram showing the essential apparatus used in paralleling two alternators.

stant, the machines are in phase with each other with respect to the bus bars or to the external circuit, and there is no emf acting in the series circuit between them. Therefore, **the alternators are synchronized by closing switch S_2 at the middle of a dark period.** This calls for accurate timing on the part of the operator.

While the lamps are bright, when connected as in Fig. 33–7, they indicate that the resultant voltage in the local circuit of A_1 and A_2 in series is **high** and, therefore, the polarity of the incoming machine is reversed. If the switch S_2 is now closed, the alternators are short circuited on each other, the circuit breakers will open and damage to the machines may result.

In synchronizing the alternators, as described above, the speed of the incoming machine might be so adjusted that the lamps **remain dark.** This would indicate that the alternators were **in**

phase, running at exactly the same frequency and in proper position for synchronizing. However, it **may** mean that **the filament of one of the lamps is broken**; therefore, it is preferable to adjust the speed of the incoming machine so that the lights brighten and darken slowly. Thus there is **more recent evidence** that the lamps are indicating properly. Alternators are generally synchronized with the incoming machine running slightly fast.

The fact that the alternators remain in synchronism, when paralleled at slightly different frequencies, is due to the drooping speed characteristics of the prime movers and to the interaction between the machines, which is discussed later.

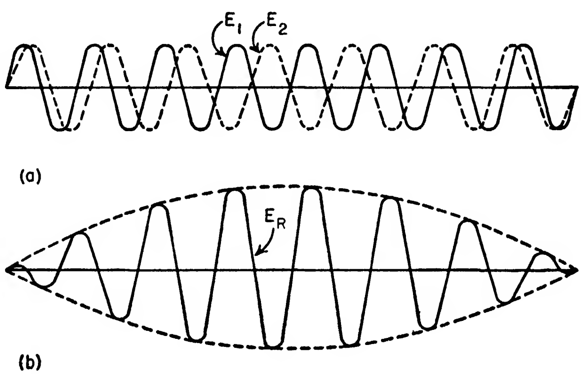


FIG. 34–7. (*a*) Emf waves of two alternators drawn with reference to their relation to the bus bars, or external circuit. Wave from E_2 completes 8 cycles while E_1 completes 7 cycles. (*b*) The *difference* of the waves in (*a*) produces the resultant emf wave.

The sequence of operations in synchronizing is thus:

First: bring the incoming machine approximately to the same frequency:

Second: excite its field and adjust the voltage to that of the bus bars;

Third: adjust its speed until the light beats are very slow;

Fourth: close the synchronizing switch in the middle of a period when the lamps are dark.

The reason why the lamps brighten and darken, when the frequencies are not the same, is illustrated in Fig. 34–7. Figure 34–7(*a*) shows the emf waves of the two machines having the same wave form and effective value. Curve E_1 is the emf of alternator A_1 and E_2 that of A_2. Alternator A_2 is running fast and E_2 goes through 8 cycles, while E_1 is going through 7. These curves are

drawn with reference to their relation in the external circuit, or to the bus bars. The resultant voltage acting on the lamps L_2L_2 (Fig. 33–7) is thus the **difference** of these two curves and their resultant in the curve, E_R, in Fig. 34–7(*b*). During the time in which E_2 completes one more cycle than E_1, the emf acting on the lamps is alternating rapidly, but its maximum value increases from zero to a maximum and back to zero again, as indicated by the dotted line in (*b*). This causes the lamps to brighten and darken correspondingly. Thus, if the frequency of E_1 in (*a*) is 60 cycles and that of E_2 is $\frac{8}{7}$ of 60 or about 67 cycles, the lights will flicker 67–60, or 7 times per second. This difference in frequency is much too great for synchronizing but is here used for clearness in

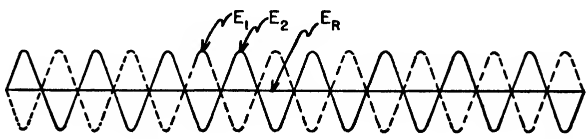


Fig. 35–7. Waves E_1 and E_2 are completing the same number of cycles. They have a phase difference of 180° in the local circuit between the machines and the resultant voltage is zero. The synchronizing lamps are therefore dark.

the figures. The duration of each dark period should be several seconds.

When the two alternators are running at exactly the same frequency with E_2 equal and opposite to E_1 at every instant, the resultant emf, E_R, acting at every instant in the circuit between the machines is zero, as shown by the curves of Fig. 35–7, drawn with reference to their relation in the local circuit.

The variation in voltage across the synchronizing lamps is also indicated by the vector diagram of Fig. 36–7. In the middle of a bright period, the emfs of the two alternators are in the same direction and in phase in the series circuit between the machines. These emfs are shown, as OE_1 and OE_2, and their vector sum (equal to their arithmetic sum at this instant) is OE_R, which represents the greatest effective voltage across the lamps. The vector, OE_2, having the higher frequency, rotates faster than OE_1. When OE_2 reaches the position OE_2^{I}, leading OE_1 by ϕ_1° the resultant emf across the lamp drops to OE_R^{I}; at position ϕ_{11}°, the resultant emf drop to OE_R^{II}; at position ϕ_{111}, the resultant emf is again reduced to OE_R^{III} and finally, when OE_2 reaches the position OE_R^{IV}, 180° leading OE_1 in the local circuit, the resultant emf across the lamps

is zero. The synchronizing lamps, at this instant, are in the middle of a dark period and the machines are synchronized and in the proper position for switch S_2 to be closed. If this is not done at approximately this instant, the vector OE_2 continues to advance, the resultant emf across the lamp increases until OE_2 and OE_1 are again in phase, and the lamps are in the middle of a bright period.

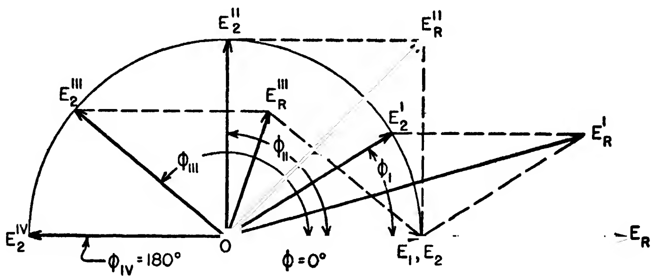


FIG. 36–7. Vector diagram showing how the voltage across the synchronizing lamps varies from a maximum to zero, and back to a maximum again, when the frequencies of the alternators are not the same.

Example 7. Alternator A_1, in Fig. 33–7, has a terminal voltage of 120 volts of sine wave form and a frequency of 60 cycles. A_2 has a terminal emf of 122 volts of sine wave form and a frequency of 61 cycles per second. When S_1 is closed and S_2 is open:

(*a*) What is the greatest effective voltage across each lamp, assuming they are all alike? (*b*) How many light beats occur each second?

Solution: (*a*) Greatest effective voltage across the lamps is always the sum of the effective terminal emfs of the two alternators, or

$$E_1 + E_2 = 120 + 122 = 242 \text{ volts.}$$

But, when S_1 is closed, two lamps (L_2L_2) are in series, so that effective voltage across each lamp $= \dfrac{242}{2} = 121$ volts.

(*b*) Number of light beats per second equals the difference in frequency of the two alternators, or, 61–60 = 1 light beat per second.

Prob. 22–7. Each alternator of Fig. 33–7 generates a terminal emf of 230 volts. The frequencies are 60 and 58 cycles per second. S_1 is closed and S_2 open. The synchronizing lamps are all alike. (*a*) What is the greatest effective voltage across each lamp L_2L_2? (*b*) How many of these maxima occur per minute?

Prob. 23–7. Each alternator in Fig. 33–7 generates a sine wave of emf of 60 cycles; S_1 is closed and S_2 open; all lamps are alike. The terminal emf of A_1 is 240 volts, and of A_2 is 200 volts. What is the effective voltage across each synchronizing lamp L_2; (*a*) when the phase

difference between E_1 and E_2 is such that this voltage is the greatest? (*b*) When this voltage is a minimum?

Prob. 24–7. Alternator A_1, in Fig. 33–7, generates an effective terminal emf of 240 volts at 60 cycles and A_2 generates an effective emf of 200 volts at 62 cycles, both of sine wave form. Both switches S_1 and S_2 are open. Describe the behavior of each lamp L and compute the maximum and minimum values of effective voltage across it.

21–7. Synchronizing Three-Phase Alternators. Figure 37–7 shows the connections for synchronizing a three-phase alternator. Lamps, L, are connected across the three blades of a three-pole switch, as shown. If the incoming alternator is properly connected the three lamps will all brighten and darken in unison. The

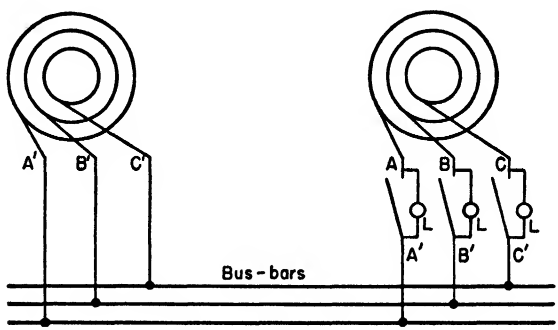
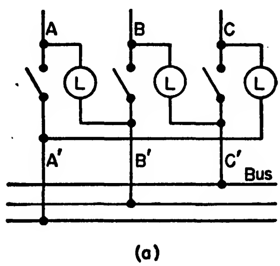


FIG. 37–7. Connections for synchronizing three-phase alternators. Arrangement of the lamps for the " three-dark " method of synchronizing.

switch is closed when all the lamps are in the middle of a dark period. This is called the " three-dark " method of synchronizing.

When alternators are first connected for paralleling, it may happen that the light beats do not all come at the same time, but follow each other in succession. This means that the relative phase rotation of the two machines is not the same. That is, the emf in each of the phases in one alternator may reach its maximum value in a positive direction in the order A, B, C, while the other phase sequence is in the order A', C', B'. The phase rotation of the incoming alternator can be reversed with respect to the other by reversing its direction of rotation, but this is not advisable for several reasons. The easier way is to **reverse the connections of any two of the alternator leads** to the synchronizing switch. After the phase rotation is known to be correct, lamps across only two blades of the switch are necessary.

The synchronizing lamps, L, may be all cross connected on the switch blades, as indicated in Fig. 38–7(a), and the machine synchronized at the middle of a bright period. This is called the "three-bright" method of synchronizing and is little used in this country. Another arrangement of the lamps is shown in Fig. 38–7(b), in which only two of the lamps are cross connected. Lamps across blades B and C are now brighter when the third lamp is dark, at the moment for synchronizing. At this instant one of the bright lamps is increasing and the other is decreasing in brilliancy. The proper instant for synchronizing can thus be more accurately determined. This arrangement is called the "two-bright-one-dark" method of synchronizing.



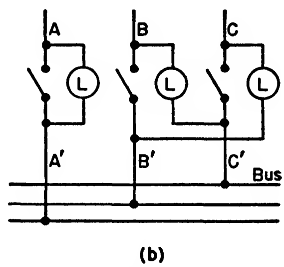


FIG. 38–7. (a) Connections of the lamps for the "three-bright" method of synchronizing. (b) Connection of the lamps for the "two-bright-one-dark" method of synchronizing.

The rated voltage of each synchronizing lamp should be somewhat greater than the normal voltage of the alternators, or two or more lamps may be connected in series across the switch blades. For higher voltage machines the lamps are connected through potential transformers.

In most power plants a "synchroscope" is used for the paralleling of alternators.

22–7. The Synchroscope. A synchroscope is an instrument which indicates when an alternator is in the proper position for synchronizing with the bus bars, or with another alternator. It also shows whether the incoming machine is running fast, or slow. It operates on the same principle as the power factor meter, described in Ch. V, Art. 13. In this case, however, the moving coils are free to rotate and are connected to the bus bars through slip rings mounted on the shaft. These coils rotate in the field of a bipolar iron magnet which is excited by a winding connected across the terminals of the incoming machine. The appearance of

the instrument, for switchboard mounting, is shown in Fig. 39–7. The lamps are also connected in circuit as a check on its indications. If the frequency of the incoming machine is exactly the same as that of the bus bars, a pointer, fastened to the rotating element remains stationary. If the incoming machine is fast, the pointer rotates clockwise over a circular scale; if slow, it rotates in the opposite direction. The number of revolutions per second of the pointer is equal to the **difference in frequency** between the incoming machine and the bus bars. When the machines are in synchronism, the pointer stands vertically over an index on the dial. The incoming machine is generally synchronized when the pointer is moving slowly in the "fast" direction and approaching closely to

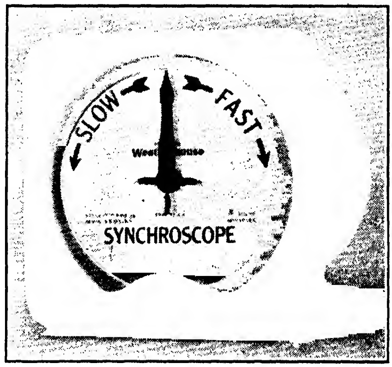


FIG. 39–7. A synchroscope for switchboard mounting. (*Westinghouse Electric Corp.*)

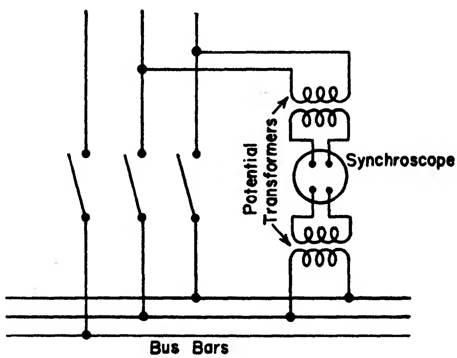


FIG. 40–7. Connections of a synchroscope through potential transformers.

the index on the dial. On higher voltage generators the instrument leads are connected through potential transformers, as in Fig. 40–7. The relative phase rotation is not indicated by the instrument and must be determined by lamp or by other means. However, after this is once determined for all alternators in a plant, it does not change. The synchroscope is a more accurate

device for the paralleling of alternators than any synchronizing lamp.

23–7. Reactive Circulating Current. It already has been shown that, when two alternators with equal terminal voltages of the same wave form are properly synchronized, there is no resultant emf acting between the two machines. Since these voltages are

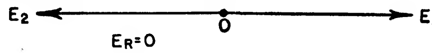


FIG. 41–7. When the alternator emfs are the same, and the machines are in synchronism, there is no resultant emf acting in the local circuit and $E_R = 0$.

directly opposed, they are 180 electrical degrees from each other, as indicated in Fig. 41–7. However, if the terminal voltages differ in wave form, or in effective value, a resultant emf is set up between the machines and a circulating current will flow as stated in Art. 19.

Assume the feeders in Fig. 33–7, Chap. VII, are disconnected and the two alternators are operating without load. Also assume they

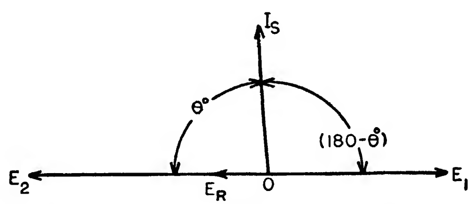


FIG. 42–7. If the emf, E_2, of one machine is increased there is a resultant emf, E_R, acting in the local circuit. A synchronizing current, I_S, will be circulated between the machines, which must lag E_R by an angle θ, whose value is determined by the ratio of the reactance to the resistance of the armature circuits.

are properly synchronized, but assume the emf of A_2 is higher than that of A_1. E_2 is thus greater than E_1 and 180° from it, as shown in Fig. 42–7. There is now a resultant emf, E_R, acting in the local circuit between the machines. This emf is opposed only by the synchronous impedance of the local circuit. Thus a current, I_S, called the **synchronizing current** will circulate between the machines of such value that, neglecting the slight impedance of the machine leads and bus bars,

$$I_S = \frac{E_R}{\sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2}}, \quad \text{or} \quad I_S = \frac{E_R}{Z_1 \oplus Z_2}, \qquad (9\text{–}7)$$

where R_1, R_2, X_1, X_2, Z_1 and Z_2 are the resistances, reactances and impedances of the two armatures.

Since the reactance of an armature usually is high with respect to its resistance, this current will **lag** by an angle θ, nearly 90°, behind E_R, the emf which produced it. The current, I_S, also lags behind E_2, the emf of the higher voltage machine, by the angle θ, and the power, P_2, generated in A_2 is

$$P_2 = E_2 I_S \cos \theta.$$

This is generator action, as θ is less than 90°.

The current, I_S, also **leads** E_1, the emf of the lower voltage machine, by an angle $(180 - \theta)$, and the power, P_1, generated in A_1 is,

$$P_1 = E_1 I_S \cos (180 - \theta).$$

As the angle $(180 - \theta)$ is greater than 90° (Fig. 42-7), the cosine is **negative** and the power is **negative**, indicating motor action.

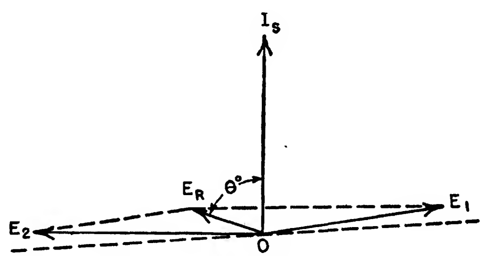


FIG. 43-7. When the alternators adjust their phase positions, the synchronizing current, I_S, takes such a position that there is no motor action in either machine.

That is, the current and voltage relations in machine A_1 are such as to set up torque in the direction it is being driven by its prime mover.

The difference in the power P_1 and P_2 generated in each machine, represents the power transformed into heat, or the I^2R losses in the two armatures, which must be supplied mechanically.

Both machines are being driven at the same frequency by prime movers supplying just enough power, when synchronized, to overcome the rotational losses. Since both prime movers must have drooping speed characteristics, E_2 will drop back and E_1 will advance slightly in phase position with respect to each other, as indicated in Fig. 43-7. This increases both E_R and I_S, and

additional power input to each alternator is just sufficient to supply the I^2R losses due to the circulating current in that machine. There will then be no motor action and I_S will be constant.

Since the current, I_S, lags nearly 90° behind E_2, the emf of the higher voltage machine, and leads E_1, the emf of the lower voltage machine, by approximately the same angle, the effect of the circulating current upon these voltages is always such as to reduce E_R. It has been shown that a lagging current weakens, and a leading current strengthens, the fields of an alternator. Therefore, I_S will weaken the field of A_2 and reduce E_2; and, at the same time, strengthen the field of A_1 and increase E_1. Thus, E_2 and E_1

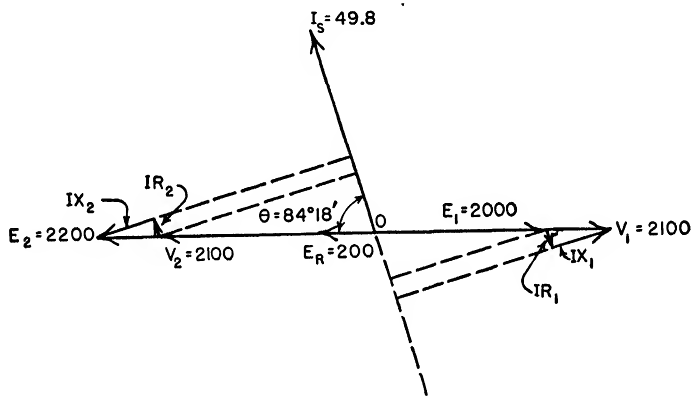


FIG. 44–7. Vector diagram showing voltage and current relations in Example 8.

will automatically be made more nearly equal to each other by the armature reaction due to I_S; and as E_2 approaches E_1 in value, E_R is reduced. Thus, the synchronizing current limits itself. Of course the synchronizing current brings the **terminal voltages** of the two machines to the same value.

The alternators, therefore, automatically adjust themselves to inequalities in voltage without any appreciable transfer of load. They are, thus, not sensitive to variations in voltage as are d-c generators. However, the circulating current is superimposed on any load current the armatures may be supplying to the external circuit and reduces the load capacity of the machines.

It should be borne in mind that the vector diagrams above are drawn with reference to the local circuit between the machines and refer to one phase only in a polyphase alternator.

Example 8. Two single-phase alternators, A_1 and A_2, each having an effective resistance of 0.2 ohm and a synchronous reactance of 2.0 ohms, are running without load on the bus bars. They are properly synchronized with their emfs, E_1 and E_2, opposed 180° to each other in the local circuit, as indicated in Fig. 44–7, but the emf of A_1 is adjusted to 2000 volts, while that of A_2 is 2200 volts. Immediately upon closing the synchronizing switch, and before the machines adjust their phase positions determine:

(*a*) The resultant voltage, E_R, and the synchronizing current, I_S.
(*b*) Angle θ between E_R and I_S.
(*c*) The generator action, or power, P_2, developed in alternator, A_2.
(*d*) The motor action, or power, P_1, developed in alternator, A_1.
(*e*) The I^2R loss in both armatures.
(*f*) The terminal voltage of each machine.

(Neglect the slight resistance and reactance of bus bars and leads between the two machines.)

Solution:

(*a*) Resulting voltage, $E_R = E_2 - E_1 = 2200 - 2000 = 200$ volts.

$$\text{Impedance of local circuit} = \sqrt{(0.2 + 0.2)^2 + (2.0 + 2.0)^2}$$
$$= \sqrt{0.4^2 + 4.0^2} = 4.02 \text{ ohms.}$$

$$\text{Synchronizing current} = \frac{200}{4.02} = 49.8 \text{ amperes.}$$

(*b*) Tangent of angle θ, between E_R and $I_S = \dfrac{4.0}{0.4} = 10$,

$$\theta = 84° \ 18'; \ \sin\theta = 0.995; \ \cos\theta = 0.09932.$$

(*c*) $P_2 = E_2 I_S \cos\theta = 2200 \times 49.8 \times 0.09932 = 10{,}890$ watts.
(*d*) $P_1 = E_1 I_S \cos(180 - \theta) = 2000 \times 49.8 \times (-0.09932) = -9900$ watts.
(*e*) $I^2R = 49.8^2 \times 0.4 = 990$ watts. $(10890 - 9900) = 990$ watts. Check.
(*f*) (From Fig. 44–7) Terminal voltage of alternator A_2,

$$V_2 = \sqrt{(E_2 \cos\theta - IR)^2 + (E_2 \sin\theta - IX)^2}$$
$$= \sqrt{[(2200 \times 0.09932) - (49.8 \times 0.2)]^2 + [(2200 \times 0.995) - (49.8 \times 2.0)]^2}$$
$$= \sqrt{209^2 + 2090^2} = 2100 \text{ watts.}$$

Terminal voltage of alternator A_1,

$$V_1 = \sqrt{(E_1 \cos\theta + IR)^2 + (E_1 \sin\theta + IX)^2}$$
$$= \sqrt{[(2000 \times 0.09932) + (49.8 \times 0.2)]^2 + [(2000 \times 0.995) + (49.8 \times 2.0)]^2}$$
$$= \sqrt{209^2 + 2090^2} = 2100 \text{ volts.} \quad \textit{Ans.}$$

It is to be noted in the example above, the terminal voltage of each machine equals $\dfrac{2200 + 2000}{2}$, or 2100 volts, the average of the two voltages. This is only true when they have the same resistance and

reactance. Had these values been unequal, an entirely different terminal voltage would have resulted.

Prob. 25–7. Two delta-connected alternators, Fig. 45–7, each rated at 300 Kva, 240 volts, are running without load; A_1 is generating 240 volts and A_2 is generating 210 volts. The synchronizing switch was closed at the middle of a dark period of the lamps, but the difference in voltage of the two machines was not noted on the voltmeter, as it should have been. Each alternator has a resistance of 1 per cent and a reactance of 10 per cent per phase. Calculate for the instant immediately following the switching, with no load on the bus bars: (*a*) Amperes per phase flowing in both armatures after the switch is closed. (*b*) Terminal voltage of each alternator. (Note that these two terminal voltages should be equal. This furnishes a check on your work.) Draw vector diagrams showing how the terminal voltage of each alternator is obtained.

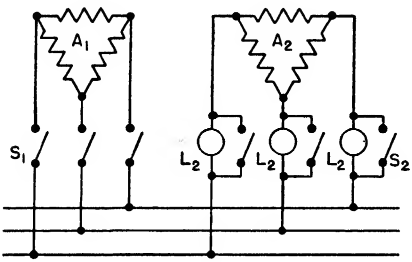


FIG. 45–7. Simple connections for synchronizing two delta-connected alternators.

Prob. 26–7. From the data and results of Prob. 25–7, compute the values of the following quantities the moment after the switch is closed, and before the alternators adjust their phase positions.

(*a*) Electrical power generated in alternator A_1.

(*b*) Electrical power output from the terminals of A_1, or input to the terminals of A_2.

(*c*) Electrical power used to overcome induced (counter) emf in A_2, or to develop mechanical power in its rotor.

(*d*) I^2R loss in each armature. Check your work by comparing (*d*) with the difference between (*a*) and (*b*), or between (*b*) and (*c*).

Prob. 27–7. The alternators in Prob. 25–7 are both running without load and are synchronized at the middle of a dark period of the lamps, but by mistake alternator A_2 is generating 330 volts per phase, while that of A_1 is operating at its normal voltage of 240 volts per phase. Compute, for the instant immediately after synchronizing, the results called for in Probs. 25–7 and 26–7.

24–7. Synchronizing Power. If one alternator is ahead of another in phase, when the synchronizing switch is closed; that is, if the machines are synchronized at an instant other than the middle of a dark period of the lamps, a resultant emf, E_R, is set up in the local circuit even though the alternator emfs, or terminal voltages, are equal. The synchronizing current I_S set up by this

emf takes power from the machine that leads and delivers it to the machine that lags. Thus the leading machine is pulled back, or retarded, while the other is pushed ahead, or accelerated, by motor action, until their voltages come more nearly into phase. This action reduces E_R and therefore reduces I_S, so that again, the synchronizing current limits itself. The greater the phase displacement of the armatures, when the synchronizing switch is closed, the greater are E_R and I_S and the more violent is the action pulling the machines into phase. This sets up dangerously high values of torque on the shafts against the driving torque of the prime movers.

Assume that the machines in Fig. 33–7 are running without load and are synchronized when E_1, of alternator A_1, is $\phi°$ in advance

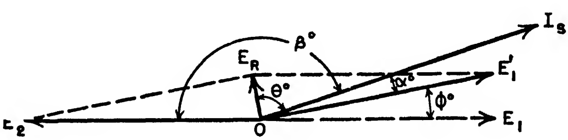


FIG. 46–7. Vector relation when two alternators are synchronized with equal emfs, by when E_1 of one alternator is advanced $\phi°$ ahead of its proper phase position. E_R and I_S (lagging E_R by $\theta°$) are thrown more nearly in phase with E_1. Angle α is small and β large, thereby setting up great synchronizing power.

of its proper phase relation to E_2, of alternator A_2, shown as E_1^{I} in Fig. 46–7. The resultant emf, E_R, is now the diagonal of a parallelogram, the sides of which are E_2 and E_1^{I}. The current, I_S, again lags behind E_R by the angle θ, determined by the ratio of reactance to resistance in the circuit between the machines. I_S is now displaced from E_1^{I} by the angle α, and from E_2 by the angle β.

The power, P_1, generated in machine A_1 is,

$$P_1 = E_1^{\mathrm{I}} I_S \cos \alpha$$

and the power, P_2, generated in machine A_2 is

$$P_2 = E_2 I_2 \cos \beta$$

Note that P_1 is **positive,** representing generator action, and P_2 is **negative** representing motor action. This results in pulling E_1^{I} back (clockwise) and pushing E_2 ahead (counter-clockwise), thus bringing them more nearly into phase (180° to each other in the local circuit), as they should have been when synchronized.

The mechanical power which is exchanged between the machines while they are out of phase, and which brings them into phase, is very important for the successful operation of alternators in parallel. It is called **synchronizing power**; and the circulating, or synchronizing, current is so called because it keeps the machines in synchronism.

Synchronizing power depends primarily upon the angle θ. If this is large, as indicated in Fig. 46–7, angle α is small and its cosine is high. Also the cosine of β is high. Thus the synchronizing power is great. If θ is small, as indicated in Fig. 47–7, the same phase displacement of E^{I} and E_2 will set up the same

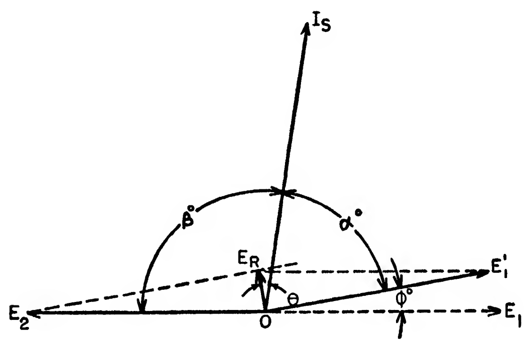


FIG. 47–7. Vector relations, as in Fig. 46–7, when θ is small. The angle α is larger and β smaller, and the synchronizing power is reduced. The angle θ is large when armature reactance is large with respect to its resistance.

current I_S (assuming the same impedance between the machines), but the angle α will be increased and β decreased, thereby decreasing the synchronizing power. The angle, θ, is large when the ratio of reactance to resistance between the machines is high. Therefore, in order that alternators may operate satisfactorily in parallel, the reactance of their armatures must be high with respect to the resistance.

Thus, if difficulty is experienced in keeping alternators in parallel, conditions may be improved by inserting reactances in the armature leads. In fact, the use of current limiting reactors, discussed in Art. 16, not only protects the alternator on short circuit, but increases the synchronizing power. However, if the reactance is made too large, the value of the synchronizing current may be reduced more than angle θ is increased and the synchronizing power decreased. Since reactance in an alternator

armature causes poor regulation, particularly on lagging loads, alternators having poor voltage regulation may be expected to operate more satisfactorily in parallel than those having good regulation.

The conditions for satisfactory operation of alternators in parallel, thus, require that:

(*a*) Small changes in phase relation between their voltages produce a large value of synchronizing current.

(*b*) The ratio of reactance to resistance in the local circuit between them must be high, so that the synchronizing current lags the resultant emf by a large angle.

Drooping speed characteristics of prime movers. It is shown above that generator action takes place in the alternator which is advanced in phase with respect to the other. It was also stated in Art. 19, that parallel connected alternators must be driven by prime movers having drooping speed characteristics. If this condition were not fulfilled, the leading machine would continue to advance in phase and the other continue to drop back, thereby increasing the resultant voltage and circulating current, and also the generator and motor action between them. If an external load were connected, the leading alternator would take all the load. Thus, the machines would be in unstable equilibrium, much the same as two overcompounded d-c generators connected in parallel without an equalizer.

It has been previously stated, that alternators are generally synchronized with the incoming machine running slightly fast. As soon as the synchronizing switch is closed, this machine attempts to advance in phase position with respect to the other, but the synchronizing power here comes into action and pulls the leading machine back and forces the other ahead, thus holding them in synchronism.

Example 9. Two 1500-Kva three-phase alternators, each rated at 2300 volts per phase, are operating without load and are synchronized when alternator A is 20° ahead of its correct 180° position with respect to alternator B. Each alternator has 10.5 per cent synchronous reactance and 1.5 per cent effective resistance per phase.

(*a*) What is the synchronizing current per phase?

(*b*) Which generator delivers synchronizing power to the other and how much is this power?

(*c*) What is the total I^2R loss per phase in the two armatures?

(*d*) Before the machines adjust themselves in phase position, how much power per phase is transferred through the bus bars from one alternator to the other?

Solution: Construct the vector diagram of Fig. 48-7 with E_B representing the emf of alternator B, and E_A that of alternator A, 20° ahead of its 180° position, E_A^1, as shown.

(*a*) Phase difference between E_A and E_B = 180 − 20 = 160°.

$$E_R = \sqrt{E_A^2 + E_B^2 + 2 \times E_A E_B \cos 160°}$$
$$= \sqrt{2300^2 + 2300^2 + 2 \times 2300 \times 2300 \times (-\cos 20°)} = 798 \text{ volts}$$

$$\text{Full load current per phase} = \frac{50,000}{2300} = 217 \text{ amperes.}$$

Reactance drop per phase at full load current

$$= 10.5 \text{ per cent of } 2300 = 241.5 \text{ volts.} \quad \text{(See Art. 13)}$$

$$241.5 = 217X; \ X = \frac{241.5}{217} = 1.275 \text{ ohms.}$$

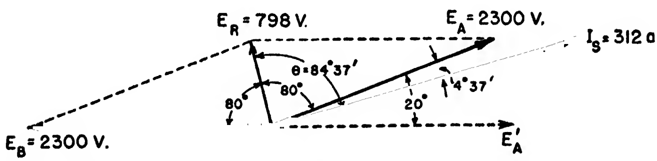


FIG. 48-7. Vector diagram of voltage and current relations in Example 9.

Resistance drop per phase at full load current

$$= 1.5 \text{ per cent of } 2300 = 26 \text{ volts.}$$

$$26 = 217R; \ R = \frac{26}{217} = 0.12 \text{ ohm.}$$

Synchronous impedance per phase in series circuit between armatures
$$= \sqrt{(0.12 + 0.12)^2 + (1.275 + 1.275)^2} = \sqrt{0.24^2 + 2.55^2} = 2.56 \text{ ohms}$$

$$\text{Synchronizing current per phase} = \frac{798}{2.56} = 312 \text{ amperes.} \quad \textit{Ans.}$$

(*b*) Tangent of angle θ between E_R and $I_S = \frac{2.55}{0.24} = 10.625$

$\theta = 84° 37'$
Phase angle between E_A and I_S = 84° 37′ − 80° = 4° 37′
Phase angle between E_B and I_S = 160 + 4° 37′ = 164° 37′
Power generated per phase in alternator A equals

$$P_A = E_A I_S \cos 4° 37'$$
$$= 2300 \times 312 \times 0.997 = 715,000 \text{ watts} = 715 \text{ kw.}$$

Power used by B per phase tending to advance B and A equals

$$P_B = E_B I_S \cos 164° 37'$$
$$= 2300 \times 312 \times (-\cos 15° 23') = -692,000 \text{ watts} = -692 \text{ kw}$$

Generator A thus delivers 692 kw of synchronizing power to generator B.
Generator A is pulled back by 715 kw per phase, and B is pushed ahead by 692 kw per phase. *Ans.*

(c) Total I^2R loss per phase in both machines $= 715 - 692$, or 23 kw.
$= 2 \times 312^2 \times 0.12 = 23{,}000$ watts (check). *Ans.*

(d) The power per phase transferred through the bus bars

$$= \left(715 - \frac{23}{2}\right) = \left(692 + \frac{23}{2}\right) = 703.5 \text{ kw.} \quad Ans.$$

Prob. 28–7. (a) What is the terminal voltage of each alternator in Example 9? (b) From the value of the terminal voltage found in (a) check the value of the power transferred through the bus bars, as determined in part (d) in Example 9.

Prob. 29–7. Each three-phase alternator in Fig. 45–7, rated at 300 kva, 240 volts, with 10 per cent reactance and 1 per cent resistance per phase, is running without load. Each lamp L_2 is rated at 240 volts and its filament does not glow until the voltage across its terminals is 90 volts. (a) What is the maximum phase difference that may exist between the two alternators, each generating 240 volts, while the lamps L_2 still appear to be in their dark period with S_1 closed?

If the switch S_2 is closed when this phase difference exists, compute: (b) the synchronizing current per phase; (c) voltage of the bus bars; (d) power per phase transferred through the bus bars from one alternator to the other.

Prob. 30–7. Repeat Prob. 29–7, on the assumption each alternator has a reactance of 5 per cent and a resistance of 1 per cent. Compare with corresponding results of Prob. 29–7. (a) By what percentage is the synchronizing power increased or diminished?

Prob. 31–7. Repeat Prob. 29–7 on the assumption each alternator has the **same** impedance, but only **half** of the reactance. Compare with corresponding results in Probs. 29–7 and 30–7. By what percentage has the synchronizing power been decreased from that in Prob. 29–7?

25–7. Hunting of Alternators. If the driving force on parallel connected alternators is not uniform throughout the revolution, as is the case with reciprocating engines, first machine A_1 surges ahead of A_2, increasing the resultant emf, E_R, and the synchronizing current, I_S. This, due to the synchronizing power, retards A_1 and tends to slow it down; at the same time A_2 is accelerated and tends to speed up. The relative positions of the machines is now reversed and the synchronizing current, I_S, is also reversed with respect to the alternator emfs. This action is repeated and

results in " swings " of I_S, first in one direction and then the other. Thus, momentary impulses are given to the rotors of the alternators and they **oscillate** with respect to their normal speed of rotation. This action is called "**hunting.**"

If the governors of the engines have a natural period of oscillation close to that of the alternator rotors, this hunting action may be severe enough to cause the machines to pull out of synchronism and may be transmitted to synchronous motors and other apparatus on the system.

Hunting may be reduced, or eliminated, by heavier fly-wheels on the engines and by "amortisseur," or squirrel-cage, windings on the field structure of the alternator (see Chap. XI). Turbine driven alternators are less liable to hunt than those driven by reciprocating engines.

26–7. Distribution of Load on Alternators in Parallel. In practice, an alternator is well loaded before another is parallel with

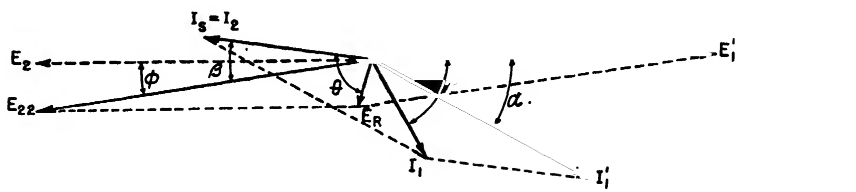


FIG. 49–7. Vector diagram showing result of connecting an alternator, with voltage E_{22}, in parallel with an alternator of voltage E'_1, when the latter is already delivering a load of current I'_1 at a power factor of cos α.

it. Consider alternator A_1 in Fig. 33–7 to be carrying such a load that it is necessary also to connect A_2 to the bus bars. Thus in Fig. 49–7, E_1^{I} represents the terminal voltage of A_1, which is delivering a current, I_1^{I}, to the external circuit at a power factor cos α. The power output of A_1 equals $E_1^{\mathrm{I}} I_1^{\mathrm{I}}$ cos α. Alternator A_2 is now brought up to speed, its emf E_2 is adjusted to the same value as E_1^{I} of alternator A_1, or to that of the bus bars, and the switch is closed when they are in synchronism, that is when E_2 is 180° to E_1^{I}, as shown. Alternator A_2 will now neither take nor deliver any current or power. To make A_2 take some of the load from A_1, it must be made to advance in relative phase position. Therefore, the throttle is adjusted to let a little more steam, or gas, or water, into the prime mover driving A_2, increasing its driving torque. E_2 now advances on E_1 to a new phase position, shown as E_{22} in the figure. Thus a resulting emf, E_R, and a synchronizing

current I_S is produced lagging θ degrees

$$\left(\tan\theta = \frac{\text{total } X \text{ in local circuit}}{\text{total } R \text{ in local circuit}}\right)$$

behind E_R. Now, I_S is the only current flowing in A_2, while both I_S and I_1^{I} flow in A_1, the vector sum of which is I_1, as shown.

The phase angle, β, between E_{22} and I_S is **less** than 90°, therefore, A_2 is now **generating electric power** equal to

$$P_2 = E_{22}I_2 \cos\beta$$

The current I_1, now flowing in A_1, is less than before and is lagging behind E_1^{I} by a larger angle γ, thus the power generated in A_1 is **less.** That is,

$$P_1 = E_1^{\text{I}}I_1 \cos\gamma \text{ is less than } P_1 = E_1^{\text{I}}I_1^{\text{I}} \cos\alpha.$$

Thus, as the driving force on the incoming machine is increased, E_{22} is made to advance in phase position with regard to E_1^{I}, thereby forcing it to assume part of the load. The amount of the load assumed depends entirely upon how far E_{22} is made to advance on E_1^{I} and can be controlled by the operator. It should be noted that the actual speed of the incoming machine is unchanged; it is simply made to advance in phase position with respect to the other.

The vector diagram of Fig. 49–7 is drawn with reference to the phase relations in the local circuit between the machines. To show the diagram with respect to the bus bars, or outside circuit, $I_S = I_2$, must be reversed, and the vector sum of I_2 and I_1, the total current output of the two alternators, should be equal to I_1^{I}, the current delivered by A_1 before paralleling.

For the sake of clearness in the diagram, the terminal voltage of the two machines is not shown. The current in A_2 and the change of current and power factor in A_1 causes the terminal voltages to differ from E_{22} and E_1^{I}, both in value and in phase position. The current will continue to change and the phase position shift until the terminal voltages are exactly the same and equilibrium is established.

The voltage at the bus bars is kept constant, either by an automatic voltage regulator or by field rheostat adjustment; also the power factors of both machines can be adjusted to that of the external circuit by field adjustment, explained in Art. 27.

Removing an Alternator from the Bus Bars. When the load on a station has dropped sufficiently, one of the alternators is removed from service. The power supply to this machine is reduced by adjusting the governor of its prime mover in the direction of lower speed. The vector E_{22} of Fig. 49–7 is allowed to drop back in phase position until both the current and power it supplies is practically, or almost, zero. The switch is then opened, disconnecting it from the bus.

Prob. 32–7. Each of the three-phase alternators of Fig. 45–7 is rated at 300 Kva, 240 volts and has an effective resistance per phase of 2 per cent. When the field current is such as will produce a terminal voltage per phase of 240 volts at full-non-inductive load, the sustained short-circuit current is five times the full-load value. Compute for each machine:

(*a*) The effective armature reactance per phase in ohms.
(*b*) The synchronous reactance per phase in ohms.
(*c*) The synchronous impedance per phase in ohms.

Prob. 33–7. Alternator A_1 of Prob. 32–7 and Fig. 45–7 is delivering 125 per cent of its rated load in Kva to an external circuit of 240 volts and 0.8 lagging power factor. Alternator A_2 is synchronized perfectly and connected to the bus bars. Draw a vector diagram similar to Fig. 49–7 showing this condition. From the IR and IX drops in A_1 compute the value and location on the diagram, the total induced emf, or no-load voltage.

Prob. 34–7. Now let the driving force behind A_2 in Prob. 33–7 be increased enough to advance its induced emf vector by 10 electrical degrees (that is, $\theta = 10°$ in Fig. 49–7). Assume the current delivered to the external circuit and its power factor remain unchanged. Compute, for the instant before the machines adjust their phase positions: (*a*) The current per phase flowing in A_2. (*b*) The current per phase flowing in A_1. (*c*) The power per phase generated in A_2. (*d*) The power per phase generated in A_1. Place these current vectors on the vector diagram of Prob. 33–7.

Prob. 35–7. On the vector diagram of Prob. 34–7 (similar to Fig. 49–7) add to E_{22} the IR and IX drops due to I_2 and determine the vector representing the **terminal** voltage of generator A_2. Notice this is not equal to the terminal voltage of generator A_1 as it was before the distribution of the load.

Prob. 36–7. The two alternators of Prob. 32–7, operating together in parallel, are each loaded to rated Kva at 0.8 lagging power factor, the external load being 200 Kva per phase at 0.8 power factor. Draw a complete vector diagram showing the terminal and total induced, or excitation, emf for each alternator. Also the current per phase in each alternator and the total current per phase delivered from the bus bars.

Compute: (*a*) The value of generated emf for each alternator. (*b*) The value of synchronizing component of armature current per phase.

27-7. Field Excitation Controls Power Factor but Not Load Distribution. It was shown in Art. 23 that when alternators having different emfs are synchronized, a reactive current flows between them. Similarly, in parallel connected alternators supplying a load, the adjustment of field excitation, or change in field current, in one alternator sets up a reactive circulating current between them. This current **does not alter their division of the load,** but **does** affect their power factors.

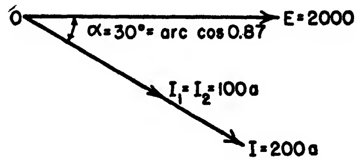


FIG. 50-7. Vector E represents the induced voltage of each generator with respect to the bus bars. I represents the current delivered by both generators.

Assume two alternators, A_1 and A_2 in parallel, are each generating 2000 volts per phase and that each delivers 100 amperes per phase to the line at 0.87 lagging power factor, as indicated in Fig. 50-7. The internal drop in the armature is here neglected. This figure is drawn with reference to the external circuit, or to the bus bars. In Fig. 51-7 the vectors of A_2 are reversed to indicate the relation in the circuit between the machines.

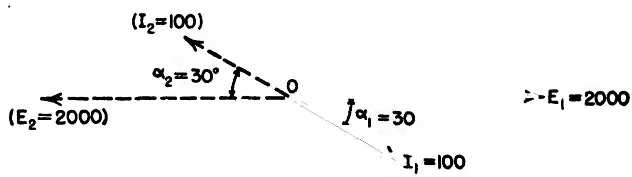


FIG. 51-7. The currents and voltages of Fig. 50-7 now drawn with respect to the local circuit between the two armatures.

If the induced voltage of A_2 is raised by field rheostat adjustment to 2200 volts, shown as E_2^{I} in Fig. 52-7, a resultant emf E_R equal to 2200 − 2000, or 200 volts, is induced. Assuming a synchronous impedance for both machines of approximately 4 ohms, E_R produces a synchronizing current of $\frac{200}{4}$, or 50, amperes lagging practically 90° behind E_R and E_2^{I}. The total current in A_2 is now I_2^{I} and in A_1 it is now I_1^{I}.

This adjustment of the field current has **increased** the reactive component and total current in A_2 and **reduced the power factor**

of A_2. At the same time it has **reduced** the reactive component and the total current in A_1 and **increased the power factor** of A_1. In other words, this change of field current in A_2 has merely

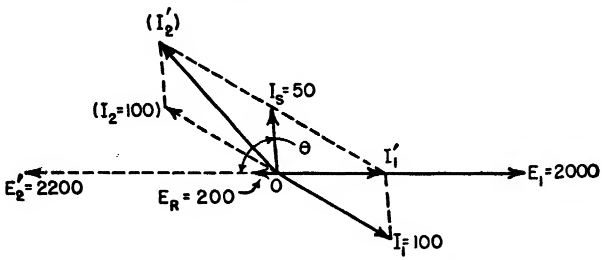


FIG. 52–7. The emf of alternator A_2 has been increased to 2200 volts by field adjustment. The current in alternator A_1 becomes I_1', and in alternator A_2 becomes I_2'.

shifted the reactive component of the load current from one alternator to the other. It has not altered the distribution of the load between the machines, because the power component of I_2^{I} is practically the same as that of I_2, and the power component of I_1^{I} is practically the same as that of I_1. In Fig. 53–7, I_2^{I} of Fig. 52–7 is reversed to indicate the relations with respect to the bus bars, or external circuit, and shows that the resultant of I_1^{I} and I_2^{I}, or the current I, remains the same as before the change in field current.

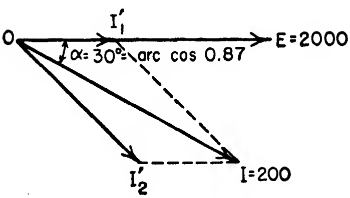


FIG. 53–7. The vectors of Fig. 52–7 are now drawn with respect to the bus bars, instead of to the local circuit. Note that E and I are exactly as in Fig. 50–7.

28–7. Synchronous Motor. If the driving force on one of two (or more) alternators connected in parallel is removed, this alternator **will not stop**, but will continue to rotate exactly at synchronous speed. It, therefore, becomes a "synchronous motor," taking just enough power from the other alternators to supply all its losses.

This action is illustrated in Fig. 54–7. When the mechanical power driving alternator A_2 is removed, it tends to slow down and actually drops back in phase position relative to A_1, and its voltage vector E_2 falls back in phase position with A_1 to the position E_2^{I}. This produces the resultant emf E_R^{I} and the current I_2^{I} ($= I_S$) supplied by A_1 and lagging E_R^{I} by the angle θ. Current I_2^{I} is now

in such a phase position that it produces strong motor action in A_2, tending to prevent it from slowing down and to hold it in synchronism. E_2^{I} thus takes such a position that the power input is just sufficient to supply the losses and A_2 continues to rotate at synchronous speed.

If A_2 is now loaded, that is, used to deliver mechanical power to some other machine through its shaft, the additional load causes the vector E_2^{I} to fall back further on E_1 to the position $E_{\mathrm{R}}^{\mathrm{II}}$. In consequence, the resulting emf $E_{\mathrm{R}}^{\mathrm{I}}$ is increased to $E_{\mathrm{R}}^{\mathrm{II}}$; and I_S, again lagging behind E_2^{II} by the angle θ is correspondingly in-

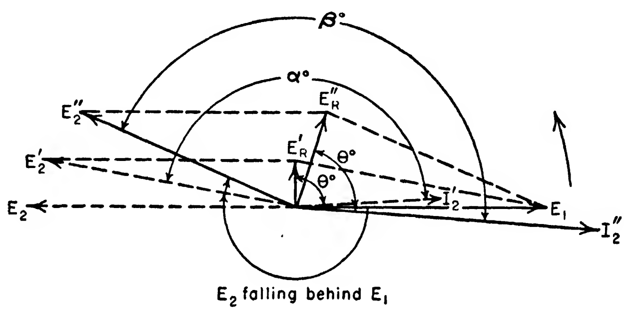


FIG. 54–7. Vector diagram representing conditions when the driving force of A_2 is removed. As its voltage vector, E_2, drops back, A_2 is driven by A_1 as a synchronous motor.

creased from I_2^{I} to I_2^{II}. The mechanical power, or motor action developed in A_2 is thus increased, for it is evident that,

$$(E_2^{\mathrm{II}} I_2^{\mathrm{II}} \cos \beta) \text{ is greater than } (E_2^{\mathrm{I}} I_2^{\mathrm{I}} \cos \alpha)$$

The rotor of R_2 continues to fall back until the value of the electrical input $(E_{\mathrm{R}}^{\mathrm{II}} I_{\mathrm{R}}^{\mathrm{II}} \cos \beta)$ has become equal to the output plus the losses, and then continues to rotate at synchronous speed.

If the field of the motor A_2 is over-excited, so that its emf E_2^{I} is considerably higher than the bus bar voltage, or E_1, as indicated in Fig. 55–7, the resultant emf $E_{\mathrm{R}}^{\mathrm{I}}$ and the current I_2^1 are both increased and take such a position that I_2^{I} **leads** E_1 by the angle ϕ. Thus the alternator is made to supply a heavy **leading** current to the over-excited synchronous motor.

Over-excited synchronous motors have been largely used to draw a leading reactive current from alternators supplying a lagging industrial load, thereby improving the power factor of the plant. However, in recent years, static condensers, due to im-

provement in quality and decrease in cost of manufacture, have largely replaced the synchronous motor for this purpose. When the synchronous motor is operated without load to improve power factor, it is called a "synchronous condenser." Synchronous condensers are mainly operated today at the receiving end of long high-voltage transmission lines to control the power factor of the line. See Art. 13–13. The Hoover Dam — Los Angeles Line.

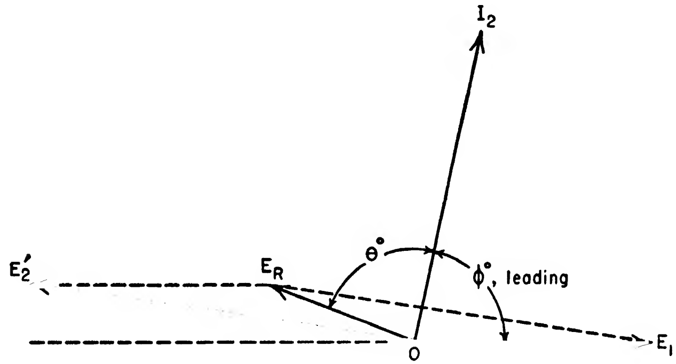


FIG. 55–7. When the emf E_2' of the synchronous motor, A_2, is increased by field adjustment (overexcited), the synchronous motor draws a heavy *leading* current, I_2, from alternator, A_1, to which it is connected.

29–7. Rating and Load Capacity of the Alternator. It has been previously stated that alternators are usually rated in kilovolt-amperes, at a definite voltage and frequency. They also may be rated in kilowatts at specified power factor. In the latter case, a lagging power factor of 0.8 is usually specified. The rating of polyphase machines is given for a balanced load; that is, a load equally balanced among the phases. It is a maximum for balanced load and a minimum for a single-phase load, as shown in Ch.VI.

Alternating current generators are almost universally classified by a combination of letters, as "*ASB*," "*AQB*" and "*ATB*"; where *A* stands for alternator; *S* for single phase; *Q* for quarter, or two phase; *T* for three phase; and *B* for revolving field.

The maximum output of an alternator is limited by the temperature of its parts. All the losses in any electrical machine appear as heat in the various parts. These losses are affected, either by the flux density, the frequency, the power factor, or the load. Therefore the limit of output is generally obtained under load conditions.

The maximum voltage any alternator can deliver at a specified frequency depends upon the allowable flux density and the heat loss in the field circuit. The maximum allowable armature current depends upon the safe temperature of the "hot spots" in the armature winding, and is independent of the voltage and power factor. Since the field current, and therefore the heat loss in the field for a given armature current and terminal voltage, must be greater for lagging loads than at unit power factor, the temperature of the field winding is dependent upon the power factor.

Since all the losses develop heat energy, the temperature of the machine rises. The machine tends to lose heat energy to the surrounding air in proportion to the difference between its own temperature and that of its surroundings. The temperature of the machine will rise until the rate of losing heat is equal to rate at which heat is developed. When this condition is attained the temperature of the machine ceases to rise and remains steady. If the load on the machine is increased, thereby increasing the total losses and the heat energy developed, the temperature again rises until the heat dissipated is equal to the heat developed, and the temperature again becomes steady. The excess in temperature of the machine above its surroundings, then, is a measure of the rate at which heat is being developed and dissipated.

This action is somewhat the same as that of water pumped into a tank with a hole in its base; the level of water will rise until its pressure or "head" forces water out through the hole as fast as it is being pumped in. Then the water level in the tank becomes stationary. If the rate of pumping is increased, the level, or head, rises until the water again is forced out as fast as it is being pumped in. Then the water level again becomes stationary.

If the rate of pumping is decreased, the water level falls until the rate of outflow is just equal to the rate of pumping. Similarly, if the load on the alternator is decreased, decreasing the losses and the heat energy developed, the temperature of the machine falls until a balance between heat developed and heat generated is again obtained.

In most machines a great increase in output for a given temperature rise is obtained by providing "ventilating ducts" in the cores. Vanes, or fans, are also used on the rotors of many machines to force cooling air through these ducts and along the air gaps.

The allowable temperature rise in windings with different

classes of insulation has already been given in Art. 17, of this chapter.

30–7. Losses and Efficiency. The efficiency of an alternator, as of a d-c generator, is not readily obtained by loading it, due to the difficulty of determining the mechanical power input. Also, with large machines, there is the difficulty and expense of providing the necessary power and a suitable load. Therefore, the efficiency is usually obtained from a determination of the losses.

The losses in the alternator are:

(*a*) **Friction and Windage Loss.** Includes bearing and brush friction and power required to circulate the cooling air through the ventilating ducts. This loss is constant at all loads.

(*b*) **Core Losses.** Hysteresis and eddy current losses in the core, teeth and pole faces due to the **resultant** field flux. These losses vary slightly with load but are usually considered constant.

(*c*) **Field, or Excitation Loss.** I^2R losses in the field, due to the field current necessary to produce normal terminal voltage at a given load and power factor. This may, or may not, include the losses in the field rheostat. This loss varies considerably with change in load.

The losses in (*a*), (*b*) and (*c*) above are usually called the no-load losses.

(*d*) **Armature I^2R Loss.** Equal to the number of phases times the square of the current per phase multiplied by the ohmic, or d-c, resistance per phase. This loss, of course, varies with the square of the load current per phase.

(*e*) **Load Loss.** Due to the leakage flux which causes eddy current in the armature conductors and hysteresis and eddy currents in the surrounding iron. This loss is difficult to obtain accurately, but varies approximately with square of the armature current. This loss is included in the armature I^2R loss in (*d*) above if the **effective** resistance, instead of ohmic resistance, is used in the computation.

The losses in (*d*) and (*e*) are called the losses due to load.

Determination of Losses. The alternator to be tested is driven, unloaded, at rated speed by a small d-c shunt motor. The motor need be large enough to supply only the losses in the alternator.

(1) The alternator is first driven at rated speed with no field and the armature on open circuit. The power input to **motor armature** is measured and its speed noted.

(2) The **motor** is disconnected from the alternator and the motor's stray power determined (see Vol. I, Chap. XII, Art. 7) at its speed in (1) above.

(3) The input to the **motor armature** less its I^2R losses in (1), minus the motor stray power loss in (2), is the output of the **motor,** or the **friction and windage** loss in the alternator.

(4) The alternator is now driven with its field excited to give rated terminal voltage on open circuit, and the field current I_o noted. The input to the **motor armature** is again measured and corrected for output, as in (3) above. The **increase** in motor output over that in (3) is now the core loss at this excitation. These readings should be repeated for increasing values of terminal voltage and field current over a wide range, and a curve plotted between core loss (as ordinates) and field current.

(5) The alternator is now short circuited through ammeters with reduced field current (as in Fig. 18–7). Field current is adjusted to give short-circuit current through a wide range and motor input measured. For a given armature current and field current, I_S, the **increase** in motor output above that in (3) is the approximate value of the **effective armature** I^2R loss for this current, and **includes the load loss.** The slight core loss due to the small field current is here neglected. The effective resistance of the armature is difficult to measure, but can be expressed approximately for a three-phase machine as,

$$R_{\text{eff}} = \frac{W_a}{3I^2} \tag{10–7}$$

where W_a is the armature I^2R loss as obtained above, and I, the corresponding current per phase.

If the ohmic, or d-c, I^2R for the given current is computed and subtracted from the loss, as found above, the **load loss** is obtained.

(6) Since the core loss under load is that of the resultant field, an approximate value of the field current necessary to obtain rated terminal voltage for a given current output may be computed as $I_f = \sqrt{I_o^2 + I_s^2}$, where I_o is the open circuit field current as obtained in (4) above and I_s is the field current for this armature current on short circuit. The **core loss at this load** and for **this field current** is now found from the core loss, as plotted.

(7) The excitation, or field, I^2R loss is now $I_f{}^2R$ where I_f is the resultant field current as determined in (6) above, and R the

resistance of the field circuit; or EI_R where E is the voltage impressed on the field.

The efficiency of the alternator can be expressed as:

Per cent efficiency

$$= \frac{\text{watts output}}{\text{watts output} + W_w + W_c + W_f + W_a + W_e} \times 100 \quad (11\text{–}7)$$

where, W_w = friction and windage loss in watts; W_c = core loss due to resultant field; W_f = field or excitation loss; W_a = armature ohmic I^2R loss ($= nI^2R_o$, where n = number of phases, I and R_o = current and ohmic resistance per phase); W_e = load loss.

If the total armature loss on short circuit, as found in (5) above, is used, W_a includes the load loss and the term W_e is dropped. Thus the efficiency of a three-phase alternator on a load at power factor, $\cos\theta$, is expressed as

Per cent efficiency

$$= \frac{\sqrt{3}\,EI\cos\theta}{\sqrt{3}\,EI\cos\theta + W_w + W_c + W_f + W_a} \times 100 \quad (12\text{–}7)$$

Example 10. The losses in a 500-Kva, 2300-volt, Y-connected alternator, as determined by the method explained above, are as follows. Friction and windage, 7 Kw; Core loss, 11 Kw; Field I^2R, 7.5 Kw; Armature I^2R and Load loss, 6 Kw. (*a*) What is the efficiency at full non-inductive load? (*b*) What is the effective armature resistance per phase? (*c*) Assuming the field I^2R loss as constant, what is the efficiency at half load non-inductive?

Solution:

(*a*) Efficiency $= \dfrac{500}{500 + 7 + 11 + 7.5 + 6} \times 100 = 94$ per cent.

(*b*) Full load line (or phase) current $I = \dfrac{500{,}000}{\sqrt{3} \times 2300} = 125.6$ amperes

From Eq. (10–7), $R_{\text{eff}} = \dfrac{W_a}{3I^2} = \dfrac{6000}{3 \times 125.6^2} = 0.127$ ohm.

(*c*) Efficiency $= \dfrac{250}{250 + 7 + 11 + 7.5 + 6 \times (\frac{1}{2})^2} \times 100 = 90.3$ per cent.

Table I gives the results of a test for losses and efficiency on a medium sized alternator. Note that the friction and windage loss is constant at all loads, while the core loss is nearly so. These are the constant losses. It will be noted that the field copper, or excitation, loss nearly doubles between no load and full load. This

is due to the very poor regulation of this machine. The armature I^2R loss here, also, includes the load loss. In an alternator, as in the d-c dynamo, the maximum efficiency occurs at the load at which the constant losses are equal to the variable losses. An

TABLE I

Load, per cent of rated	0.0	25.0	50.0	75.0	100.0	125.0
Line amperes per terminal	0.0	49.0	98.0	147.0	196.0	245.0
Field amperes	77.0	81.5	86.5	94.0	101.5	112.0
Terminal volts	6300.0	6300.0	6300.0	6300.0	6300.0	6300.0
Core loss, in kw	39.4	39.5	39.6	39.7	39.8	39.9
Field copper loss, kw	4.68	5.24	5.92	6.98	8.13	9.92
Armature copper loss, kw	0.0	0.42	1.68	3.78	6.72	10.51
Friction and windage, kw	30.25	30.25	30.25	30.25	30.25	30.25
Total losses, kw	74.33	75.41	77.45	80.71	84.90	90.58
Kv-a output	0.0	534.0	1071.0	1604.0	2140.0	2672.0
Real kw output	0.0	427.0	857.0	1283.0	1712.0	2137.0
Real kw input	74.33	502.4	934.4	1363.7	1796.9	2227.6
Efficiency, per cent	0.0	85.0	91.7	94.1	95.3	95.9

inspection of the losses in Table I shows that this alternator does not reach maximum efficiency even at twenty per cent overload.

Prob. 37–7. An alternator is delivering 250 Kva at 85 per cent power factor and its efficiency at this load is 92 per cent. How many horse power must the direct-connected engine deliver to the shaft to drive this alternator?

Prob. 38–7. Is the alternator whose performance is given in Table I a three-phase or a two-phase machine? Prove your answer.

Prob. 39–7. (*a*) Assuming the alternator of Table I to be three-phase, Y-connected calculate the effective armature resistance. (*b*) Repeat the calculation, assuming it to be delta-connected.

Prob. 40–7. From Table I draw a set of curves, using per cent of rated output as abscissa, and as ordinates the following: (*a*) friction and windage loss; (*b*) friction and windage plus core loss; (*c*) friction and windage plus core loss plus field I^2R loss; (*d*) total losses; (*e*) efficiency. Plot ordinates of loss curves in kilowatts, and efficiency curve in percentage.

Prob. 41–7. The losses in a 500-Kva, 25-cycle alternator **in per cent of output at full non-inductive load** are as follows: friction and windage, 0.5 per cent; core loss, 1.42 per cent; field copper loss, 1.25 per cent; armature copper loss, 1.0 per cent. Assuming the armature loss varies with the square of the armature current and all other losses are constant, compute the efficiency of the machine at $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$ and $1\frac{1}{2}$ times rated full-load output.

31–7. Hydrogen Cooling of Large Synchronous Machines. Hydrogen gas for cooling large turbo-alternators and synchronous condensers has come into considerable use in recent years to reduce the windage losses. A large volume of air must be passed through the ventilating ducts in these machines and their windage losses are high in comparison to the other losses. With hydrogen cooling, the frames of the machines are enclosed and sealed. The gas, under pressure slightly above that of the outside air, is circulated by fans, and passed over water-cooled coils of pipe sealed within the enclosure.

The windage loss of a rotating body, such as a machine rotor, varied directly with the density of the medium in which it rotates. Hydrogen is lighter than air, has less internal friction (viscosity), and about the same heat capacity per unit volume. It also conducts heat several times better than air. Therefore, cooling with the same flow of hydrogen gas, as of air, is more efficient and requires less power.

Certain mixtures of hydrogen gas and air are explosive, but it has been found that a mixture of hydrogen and air in a ratio of 3 to 1 is not dangerous. A mixture of 90 per cent hydrogen is considered safe for the temperatures met with in practice.

The use of hydrogen gas for cooling increases the load capacity of a machine by about 20 per cent, with the same rise in temperature, and increases its efficiency by a very small percentage.

SUMMARY OF CHAPTER VII

LOADING AN ALTERNATOR generally causes the TERMINAL VOLTAGE TO FALL. This is due to three effects:

(1) ARMATURE REACTANCE. Current in the armature coils sets up an alternating LEAKAGE FLUX, largely in the armature teeth, which causes a REACTANCE DROP in the armature.

(2) ARMATURE RESISTANCE. Alternating current in the armature coils sets up hysteresis and eddy current losses in the teeth which must be supplied by the current. The armature thus acts like an iron cored coil and its resistance to the flow of a-c is greater than the ohmic resistance. This is the EFFECTIVE RESISTANCE and varies from 115 to 175 per cent of the ohmic resistance. ARMATURE RESISTANCE AND REACTANCE cause an IMPEDANCE DROP in the winding.

(3) ARMATURE REACTION. The armature current sets up an mmf, or a flux (exclusive of the leakage flux) which

(a) ON UNITY POWER LOAD, distorts the distribution of the field flux under the poles, crowding the magnetic lines to the trailing pole tips.

(b) ON LAGGING POWER FACTOR, weakens the field flux.

(c) **ON LEADING POWER FACTOR,** strengthens the field flux.

ARMATURE REACTION IN SINGLE-PHASE ALTERNATORS is **PULSATING,** and may both strengthen and weaken the field, alternating at double the frequency of the emf. This induces an a-c voltage in the field winding.

ARMATURE REACTION IN POLYPHASE MACHINES is **CONSTANT** in value and direction for a given armature current and power factor.

THE VOLTAGE REGULATION of an alternator determines to large extent the current which will flow on short circuit and how satisfactorily it will operate in parallel with other alternators.

$$\text{Regulation in per cent} = \frac{(\text{no load volts}) - (\text{full-load volts})}{(\text{full-load volts})} \times 100.$$

Regulation may be **NEGATIVE** on loads with leading power factors. That is, the voltage may **RISE** with increase in load.

The Regulation of an alternator is determined from no-load tests by several methods.

IN THE SYNCHRONOUS IMPEDANCE METHOD, ARMATURE REACTION is combined with armature reactance. This is called the **SYNCHRONOUS REACTANCE. SYNCHRONOUS REACTANCE** combined vectorially with the Effective Resistance gives the **SYNCHRONOUS IMPEDANCE** of the armature. It equals

$$\frac{\text{Open-circuit volts per phase}}{\text{Short-circuit amperes per phase}}$$

both values corresponding to the same field current and speed.

ALTERNATOR EXCITATION. Alternator voltage is adjusted by a rheostat in its field and also by a rheostat in the shunt field of the exciter. Exciters may be shunt or compound wound. Each alternator in a power station may have its own direct-connected exciter, or it may be excited from a d-c bus to which several exciters are connected in parallel. Storage batteries are often installed as a reserve in case of failure of the exciters.

THE DIACTOR REGULATOR automatically regulates the terminal voltage of an alternator by rapid smooth changes in the d-c voltage across the field of the a-c generator.

ON SHORT CIRCUIT, the first rush, or momentary, current in an alternator is very great and sets up dangerously high mechanical forces in the armature winding. The alternator and its oil switch are often protected by **CURRENT LIMITING REACTORS** in the alternator leads, which reduce the current. The oil switch may also be protected by **OVER-LOAD-INVERSE-TIME-LIMIT RELAYS** to delay its opening before the current has dropped to the steady, or sustained, short-circuit value. This prevents disconnecting the alternator from the line before the short has had a chance to clear itself.

HOT SPOT TEMPERATURE. The safe load on an alternator is often determined by the temperature of the "hot spots" in the armature winding. Thermocouples placed in the bottom of several slots in the core indicate this temperature.

SYNCHRONIZING is the process of connecting alternators in parallel.

ALTERNATORS MAY BE SAFELY SYNCHRONIZED, WHEN

(1) Their terminal voltages are the same;

(2) Their frequencies are practically the same;

(3) Their terminal voltages are in phase, or at 180° to each other in the local circuit between the machines.

(4) Their relative phase rotation is the same.

SYNCHRONIZING LAMPS, connected across the blades of the synchronizing switch are used to indicate when alternators are in synchronism. The speed of the incoming machine is adjusted until the lamps brighten and darken in slow beats. The machines are in phase and in synchronism and the synchronizing switch is closed at the middle of a dark period of the lamps.

THE SYNCHROSCOPE is an instrument which accurately indicates when the alternators are in synchronism. They are synchronized when the incoming machine is running slightly fast as indicated by this instrument.

A SYNCHRONIZING CURRENT flows between two alternators connected in parallel when their emfs do not fufill the above conditions for parallel operation. This synchronizing current tends to bring the two emfs into proper relation.

SYNCHRONIZING POWER is the power transferred from one alternator to the other by means of the synchronizing current. It is supplied by one alternator to the other, which lags behind its proper phase position with respect to the first. By means of motor action the lagging machine is caused to advance in phase position and holds the two machines in synchronism. A high reactance in the armature circuit increases this synchronizing power.

HUNTING. When the speed of parallel connected alternators is not uniform throughout the revolution, their rotors alternately surge ahead and fall behind in phase position with respect to each other. The alternators, therefore, "oscillate" with respect to their normal speed of rotation and may pull out of synchronism.

THE LOAD CAPACITY of an alternator is limited by the temperature rise in its parts. The losses in the machine develop heat energy which is dissipated to the surrounding air. The temperature of the machine will rise until the rate at which heat is developed is equal to the rate at which it is dissipated. An increase of load and of losses increases the temperature; a decrease in load reduces the temperature.

LOSSES AND EFFICIENCY. The efficiency of an alternator is usually obtained from a determination of the losses, and is expressed by the equation

Efficiency in per cent

$$= \frac{\text{watts output}}{\text{watts output} + W_w + W_c + W_f + W_a + W_l} \times 100.$$

where, W_w = Friction and windage loss in watts; W_c = Core loss (hysteresis and eddy currents in the magnetic circuit); W_f = Field I^2R, or excitation, loss; W_a = armature ohmic I^2R loss $(= nI^2R_o,$

where n = number of phases, I and R_o = current and ohmic resistance per phase); W_ρ = load loss.

FRICTION, WINDAGE, AND CORE LOSSES are practically CONSTANT, while the other losses change with the load or the power factor. The LOAD LOSSES are equal to the difference between the EFFECTIVE I^2R_e loss and the ohmic I^2R_o in the armature.

IN ORDER TO CAUSE A CERTAIN ALTERNATOR TO TAKE A CERTAIN SHARE OF THE LOAD, the governor on the prime mover is adjusted by hand in a way which would cause it to run at a higher speed if it were operating alone. The alternator forges ahead and slightly advances the phase of its emf which causes it to take a larger share of the load. INCREASING THE FIELD OF A GENERATOR TO RAISE ITS VOLTAGE WILL NOT CAUSE THE GENERATOR TO TAKE A GREATER SHARE OF THE LOAD, it merely changes the phase relation between the currents and emf's of the several alternators.

TO RAISE THE VOLTAGE OF THE BUS-BARS, the field strength of all the generators must be increased.

TO RAISE THE FREQUENCY OF THE BUS-BARS, the speed of all the alternators must be increased.

TO DISCONNECT AN ALTERNATOR FROM THE BUS-BARS, decrease the power from the prime mover until the wattmeter shows that the generator is delivering practically no power. Then adjust the field current until the ammeter shows no current through the armature of the alternator; open switches and shut off all power from the prime mover.

AN ALTERNATOR WILL CONTINUE TO ROTATE AS A SYNCHRONOUS MOTOR if, when running as a generator in parallel with other generators, the power is shut off from its prime mover and it remains connected to the bus-bars.

OVER-EXCITING THE FIELDS OF A SYNCHRONOUS MOTOR causes it to take currents which are leading with respect to the emf of the line and thus improves the power-factor of a line supplying lagging loads. When so used the machine is called a SYNCHRONOUS CONDENSER.

The MAXIMUM EFFICIENCY occurs for that load at which the constant losses are equal to the variable losses.

HYDROGEN COOLING. In large synchronous machines, the windage losses are reduced and the load capacity is increased by using HYDROGEN GAS for cooling. The frame of a machine so cooled is sealed and the gas forced through the ventilating ducts in the core and over water cooled coils by means of fans.

PROBLEMS ON CHAPTER VII

Prob. 42-7. A 60-Kva, 600-volt, single phase alternator has an effective armature resistance of 0.16 ohm and an inherent, or leakage, reactance of 0.75 ohm. What is the induced emf when it is delivering rated current at unity power factor and at rated terminal voltage?

Prob. 43–7. (*a*) Repeat Prob. 42–7 for a lagging current at 0.7 power factor. (*b*) For a leading current at 0.7 power factor.

Prob. 44–7. The synchronous reactance of the alternator in Prob. 42–7 is 3.5 ohms. If it is supplying rated full load at unity power factor and at rated terminal voltage, find the **no-load** voltage when the load is removed. Show vector diagram and compute the regulation.

Prob. 45–7. Repeat Prob. 44–7 and find the no load voltage and per cent regulation of the machine in Prob. 42–7: (*a*) for a power factor of 0.7 lagging; (*b*) 0.7 leading.

Prob. 46–7. What is the efficiency of the alternator of Prob. 42–7 at rated load and unity power factor, if the friction, windage and core losses are 2800 watts and the field takes 25 amperes at 125 volts?

Prob. 47–7. A 40-Kva, 460-volt, single-phase alternator has an effective armature resistance of 0.175 ohm and a synchronous impedance of 2 ohms. (*a*) What is the synchronous reactance? (*b*) What is the regulation at unity power load? (*c*) At 0.7 power factor lagging? (*d*) At 0.6 power factor leading?

Prob. 48–7. A 240-Kva, 2300-volt, 25-cycle, 750 rpm delta-connected alternator has an effective armature resistance per phase of 1.8 ohms and a synchronous reactance of 15 ohms. If the field is adjusted to give rated terminal voltage at rated load and unity power factor: (*a*) What is the induced voltage at no load? (*b*) What is the regulation? (*c*) How many poles has this machine?

Prob. 49–7. Repeat (*a*) and (*b*) of Prob. 48–7 for 0.8 power factor lagging and leading loads.

Prob. 50–7. What is the efficiency of the alternator in Prob. 48–7 at rated Kva, if the friction and windage loss is 3.4 kw; core loss 6.25 kw; field current 60 amperes at 125 volts?

Prob. 51–7. (*a*) What is the efficiency of the alternator in Prob. 48–7 at rated Kva and 0.8 lagging power factor, if the field current is 72 amperes at 125 volts? Other losses as in Prob. 50–7. (*b*) What is the efficiency at rated Kva and 0.8 leading power factor if the field current under the condition is 56 amperes at 125 volts?

Prob. 52–7. A 50-Kva, 240-volt, Y-connected alternator has a d-c armature resistance of 2 per cent and a synchronous reactance of 15 per cent. Effective resistance equals 1.5 times the ohmic resistance. (*a*) What is the effective armature resistance in ohms per phase? (*b*) What is the synchronous reactance in ohms per phase? (*c*) What is the regulation at unity power factor? At 0.75 power factor lagging? At 0.75 power factor leading?

Prob. 53–7. The following data are obtained from a 1000-Kva, 4000-volt three-phase alternator. On short circuit the current in each line is 200 amperes with 32 amperes field current. On open circuit, with 32 amperes in the field, the terminal voltage is 2100 volts. The resist-

ance between armature terminals, as measured with direct current is 0.693 ohm. Ratio of effective to ohmic resistance is 1.5 to 1.

Assuming the machine to be delta connected, compute: (*a*) Effective resistance per phase. (*b*) Synchronous impedance per phase. (*c*) Synchronous reactance. (*d*) Voltage regulation at 0.87 power factor lagging; at 0.87 power factor leading.

Prob. 54–7. Repeat Prob. 53–7 assuming the alternator to be Y-connected.

Prob. 55–7. Open-circuit, short-circuit tests and armature resistance measurements on a 25-Kva, 240-volt, 60-cycle, Y-connected alternator give the following results:

OPEN-CIRCUIT, SHORT-CIRCUIT TESTS

Field Current	0	1	3	4	5	6	7	8	9	10	11	12
Line Amperes	0	20.5	30	40	50	60.5						
Terminal Volts	0	69	102	140	180	218	249	274	293	307	313	320

DIRECT-CURRENT RESISTANCE

Volts between Terminals	2	3.7	5.9	7.6	9.3
Current per Terminal	14.5	26	41.5	55	65.5

Compute: (*a*) The average value of the synchronous impedance.
(*b*) Effective armature resistance per phase, assuming ratio of effective to ohmic resistance is 1.5.
(*c*) Synchronous reactance per phase.
(*d*) Voltage regulation on loads of unity; 0.6 lagging and 0.6 leading power factors. Show vector diagrams.

Prob. 56–7. A test of a 2000-Kva, 2200-volt, 60-cycle, delta-connected alternator gives the following data:

OPEN-CIRCUIT, SHORT-CIRCUIT TESTS

Field Amperes	0	10	20	30	40	50	60	62.5	70	80	90	100
Line Amperes	0	135	268	400	535	665						
Terminal Volts	40	360	720	1110	1490	1850	2140	2200	2365	2535	2660	2770

DIRECT-CURRENT RESISTANCE

Volts between Terminals	6.7	14.5	20	25	30
Current per Terminal	100	220	310	380	450

Compute: (*a*) The average value of the synchronous impedance.
(*b*) The effective armature resistance per phase ($R_{eff} = 1.5 \times R_{d\text{-}c}$).
(*c*) The synchronous reactance.
(*d*) Voltage regulation for loads at unity, 0.8 lagging and 0.6 leading power factors.

Prob. 57-7. Two 600-Kva, 1100-volt, Y-connected alternators, operating without load are synchronized at the correct phase position, but when the synchronizing switch is closed, the terminal voltage of machine A_1 is 1100 volts, while that of the incoming machine, A_2, is only 900 volts. Alternator A_2 has a reactance of 0.302 ohm and an effective resistance of 0.0202 ohm per phase, while alternator A_2 has a reactance of 0.202 ohm and an effective resistance of 0.0404 ohm. Immediately after closing the synchronizing switch, and before the machines adjust their phase positions, compute: (*a*). The synchronizing current per phase flowing through each armature. (*b*) Terminal voltage per phase of each armature.

Prob. 58-7. From data and results of Prob. 57-7, compute the values of the following quantities the moment after the switch is closed and before the machines adjust their phase positions. (*a*) Electrical power generated per phase in alternator A_1. (*b*) Electrical power output per phase from the terminals of A_1 or input to the terminals of A_2. (*c*) Electrical power per phase used to overcome induced (counter) emf in A_2, or to develop mechanical power in its rotor. (*d*) I^2R losses per phase in each armature.

Prob. 59-7. The alternators of Prob. 57-7 are running without load at rated speed and terminal voltage. The machines are synchronized when A_2 is 15° in advance of its proper phase position. Before the alternators adjust their phase positions: (*a*) What is the resultant voltage per phase in the local circuit? (*b*) The synchronizing current per phase? (*c*) The total synchronizing power tending to pull the machines into position. (*d*) The terminal voltage of the two machines. (*e*) The total power transferred through the bus bars.

Prob. 60-7. Repeat Prob. 59-7 if the machines are 5 electrical degrees out of proper phase position when the synchronizing switch is closed. Compare results with those of that problem.

Prob. 61-7. Alternator A_1 of Prob. 57-7 is supplying a constant lagging load of 0.87 power factor at normal terminal voltage. Alternator A_2 is adjusted to rated terminal voltage and properly synchronized. (*a*) Draw a vector diagram showing this condition. Compute and locate on the diagram the vector representing the total induced emf per phase in A_1. A_2 is now made to advance 5 electrical degrees (that is, $\phi = 5°$ as in Fig. 49-7). Assuming the current delivered to the external circuit and its phase relation to the induced emfs of both machines remain unchanged, calculate for the instant before any phase adjustment of the machines occurs, and locate on the diagram. (*b*) The current per phase flowing in A_2. (*c*) The current flowing in A_1. (*d*) The power per phase generated in A_2. (*e*) The terminal voltage per phase of A_2.

Prob. 62-7. Two 2300-volt, 60-cycle, Y-connected alternators have rated full load capacities of 500 and 1000 Kva respectively. The first alternator has 5.27 ohms reactance and 0.527 ohm effective resistance per phase, while the second has 1.31 ohms reactance and 0.131 ohm

resistance. If these alternators are synchronized when the terminal voltage of each is 2300 volts, but they are 10 electrical degrees out of phase, compute: (*a*) Resultant voltage in the local circuit between the machines. (*b*) Synchronizing line current. (*c*) Total synchronizing power in watts. (*d*) Synchronizing torque in per cent of full load torque of the smaller machine.

Prob. 63–7. Repeat Prob. 62–7 if the two alternators are paralleled when only 2 electrical degrees out of phase. Is the synchronizing power directly proportional to the phase difference between the machines?

CHAPTER VIII

THE TRANSFORMER

The transformer is a machine for transferring energy from one alternating electric circuit to another alternating circuit. It consists essentially of two coils, wound on an iron core, which forms a closed magnetic circuit. When power is supplied to one coil at a definite frequency and voltage, power can be taken from the other coil at the **same** frequency and at the same or at a **different,** voltage. A transformer, which receives power at one voltage and delivers it at the **same** voltage, is called a **one-to-one** transformer. When it receives power at one voltage and delivers power at a **higher** voltage, it is called a **step-up** transformer. And when it receives power at one voltage and delivers power at a **lower** voltage, it is called a **step-down** transformer.

The coil, or winding, **to which power is supplied** is called the **primary;** and the coil, or winding, **from which power is taken** is called the **secondary.** In the ordinary transformer, the coils, wound of insulated copper wire or strap, are insulated from the iron core and from one another, so that the primary and secondary electric circuits are entirely separate and insulated from each other.

Since there are no moving parts in the machine it is called a static, or stationary, transformer. The cost per kilowatt of capacity is low compared to that of other electrical apparatus. The losses are small and the efficiency is correspondingly high — higher than that of any other electrical power machine. Efficiencies of large transformers may be 99 per cent or more. They require little attention and maintenance costs are low.

The transformer is one of the most important single electrical power devices in use today. It can transform electrical energy from low to high voltage for long distance transmission of power; and from high voltage to low voltage for distribution and for commercial and domestic use. (See Ch. I, Art. I, and Figs. 1–1 and 2–1.)

1–8. Transformer Action. An electromotive force is induced in a coil by a change in the value of flux interlinking its turns. In the generator, the flux is approximately constant and the change

in interlinkages or the induced emf, in the armature coils is obtained by the **relative motion** of the field poles and armature. In the transformer, both the coils and core are stationary and an emf is induced in the secondary winding by **electromagnetic induction.** That is, each winding is essentially an induction coil, both windings being mutually inductive and closely coupled by means of the closed magnetic path of the iron core. (See Vol. I, Ch. IX, Art. 17.)

Figure 1–8 represents a simple transformer. The iron core is **laminated** and made up of punchings of sheet steel held in place by bolts or clamps. One winding, P, is shown wound on one leg of the core, while the other winding, S, of a different number of turns, is wound on the opposite leg. (In commercial transformers,

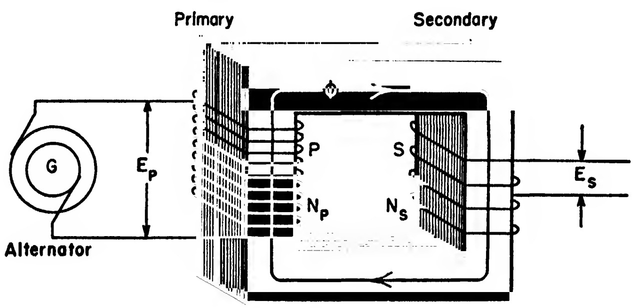


FIG. 1–8. Elementary step-down transformer.

half of each coil is wound on each leg to reduce magnetic leakage, as explained later.)

The alternator G supplies an alternating current to the primary winding, P, which sets up an mmf and an **alternating flux,** ϕ, in the iron core. The flux, ϕ, also links the turns of the secondary winding, S, and sets up an alternating emf of the same frequency in this winding. Because of the voltage induced in the secondary winding, it can be used to **supply** current and power. This power, or energy, is transferred from the primary to the secondary winding by means of the changing magnetic flux, ϕ, in the iron core.

2–8. Induced emf. Fundamental Equation. The flux, ϕ, in the core of Fig. 1–8 links both the turns in the primary winding, P, and the secondary winding, S, as previously stated. Thus flux, called the mutual flux, must, therefore, set up an emf in both windings. Since the turns in each winding are linked by the **same** flux, **the emf, induced per turn, must be the same for each winding.**

Therefore, the total induced voltage in each winding must be proportional to the number of turns, or

$$\frac{E_p}{E_s} = \frac{N_p}{N_s} \tag{1-8}$$

where E_p, E_s and N_p, N_s are the total induced emfs and number of turns in the primary and secondary windings, respectively.

Equation (1) may also be written

$$E_p = \frac{N_p}{N_s}E_s, \quad \text{or} \quad E_s = \frac{N_s}{N_p}E_p$$

The voltage induced in the primary winding is the counter emf of self-induction in this winding. When the secondary circuit is open, that is, when the transformer operates at no-load, the induced, or counter, emf in the primary is practically equal to the impressed voltage. In fact, when the transformer is supplying a load, there is only a slight difference between the impressed and induced voltages in the primary, because the resistance and reactance drops are small. Therefore, the ratio $\frac{E_p}{E_s}$ or $\frac{N_p}{N_s}$ is called the voltage ratio of the transformer. In Fig. 1–8, there are half as many turns in the secondary as in the primary. So this is a step-down transformer with a voltage ratio of 2 : 1 (2 to 1).

In Vol. I, Ch. IX, Art. 1, the **average value** of an induced voltage is expressed as:

$$E_{av} = \frac{N\phi_m}{10^8 t} \tag{2-8}$$

where ϕ is the maximum flux linking the coil; N, the number of turns, and, t, the time in seconds for a complete change in flux from zero to a maximum. $\frac{\phi_m}{t}$ is, therefore, the rate of change in flux per second. If the impressed voltage is of sine wave form, the flux wave and, also, the wave of induced, or counter, emf will also have the same form. The flux wave, shown in Fig. 2–8, varies from zero to a maximum, or from a maximum to zero, four times in a cycle. The time required for a change of flux from zero to a maximum is thus $\frac{1}{4f}$ seconds, or $t = \frac{1}{4f}$, where f is the fre-

quency in cycles per second. Equation (2) can therefore be written as,

$$E_{av} = \frac{N\phi_m}{10^8 \frac{1}{4f}} = \frac{4fN\phi_m}{10^8} \tag{3-8}$$

The effective value of the induced emf can be written as,

$$E_{eff} = \frac{.707}{.636} \times \frac{4fN\phi_m}{10^8}$$

$$= \frac{1.11 \times 4fN\phi_m}{10^8} \quad \text{or} \quad \frac{4.44fN\phi_m}{10^8} \tag{4-8}$$

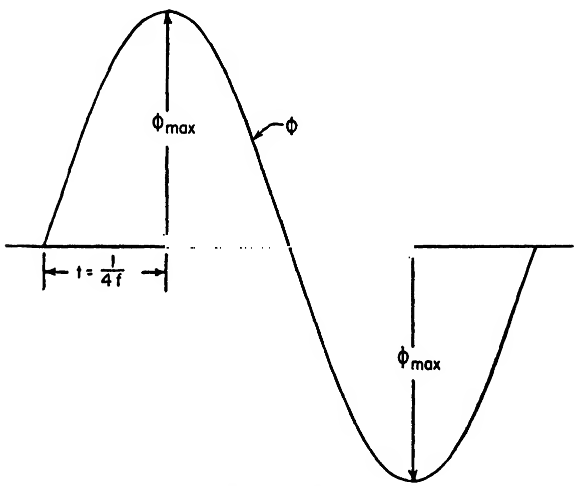


FIG. 2–8. A sine wave of voltage impressed on the primary produces a sine wave of flux in the transformer core.

From the equations above, it is seen that **the induced voltage in the transformer varies directly with the maximum flux in the core, the number of turns in the winding and the frequency of the circuit.** The flux in the magnetic circuit is generally expressed in terms of allowable flux density. The maximum flux $\phi_m = B_m \times A$, where B_m is the maximum flux density in lines per square inch and A is the cross section area of the core. Thus, Eq. (4) becomes:

$$E_{eff} = \frac{4.44fNB_mA}{10^8}. \tag{5-8}$$

Equation (5) is known as the **fundamental equation** of the transformer. The effective voltage induced in the primary (or primary impressed voltage) is thus,

$$E_p = \frac{4.44fN_pB_mA}{10^8} \tag{6-8}$$

And in the secondary,

$$E_s = \frac{4.44fN_sB_mA}{10^8}. \tag{7-8}$$

Example 1. There are 900 turns in the primary, or high side, and 90 turns in the secondary, or low side, of a transformer. If the net cross section of the core is 16 square inches, the maximum flux density is 60,000 lines per sq. in. and the frequency is 60 cycles, compute:

(*a*) The **rated** voltage of the primary, or high side.
(*b*) The rated voltage of the secondary, or low side.
(*c*) The effective voltage induced per turn in the primary.
(*d*) The effective voltage induced per turn in the secondary.
(*e*) The ratio of the transformer.

Solution:

$$(a)\ E_p = \frac{4.44 \times 60 \times 900 \times 60{,}000 \times 16}{10^8} = 2300 \text{ volts.}$$

$$(b)\ E_s = \frac{4.44 \times 60 \times 90 \times 60{,}000 \times 16}{10^8} = 230 \text{ volts.}$$

$$(c)\ \frac{2300}{900} = 2.55 \text{ volts.} \qquad (d)\ \frac{230}{90} = 2.55 \text{ volts.}$$

$$(e)\ \frac{2300}{230} = \frac{10}{1}, \qquad \text{or } \frac{900}{90} = \frac{10}{1}.$$

Prob. 1-8. Answer the five parts of Example 1 if the frequency is 40 cycles, all other data remaining the same.

Prob. 2-8. (*a*) If the turns in the primary of Example 1 are reduced to 650 and the rated voltage of the secondary is to be 125 volts, what is the rated voltage of the primary and the number of turns in the secondary, all other data remaining the same?
(*b*) What is the ratio of the transformer?

Prob. 3-8. With the same rated voltage and number of turns as in Example 1 above, what must be the cross section of the transformer core if the flux density is 65,000 lines at 40 cycles?

Prob. 4-8. The primary, or high side, of a transformer has 1250 turns.
(*a*) What must be the core flux, if the rated voltage of the high side is 4000 volts at 25 cycles?
(*b*) How many turns must the low side have if it is to generate 460 volts?

(*c*) If this transformer has two low-side coils, each rated at 230 volts, how many turns are there in each coil?

3–8. No-Load Conditions.—Vector Diagram. When an a-c voltage is applied to the primary of a transformer with the secondary open, as in Fig. 1–8, the primary current is very small, generally 5 per cent or less of the full-load value. This no-load current performs two functions. It supplies the no-load, or iron losses in the core and also sets up the magnetic flux. It can, therefore, be resolved into two components; one, a power component in phase with the impressed voltage, and the other, lagging 90° from the first, called the "magnetizing current," which produces and is in phase with the flux. The phase relation of the magnetizing

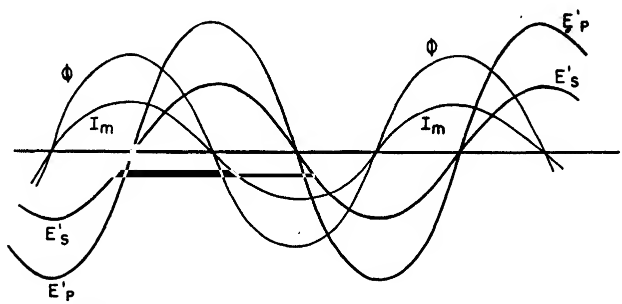


Fig. 3–8. Phase relations of magnetizing current, flux and induced voltage in the primary and secondary coils of a transformer.

current, the flux and the induced emfs in a 2:1 transformer are shown by the curves of Fig. 3–8. Note that the induced voltages, E'_p and E'_s, in both primary and secondary lag 90° behind the flux, ϕ and the magnetizing current I_m. This is in accordance with Lenz's law, for as the current and flux decrease from a positive value at their maximum rate, which is when they are passing through their zero values in a negative direction, the induced voltage E'_p in the primary (and also E'_s in the secondary) reaches a maximum positive value, tending to prevent this change. (See also Ch. IV, Art. 2–4.) Since the flux and the current, ϕ and I_m, have reached their maximum positive values 90° earlier, the induced voltages E'_p and E'_s lag 90° behind ϕ and I_m.

The above relations are also shown in Fig. 4–8(*a*), which represents the same 2:1 transformer in Fig. 1–8. The conditions are indicated at the instant when the lower primary terminal is positive (+) and the current is increasing. The direction of the

flux, ϕ, according to the right hand rule for coils is also indicated. Note that the current and impressed voltage are in a direction up in the primary coil, while the induced, or counter, emf, E'_p, in this coil is **down,** in a direction to oppose the current: also, in accordance with Lenz's law. Note, also, that the secondary induced voltage, E'_s, is in a direction **up** at this instant, or in the same

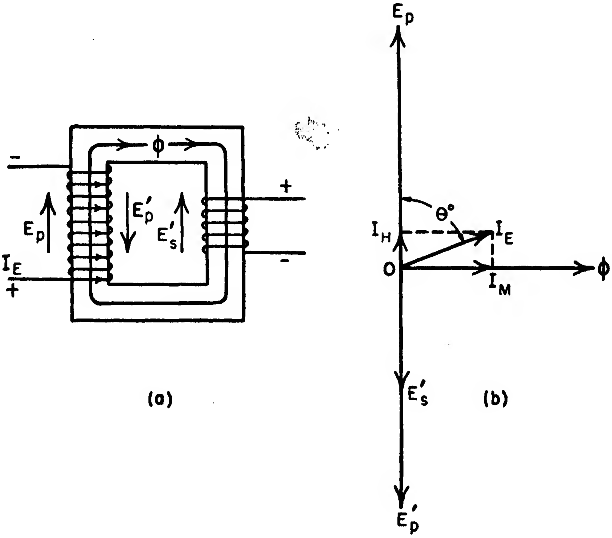


FIG. 4–8. (*a*) Relative direction of primary current, flux and induced emfs in the coils of an unloaded transformer at the instant the lower primary terminal is positive and the current is increasing. (*b*) Vector diagram of an unloaded transformer. The primary current, I_E, called the exciting current, consists of two components, I_M, the magnetizing current in phase with the flux, and the core-loss current, I_H, in phase with E_p.

direction, with respect to the flux, as the primary induced emf. Thus the lower secondary terminal is negative ($-$) at this instant.

Figure 4–8(*b*) is a vector diagram representing the above relations in the same 2:1 step-down transformer. The flux, ϕ, is laid off as the reference vector, since it is common to both coils. The impressed voltage, E_p, leads the flux by 90°, and I_E, the no-load current, lags the impressed voltage by the angle θ. This no-load current is called the **exciting current** and cos θ is the power factor of the transformer at no-load. The exciting current is resolved into the two components, I_M in phase with the flux, which it produces, and I_H, the core loss or hysteresis current in phase with

the impressed voltage which supplies the iron losses. At no-load, the I^2R losses in the primary are negligible, since the current is so small. The vectors, E_p' and E_s', in phase with each other, represent the induced emfs in the two windings, and lag 90° behind the flux vector, as already explained. It is important to note that the **induced** voltage in both primary and secondary is 180° out of phase with the voltage **impressed** on the primary.

The power factor of the transformer equals $\dfrac{I_H}{I_E}$ or $\dfrac{I_H}{\sqrt{I_H^2 + I_M^2}}$. The magnetizing current I_M is large with respect to I_H, so the power factor at no-load is low, generally from 10 to 20 per cent. Thus the magnetizing current is very nearly equal to the exciting current, as shown in the vector diagram; so that the total current at no-load is often called the magnetizing current.

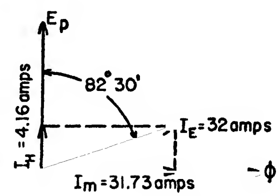


Fig. 5–8. Relations in the transformer of Fig. 1–8 when the secondary is connected to a load.

Example 2. The low-side of a 2400-240-volt, 200-kva, 60-cycle transformer takes an exciting current of 32 amperes at 0.13 power factor from 240-volt mains, when the high-side, a high-voltage winding, is open. Compute:

(*a*) The hysteresis, or core-loss, current.
(*b*) The magnetizing current.
(*c*) Power taken by the transformer.
(*d*) The exciting current in per cent of full load current.

Solution: See Fig. 5–8.

$$0.13 = \cos 82.5°. \qquad \sin 82.5° = .9914$$

(*a*) $32 \cos 82.5° = 32 \times 0.13 = 4.16$ amperes.
(*b*) $32 \sin 82.5° = 32 \times .9914 = 31.73$ amperes.
(*c*) $EI \cos \theta = 240 \times 32 \times 0.13 = 998.4$ watts.
(*d*) Full load current, low side $= \dfrac{200{,}000}{240} = 833$ amperes.

$$\frac{32}{833} \times 100 = 3.84 \text{ per cent.}$$

Show vector diagram of impressed voltage flux and currents in solution of each of the following problems:

Prob. 5–8. A 15-kva, 2400-120-volt, 60-cycle transformer takes 125 watts from 120-volt mains, when the secondary, or high side, is on open circuit.

(*a*) What is the core loss current?
(*b*) The magnetizing current?
(*c*) The exciting current?

Prob. 6–8. A transformer rated at 10 kva, 2200-220-volts, 60 cycles, takes 100 watts from a 2200-volt, 60-cycle line, when its secondary, or low side, is on open circuit.

(*a*) If the magnetizing current is 90 per cent of the exciting current, what is the zero load power factor of the transformer?

(*b*) What is the exciting current in amperes and in per cent of full load current?

Prob. 7–8. A 5-kva, 2300-230-volt, 60-cycle transformer takes 0.10 ampere at 0.40 power factor from 2300-volt mains, when the secondary circuit is open. Compute:

(*a*) Exciting current in per cent of full load current.

(*b*) Magnetizing current in amperes and in per cent of full load current.

(*c*) Core loss current in amperes and in per cent of full load current.

Prob. 8–8. At zero load, the transformer of Prob. 7-8 takes how many watts from the line?

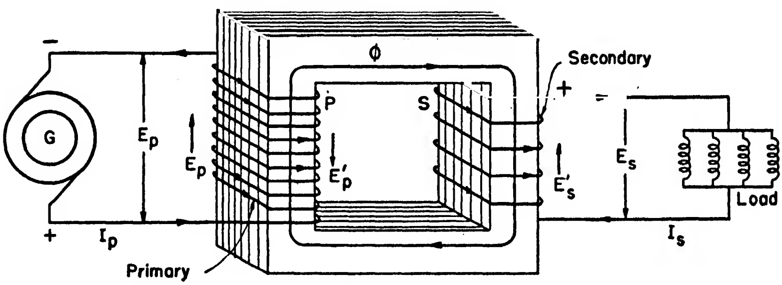


FIG. 6–8. The transformer of Fig. 1–8 with a load on the secondary coil.

4–8. Conditions Under Load. When the secondary of a transformer is connected to a load, the secondary induced emf causes a current to flow in the secondary winding, as indicated in Fig. 6–8. This figure represents the same 2 : 1 transformer shown in Figs. 1–8 and 4–8(*a*). It again shows the conditions at the instant when the lower primary terminal is positive (+) and the current is increasing. At this instant, the direction of the flux, ϕ, according to the right-hand rule for coils, is also indicated. Since according to Lenz's law, both the secondary induced emf, E'_s and secondary, or load, current, I_s, must be in a direction to oppose the action which produced them, they are also, according to the right-hand rule, in the direction shown.

Thus, when a load is connected to the transformer, the secondary ampere-turns, N_sI_s, set up an mmf which opposes and tends to decrease both the flux, ϕ, and the induced, or counter, emf in the

primary. This results in an increased flow of current in the primary, until the additional primary ampere turns are exactly equal to the secondary ampere-turns, and the flux is restored to its original value. The flux, by its rate of change, must balance the primary impressed voltage, and since the primary counter emf varies only slightly from no-load to full-load current, the flux remains practically constant regardless of the load.

Neglecting the small no-load primary current and ampere-turns, necessary to set up the flux, the primary ampere-turns, N_pI_p, due to the load, are equal to the secondary ampere-turns, N_sI_s, and we can write:

$$N_pI_p = N_sI_s, \quad \text{or} \quad \frac{N_p}{N_s} = \frac{I_s}{I_p}. \tag{8-8}$$

Thus, **the currents in the primary and secondary** are seen to be **inversely proportional to their turns.**

$$\text{Also from Eq. (1),}\ \frac{E_p}{E_s} = \frac{N_p}{N_s}.$$

Therefore,

$$\frac{E_p}{E_s} = \frac{I_s}{I_p} \quad \text{or} \quad E_pI_p = E_sI_s. \tag{9-8}$$

And the currents in the primary and secondary coils are also inversely proportional to their respective voltages. (Note that the relations in Eq. (9) neglect the losses in the transformer.)

Prob. 9-8. Neglecting the exciting current and losses in the transformer of Example 2:

(*a*) What will be the primary current if the high-side secondary delivers 80 amperes to a load?

(*b*) If the transformer generates 4 volts per turn, what are the secondary ampere turns in (*a*)?

(*c*) The primary ampere turns?

Prob. 10-8. The low-side coil of a 50-kva, 5:1 ratio transformer is rated at 460 volts and has 160 turns.

(*a*) What is the rated voltage and number of turns in the high-side coil?

(*b*) What is the full-load current output of the low-side coil and the corresponding current in the high-side coil?

(*c*) What are the ampere-turns at full load in the high-side? In the low-side?

5-8. Elementary Vector Diagram of the Transformer. Figure 7-8 is the vector diagram of a 2:1 ratio, step-down transformer,

showing the relations discussed in the preceding section. The vectors representing the mutual flux, ϕ, exciting current, I_E, impressed primary voltage, E_p, and the induced voltages E'_p and E'_s are drawn exactly as in Fig. 4–8(b). The secondary current I_s is here shown lagging the secondary emf by the angle θ_s, which depends upon the power factor of the load. Since the secondary ampere-turns must be exactly balanced by an equal number of **additional** primary ampere-turns, a current, I'_p $\left(\text{equal to } I_s \dfrac{N_s}{N_p}\right)$ flows in the primary 180° from I_s, and lags the impressed voltage by the same angle θ_s. I'_p is called the **primary load current.** The total primary current is now the vector sum of I'_p and I_E, shown as I_p in the diagram. This current lags the impressed voltage by the angle θ_p and cos θ_p is the power factor of the primary circuit for this load.

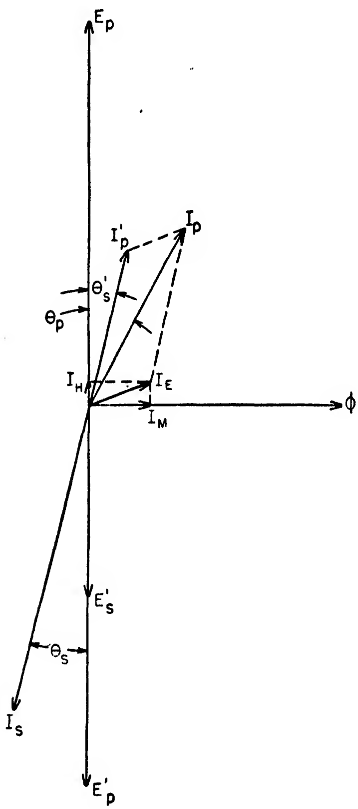


FIG. 7–8. Elementary vector diagram of a loaded transformer.

If the load on the transformer I_s is doubled, the primary load current I'_p is doubled, but the exciting current, I_E, remains unchanged. The total primary current is again the vector sum of I'_p and I_E. Thus the primary current (except in particular cases of leading load) can never be exactly proportional to that in the secondary, and the primary power factor will differ slightly from that of the secondary under normal conditions of flux. However, in the commercial transformer, I_E is so small in comparison to I_p at full load, that in many practical calculations it is assumed that the amperes input to the primary at full load are directly proportional to the secondary amperes output, and that the power factor of the primary is equal to that in the secondary.

The constant-potential transformer thus operates under the following conditions:

(1) The core flux is practically constant throughout its range of load.

(2) As the secondary current increases or decreases, the primary current increases or decreases almost proportionally.

(3) The primary and secondary currents differ in phase by practically 180°.

(4) The power-factor of the primary is substantially the same as that of the secondary circuit.

It should be noted that the effect of resistance and reactance in the windings has been so far neglected. This is considered in a following article.

Example 3. At no load, a 5-kva, 460-230-volt, 60-cycle transformer takes 1.3 amperes at 0.3 power from 230-volt mains. Compute the total primary current when the secondary, or high side, supplies rated kva and current to a lagging load of 0.8 power factor.

Solution: See Fig. 8–8.

$$\text{Full load secondary current} = \frac{5000}{460} = 10.87 \text{ amperes.}$$

$$\text{Load component of primary current} = 2 \times 10.87 = 21.74 \text{ amperes.}$$

$$0.8 = \cos 37°. \qquad 0.3 = \cos 72° 30'.$$

$$72° 30' - 37° = 35° 30'. \quad \cos 35° 30' = 0.814. \quad \sin 35° 30' = 0.58.$$

$$\begin{aligned} I_p &= \sqrt{(21.74 + 1.3 \cos 35° 30')^2 + (1.3 \sin 35° 30')^2} \\ &= \sqrt{(21.74 + 1.06)^2 + (0.754)^2} = \sqrt{519.84 + .5625} = 22.81 \text{ amperes.} \end{aligned}$$

Prob. 11–8. Determine the primary current in the transformer of Example 3, if it delivers rated kva and current at unity power factor. Show vector diagram.

Prob. 12–8. Repeat Prob. 11–8 for rated load at 0.8 power factor leading current.

6–8. Classification of Transformers. Fundamental differences in the design and construction of transformers separate them into definite and distinct types. Transformers are classified with respect to:

(*a*) The shape in which the laminated iron core is formed in relation to the coils;

(*b*) The method used in cooling;

(*c*) The type of service for which they are designed.

Transformer cores are of two fundamental types, the **core** type and the **shell** type. There are also modifications of these types. They may be oil cooled, which is the more common type; air cooled; or oil insulated water cooled. Transformers are classified for convenience as power or station transformers, distribution transformers, lighting transformers, constant-current transformers, compensators and instrument transformers, etc.

Power, distribution and lighting transformers are known as constant-potential transformers, since they are designed to give, as nearly as possible, a constant secondary terminal voltage under varying loads. There is no sharp line of demarcation among these three types. Power transformers are used in generating plants, substations, and on primary transmission lines for the transformation of relatively large amounts of power. Most high-voltage transformers fall into this class. Distribution transformers are usually of moderate voltage and are used to distribute power from transmission lines and distribution systems. Lighting transformers are generally used to supply relatively small amounts of power, in capacities of 10 kw or less, for commercial and domestic use. Their high-side rating is seldom more than 2300 volts.

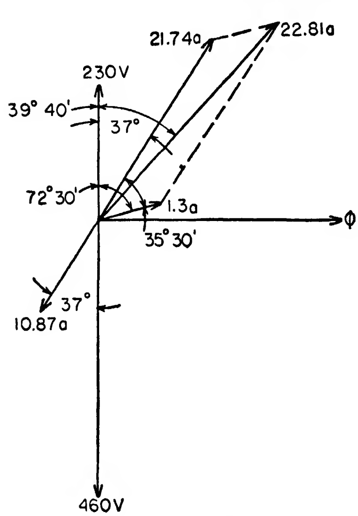


FIG. 8–8. Vector diagram of the transformer in Example 3.

7–8. Transformer Cores. The magnetic circuit, or core, of the transformer is usually made of high-grade silicon steel, containing about 4 per cent silicon. The use of silicon improves the magnetic characteristics and reduces the iron losses. The laminations are about 14 mils in thickness for 60-cycle transformers and slightly thicker for 25-cycle machines.

If each layer of the laminated core of a transformer, similar to that in Fig. 1–8, were made in one continuous section, the coils would have to be wound laboriously by hand, and each turn passed through the " window " in the core. Each layer is, therefore, made up of several sections, generally rectangular, or L shaped,

the joints of which are butted together. The sections are so assembled that they overlap the joints in alternate layers as indicated in Fig. 9–8. A core built in this manner permits the completed form wound coils to be placed on the legs, before the top sections are put in place. When the assembled core is bolted, or clamped, together, the lapped joints form a continuous magnetic path of little more reluctance than if each layer were one continuous piece.

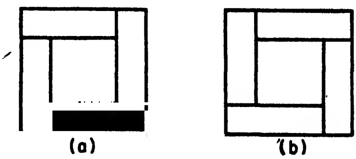


FIG. 9–8. Arrangement of joints in adjacent layers of the laminated core.

8–8. Core-Type Transformer. Figure 10–8 represents a core-type transformer. Note in Fig. 10(*a*) that the core is in the form of a hollow square, and that the coils are wound on opposite legs and fill the entire "window" in the core. Figure 10(*b*) shows that both high- and low-voltage coils are divided, and half wound on each

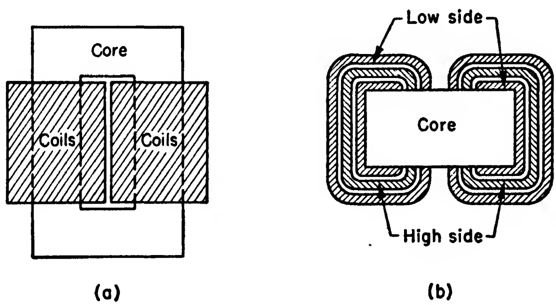


FIG. 10–8. Core-type transformer. (*a*) Elevation. (*b*) Top view.

leg; this improves performance and reduces leakage. Also note in Fig. 10(*b*) that part of a low-voltage coil is wound next to the iron core. If the high-voltage coil were placed next to the core, this winding would have to be insulated both from the core and from the low-voltage winding, thereby using an additional layer of "heavy" insulation. By placing the low-voltage winding next to the core, as shown, one less layer of "heavy" insulation is needed and less space in the window is required. Therefore, the amount of iron, the cost, and the iron losses are reduced. Figure 11–8 shows a small three-phase core-type transformer removed from its case.

The distinguishing feature of the core-type transformer is that

the copper winding surrounds the iron core. Also the length of the magnetic circuit is longer and the average length per turn of the winding is shorter than in the shell-type transformer. The high-voltage coils in the core type are more easily insulated, so that it is particularly adapted for use on high-voltage circuits, particularly in small and medium capacities.

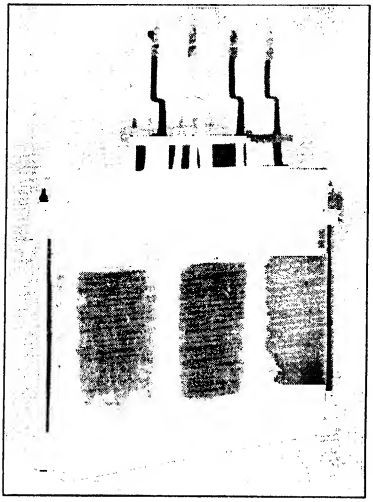


FIG. 11-8. Small three-phase core-type transformer removed from its case.

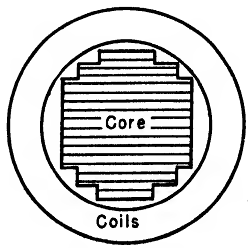


FIG. 12-8. Section of cruciform transformer core.

Cruciform core. In the transformer shown above, the cross section of the core and coils are rectangular in form. By building the core in " cruciform " section, the coils may be circular in form,

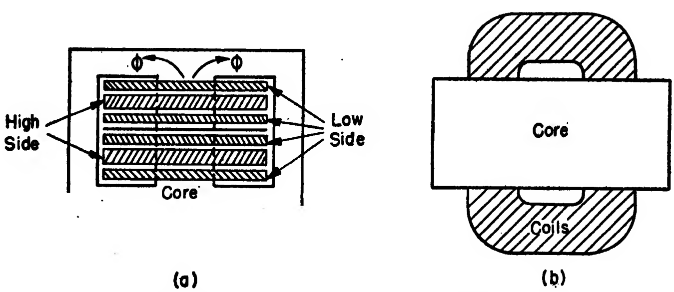


FIG. 13-8. Shell-type transformer. (*a*) Elevation. (*b*) Top view.

as indicated in Fig. 12-8. While the cruciform core is more costly to manufacture, since the laminations must be cut in several sizes, the circular coils are easier to wind and are not subject to de-

struction under short circuit. Cruciform cores are commonly used in the construction of medium sized core-type transformers.

9–8. Shell-Type Transformers. The core of the shell-type transformer consists of three legs and both primary and secondary coils are wound on the center leg. The construction is indicated in Fig. 13–8, and a small transformer of this type, removed from its case, is shown in Fig. 14–8. Note that the coils occupy the entire space in both "windows" of the core. The entire flux passes through the center leg, but divides between the outside legs. The coils are flat or "pancake" in shape, generally wound of copper strap, and sections of the primary and secondary coils are placed in alternate layers to reduce magnetic leakage. Sections of the low-voltage coils are placed next to the core at the top and bottom of the stack, as indicated in Fig. 13–8.

In the shell-type transformer, the iron surrounds the copper winding. The length of the magnetic circuit is shorter, and the average length per turn of coil is greater than in the core-type transformer.

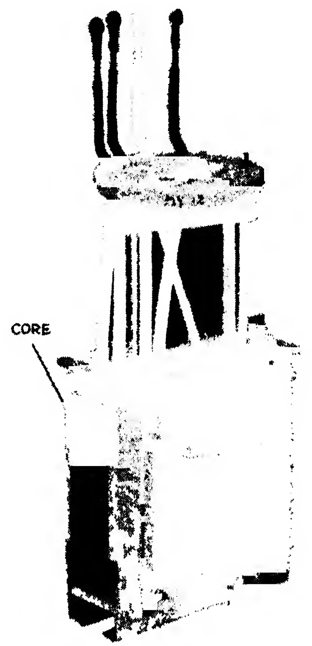


Fig. 14–8. Small Spiracore shell-type transformer removed from its case. (*General Electric Co.*)

10–8. Distributed Shell-Type Transformer. Both the Westinghouse and the General Electric Company build a **distributed** shell-type transformer in which there is a center core and **four** outside legs. The Westinghouse transformer is known as the **Type S,** while that manufactured by the General Electric Co. is called the **Type H.** Figure 15–8 shows a Type H transformer removed from its case. The high-side terminals are brought out near the top of the case, and the low-side terminals are connected to a porcelain terminal block mounted on the core. Figure 16–8 is a diagram of a top view of the Type H transformer and shows how the four sections of the core are fitted together. All the coils are wound on the central core, as shown in Fig. 15–8, with the

low-side coils next to the core and the outside legs; while the high-side coils are placed in the intervening space and insulated by mica shields. Space is left between the coils, as indicated, to provide channels for circulation of the oil. By this design, both a shortened magnetic circuit and a shorter average length per turn of coil is obtained; and thereby makes a smaller and more compact machine per kw of capacity.

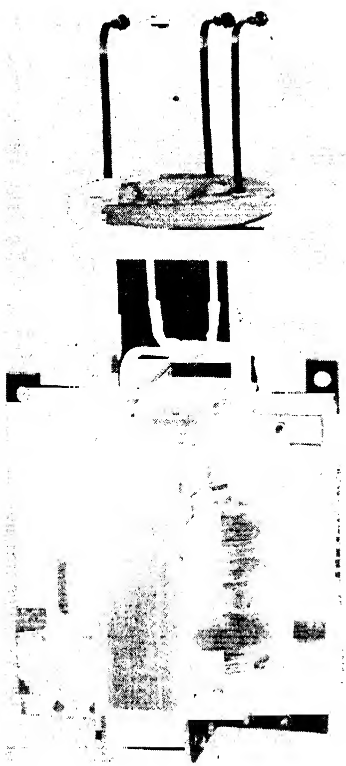


FIG. 15–8. Type H Spiracore transformer removed from its case. (*General Electric Co.*)

These transformers are used mostly in small sizes for stepping the voltage from 2300 or 1150 volts to 230 or 115 volts.

11–8. Spirally Wound Cores. Certain grades of silicon steel show an increase in permeability at high flux densities, if the path of the flux is in the same direction in which the steel sheet is rolled. This has led to the construction of transformers with cores spirally wound from a continuous strip of iron, as indicated in Fig. 14–8.

In order that form-wound coils may be placed on the spirally wound core, the laminations are cut. After the coils are put in place, the core is butted together and held in place by clamps. This construction, even with the butted joints, results in reduced reluctance of the magnetic circuit and improved transformer performance.

12–8. Cooling of Transformers. All the losses in the transformer appear as heat in the coils and core. While these losses are small in comparison to the capacity of the machine, this heat must be transferred to and dissipated from the surrounding surfaces, in order to keep the temperature within safe limits.

Oil Cooled. The most common form of cooling is to set the transformer in a steel tank filled with oil. The oil also increases the dielectric strength of the insulation between the coils and, therefore, acts as an insulator in the tank. Vertical ducts, or spaces, are provided between sections of the coils, as indicated in Fig. 16–8. The heat generated in the coils and core is transferred to the oil, which rises through these ducts, as its temperature increases. The oil circulates up through the ducts and down next to the tank walls, thus transferring the heat to the tank surfaces

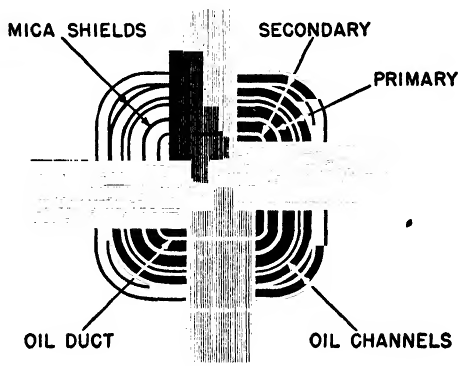


FIG. 16–8. Top view of Type H transformer.

from which it is dissipated to the surrounding air. For small transformers, the side walls are generally flat surfaces.

The rate at which the heat may be dissipated depends upon the outside surface area of the tank. The volume, or cubical content, of the transformer and tank is approximately proportional to the kva capacity. The surface area of the tank, however, does not increase in proportion to its cubical dimensions, so that, as the kva capacity of transformers is increased, additional tank surface, or dissipating area, must be provided.

For medium sized transformers the side walls of the tank are generally fluted, or corrugated, to increase the surface area, as indicated in Fig. 17–8. For larger transformers, vertical return tubes are welded into the top and bottom of the side walls, thereby, increasing the effective dissipating area, as illustrated in Fig. 18–8.

In the design of transformers of large kva capacity, the overall dimensions are limited by railroad clearances. Therefore, in place of return tubes, radiators are bolted to the tank, as shown in Fig. 19–8. These radiators can be removed for shipment, and

replaced when the transformer is installed. Electric fans, not shown in the figure, are often installed to blow cooling air upward through these radiators. These fans may be arranged to operate automatically when the transformer temperature rises above normal value, as in the case of an overload.

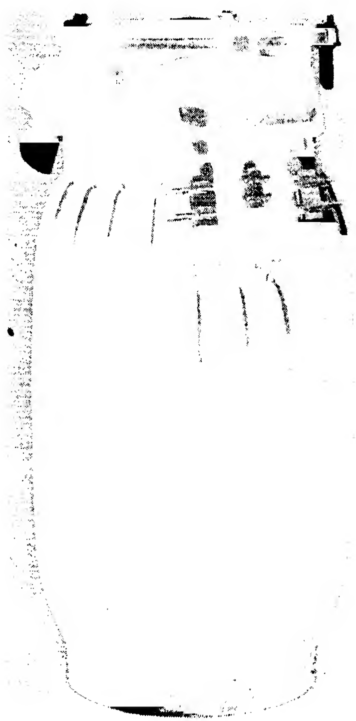


FIG. 17–8. Transformer with fluted or corrugated walls. (*General Electric Co.*)

Oil-Insulated, Water-Cooled. Where space is limited and cooling water is available, oil-filled transformers may be constructed with a coil of copper tubing, placed in the top of the tank above the windings, as shown in Fig. 20–8. Cooling water is pumped through this tubing, which is in contact with the hottest oil at the top of the tank. The transformer is thus cooled, both by the natural circulation of the oil and by the artificial cooling of the water in the tubing. Such transformers require less space per kva of capacity than the oil-cooled type, but cost more to

manufacture and maintain. The main disadvantage is the danger of leaks in the copper tubing, which must be checked at intervals, as even a very small amount of moisture in the oil greatly reduces

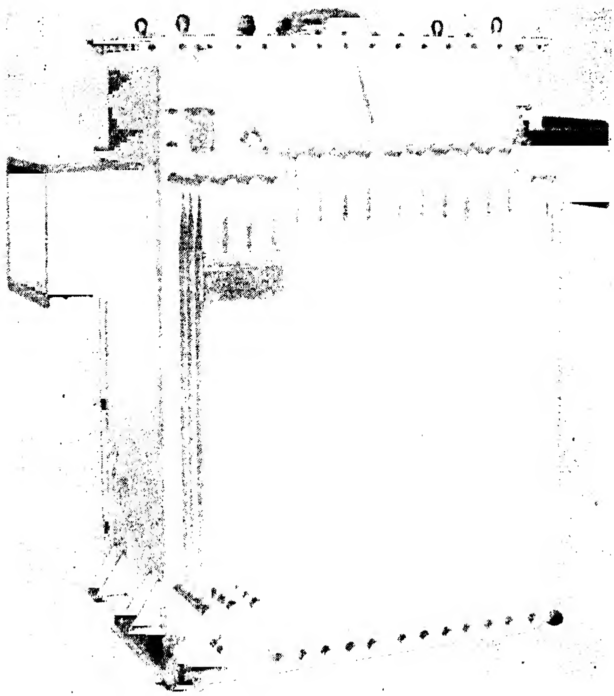


FIG. 18–8. A 750-KVA "Pyranol" transformer with vertical return tubes attached to the side walls for cooling the Pyranol fluid. (*General Electric Co.*)

its insulating qualities. This method of cooling is not in general use today.

Air Cooled. The installation of oil filled transformers inside public and private buildings is considered a fire hazard, and is not allowed by the Fire Underwriters, unless the transformers are located in fireproof vaults. Small air-cooled transformers, however, are allowed in buildings without being so enclosed. These transformers have no containing tank and are liberally designed with respect to surface area of core and coils. Heat is dissipated directly to the surrounding air and cooling is accomplished by the

natural circulation of air over the exposed surfaces. Instrument transformers for moderate voltage circuits are also air cooled.

Air Blast. Transformers, which are cooled by forced circulation of air through their windings, are called **air-blast** transformers. These are generally power transformers, installed in thickly populated districts, where the presence of large amounts of oil is

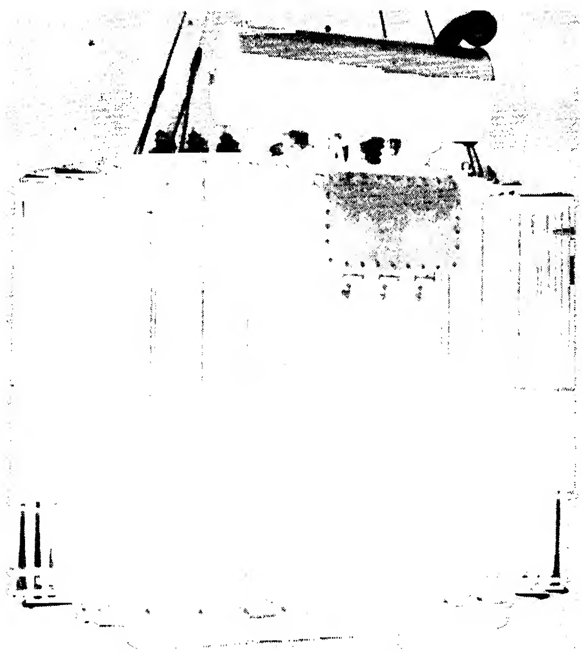


Fig. 19–8. The oil is cooled by circulating through the radiator tubes attached to the sides of the transformer case. (*General Electric Co.*)

considered a fire hazard, as in a city substation. The transformer is enclosed in a shell, open at the top and bottom. It is generally placed over a pressure chamber and is supplied by a blower from which air is forced upward through ducts in the core and coils and discharged through the top, as indicated in Fig. 21–8. The air pressure and flow of air is regulated according to the temperature, or load, on the transformer. The intake air to the blower is filtered and cleaned to prevent dirt and other foreign matter from clogging the ventilating ducts and injuring the insulation of the coils. Since the air-blast transformer does not

have the added protection of the insulating oil it is not used on circuits much above 20,000 volts. One disadvantage is the fact that an arc formed by the failure of insulation is quickly fanned into flame by the strong air current.

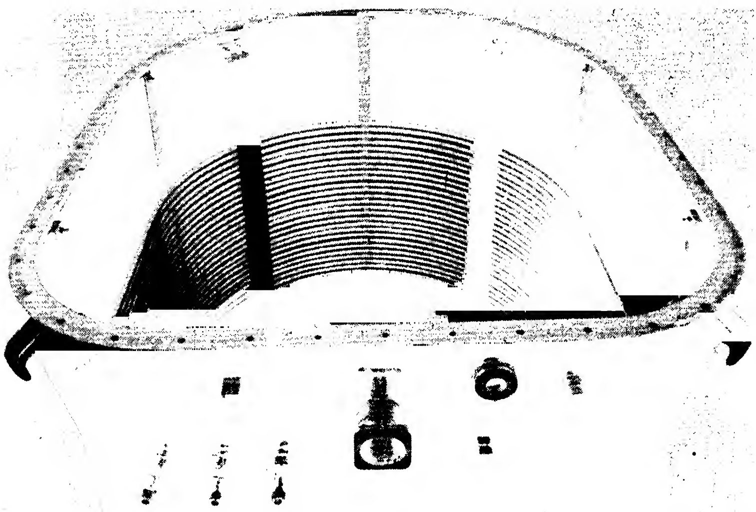


FIG. 20–8. View into the top of the tank of a 23,333 KVA transformer showing the coils for the cooling water. (*General Electric Co.*)

13–8. Transformer Oil Breathers. Oil used in transformers is a high grade mineral oil which must have high dielectric strength; that is, good insulating properties, with a high flash temperature to prevent its ignition in case of coil failure. It must contain no chemical impurities which will attack the copper windings and insulation. It must also be free of **moisture** and **sludge.**

Transformer oil absorbs moisture from the atmosphere, and the presence of even $\frac{1}{100}$ of one per cent of water seriously reduces its insulating properties. Also, oxygen in the air, in contact with the oil, oxidizes it and forms a thick sludge, which clogs the oil ducts and causes overheating. Therefore, the oil in transformers must be effectively sealed against the surrounding air.

In large transformers, a space for air or gas must be provided above the oil level to allow for the expansion and contraction of the oil. This also allows for expansion of gases in case of internal explosion. As the temperature of the transformer rises, the oil

and gas expand and the gas is forced out. When the transformer cools, air is drawn in. Thus the transformer is said to **breathe.**

To exclude moisture, the Westinghouse Company employs an "Inertaire" **breather,** mounted on the transformer case, through which passes the out-going and incoming air. This breather

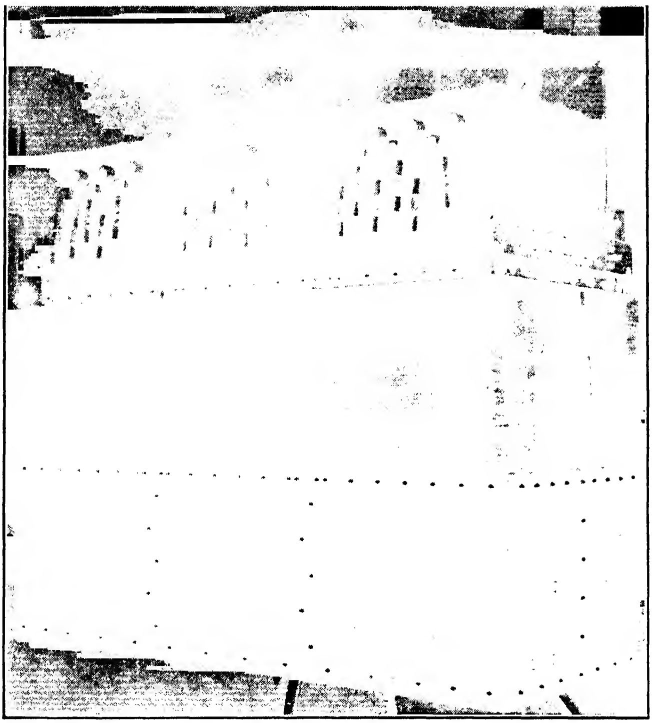


FIG. 21–8. A large high-voltage indoor air-blast transformer. (*General Electric Co.*)

filters the incoming air and removes from it both the moisture and oxygen.

The General Electric Company installs a small expansion tank, called a Conservator, at the top of the transformer, and connected to it by a pipe, as shown in Fig. 19–8. This tank is always partially filled with oil and provides for the expansion and contraction due to temperature changes. Thus the transformer is always full, and the oil in it is not in contact with the air. An orifice or breather on the expansion tank is packed with a drying chemical which removes the moisture from the entering air.

In small transformers, it is unnecessary to provide for the expansion and contraction of the oil. Of course, the oil level must at all times cover the core and coils. A strip of felt or other material effectively seals the transformer when the top is screwed or bolted to the case.

In time, the oil in transformers deteriorates. It is common practice to test the oil in large high voltage transformers at regular intervals for sludge and moisture content. If necessary, the oil is drawn off and filtered, or the transformer is refilled with new oil.

14–8. Pyranol. A synthetic insulating liquid called "Pyranol," which has most of the desirable qualities of transformer oil, has recently been developed by the General Electric Company. The outstanding characteristic of pyranol is that it is non-inflammable. Thus, it is possible to install liquid-cooled transformers in the interior of buildings without enclosing them in fire-proof vaults. Pyranol, also, will not oxidize, form sludge, or explosive or inflammable gases.

However, pyranol acts as a solvent of most of the resins, gums and paints, etc., customarily used for insulating the coils in oil-filled transformers. Pyranol-cooled transformers, therefore, are insulated only by paper, pressboard, porcelain and other materials, which are not affected by the liquid.

The cost of pyranol is considerably greater than that of transformer oil but the manufacturers claim this is compensated by the longer life of its insulating and dielectric properties.

15–8. Relations Among Magnetizing Current, Flux, Voltage and Frequency. From the fundamental equation,

$$E = \frac{4.44 f N B_m A}{10^8}, \tag{10–8}$$

if the frequency remains constant, it is evident the flux density varies directly with the impressed voltage. Thus, a given increase in the voltage applied to the primary, results in a corresponding increase in the flux density. This requires an increase in primary NI and in exciting or magnetizing current. Below the knee of the saturation curve, the increase in the current corresponds approximately with the increase in B_m. As the saturation of the core is approached, however, the magnetizing current increases much more rapidly. Thus in Fig. 22–8, with an impressed voltage, or flux density, OB_m, the magnetizing current is proportional to OI_m. When the impressed voltage, or flux density, is increased

by about 50% to OB'_m, the magnetizing current, due to the shape of the saturation curve, increases by 250% to OI'_m.

At normal impressed voltage, the number of primary turns and cross section of the core in 60-cycle transformers are generally designed to give a maximum flux density approximately 65,000 lines per square inch, with an exciting current 3 to 5 per cent of full load value. Twenty-five cycle transformers are operated at somewhat higher densities. If the impressed voltage (and flux) are raised much above the normal value, the no load, or exciting current, rises rapidly, and at very high flux densities, this current may approach, or even exceed, the full-load value. From the equation above, it is also evident that **increasing** the turns, while the impressed voltage and frequency remain unchanged, **decreases** B_m; and **decreasing** the number of turns **increases** B_m. Changing the number of turns and impressed voltage in direct proportion, leaves B_m unchanged.

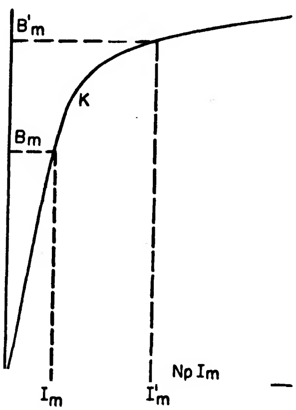


FIG. 22–8. The saturation curve for steel. The flux is practically proportional to the exciting current until the saturation point, K, is reached.

For any given transformer, in which the number of turns and area of the core are fixed, the only variables in the equation above are the impressed voltage, the frequency and the resulting flux density. Therefore, we may write

$$E_p = KfB_m \quad \text{or} \quad KB_m = \frac{E_p}{f}. \tag{11–8}$$

Thus B_m varies **directly** with the impressed **voltage,** as already noted, and **inversely** with the **frequency.** If the voltage and frequency are changed in exact proportion, B_m remains unchanged.

Thus, if a transformer is operated at normal voltage on a circuit of lower than normal frequency, the flux is correspondingly increased, and the exciting current may be several times its normal value. For instance, the maximum flux density in a 60-cycle transformer, operated at normal voltage on a 25-cycle circuit, will be $\frac{60}{25}$, or 2.4 times its permissible, or normal value. The exciting, or no-load, current in this case will be great enough to burn out

the winding. The transformer, however, can be operated at normal flux with normal exciting current, if the applied voltage is correspondingly reduced to $\frac{25}{60}$ of its rated value. The kva capacity and the iron losses will be reduced approximately in the same ratio. The transformer may also be operated at normal B_m at a higher than normal frequency, if the applied voltage is correspondingly increased. Both kva capacity and iron losses will be increased in this case; also the voltage regulation will be poorer.

It can be stated that, within the limit of insulation breakdown, a transformer can be satisfactorily operated at any frequency with normal B_m, if the ratio of impressed voltage to frequency is held approximately constant. Most transformers will operate satisfactorily at rated voltage, but with reduced efficiency, within a range of 10 per cent above or below normal frequency.

Prob. 13–8. A 50-kva, 4000-460-volt, 60-cycle transformer has 1000 turns on the high side and operates at a normal flux density of 60,000 lines per sq. in.

(*a*) What would be the maximum flux density if the turns are reduced 20 per cent?

(*b*) If the turns are increased 20 per cent?

Prob. 14–8. The turns in the coils of a 10-kva, 2400-240-volt, 60-cycle transformer are 800 and 80 respectively. The normal flux is 65,000 lines per square inch.

(*a*) What will be the flux density if the transformer is operated at rated voltage on a 40-cycle circuit?

(*b*) On an 80-cycle circuit?

(*c*) How will the magnetizing current be affected in (*a*)? In (*b*)?

Prob. 15–8. (*a*) How many turns in each coil must the transformer in Prob. 14–8 have, if it is to operate at rated voltage with normal flux density on a 40-cycle circuit? (*b*) On an 80-cycle circuit?

Prob. 16–8. What is the voltage induced per turn in Prob. 14–8?

Prob. 17–8. Neglecting the losses in the 10-kva transformer of Prob. 14–8, what will be its voltage and kva rating if operated at normal flux density: (*a*) On a 40-cycle circuit? (*b*) On an 80-cycle circuit?

Prob. 18–8. Neglecting the change in losses, what is the kva and voltage rating of a 5-kva, 2300-230-volt, 60-cycle transformer, when operated at normal flux density on a 25-cycle circuit?

Prob. 19–8. What is the voltage and kva rating of a 5-kva, 2300-230-volt, 25-cycle transformer, when operated at normal flux density on a 60-cycle circuit?

16-8. Form of Exciting Current Wave. In an unloaded transformer, the primary counter emf at every instant is practically equal to the impressed voltage, since the resistance drop and other reactions are negligible. If the impressed voltage is of sine wave form, the curve of counter emf must also be of the same form. Since the counter emf is induced by the rate of change of flux linkages in the magnetic circuit, the flux must also be of sine wave form, displaced 90° in phase from both the impressed and induced voltage waves. Thus, **if a sine wave of voltage is impressed on a transformer, the flux wave must also be of sine wave form.** Also, as

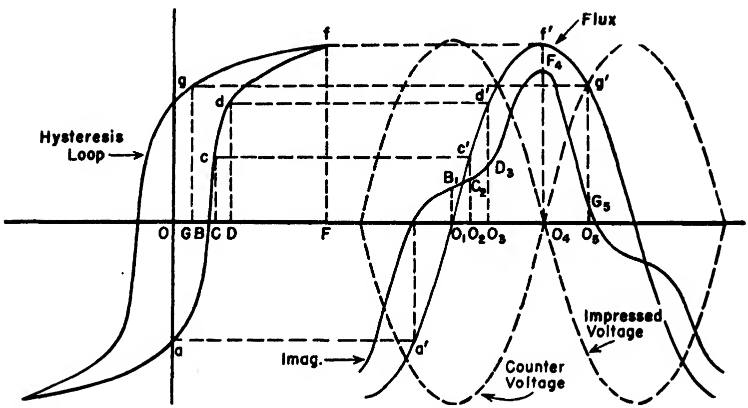


FIG. 23-8. The hysteresis loop and magnetizing current in a transformer. The magnetizing current is distorted in order to produce a sine wave of flux even though the impressed emf is a sine-curve.

the flux in the magnetic circuit varies periodically from a maximum in one direction to a maximum in the opposite direction, the core is carried through a complete hysteresis loop for each cycle of the flux wave. The hysteresis loop, therefore, indicates the relation, from instant to instant, between the flux (flux density) in the core and the magnetizing current (ampere turns).

In Fig. 23-8, sine curves of impressed and counter voltage are shown in their relative phase position, together with the flux wave and the corresponding hysteresis loop. The magnetizing current can now be determined by plotting the values of current from the hysteresis loop against corresponding values of the flux wave for successive instants over the cycle.

Thus, when the flux is passing through zero, from a negative to a positive value on the hysteresis loop, the current is *OB*. A

perpendicular O_1B_1 is now laid off, equal to OB, from the corresponding point on the flux wave, and B_1 is a point on the magnetizing current curve.

When the flux rises to a value c' on the loop, the current is OC. Erect a perpendicular O_2C' to the corresponding value of the flux wave and lay off O_2C_2 equal to OC. C_2 is another point on the current wave. Other points on the current curve are obtained in similar manner. As the flux decreases from its maximum positive value, f', to point q' on the hysteresis loop, the current decreases to OG. On the perpendicular O_5g', the corresponding point on the flux wave, lay off O_5G_5 equal to OG on the loop. Point G_5 is another point on the current curve.

From the figure, it is evident that the curve of magnetizing current, shown by the heavy line is **not** a sine curve, but is distorted. Since the hysteresis, or core-loss, current is relatively very small, the exciting, or total no-load current will be practically of the same form. In ordinary computations, however, the exciting current is treated as a sine curve and shown as a vector in the transformer diagram with an effective value equal to that indicated by the ammeter.

It can be shown that the distorted curve of exciting current is a combination of several sine curves of different frequencies. It consists chiefly of a curve of the same frequency as the impressed voltage, called the fundamental, together with one of three times the voltage frequency, called the third harmonic. Fifth, seventh, ninth, etc., harmonics are also present in decreasing amplitude. At flux densities above the knee of the saturation curve, the amplitude of the third harmonic increases rapidly. This results in greater distortion in the shape of the curve of exciting current.

17–8. Transformer Connections–Polarity-Phasing. Most small and medium sized transformers, generally called "lighting transformers" are wound with divided coils in both primary and secondary windings, as indicated in Fig. 24–8. Transformers, so constructed, can operate at rated kva capacity and normal flux, with several voltage and current ratings. The four leads from the secondary, or low side, coils are brought out through the case at one side. The four leads from the primary coils are brought to an insulated block within the case. By means of metal links on this block, these coils can be connected in series or in parallel. Two leads from this block are brought out through the opposite side of the transformer case.

Figure 24(*a*) represents a transformer with two 1150-volt primary coils, joined in series for connection to 2300-volt mains, and two 115-volt coils, likewise connected to supply a 230-volt secondary circuit. A neutral lead *n* may also be connected to the junction of the secondary coils to supply an Edison wire-circuit, as indicated. In (*b*), the primaries are joined in parallel for connection to 1150-volt mains, and the secondaries are likewise connected to supply a 115-volt circuit. The transformer may also be connected with primaries in series and secondaries in parallel; or, with primaries in parallel and secondaries in series. It may also be similarly connected as a step-up transformer.

Assuming 10-kva rating for this transformer, the full-load primary current per coil may be taken as $\frac{10,000}{2300}$, or 4.35, amperes; and full-load secondary current as $\frac{10,000}{230}$, or 43.5, amperes. When the coils are connected in parallel, the full-load current-input or -output is doubled. Voltage and current ratings for the four different connections above are as follows:

10-kva Transformer	Rated Voltage			Full Load Amperes	
Connection of Coils	Primary	Secondary	Ratio	Primary	Secondary
Series Primaries, Series Secondaries	2300	230	10:1	4.35	43.5
Parallel Primaries, Parallel Secondaries	1150	115	10:1	8.7	87.
Series Primaries, Parallel Secondaries	2300	115	20:1	4.35	87.
Parallel Primaries, Series Secondaries	1150	230	5:1	8.7	43.5

Note that with both connections of the primary coils in Fig. 24–8, the rated impressed voltage, and therefore the induced voltage, per turn, is the same. Therefore, the flux density, B_m, and the ampere-turns, necessary to set up this flux, must be the same. Since the number of series turns in (*b*) is only **half** that in (*a*), the exciting current at no-load with parallel connected coils must be **twice** that with the series connection. The current per coil, however, will be the same.

Polarity. The primary, or high-side, coils in Fig. 24–8 are so connected that the direction of the exciting current in both coils at any instant tends to set a flux in the **same direction in the core,**

as indicated. Also the induced, or counter, emf in both coils is in the **same direction** in the electric circuit. The coils are, there-

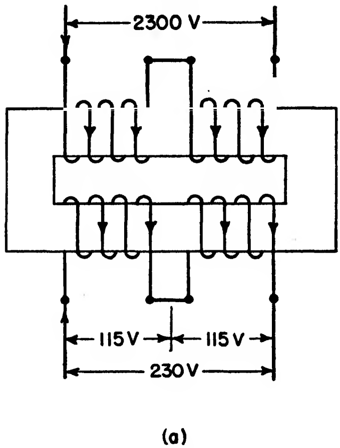


FIG. 24–8. Transformer with divided coils in both primary and secondary. (*a*) Primary coils connected in series; secondaries in series. (*b*) Primary coils connected in parallel; secondaries in parallel.

fore, joined in "**additive polarity**" with unlike terminals, connected in (*a*), and like terminals connected in (*b*).

If the primary coils of Fig. 24–8 are connected, as in Fig. 25–8(*a*) they set up mmfs in **opposite** directions in the core, and the re-

FIG. 25–8. (*a*) Primary coils connected in opposite polarity, resulting in zero flux — a short circuit. (*b*) Secondary coils connected in opposite polarity. The secondary voltage is zero.

sulting flux is zero. The coils are connected in "**subtractive polarity**"; there is no counter emf in the electric circuit and a short circuit results. Similarly, if the primary coils are properly connected and the secondaries are joined, as in Fig. 25–8(*b*),

they are connected in **subtractive polarity;** their emfs oppose each other in the electric circuit and the terminal voltage is zero.

Phasing. The determination of the polarity of the coils in a transformer is called "**phasing the windings.**" This is important for the proper connection of transformers having two or more windings in the primary or in the secondary; also, for connecting transformers in parallel, or in groups, or "banks" for polyphase circuits. There are several methods of determining polarity.

One method of phasing the coils of the $\frac{2300}{1150} - \frac{230}{115}$-volt transformer of Fig. 24–8, by means of an ordinary range voltmeter, is

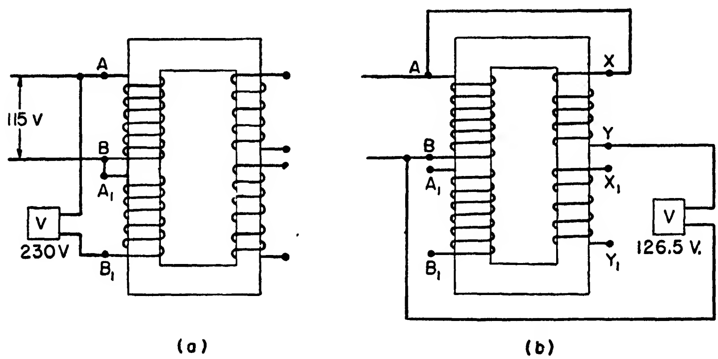


FIG. 26–8. Method for determining polarity of transformer coils. (*a*) Test for polarity in primary coils. (*b*) Test for polarity in secondary coils.

as follows. A low voltage, such as 115 volts a.c. is applied to the terminals of one of the high-voltage coils, which are arbitrarily tagged A and B, as in Fig. 26–8(a). One terminal of the other high-voltage coil is connected to terminal A or B. If a voltmeter placed across the two coils in series, as shown, now indicates 230 volts, the coils are joined in additive polarity with unlike terminals connected together. The terminals of this coil are now tagged A_1 and B_1 respectively, as shown. If the voltmeter should indicate zero, the second coil is in subtractive polarity with like terminals connected, and they are so tagged.

To phase a low-voltage coil, the connections are made as in Fig. 26–8(b). The induced voltage in this coil is 11.5 volts. If the two coils are connected in additive polarity, the voltmeter should now indicate 115 + 11.5, or 126.5 volts, and the terminals are tagged x and y, as shown. If the coil is connected in opposite

polarity, the voltmeter will indicate 115 − 11.5, or 103.5 volts. The polarity of the other low-voltage coil is tested in the same manner and its terminals tagged x_1 and y_1, respectively.

Before putting a transformer into service, it is good practice to test for polarity in case the terminals may have been interchanged after the factory test.

In the following problems, wherever possible, show a sketch with all coil terminals clearly labeled. Indicate primary and secondary terminal voltages.

Prob. 20–8. Each of the low-side coils xy and x_1y_1 in a 5-kva, 50-cycle lighting transformer has $\frac{1}{10}$ as many turns as each high-side coil AB and A_1B_1. The maximum normal flux density in the core generates 120 volts in each low-side coil. If terminal B is connected to A_1 and x to y_1, what line voltage must be applied between A and B_1 to give this flux density, and what voltage would be obtained across x_1y?

Prob. 21–8. In the transformer of Prob. 20–8, if B is connected to B_1 and the terminals A and A_1 are connected across the same high-voltage mains as in that problem, what would be the result? Low voltage connections are unchanged.

Prob. 22–8. If the terminals A and A_1 are connected to one wire of the same high-voltage mains, as in Prob. 20–8, and B and B_1 to the other, while the connection of the low-side coils remains unchanged, what will be the voltage between the terminals y and x_1? How will the values of B_m and the exciting current compare with corresponding values for Prob. 20–8?

Prob. 23–8. In the transformer of Prob. 20–8, connect x_1 to y, and then connect x and y_1 to 120-volt mains. If the high-side coils are connected as in Prob. 20–8, what will be the voltage across the terminals A and B_1?

Prob. 24–8. With the high-side coils connected as in Prob. 20–8, and to a line of the same voltage, the secondary coils are to be paralleled. Calculate the voltage across the secondary mains.

Prob. 25–8. When the low-side coils of the transformer of Prob. 20–8 are correctly connected in series across a 240-volt, 60-cycle line, the exciting current is 1.04 amperes.

(*a*) What would be the exciting current if these coils were properly connected in parallel across a 120-volt, 60-cycle line?

(*b*) Answer part (*a*) if one low-side coil only were connected across the line.

(*c*) What would be the exciting current if the two high-side coils were properly connected in series across a 2400-volt, 60-cycle line?

18–8. Losses in the Transformer. The losses of power in the transformer are classified as the **copper loss** and the **core loss.**

These two losses together may be from 1 per cent of the full load rating for large transformers to 3 or 5 per cent for the smaller sizes.

The copper loss consists of:

(a) $I_p^2 R_p$ in the primary windings
(b) $I_s^2 R_s$ in the secondary windings

where R_p and R_s are the ohmic resistances of the primary and secondary windings respectively. The copper losses are generally about equally divided between the primary and secondary, and may be exactly the same, depending upon the length and size of wire used. This loss, of course, changes with the load on the transformer.

The core loss consists of:

(a) Hysteresis loss
(b) Eddy-current loss.

Hysteresis loss, as explained in Vol. I, Ch. VIII, is due to the lag of the magnetic flux behind the applied magnetizing force. It depends upon the chemical composition and heat treatment of the iron, the maximum flux density and frequency of the applied voltage.

Hysteresis loss is given by the equation,

$$p_h = k_h W f B_m^{1.6}, \tag{12-8}$$

where

p_h = watts loss due to hysteresis.
K_h = Steinmetz's hysteresis constant, depending upon the magnetic quality of the steel core.
W = weight in pounds of the steel, acted on by the magnetic flux.
f = frequency in cycles per second of the applied voltage.
B_m = maximum flux density in lines per square inch.

Eddy-current loss is an I^2R loss set up by emfs induced in the core by the changing magnetic flux, just as emfs are induced in the copper windings. (See Vol. I, Ch. XII, p. 416.) These emfs are in a direction perpendicular to the flux path; so the layers of the laminations are laid parallel to the direction of the flux, as has been indicated in preceding figures. The laminations, or layers, are insulated from each other by natural oxidation or by varnishing their surfaces.

Eddy-current loss is given by the equation,

$$p_e = k_e W f^2 t^2 B_m^2, \tag{13-8}$$

where

p_e = watts lost due to eddy-currents.
k_e = a constant, determined by the particular kind of steel, and is inversely proportional to its specific electrical resistance.
t = thickness of the laminations in inches.

At normal flux density of about 65,000 lines per square inch, 60-cycle transformer cores, having 3 to 4 per cent silicon, have been found to have a hysteresis loss of approximately 0.5 to 0.8 watt per pound of core. The eddy-current loss under these conditions was roughly one fifth this value.

In a constant potential transformer, at normal voltage and frequency, the maximum flux density is practically constant throughout their range of load (see Art. 4); therefore, the core losses are constant at all loads.

Prob. 26–8. The hysteresis loss in a given sample of transformer steel is 0.6 watt per pound at 60 cycles and a maximum flux density of 64,500 lines per square inch for a sine wave flux.
(*a*) What would be the watts lost per pound at 25 cycles frequency?
(*b*) At 133 cycles? (Same B_m in all cases.)
(*c*) What is the flux density in lines per square centimeter?

Prob. 27–8. What would be the watts lost per pound due to hysteresis in the steel of Prob. 26–8, if the maximum flux at 60 cycles were: (*a*) half as great? (*b*) twice as great? (*c*) 45,000 lines per square inch?

Prob. 28–8. How many watts would be lost in hysteresis in a core weighing 120 lbs, made of the same steel as in Prob. 26–8, but operated at 25 cycles with a flux density of 75,000 lines per square inch?

Prob. 29–8. The eddy-current loss in a given sample of transformer steel is 0.15 watt per pound, when worked at 60 cycles with a maximum flux density of 64,500 lines per square inch, the thickness of the laminations being 14 mils. If B_m remains the same, but the steel is worked at 25 cycles, how many watts are lost per pound? By what percentage must the voltage across the primary winding be changed, when f is thus reduced, in order not to change B_m?

Prob. 30–8. What would be the eddy-current loss per pound of the same steel as in Prob. 29–8, worked at the same frequency (60 cycles) and the same flux density, but used in core sheets having a thickness of 18.7 mils?

Prob. 31-8. If the steel of Prob. 29-8 in laminations 18.7 mils thick, were used at 25 cycles, but same density (64,500 lines) what would be the watts lost per pound in the eddy currents?

Prob. 32-8. What would be the watts lost per pound in eddy currents for the steel of Prob. 29-8, if used in a 25-cycle transformer at 96,750 lines per square inch, the thickness being 18.7 mils?

Prob. 33-8. If Probs. 26 and 29-8 both refer to the same sample of silicon-steel, what per cent of the total core loss is due to hysteresis, and what per cent is due to eddy currents ($f = 25$):
(*a*) When B_m is 64,500 lines per square inch?
(*b*) When B_m is 96,750 lines per square inch?

19-8. Determination of Core Loss. Open-Circuit Test. The core loss in a transformer is readily determined by the power input at rated frequency and impressed voltage, with the secondary on

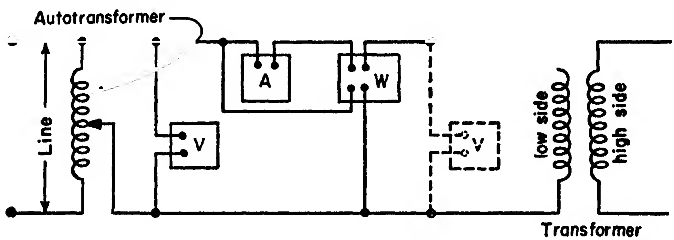


FIG. 27-8. Connections for open circuit or core-loss test.

open circuit. This input includes a small I^2R loss, which is negligible at no load, so the entire input is considered as core loss.

Figure 27-8 shows a diagram of connections for measuring core loss. The low-side is connected to an a-c source with the high-side on open circuit. Means must be provided for adjusting the applied voltage, either by a separate alternator, a potentiometer rheostat or an auto-transformer (see Art. 31, Chap. 8) as indicated in the figure. The proper arrangement of the instruments is also indicated. If the voltmeter is connected **inside** the wattmeter, as shown by the broken line, the wattmeter measures the power taken by the voltmeter. In small transformers, this may cause an error of 25 per cent or more in core loss measurement. The wattmeter reading should be taken, in this case, only when the voltmeter circuit is open. When the voltmeter is connected **outside** the wattmeter, the actual voltage on the transformer is less than indicated, due to the *IR* drop in the ammeter and current coil of the wattmeter, but this is negligible.

Core loss can be measured by applying rated voltage to either the high or low side; it is generally more convenient, however, to use the low side. In lighting transformers, particularly, instruments of ordinary voltage range can be placed directly in the circuit without the use of instrument transformers (see Art. 41, Chap. 8). In the lighting transformer of Fig. 24–8, the normal core loss can be measured by applying 115 volts to the two low-side coils in parallel, or 230 volts to these two coils in series, and can, with slight error, be measured by applying 115 volts to only one coil.

Equations (11) and (12) show that the hysteresis loss varies with $B_m^{1.6}$ and eddy-current loss with B_m^2. Thus the combined loss varies almost with the square of the flux density, and therefore with the square of the impressed voltage, as shown by the curve of Fig. 28–8. Transformers are operated at the maximum flux density which will not exceed an allowable core loss — generally below the " knee " of the saturation curve. The loss curve of Fig. 28–8 shows that the core loss rises rapidly, when the applied voltage is raised much above the rated value. An increase of 10 per cent in the applied voltage may increase the core loss as much as 25 per cent.

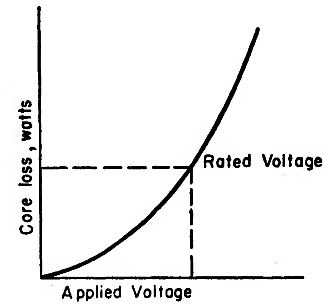


FIG. 28–8. Curve showing relation of core loss to applied voltage.

Prob. 34–8. A transformer, rated at 10 kva, 60 cycles, 1250-125 volts, takes 2.5 amperes exciting current and 120 watts from 125-volt, 60-cycle mains, when the secondary circuit is open. If the resistance of the primary and secondary coils are such that the primary copper loss is equal to the secondary copper loss at rated full load current, the total copper loss being then equal to the core loss, calculate:

(*a*) The copper loss at zero load.
(*b*) The core loss.
(*c*) The copper loss at zero load in per cent of total no load losses.

Prob. 35–8. (*a*) If the transformer of Prob. 34–8 is reversed, and 1250 volts is applied to the high-side winding (the secondary open), what will be the total zero-load losses and the exciting current?
(*b*) What will be the core loss and copper loss at zero load?

20–8. Determination of Copper Loss. Short-Circuit Test. Copper loss in the transformer, as in other electrical machines, increases with the square of the current. This is true for both

windings since, as previously shown, the primary current increases in the same ratio as the secondary current. This loss can be determined by measuring the ohmic resistance of each winding and calculating the loss in each (which is not necessarily the same) for any given current. The copper loss in both windings, however, can be measured directly by a short-circuit test.

In Fig. 29–8, the transformer of Fig. 27–8 is reversed and the low-side coils are short circuited. In most constant-potential transformers, the impressed voltage necessary to send full load current through the windings on short circuit is seldom greater than 5 per cent of the rated value. The connections are, therefore, reversed in order to obtain a more convenient voltage drop. Thus, for a 2300-230-volt transformer, a circuit of approximately

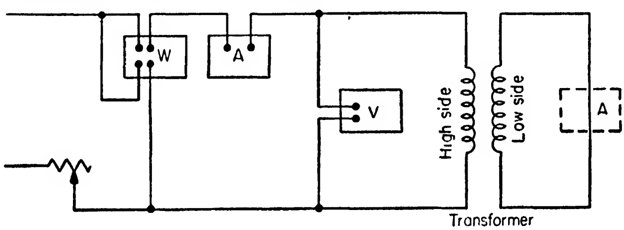


FIG. 29–8. Connections for short circuit or impedance test. Wattmeter indicates the copper loss.

2300 × .05, or 115 volts is required. Instruments of ordinary voltage range can be used for the test. Furthermore, the high-side winding draws less current from the line. Note that **all** the coils in the transformer must be connected in circuit for this test.

A very low a-c voltage of proper frequency, controlled by an auto-transformer, or rheostat, as shown, is now impressed on the primary. The voltage is raised until any desired current flows in the coils. This current can be measured by an ammeter of proper range placed either in the primary circuit as shown, or in the secondary. The wattmeter now indicates the total I^2R loss in **both windings,** plus a negligible core loss.

That this core loss on short circuit is negligible is shown as follows: At full-load current with 5 per cent of normal impressed voltage, the maximum flux density, B_m, is also 5 per cent of normal. It has been shown that the core loss varies practically with B_m^2; therefore, the core loss is 0.05^2, or $\frac{25}{100}$ of 1 per cent of normal. For instance, if the normal core loss in a 5 kva transformer is 75 watts, the loss at 5 per cent of rated voltage on short circuit is

only $0.05^2 \times 75$, or 0.1875 watts, which is too small to be read on the wattmeter.

21–8. Equivalent Resistance. Since from Art. 20, the wattmeter in the primary circuit on short circuit test indicates the I^2R loss in both primary and secondary windings of the transformer, we may write,

$$W = I_p^2 R_{ep} \quad \text{or} \quad R_{ep} = \frac{W}{I_p^2} \cdots \qquad (14\text{–}8)$$

where, W = the wattmeter reading or short circuit,
I_p^2 = the square of the primary circuit,
R_{ep} = the equivalent resistance in the transformer.

The resistance, R_{ep}, in this case, is **not** the ohmic, or d-c resistance of the primary coil, but a fictitious resistance, which, multiplied by the primary current, is equal to the resistance drop in both windings in terms of primary circuit values. It is known as the "equivalent primary resistance." In effect, it assumes that **all** the resistance in the transformer is in the primary with zero resistance in the secondary.

The calculation of voltage regulation of the transformer is much simplified by use of equivalent resistance, as will be shown later.

Equivalent primary resistance can also be expressed in terms of the ohmic resistances of both primary and secondary windings, and is written as

$$R_{ep} = R_p + R_s\left(\frac{N_p}{N_s}\right)^2 \cdots \qquad (15\text{–}8)$$

Also, the equivalent resistance in terms of secondary circuit values, called the "equivalent secondary resistance," R_{es}, can be similarly expressed as,

$$R_{es} = R_s + R_p\left(\frac{N_s}{N_p}\right)^2 \cdots \qquad (16\text{–}8)$$

The relations of the quantities involved is illustrated in the example below.

Example 4. The copper loss in a 5-kva 2300-230-volt transformer is 80 watts at full load current, as determined by the short-circuit test. Assume, for convenience, that the copper loss is equally divided between primary and secondary, or 40 watts in each coil. **Compute:**

(*a*) The ohmic resistance in the primary and in the secondary coils.
(*b*) The equivalent primary resistance from the short-circuit data.

(*c*) The equivalent primary resistance from ohmic resistance values (Eq. 15).

(*d*) The equivalent secondary resistance from ohmic resistance values (Eq. 16).

Solution:

$$\text{Full-load primary current} = \frac{5000}{2300} = 2.13 \text{ amperes.}$$

$$\text{Full-load secondary current} = \frac{5000}{230} = 21.3 \text{ amperes.}$$

$$(a)\ \text{Ohmic resistance, primary} = \frac{40}{2.13^2} = 8.81 \text{ ohms.}$$

$$\text{Ohmic resistance, secondary} = \frac{40}{21.3^2} = 0.0881 \text{ ohm.}$$

(The values in (*a*) are ordinarily obtained by d-c measurement.)

$$(b)\ \text{Equivalent primary resistance} = \frac{80}{2.13^2} = 17.62 \text{ ohms.}$$

$$(c)\ \text{Equivalent primary resistance} = 8.81 + 0.0881\left(\frac{10}{1}\right)^2$$

$$= 8.81 + 8.81 = 17.62 \text{ ohms (check).}$$

$$(d)\ \text{Equivalent secondary resistance} = 0.0881 + 8.81\left(\frac{1}{10}\right)^2$$

$$= 0.0881 + 0.0881 = 0.1762 \text{ ohm.}$$

Note that the equivalent primary and secondary resistances are to each other as the square of the ratio of turns.

In the actual transformer, due to eddy currents in the copper windings, and the slight core loss, already mentioned, the equivalent resistance, determined by short-circuit test, will be slightly greater than that computed from d-c measurements.

Prob. 36–8. In a short-circuit test of a 10-kva, 2200-220-volt transformer, the wattmeter connected in the high side, as in Fig. 29–8, indicates 100 watts, when the ammeter registers full-load current.

(*a*) What is the equivalent resistance in terms of the high-side winding?

(*b*) In terms of the low side?

Prob. 37–8. The resistance of the high side of a 5-kva, 500-100-volt transformer, as measured with d-c, is 0.7 ohm; and that of the low side is 0.036 ohm. Compute:

(*a*) The equivalent resistance of the transformer referred to the high side.

(*b*) Referred to the low side.

(c) The total I^2R loss at full load, using the value of equivalent resistance determined in (a).

(d) Total I^2R loss, using the value in (b).

(e) Total I^2R loss using the d-c resistances of the two windings.

Prob. 38–8. The resistance of the high and low sides of a 20-kva, 2000-100-volt transformer, as measured with d-c, are 1.45 and 0.0025 ohm, respectively. Repeat the calculations called for in Prob. 37–8.

22–8. Efficiency. The efficiency of the transformer is the ratio of the power output to the power input, or

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{Losses}}.$$

Since the efficiency is high, the losses are correspondingly small, generally from 1 to 3 or 5 per cent at rated full load. There is very little difference between the measured input and output under load. Thus, even a small error in instrument readings of input or output causes a correspondingly large error in the calculation of efficiency. (See Vol. I, Ch. XII, Art. 10.) Therefore, the efficiency of the transformer is seldom determined under load but is almost universally computed from a determination of the losses.

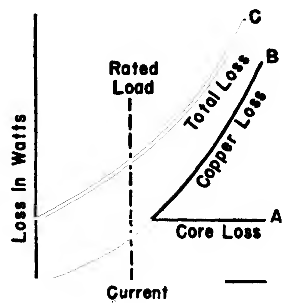


FIG. 30–8. Relation of losses to load on the transformer.

Figure 30–8 shows the form of the loss curves of a constant-potential transformer, plotted with load current, as abscissae, and losses in watts, as ordinates. Curve A is the core loss, which, as we have seen, is constant at all loads, and determined from the open-circuit test at normal voltage and frequency. Curve B is the copper loss in both primary and secondary windings, determined from the short-circuit test, or computed from the d-c resistance measurements for various load currents. Curve C is the sum of curves A and B, or the total loss at various load currents.

The efficiency of a step-down transformer at any power factor, computed from the open-circuit and short-circuit tests, is

$$\text{Efficiency} = \frac{E_sI_s \times (\text{PF})}{E_sI_s(\text{PF}) + \text{Core Loss(open circuit)} + \text{Copper Loss(short circuit)}} \tag{17–8}$$

The efficiency from the open-circuit and measured resistance tests, is

$$\text{Efficiency} = \frac{E_s I_s \times (\text{PF})}{E_s I_s(\text{PF}) + \text{Core Loss} + (I_p^2 R_p + I_s^2 R_s)} \quad (18\text{–}8)$$

The efficiency for a step-up transformer can be computed by using the current and voltage for the high side in the equations above.

Prob. 39–8. The core loss, in a 50-kva, 1100-220-volt, 60-cycle transformer, is 240 watts, as indicated by the wattmeter, when 220 volts at 60-cycles is applied to the low-side winding, high-side open circuited. On short-circuit test with full-load current, the wattmeter indicated 690 watts:

(*a*) Compute the efficiency at rated kva output.
(*b*) At half load.
(*c*) At $\frac{1}{4}$ load.
Unity power factor in each case.

Prob. 40–8. Compute the efficiency of the transformer in Prob. 39–8 at 0.7 power factor for conditions (*a*), (*b*), and (*c*) of that problem.

Prob. 41–8. The measured core loss in a 20-kva, 2300-230-volt transformer is 100 watts. The resistance of the high-side winding, as measured with d-c, is 1.98 ohms, and that of the low side is 0.025 ohm.

(*a*) Compute the efficiency at rated kva and 0.8 power factor.
(*b*) At $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ rated kva, and the same power factor.

23–8. All-Day Efficiency. The all-day efficiency of a transformer is the ratio of the watthours output to the watthours input, generally computed over an average 24-hour day, but may be computed for a period of a month, or a year. The all-day efficiency is much more important for distribution or lighting transformers, which operate for considerable periods with little or no load, than is the efficiency as ordinarily computed. It is really a measure of the performance of the transformer under service conditions, as a profit-making machine.

A transformer, or other machine, which operates continuously at full load over an ordinary 24 hour day is said to have a 100 per cent "load factor." Load factor of a machine, plant, or system is generally defined as the ratio of the average power to the maximum power, supplied during a specified period of time. In machines, such as lighting transformers, the load factor is generally low and the all-day efficiency is thereby reduced.

A transformer delivers energy only during the time it supplies a load. Similarly, energy is expended in I^2R losses, only when it

delivers power. Since the primary, however, is generally connected permanently to the supply line, energy is expended in core losses 24 hours a day, regardless of the load on the transformer.

$$\text{All-day efficiency} = \frac{\text{Watthours Output}}{\text{Watthours Output} + \text{Watthours Core Loss} + \text{Watthours Copper Loss}}$$

Example 5. What is the all-day efficiency of a 15-kva, 2300-230-volt, 60-cycle lighting transformer, with normal core loss of 200 watts, determined by the open-circuit test; and also a copper loss of 200 watts, determined, at full load current, by the short-circuit test? Compute the all-day efficiency, assuming the transformer operates 3 hours a day at full-non-inductive load, and 21 hours at zero load.

Solution: During the 24-hour period:

Energy output $= 15{,}000 \times 3 = 45{,}000$ watthours.
Energy expended in core losses $= 200 \times 24 =$ ~~4800 wat~~thours.
Energy expended in copper losses $= 200 \times 3 = 600$ watthours.

$$\text{All-day efficiency} = \frac{45{,}000}{45{,}000 + 4800 + 600} \times 100$$

$$= \frac{45{,}000}{50{,}400} \times 100 = 89.28 \text{ per cent.}$$

This is considerably less than the conventional efficiency of the transformer $\left(\frac{15{,}000}{15{,}000 + 200 + 200} \times 100, \text{ or } 97.5 \text{ per cent}\right)$.

However, since the lighting transformer generally operates under load, only a comparatively small part of the 24-hour period, it can be designed to give the **same** full-load efficiency and a **higher** all-day efficiency by **increasing** the copper losses (within limits) and **decreasing** the core losses. The I^2R losses can be increased by using a smaller wire for the coils. This permits a smaller window in the core and less weight of iron, while the cross section of the magnetic path and B_m remain the same, thereby decreasing the core loss.

If the core losses in the transformer of Example 5 above are reduced to 100 watts and the copper losses are increased to 300 watts, the total losses and efficiency are unchanged.

Now the energy lost in:

$$\text{Core loss} = 100 \times 24 = 2400 \text{ watthours.}$$

$$\text{Copper loss} = 300 \times 3 = 900 \text{ watthours.}$$

$$\text{All-day efficiency} = \frac{45{,}000}{45{,}000 + 2400 + 900} \times 100$$

$$= \frac{45{,}000}{48{,}300} \times 100 = 93.17 \text{ per cent.}$$

Prob. 42–8. The copper loss of a 10-kva transformer, as determined by short-circuit test, is 136 watts, while the core loss, determined on open-circuit, is 120 watts.

(*a*) Compute the all day efficiency on the assumption that the transformer operates 5 hours a day at full non-inductive load and the remainder of the 24-hour period at zero load.

(*b*) Repeat the computation in (*a*) on the basis of rated kva load and full-load current at 0.8 power factor for 5 hours.

Prob. 43–8. If the 10-kva transformer in Prob. 42–8 had been designed with the same total losses and efficiency, but with the core loss reduced to 86 watts and the copper loss at full-load current increased to 170 watts, what would be the all-day efficiency:

(*a*) Under condition (*a*) of that problem? (*b*) Under condition (*b*)? Compare the values of all-day efficiency, obtained by a redistribution of the constant and variable losses, with those obtained in Prob. 43–8.

Prob. 44–8(*a*). What will be the all-day efficiency of the transformer of Prob. 43–8, if it operates 5 hours a day at one-half rated kva ($\frac{1}{2}$ full load current) on non-inductive load, and remainder of the period at zero load?

(*b*) Repeat the computation in (*a*) on the basis of the same kva load at 0.8 power factor.

Prob. 45–8. From careful measurements over a 12-month period, a certain 1500-kva power transformer had a core loss of 102,890 kwhrs and a copper loss of 22,220 kwhrs. During this time, the input to the transformer was 3,183,000 kwhrs. Compute:

(*a*) The all-day efficiency for the average day.
(*b*) The average kw iron loss.
(*c*) Average kw copper loss.
(*d*) Average kw input.
(*e*) Average input as a per cent of rated load (1500 kw at unity power factor).
(*f*) Average core loss as a per cent of total loss.
(*g*) Average copper loss as a per cent of total loss.

24–8. Leakage Reactance. When the commercial transformer is supplying a load, the ratio of primary impressed voltage to secondary terminal voltage generally differs from the turn ratio. This is due to both resistance and reactance drop in the two windings.

It has, so far, been assumed that **all** the flux set up linking the primary turns also links the secondary turns. This is practically true at no load. This condition, however, does not exist under load, when the secondary is carrying current. It has previously been shown that the secondary current is practically 180° from the primary current, and that the secondary ampere-turns, N_sI_s, set up an mmf in the magnetic circuit opposing that of the primary. This counter mmf cannot reduce appreciably the total value of the flux, linking the primary winding, because enough more current will automatically flow in the primary to set up sufficient flux and a counter emf, nearly equal the line voltage. But the opposing mmf of the secondary ampere-turns results in forcing some of the primary flux, shown as ϕ_p, in Fig. 31–8, out of the core, so that it

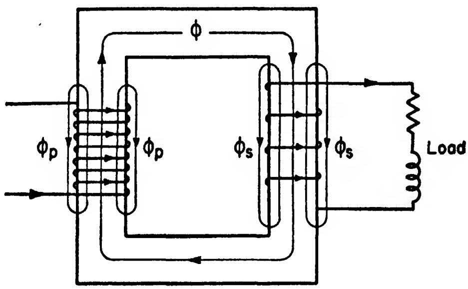


FIG. 31–8. Leakage flux in the transformer; mutual flux, ϕ; primary leakage flux, ϕ_p; secondary leakage flux, ϕ_s.

completes its circuit through the air without linking the secondary turns. As this flux links the primary turns only, it is called the **primary leakage flux.** This flux, ϕ_p, is proportional to the total ampere turns of the primary alone, and, therefore, induces an emf in the primary but not in the secondary. Since ϕ_p is set up by the primary ampere-turns, N_pI_p, it must be in phase with the total primary current, I_p. Also the emf, induced by this flux, is a counter emf, proportional to and lagging 90° behind the current. Since the counter emf must be balanced by an equal and opposite emf 90° ahead of the current, it is actually a reactance voltage, or an I_pX_p drop, in the winding, which must be overcome by a component of the impressed voltage.

Thus the effect of the primary leakage flux, ϕ_p, is to produce reactance in the primary circuit, known as the **primary leakage reactance,** which tends to limit the primary current and causes a voltage drop in the primary winding.

It should be noted that the mutual flux, ϕ, linking both coils is in phase with the no-load magnetizing current, while the primary leakage flux, ϕ_p, is in phase with the total primary current. Therefore, ϕ and ϕ_p are never in phase with each other. Since two fluxes cannot exist at the same instant in the same core, ϕ and ϕ_p are two components of the total flux in that part of the core, surrounded by the primary winding.

Also, since the mmf set up by the current in the secondary winding is opposite in direction to that of the mutual flux, ϕ, a leakage flux, ϕ_s, called the **secondary leakage flux** is forced out of the core and completes its circuit through the air, without linking the primary turns, also shown in Fig. 31-8. The flux, ϕ_s, is set up by the secondary ampere-turns, N_sI_s, acting alone, and is, therefore proportional to and in phase with the secondary current. This

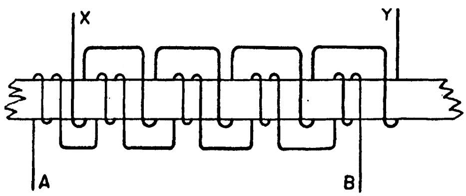


FIG. 32-8. The leakage flux is reduced by interleaving the primary winding AB and the secondary winding XY.

flux also sets up an emf, lagging 90° behind the current, which must be balanced by an equal and opposite emf, leading the secondary current by 90°. Thus a reactance-voltage, or I_sX_s drop, is produced in the secondary winding which tends to oppose the flow of current **from** the secondary, just as a reactance voltage drop exists in the armature coils of an alternator. This reactance, produced by the flux, ϕ_s, is called the **secondary leakage reactance.**

The mutual flux, ϕ, and the secondary leakage flux, ϕ_s, are also components of the total flux in that part of the core, surrounded by the secondary turns.

The effect of these leakage fluxes is to increase the voltage regulation of the transformer. To reduce the leakage flux and improve the regulation, the windings are divided, and alternate layers of primary and secondary turns are wound on the core, as previously described. The arrangement is represented diagrammatically in Fig. 32-8 and illustrated in Figs. 10-8 and 13-8. The paths of the leakage fluxes, ϕ_p and ϕ_s, are, therefore, much more complicated than shown in Fig. 31-8. Also the leakage flux

of each coil does not link all the turns of that coil. The effect of each of these leakage fluxes cannot readily be computed separately, but their combined effect can be obtained from the short-circuit test described in Art. 20. The method of computing leakage reactance is explained in Art. 27.

25–8. Complete Vector Diagram of the Transformer. The voltage and current relations, discussed in the previous article, are shown for a 2 : 1 ratio transformer in the vector diagram of Fig. 33–8.

The secondary current, I_s, is shown, lagging the secondary terminal voltage E_s, by the angle ϕ_s°; cos ϕ_s° being the power factor of the load. I_sR_s represents the secondary resistance drop in phase with I_s. And I_sX_s represents the secondary reactance drop, or the voltage, necessary to overcome the emf induced by the secondary leakage flux, ϕ_s, and leading both ϕ_s and I_s by 90°. The vector sum of E_s, I_sR_s and I_sX_s is equal to E_s', the emf induced in the secondary by the mutual flux, ϕ, and lagging it by 90°. The primary emf, E_p', is also induced by the mutual flux and lags it by the same angle. Since E_p' must be balanced by an equal and opposite emf before any current can flow in the transformer, it is revolved 180° and drawn as $-E_1'$.

Note that in any transformer diagram, the vectors $-E_p'$ and E_s' are always directly proportional to the turn ratio, and at 90° to the mutual flux, ϕ, which is common to both coils. The vector, ϕ, therefore, is always taken as the reference vector.

The exciting current, I_E, is shown with its magnetizing component, I_m, in phase with ϕ. Since the secondary ampere-turns, N_sI_s must be balanced by an equal and opposite number of primary ampere-turns, the primary load current, I_p', is drawn 180° from I_s (in this case equal to $\frac{1}{2}$ of I_s). The total primary current is now the vector sum of I_p' and I_E, shown as I_p, in phase with the primary leakage flux, ϕ_p. The primary resistance drop, I_pR_p, is drawn in phase with the total primary current, and I_pX_p, the voltage necessary to overcome the emf, due to the primary leakage flux, is drawn leading I_p by 90°. The vector sum of the three voltages, $-E_p'$, I_pR_p and I_pX_p is equal to the impressed primary voltage, E_p, necessary to supply a current of I_s amperes from the secondary to a load at a terminal voltage E_s.

In the commercial transformer, the values of I_E, the resistance and reactance drops are usually so small, compared to the other vector quantities, they are purposely exaggerated in Fig. 33–8 for

clarity in the diagram. Therefore, the values of E_p and E_s, as shown, differ more widely from the turn ratio than is the case in the actual constant-potential transformer.

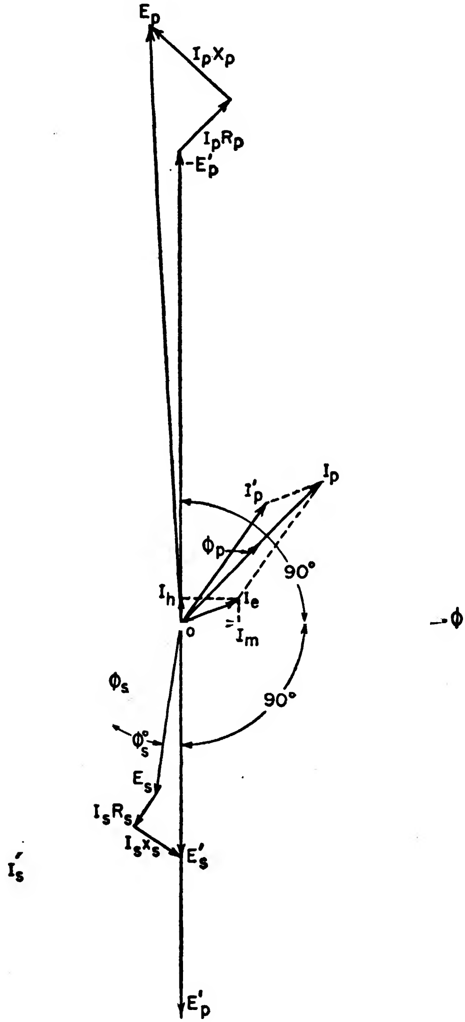


FIG. 33–8. Complete vector diagram of the transformer.

26–8. Practical Transformer Diagram. A simplified diagram of Fig. 33–8 is shown in Fig. 34–8, in which the exciting current is omitted. This causes only negligible error in computing the performance of the machine, since I_e usually differs in phase from

I_p by a relatively large angle and is a very small percentage of the full-load primary current. Therefore, in Fig. 34–8, I_p is drawn exactly 180° from I_s, its value being in inverse ratio to the number

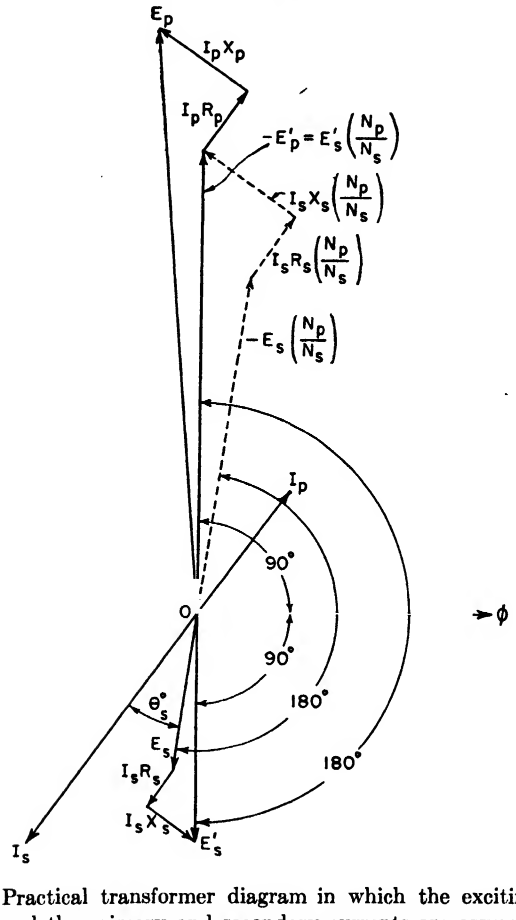


FIG. 34–8. Practical transformer diagram in which the exciting current is neglected and the primary and secondary currents are assumed to be displaced exactly 180°. The secondary *IR* and *IX* drops are revolved 180° and represented as though they took place in the primary.

of turns, or $I_p = I_s \times \dfrac{N_s}{N_p}\cdot$ The resistance and reactance drops in both coils are drawn respectively in phase and 90° leading their respective currents. Thus the *IR* drops, and also the *IX* drops, in both primary and secondary windings are exactly 180° to each other respectively.

If now the secondary voltage vectors E_s, I_sR_s, I_sX_s and E'_s are multiplied by ratio of turns, $\left(\frac{N_p}{N_s}\right)$, and are rotated 180°, as indicated in Fig. 34–8, E'_s becomes equal to and takes the position of $-E'_p$. I_sR_s and I_sX_s are also correspondingly increased, and are parallel respectively to I_pR_p and I_pX_p. Thus in Fig. 34–8, the secondary IR and IX drops and the secondary terminal voltage are shown in terms of primary values. This simplifies both the vector diagram and the method of computing the voltage regulation, regardless of the voltage, or turn, ratio of the transformer.

Conversely, if each of the primary voltages and voltage drops is multiplied by the inverse ratio of turns, $\left(\frac{N_s}{N_p}\right)$, and rotated 180°, $-E'_p$ becomes equal to and takes the position of E'_s. The primary voltages and voltage drop are thus shown in terms of secondary values. And the voltage regulation can now be computed in terms of the secondary.

27–8. Equivalent Resistance, Reactance and Impedance. The secondary resistance and reactance drops in Fig. 34–8 are shown in terms of the primary, and are in phase respectively with the primary resistance and reactance drops. The two resistance drops, therefore, can be combined into a single resistance drop, and the two reactance drops likewise can be combined into a single reactance drop, without changing the values, E_p and $E_s\left(\frac{N_p}{N_s}\right)$ in the diagram, **provided they are expressed in terms of the same current.**

Since $I_s = I_p\left(\frac{N_p}{N_s}\right)$, **the secondary resistance drop, in terms of the primary current,** equals

$$I_p\left(\frac{N_p}{N_s}\right) \times R_s\left(\frac{N_p}{N_s}\right) = I_pR_s\left(\frac{N_p}{N_s}\right)^2.$$

The combined resistance drop in the primary, shown in Fig. 35–8, equals

$$I_pR_p + I_pR_s\left(\frac{N_p}{N_s}\right)^2 = I_p\left[R_p + R_s\left(\frac{N_p}{N_s}\right)^2\right].$$

But,

$$R_p + R_s\left(\frac{N_p}{N_s}\right)^2 = R_{ep},$$

the equivalent primary resistance, as given in Eq. (14), Art. 21.

Therefore, the combined resistance drop, in terms of primary values, equals I_pR_{ep} as shown.

The **secondary reactance drop in terms of primary current,** equals

$$I_p\left(\frac{N_p}{N_s}\right) \times X_s\left(\frac{N_p}{N_s}\right) = I_pX_s\left(\frac{N_p}{N_s}\right)^2$$

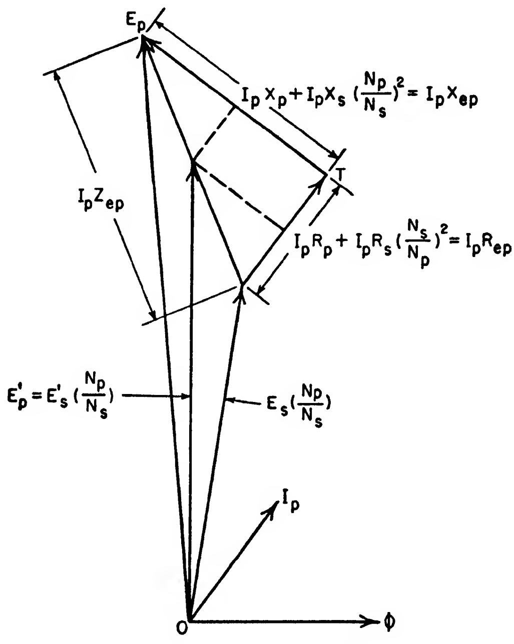


FIG. 35–8. The equivalent primary voltage drops are represented by a single triangle.

Similarly, the combined reactance drop in terms of primary values, also shown in Fig. 35, equals

$$I_pX_p + I_pX_s\left(\frac{N_p}{N_s}\right)^2 = I_p\left[X_p + X_s\left(\frac{N_p}{N_s}\right)^2\right].$$

The expression $\left[X_p + X_s\left(\frac{N_p}{N_s}\right)^2\right]$ is similar in form to that for the equivalent primary resistance, and is called the "**equivalent primary reactance,**" X_{ep}, or

$$X_{ep} = X_p + X_s\left(\frac{N_p}{N_s}\right)^2 \cdots \quad (19\text{–}8)$$

The combined reactance drop in terms of primary values is, therefore, I_pX_{ep}, as shown.

It should also be noted here that the reactance of both coils, expressed in terms of the secondary, and called the "**equivalent secondary reactance,**" is expressed as,

$$X_{es} = X_s + X_p\left(\frac{N_s}{N_p}\right)^2 \cdots \quad (20\text{–}8)$$

Compare equations (18) and (19) above with equations (14) and (15), Art. 21–8.

The **impedance drop** in both coils, in terms of the primary, is the vector sum of the equivalent primary resistance and reactance drops. This is known as the **equivalent primary impedance drop,** I_pZ_{ep}, or $I_pZ_{ep} = \sqrt{I_p^2R_{ep}^2 + I_p^2X_{ep}^2}$. And the equivalent primary impedance,

$$Z_{ep} = \sqrt{R_{ep}^2 + X_{ep}^2} \cdots \quad (21\text{–}8)$$

Similarly, the impedance drop in both coils, in terms of the secondary, is called the **equivalent secondary impedance drop,** I_sZ_{es} or $I_sZ_{es} = \sqrt{I_s^2R_{es}^2 + I_s^2X_{es}^2}$; and the **equivalent secondary impedance,**

$$Z_{es} = \sqrt{R_{es}^2 + X_{es}^2} \cdots \quad (22\text{–}8)$$

When the transformer is on short circuit test, called the "impedance test," the secondary terminal voltage is zero; and the impressed voltage is just sufficient to overcome the impedance drop in both primary and secondary windings. This is represented in Fig. 36–8, the equivalent diagram of Fig. 35–8, but in which the secondary voltage, or $E_s\left(\frac{N_p}{N_s}\right)$, is zero.

The impressed voltage,

$$E_p = I_pZ_{ep} = \sqrt{I_p^2R_{ep}^2 + I_p^2X_{ep}^2} \cdots \quad (23\text{–}8)$$

Thus the equivalent primary impedance, $Z_{ep} = \dfrac{E_p}{I_p} \cdots$ (24–8)

Also $$I_pX_{ep} = \sqrt{I_p^2Z_{ep}^2 - I_p^2R_{ep}^2}.$$

And $$X_{ep} = \sqrt{Z_{ep}^2 - R_{ep}^2} \cdots \quad (25\text{–}8)$$

The voltmeter indication, E_p, on short-circuit test, Fig. 29–8, is the equivalent-primary-impedance drop (Eq. 23 above). Since

the wattmeter indication W, on short circuit is the total I^2R loss in both windings, the equivalent-primary resistance drop,

$$I_pR_{ep} = \frac{W}{I_p} = \frac{I_p^2R_{ep}}{I_p} \cdots \tag{26-8}$$

Also from the instrument reading on short circuit, the equivalent primary-reactance drop can now be expressed as

$$I_pX_{ep} = \sqrt{E_p^2 - \left(\frac{W}{I_p}\right)^2} \cdots \tag{27-8}$$

The voltage drops at full-load current, E_p, $\frac{W}{I_p}$ and I_pX_{ep}, are called, respectively, the equivalent primary impedance, resistance and reactance in **volts**. The voltmeter and wattmeter indications are also spoken of as the "impedance volts" and "impedance watts." Dividing the voltage drop by I_p, or $\frac{E_p}{I_p}$, $\frac{W}{I_p^2}$ and $\frac{I_pX_{ep}}{I_p}$ are, respectively, the equivalent primary impedance, resistance and reactance in **ohms.** If the voltage drops above are multiplied by tne inverse ratio of turns, $\left(\frac{N_s}{N_p}\right)$, the corresponding values in terms of the secondary, or low side, are obtained. Also, multiplying the above values of impedance, resistance and reactance in ohms by the square of the inverse ratio of turns, $\left(\frac{N_s}{N_p}\right)^2$, converts them to corresponding values in terms of the secondary.

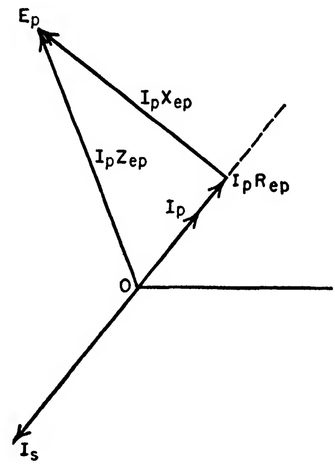


FIG. 36–8. On short circuit the secondary voltage is zero and the impressed voltage E_p is equal to the equivalent primary impedance drop at full-load current.

Example 6. From the short-circuit test of the 15-kva, 2300-230-volt transformer of Example 5, at rated full-load current, the readings of the instruments in the high side are: $I_p = 6.52$ amperes; $E_p = 126.5$ volts; $W = 200$ watts. Compute:

(*a*) The equivalent primary impedance, resistance and reactance in volts.

(*b*) In ohms.

(*c*) The equivalent-secondary impedance, resistance and reactance in volts.
(*d*) In ohms.

Solution:

(*a*) Impedance in volts $= I_pZ_{ep} = E_p = 126.5$ volts.

$$\text{Resistance in volts} = I_pR_{ep} = \frac{W}{I_p} = \frac{200}{6.52} = 30.67 \text{ volts.}$$

$$\text{Reactance in volts} = I_pX_{ep} = \sqrt{E_p^2 - \left(\frac{W}{I_p}\right)^2}$$

$$= \sqrt{126.5^2 - \left(\frac{200}{6.52}\right)^2} = 122.72 \text{ volts.}$$

(*b*) $$\text{Impedance in ohms} = Z_{ep} = \frac{E_p}{I_p} = \frac{126.5}{6.52} = 19.4 \text{ ohms.}$$

$$\text{Resistance in ohms} = R_{ep} = \frac{W}{I_p^2} = \frac{200}{6.52^2} = 4.7 \text{ ohms.}$$

$$\text{Reactance in ohms} = X_{ep} = \frac{I_pX_{ep}}{I_p} = \frac{122.72}{6.52} = 18.82 \text{ ohms.}$$

(*c*) $$\text{Impedance in volts} = E_p\left(\frac{N_s}{N_p}\right) = 126.5 \times \frac{1}{10} = 12.65 \text{ volts.}$$

$$\text{Resistance in volts} = \frac{W_p}{I_p}\left(\frac{N_s}{N_p}\right) = 30.67 \times \frac{1}{10} = 3.067 \text{ volts.}$$

$$\text{Reactance in volts} = I_pX_{ep}\left(\frac{N_s}{N_p}\right) = 122.72 \times \frac{1}{10} = 12.272 \text{ volts.}$$

(*d*) $$\text{Impedance in ohms} = Z_{ep}\left(\frac{N_s}{N_p}\right)^2 = 19.34 \times \left(\frac{1}{10}\right)^2 = 0.194 \text{ ohm.}$$

$$\text{Resistance in ohms} = R_{ep}\left(\frac{N_s}{N_p}\right)^2 = 4.7 \times \left(\frac{1}{10}\right)^2 = 0.047 \text{ ohm.}$$

$$\text{Reactance in ohms} = X_{ep}\left(\frac{N_s}{N_p}\right)^2 = 18.82 \times \left(\frac{1}{10}\right)^2 = 0.1882 \text{ ohm.}$$

Prob. 46–8. A transformer, rated at 350 kva, 60 cycles, 1100 to 2300 volts, with the high side short circuited, has a total I^2R loss of 1792 watts, and requires an applied voltage of 38.4 volts at full load current. Compute the equivalent impedance in ohms, referred to (*a*) the low side; (*b*) the high side.

Prob. 47–8. For the transformer in Prob. 46–8, compute the equivalent resistance in ohms, referred to (*a*) the low side; (*b*) the high side.

Prob. 48–8. Compute the equivalent reactance of the transformer of Prob. 46–8: (*a*) In ohms, referred to the low side. (*b*) In ohms, referred to the high side.

Prob. 49–8. The low side of a transformer, rated at 5 kva, 2400-240-volts, 60-cycles is short circuited through an ammeter. The voltage, applied to the high side, necessary to produce rated full load current, is 72 volts. The resistance of high and low windings is 11.53 and 0.1153 ohm respectively.

(*a*) Compute the equivalent high-side reactance in volts and in ohms.
(*b*) Compute the values in (*a*) referred to the low side.

Prob. 50–8. On "impedance test," the low-side of a 200 kva, 10,000-2300-volt 60-cycle transformer is short circuited. When the voltage applied to the high side is adjusted to force full load current through the windings, the voltmeter and wattmeter indicate 450 volts and 3000 watts, respectively. Compute for the high side winding:

(*a*) The equivalent impedance, resistance and reactance in volts.
(*b*) In ohms.
(*c*) Repeat the computations in (*a*) and (*b*) for the low side winding.

28–8. Regulation. The voltage regulation of a constant potential transformer is defined as the difference between the no-load and rated full-load values of the secondary terminal voltage, expressed in per cent of the rated full-load secondary voltage. That is,

$$\text{Per Cent Regulation} = \frac{\text{No-load secondary voltage} - \text{full load secondary voltage}}{\text{full-load secondary voltage}} \times 100$$

The primary impressed voltage is assumed to be held constant at such a value that the transformer delivers rated kva at rated secondary voltage and at a specified power factor.

The regulation can, however, be computed in terms either of the secondary, or of the primary winding, provided the equivalent IR and IX drops for that winding are known.

Thus in Fig. 37–8, if OE_t is the rated secondary voltage, IR and IX represent the equivalent secondary resistance and reactance drops respectively, and OI the rated full-load secondary current, then OE_i represents the no-load secondary voltage. If OE_t is the assumed rated primary voltage, IR and IX are the voltage drops in terms of the primary, and OI the rated primary current, then OE_i is the required voltage impressed on the primary. Figure 37–8(*a*) shows the diagram for unity-power factor load, and (*b*) for a lagging power factor, cos θ. The method of computing voltage regulation from the short-circuited test data is shown in the example below.

Example 7. In testing a 20-kva, 2300-230-volt, 60-cycle transformer for regulation, the low side is short circuited and instruments of proper range are connected in the primary, or high side, as in Fig. 29–8. When sufficient voltage is applied to send full-load current through the windings, the instrument readings are 8.7 amperes, 92 volts and 390 watts. Compute the voltage regulation on 0.8 power factor load:

(*a*) In terms of the primary winding. (*b*) In terms of the secondary winding.

Solution:

(*a*) High-side winding.

Equivalent primary impedance in volts $= I_p Z_{ep} = 92$ volts.

Equivalent primary resistance in volts $= \dfrac{W}{I_p} = \dfrac{390}{8.7} = 44.82$ volts

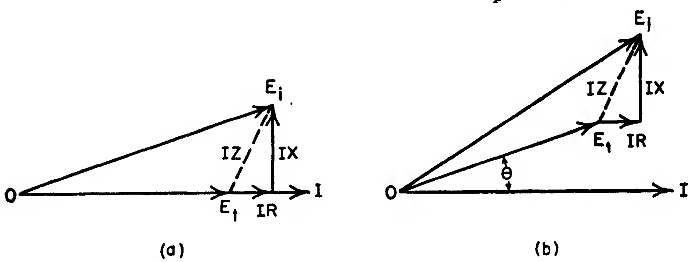


FIG. 37–8. Regulation of the transformer in which E_t represents the rated voltage (primary or secondary) at full-load current; IR and IX the equivalent resistance and reactance drops, and E_i the no-load voltage. (*a*) Unity power factor. (*b*) Lagging power factor.

$$\text{Equivalent primary reactance in volts} = \sqrt{92^2 - 44.82^2} = 80.35 \text{ volts.}$$

$$OE_i = \sqrt{(E_t \cos\theta + IR)^2 + (E_t \sin\theta + IX)^2}.$$

$$OE_i = \sqrt{(2300 \times 0.8 + 44.82)^2 + (2300 \times 0.6 + 80.35)^2} = 2384.3 \text{ volts.}$$

$$\text{Regulation} = \frac{2384.3 - 2300}{2300} \times 100 = 3.66 \text{ per cent.}$$

(*b*) Low-side winding.

$$\text{Equivalent secondary resistance in volts} = 44.82 \times (\tfrac{1}{10}) = 4.482 \text{ volts.}$$

$$\text{Equivalent secondary reactance in volts} = 80.35 \times (\tfrac{1}{10}) = 8.035 \text{ volts.}$$

$$OE_i = \sqrt{(230 \times 0.8 + 4.482)^2 + (230 \times 0.6 + 8.035)^2} = 238.43 \text{ volts.}$$

$$\text{Regulation} = \frac{238.43 - 230}{230} \times 100 = 3.66 \text{ per cent.}$$

In the following problems, show a vector diagram for each computation for regulation.

Prob. 51–8. Compute the regulation of the transformer in Example 7: (*a*) For load at unity power factor. (*b*) For load at 0.8 leading power factor.

Prob. 52–8. Compute the regulation of the transformer in Example 7 for loads of: (*a*) 0.7 lagging power factor; (*b*) 0.7 leading power factor.

Prob. 53–8. Compute the regulation of the transformer in Example 6: (*a*) For unity power factor load. (*b*) For 0.6 lagging power factor load. (*c*) For 0.6 leading power factor load.

29–8. Per Cent Resistance and Reactance. Regulation. The computation of transformer regulation is much simplified, if the equivalent resistance and reactance drop at full-load current are expressed as a percentage of the normal, or rated, voltage. When so expressed, they are called the "per cent resistance" and "per cent reactance" of the transformer, and apply either to the high or to the low voltage side of the transformer. If the equivalent primary resistance drop at full-load current is 1.3 per cent of the normal primary voltage, the equivalent secondary resistance drop is also 1.3 per cent of the rated secondary voltage, and the transformer is said to have a resistance of 1.3 per cent.

Thus, from the short-circuit data of the 10 to 1 ratio transformer of Example 7, the primary impedance drop is 92 volts, the resistance drop is $\frac{390}{8.7}$, or 44.82 volts, and the reactance drop is $\sqrt{92^2 - 44.82^2}$, or 80.35 volts.

In terms of the primary winding:

$$\text{Impedance} = \frac{92}{2300} \times 100 = 4 \text{ per cent.}$$

$$\text{Resistance} = \frac{44.82}{2300} \times 100 = 1.948 \text{ per cent.}$$

$$\text{Reactance} = \frac{80.35}{2300} \times 100 = 3.49 \text{ per cent.}$$

In terms of the secondary winding:

$$\text{Impedance} = \frac{9.2}{230} \times 100 = 4 \text{ per cent.}$$

$$\text{Resistance} = \frac{4.482}{230} \times 100 = 1.948 \text{ per cent.}$$

$$\text{Reactance} = \frac{8.035}{230} \times 100 = 3.49 \text{ per cent.}$$

Per cent reactance can also be computed as $\sqrt{4^2 - 1.948^2} = 3.49$ per cent.

This method of computing regulation is indicated in Fig. 38–8, in which the rated voltage is taken as the reference vector and the per cent resistance and reactance are separated into components in phase, and at 90° with this voltage. In the diagram,

Power factor of the load = $\cos\theta$, lagging.
OE_t = rated full-load voltage (either high or low side) = 100 per cent.
$E_tb = q_r$ = per cent resistance.
$bE_i = q_x$ = per cent reactance.
$E_ta = q_r \cos\theta$. $ab = q_r \sin\theta$.
$bc = q_x \sin\theta$. $cE_i = q_x \cos\theta$.
OE_i = induced, or no-load, voltage in per cent.

Thus

$$OE_i = \sqrt{(OE_t + q_r\cos\theta + q_x\sin\theta)^2 + (q_x\cos\theta - q_r\sin\theta)^2}.$$

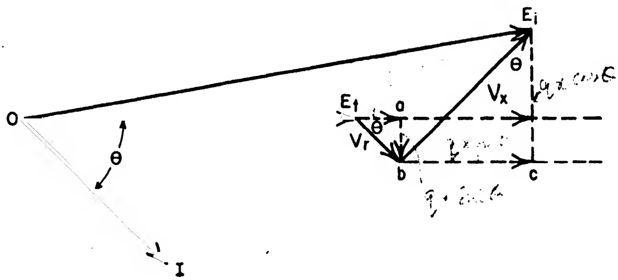


FIG. 38–8. Method of determining regulation by use of percentage values.

As the second term $(q_x\cos\theta - q_r\sin\theta)$ is very small in the constant-potential transformer, the above equation may be written with slight error,

$$OE_i = OE_t + q_r\cos\theta + q_x\sin\theta + \frac{(q_x\cos\theta - q_r\sin\theta)^2}{200} \cdots \tag{28-8}$$

Since OE_t = 100 per cent,

$$\text{Per cent regulation} = q_r\cos\theta + q_x\sin\theta + \frac{(q_x\cos\theta - q_r\sin\theta)^2}{200}. \tag{29-8}$$

At unity power factor load $\cos\theta = 1$ and $\sin\theta = 0$, so equation (28) reduces to

$$\text{Per cent regulation} = q_r + \frac{(q_x)^2}{200} \cdots \tag{30-8}$$

On leading power factor, equation (28) can be shown to be:

$$\text{Per cent regulation} = q_r \cos\theta - q_x \sin\theta + \frac{(q_x \cos\theta + q_r \sin\theta)^2}{200} \tag{31-8}$$

(Note change in signs in equation (31) from those in equation (28).)

Example 8. Using per cent resistance and reactance as computed above, determine the regulation of the 20 kva transformer of Example 7.

(*a*) For 0.8 lagging power factor.
(*b*) For unity power factor.
(*c*) For 0.8 leading power factor.

Solution:

(*a*) From eq. (28) above (remembering that $\sin\theta = 0.6$ when $\cos\theta = 0.8$),

$$OE_i = 100 + 1.948 \times 0.8 + 3.49 \times 0.6 + \frac{(3.49 \times 0.8 - 1.948 \times 0.6)^2}{200}$$

$$= 100 + 1.558 + 2.094 + \frac{(2.792 - 1.169)^2}{200} = 103.652 + .013$$

$$= 103.665 \text{ volts.}$$

$$\text{Regulation} = \frac{103.665 - 100}{100} \times 100 = 3.665 \text{ per cent.}$$

Or, from Eq. (29), $1.558 + 2.094 + .013 = 3.665$ per cent.

(*b*) From Eq. (30), $1.948 + \dfrac{(3.49)^2}{200} = 1.948 + 0.06 = 2$ per cent.

(*c*) From Eq. (31),

$$1.558 - 2.094 + \frac{(2.792 + 1.169)^2}{200} = 1.558 - 2.094 + 0.08$$

$$= 0.458 \text{ per cent.}$$

In solving the following problems show a diagram similar to Fig. 38–8, for each computation for regulation.

Prob. 54–8. (*a*) Compute the resistance and reactance of the transformer in Example 6 in per cent.

(*b*) From the results in (*a*), compute the regulation at unity power factor.

(*c*) At 0.7 lagging power factor. (*d*) At 0.7 leading power factor.

Prob. 55–8. A 500-kva transformer has a resistance of 0.52 per cent and a reactance of 4.5 per cent. (*a*) Compute the regulation at unity power factor. (*b*) At 0.8 lagging power factor. (*c*) At 0.8 leading power factor.

Prob. 56–8. The test of a 1000 kva, 110,000-22,000-volt, 60-cycle transformer gives the following data:

Impedance watts (on short circuit) = 7330.

Impedance volts = 5 per cent of rated voltage.

Compute the per cent resistance and reactance, and the regulation of the transformer on 0.8 power factor lagging load.

Prob. 57–8. An impedance test of a 100 kva, 11,000-2200-volt, 60-cycle transformer, with the low side winding short circuited, gives the following results at full load current. Applied voltage = 352 volts. Power input = 1046 watts.

(*a*) Compute the per cent resistance, reactance, and the regulation at unity power factor.

(*b*) At 0.6 leading power factor.

Prob. 58–8. The voltage applied to the high side of a 50-kva, 2400-240-volt, 60-cycle transformer on impedance test is 82.5 volts at full load current. The resistance of the high side is 0.832 ohm, and of the low side 0.012 ohm. Compute the per cent resistance and reactance and the regulation at 75 per cent lagging power factor.

30–8. Parallel Operation. When the primaries of two or more transformers are connected in parallel across a supply system of proper voltage, and their secondaries are connected to separate, or independent, circuits, as in Fig. 39–8, they operate independently of each other, if a constant supply voltage is maintained. When the secondaries are also connected in parallel, as in Fig. 40–8, the transformers must have the same rated primary and secondary voltages; that is, the turn ratio of each must be the same. Otherwise, the secondary emfs, E_{2A} and E_{2B}, will be unequal, and a current will circulate between the machines, limited only by the impedance of their windings. Since this impedance is small, such a slight difference in turn ratio, or secondary emf, will cause a large current to circulate.

Before connecting the secondary windings in parallel, their terminals must, of course, be "phased" for like polarity.

In order that the transformers may properly divide the load, the impedance drop in each must be the same at all loads. Thus each transformer must have the same per cent impedance. Furthermore, their resistances and reactances must be similar; that is, the ratio of R to X in each must be the same, or circulating currents will cause the power factor of the transformers to differ.

In general, transformers will share the load in inverse proportion to their respective impedances. For instance, if two transformers of the same rated kva are to equally divide the load, they should

have the same impedance. If one has twice the kva rating of the other, it should supply twice the current. A proportional division of the load thus requires that its impedance shall be one half that of the other, since the impedance drops must be the same.

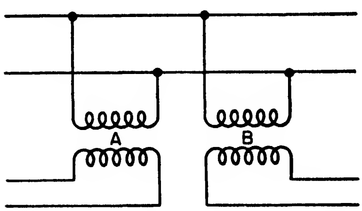


Fig. 39–8. Transformers connected in parallel to the same mains and supplying independent loads.

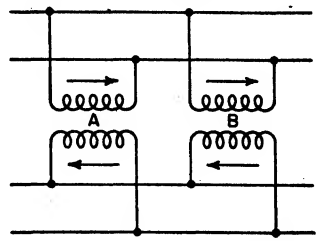


Fig. 40–8. When parallel-connected transformers are to supply the same secondary circuit, the secondaries must first be "phased" for like polarity.

31–8. The Auto-Transformer. In an ordinary transformer, the primary and secondary coils are coupled, or connected only by the magnetic flux. In an auto-transformer, the coils are magnetically coupled and electrically connected, and one of the coils is common to both the primary and secondary circuits.

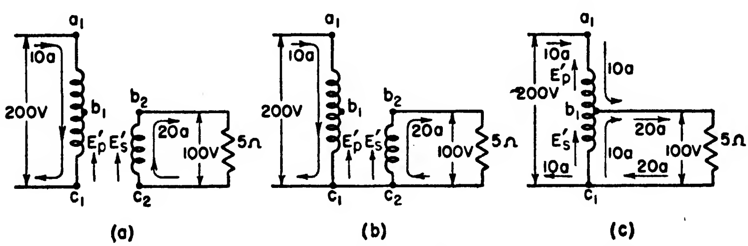


Fig. 41–8. Relation of currents and voltages in the auto-transformer.

Consider Fig. 41–8 and neglect the exciting current, the losses and the voltage drop in the windings. An ordinary transformer is indicated in Fig. 41–8(a) with the primary a_1c_1 connected to 200-volt mains. The secondary, b_2c_2, having just half the turns of the primary, supplies 20 amperes to a 5-ohm resistance load at 100 volts. The arrows indicate the direction of currents in the two coils at the same instant. Note that these currents are in opposite direction; 10 amperes flowing down in a_1c_1 and 20 amperes flowing

up in b_2c_2. Also note that the induced emfs E'_p and E'_s are in the same direction or **up** in both primary and secondary; E'_p opposed to, and E'_s in the same direction as the currents in their respective circuits.

If the coil terminals c_1 and c_2 are joined together, as in Fig. 41–8(b), there will be no difference of potential between b_1, the mid-point of the primary coil, and b_2, since the induced emfs from c_1 to b_1 and from c_2 to b_2 are **equal** and in the **same** direction at any instant. Therefore b_1 and b_2 can be electrically connected, without disturbing either the voltages, the primary current, or the current to the load. Thus the **combined current** in b_1c_1 and b_2c_2 is the **difference** of the two currents, or 10 amperes flowing **up.** But this is the normal current in the primary coil alone. The secondary coil, therefore, can be removed and the secondary circuit tapped to points c_1 and b_1, as in Fig. 41–8(c), and 10 amperes will flow **down** in a_1b_1 and **up** in b_1c_1, as indicated.

A transformer, tapped in this manner, is known as an "auto-transformer," or "compensator." That part of the coil, a_1b_1, is generally called the primary, and b_1c_1, common to both circuits in this case, the secondary.

Neglecting the losses in the auto-transformer of Fig. 41–8(c):

The power supplied to the transformer	$= 200 \times 10$, or 2000 watts.
The power delivered by the transformer	$= 100 \times 20$, or 2000 watts.
The power in the primary, a_1b_1	$= 100 \times 10$, or 1000 watts.
The power in the secondary, b_1c_1	$= 100 \times 10$, or 1000 watts.

Note that the power **transformed** by the two coils, is only 1000 watts, while the power **transferred** directly from the primary to the secondary circuit is the remainder, or 1000 watts.

Assuming the same I^2R loss in each winding of the 2 : 1 transformer of Fig. 41–8(a), there is a saving of one half the copper losses in the auto-transformer connection of Fig. 41–8(c), while the core loss remains the same. The smaller the voltage ratio, the greater is the saving in copper loss, and the greater is the amount of power transferred.

An ordinary transformer may also be connected as an auto-transformer. Consider a 5-kva 1000-100-volt transformer, so connected that the low-voltage, or secondary, coil is joined in additive series to the primary; that is, with primary terminal, B, connected to secondary terminal, X (see Art. 17), as in Fig. 42–8(a). Thus the induced emfs E'_p and E'_s are in the same

direction in the circuit, as indicated. When 1000 volts is applied across the primary terminal, A, B, the secondary voltage, AY, is raised to 1100 volts.

The rated current in the primary winding equals $\frac{5000}{1000}$, or 5 amperes; and in the secondary, $\frac{5000}{100}$, or 50 amperes. The transformer, therefore, can deliver 50 amperes, at 1100 volts, to the secondary circuit, without overloading the secondary coil XY.

Neglecting the losses, $E_pI_p = E_sI_s$, or $I_p = \dfrac{1100 \times 50}{1000}$, or 55 amperes, as shown.

Since the current in the coil common to both circuits (the primary in this case), is always the **difference** between that in the

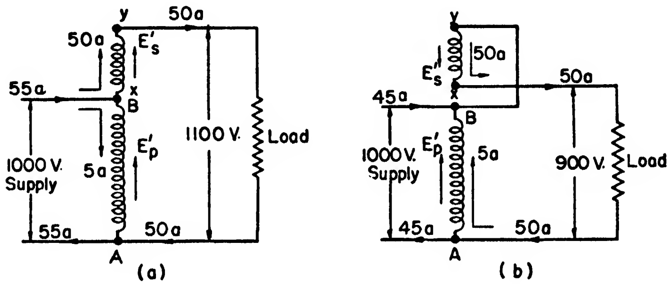


FIG. 42–8. An ordinary transformer connected as an auto-transformer. (*a*) To raise or "boost" the primary voltage. (*b*) To reduce or "buck" the primary voltage.

primary and secondary circuits, the current in AB is $55 - 50$, or 5 amperes flowing **down,** while that in the secondary, XY, is 50 amperes flowing in the opposite direction, or **up.** This must be so, since the NI in one coil must be equal and opposite to those in the other.

Thus for the transformer connected as in Fig. 42–8(*a*), on unity power factor load:

$$\begin{aligned}
&\text{The power supplied} &&= 1000 \times 55 = 55000 \text{ watts.}\\
&\text{The power delivered} &&= 1100 \times 50 = 55000 \text{ watts.}\\
&\text{The power in } AB &&= 1000 \times 5 = 5000 \text{ watts.}\\
&\text{The power in } XY &&= 100 \times 50 = 5000 \text{ watts.}
\end{aligned}$$

The power transformed, in this case, is 5000 watts, the normal rating of the transformer, while the power transferred directly from one circuit to the other, is 50,000 watts, or 50 kw. And

55 kw can be supplied to the secondary circuit at a voltage ratio of 10 to 11, without overloading the coils in the 5 kva transformer.

If the connections to the secondary coil, XY, are reversed, as indicated in Fig. 42–8(*b*), its induced emf opposes, or "bucks" that of coil AB, and the secondary voltage, AX, is reduced to 900 volts. Rated current of XY, or 50 amperes, can again be delivered to the secondary circuit. The current in primary circuit, I_p equals $\dfrac{900 \times 50}{1000}$, or 45 amperes. Current in AB equals $50 - 45$, or 5 amperes, flowing **up** in this case; while that in XY is now flowing **down,** or in the opposite direction. Note that the current in both coils in Fig. 42–8(*b*) are reversed in direction from that in (*a*). **When the connections to either coil in an auto-transformer are reversed, the direction of current in both coils is reversed.**

In Fig. 42–8(*a*), the power **transformed** is again 5000 watts, while the power **transferred** is reduced to 40,000 watts, or 8 times the normal rating. And 45,000 watts, or 45 kw, can now be supplied to the secondary circuit without overloading the transformer.

Note, in both connections of Fig. 42–8, that when one coil carries rated current, the other is similarly loaded. This is not always the case when an ordinary transformer, having several coils, is connected as an auto-transformer. The current in a primary, a secondary, or in a coil common to both circuits, may limit its rating. Thus, for any given connection, the current in **all** the coils must be computed before the rating can be determined.

Auto-transformers are seldom, if ever, used in place of the ordinary 2300-230-volt lighting transformer. In this case, the voltage ratio is high; so the saving in copper and the power transferred is relatively small, and there is little advantage in its use. Furthermore, the high side is electrically connected to the secondary; so a dangerous potential exists on the secondary circuit unless properly grounded.

Auto-transformers are generally used only where the ratio of voltage transformation is comparatively low. They are used largely as motor starters; in which case, they are called "compensators."

Prob. 59–8. A 440-110-volt auto-transformer at full non-inductive load supplies 5 kva at 110 volts to the secondary circuit.

(*a*) Compute the currents in the primary and secondary coils and the power transformed.

(*b*) What is the voltage and kva rating, if used as an ordinary transformer?

Prob. 60–8. An auto-transformer steps the voltage from 440 to 550 volts and supplies a 5-kva load at unity power factor. For this transformer, compute the values called for in (*a*) and (*b*) of Prob. 59–8.

Note: In the following problems, H_1 and H_2 are the high-side supply line, and L_1 and L_2 the low-side mains. Corresponding coil terminals are A_1, A_2, X_1 and X_2 (see Art. 17). Show diagram of connections for each case.

Prob. 61–8. An ordinary 10-kva distribution transformer has two 1100-volt coils, A_1B_1 and A_2B_2, and two 110-volt coils X_1Y_1 and X_2Y_2.

(*a*) Compute the rated full-load current for each high-side coil.

(*b*) For each low-side coil.

Prob. 62–8. The transformer of Prob. 61–8 is connected as follows: A_1 to H_1; B_1 to A_2; B_2 to X_1 and L_1; Y_2 to H_2 and L_2. When the core flux has the same value as in Prob. 61–8, compute:

(*a*) Volts between high-side mains, H_1H_2.

(*b*) Volts between low-side mains, L_1L_2.

(*c*) Largest current that can be delivered to low-side mains without overloading any coil in the transformer.

(*d*) Current taken from high-side mains corresponding to (*c*).

(*e*) Current in each 110-volt coil.

(*f*) Current in each 1100-volt coil.

(*g*) Power transferred; power transformed.

(*h*) Kva rating of the auto-transformer.

Prob. 63–8. The transformer of Prob. 61–8 is connected as follows: A_1 to H_1; B_1 to A_2; B_2 to X_1, X_2 and L_1; Y_1 and Y_2 to L_2 and H_2. Answer the questions in Prob. 62–8, assuming the same core flux. Does this make the best use of materials?

Prob. 64–8. A person standing on wet earth and touching the secondary circuit of the transformer, connected as in Prob. 62–8, would receive what voltage between contacts under the following conditions?

(*a*) H_1 grounded; hand touching L_1.

(*b*) H_1 grounded; hand touching L_2.

(*c*) H_2 grounded; hand touching L_1.

(*d*) H_2 grounded; hand touching L_2.

32–8. Power Transformation in Polyphase Systems. It is possible to raise or lower the voltage of any polyphase system without changing the number of phases. That is, a three-phase system at one voltage can be changed into another three-phase

system at the same, or at a different voltage. The same is true for a two-phase system.

It is also possible to change one polyphase system into another polyphase system of a different number of phases at the same, or at a different voltage. Thus a three-phase system can be changed into a two-phase system, or a two-phase system into a three-phase system. A three-phase system can also be changed into a (so-called) six-phase system. (See Ch. 14, Art. 9.) It is **not** possible, however, to change a single-phase system into a polyphase system, by means of ordinary transformers.

33–8. Transformer Connections in Three-Phase Banks. There are several methods of transforming three-phase power at one voltage into three-phase power at another, or at the same, voltage. This is usually accomplished by a " bank " of three exactly similar transformers, or by a single three-phase transformer. Four different combinations of connections are possible.

(*a*) Primaries in Y-secondaries in Y.
(*b*) Primaries in Δ-secondaries in Δ
(*c*) Primaries in Y-secondaries in Δ
(*d*) Primaries in Δ-secondaries in Y.

Three-phase transformation can also be obtained by using two single-phase transformers, either in a V-V, or open delta, connection (Art. 24); or by a T-connection, in which the transformer coils are specially tapped (Art. 35).

The primaries of three separate transformers in a three-phase bank may be connected in Y, or in delta, without regard to the polarity, or phasing, of the terminals. The secondaries, however, must be so connected that the proper phase relations exist at their terminals in the manner described for three-phase connections of alternator coils in Ch. II.

In a single three-phase transformer, both primary and secondary windings must be phased, since parts of the magnetic circuit are common to the several phases. This transformer is described in Art. 36–8.

In this section, the relations existing in three single-phase transformers, connected in the four combinations, (*a*), (*b*), (*c*) and (*d*) above, are considered.

Y-Y Connection. In Fig. 43–8(*a*), the terminals of the primary winding of transformers, 1, 2, and 3. are arbitrarily tagged A_1B_1, A_2B_2 and A_3B_3 respectively. In the figure, the " A " terminals

are joined to form the neutral point of the Y primary. The corresponding secondary terminals marked X_1, X_2 and X_3 (previously determined by phasing) are also joined to form the neutral point of the Y secondary. The " B " primary terminals are connected to the incoming three-phase line, H_1, H_2 and H_3; and the " Y " terminals to the outgoing secondary circuit, L_1, L_2 and L_3.

The vector diagram of induced voltages is shown in Fig. 43–8(*b*). The induced emfs in the primary and secondary of each transformer are in phase with each other, as they are wound on the same core. Thus, in transformer No. 1, the emf from A_1 to B_1 is in phase with that from X_1 to Y_1; in transformer No. 2, emf from A_2 to B_2 is in phase with that from X_2 to Y_2, etc. Since the transformers are

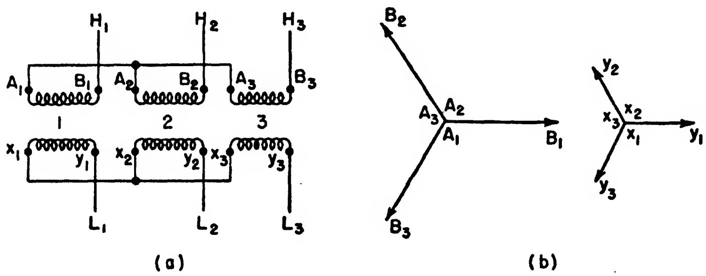


FIG. 43–8. (*a*) Y-Y connected transformer bank. (*b*) Corresponding vector diagram.

connected to a three-phase line, their voltages are all 120° to each other, and the line voltage must be $\sqrt{3}$ times that on each transformer. The current per coil is the same as the line current.

The advantage of this connection is that the windings need be wound only for $\dfrac{\text{Line voltage}}{\sqrt{3}}$, and are subjected to much less voltage strain. For instance, in transforming from 12,000 to 6000 volts, a 2 : 1 ratio, the normal voltage rating for each transformer primary is only $\dfrac{12{,}000}{\sqrt{3}}$, or 6940 volts, while that for each secondary is $\dfrac{6000}{\sqrt{3}}$, or 3470 volts. Note that the voltage ratio for each transformer is also 2 : 1, the same as the ratio of line voltages.

The neutral point can be connected back to the generator, and also, brought out to the secondary circuit, making a three-phase, four-wire system, as in Fig. 44–8. Where the secondary is used to

supply both lighting and power, 125 volts can be obtained between each line wire and neutral for lamps, and 215 volts across the Y for motors.

Unless the primary neutral is connected back to the generator neutral, this system has the following disadvantages: First — the triple frequency component of the magnetizing current, described in Art. 16, is suppressed in each transformer; the flux wave is no longer sinusoidal, and a triple frequency component appears in the secondary voltage waves, so that these are no longer of sine wave form.

Second — an unbalanced secondary load causes a "roving" or "floating," neutral. That is, the potential of the secondary neutral approaches that of the more heavily loaded secondary line. For instance, if a single-phase load only, is connected from one line to neutral, as indicated in Fig. 44–8, the secondary of trans-

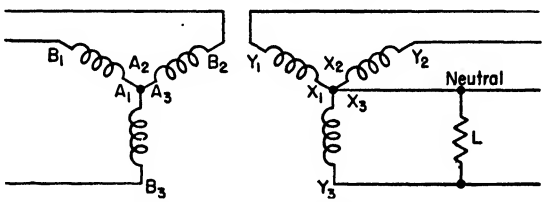


FIG. 44–8. Y-Y connected bank feeding a 3-phase, 4-wire system.

former No. 3 cannot supply the additional current, since its primary A_3B_3 is connected in series with the primaries of the other two transformers, the secondaries of which are open-circuited. These two transformers merely act as impedances in the primary circuit and the potential of the neutral X_3 becomes practically that of Y_3. In fact the secondary load L can practically short circuit X_3Y_3 with only slight increase in current.

Δ-Δ Connection. In Fig. 45–8(a), the primaries are connected in delta and the incoming three-phase lines, H_1, H_2 and H_3 are connected to the junction of the three windings, as shown. The corresponding terminals of the secondaries are similarly joined (after phasing) and the secondary line, L_1, L_2 and L_3, are brought out from their junction points. Figure 45–8(b) shows the vector diagram. Each transformer operates under normal conditions and no triple harmonic appears in the voltage waves. The current in each transformer coil, with this connection, is equal to $\dfrac{\text{line current}}{\sqrt{3}}$. The advantage of this connection is that if one

transformer is disabled, it can be removed, and the other two operated, in open delta, to supply three-phase power, provided the load is sufficiently reduced. A disadvantage is in the fact that full-line voltage is impressed on each transformer primary. This

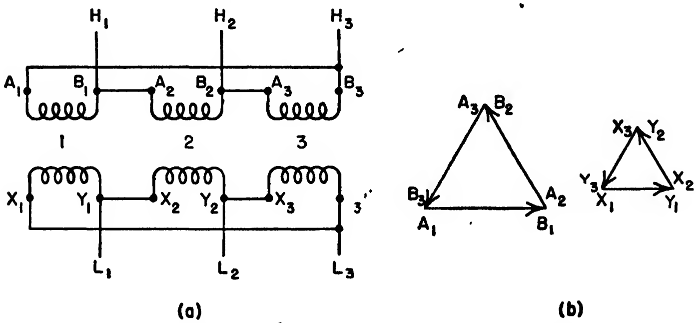


FIG. 45–8. (*a*) Δ-Δ connected transformer bank. (*b*) Corresponding vector diagram.

calls for more turns per coil and heavier insulation. For this reason, the delta-delta connection is used only on lines of moderate voltage. The voltage rating of the windings of each transformer

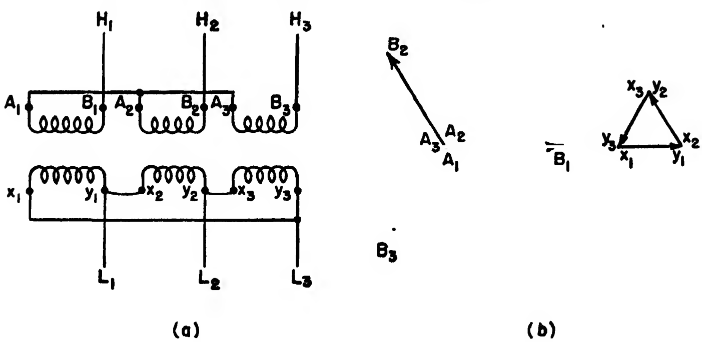


FIG. 46–8. (*a*) Y-Δ connected transformer bank. (*b*) Corresponding vector diagram.

must be the same as that of the primary and secondary lines. For instance, in transforming from 12,000 to 6000 volts, as before, the voltage rating and ratio of each transformer is also 12,000 : 6000, or 2 : 1.

Y-Δ Connection. In Fig. 46–8(*a*), the primaries are connected in Y and the secondaries in delta. Figure 46–8(*b*) shows the vector diagram. This connection is often used in transforming

the high voltage of a transmission line to a moderate voltage for distribution. In this case, the voltage ratio of each transformer differs from that of the line voltages, and equals the $\frac{\text{ratio of line voltages}}{\sqrt{3}}$. For instance, in transforming, as before, from 12,000 to 6000 volts, each primary coil rating is $\frac{12{,}000}{\sqrt{3}}$, or 6940 volts, and each secondary, 6000 volts. This is a transformer rating of $\frac{2}{\sqrt{3}}$: 1, or 1.156 : 1.

Δ-Y Connection. Transformers connected in delta-Y, the reverse of the Y-delta connection, are used for stepping up from a moderate voltage to a high voltage for transmission, as in a generating station. The transformer ratio in this case equals **ratio of line voltage $\times \sqrt{3}$.**

It should be noted that each transformer in any three-phase bank should have the same percentage of resistance and reactance — that is, the same regulation. Otherwise the voltages on the secondary system will be unbalanced.

Prob. 65–8. (*a*) What should be the rated high-side and low-side voltage of three single-phase transformers, intended to take power from a three-phase 23,000-volt line and deliver it to a three-phase 2300-volt line? Transformers are connected *Y–Y*.

(*b*) If a total of 500 kva at unity power factor is to be delivered, what will be the current in each line wire and each transformer coil on the high side?

(*c*) On the low side?

Prob. 66–8. Answer the questions in Prob. 65–8 on the basis of the same conditions, when the transformers are connected in delta-delta.

Prob. 67–8. Answer the questions in Prob. 65–8 on the basis of the same conditions, when the transformers are connected in *Y*-delta.

Prob. 68–8. Answer the questions in Prob. 65–8 on the basis of the same conditions, when the transformers are connected in delta-Y.

34–8. Open Delta, or V-V Connection. It was stated in Art. 33 that when one transformer of a delta-delta connected bank in a three-phase system is removed, the other two transformers will continue to supply three-phase power to the secondary circuit. This is known as the "**open-delta,**" or "**V-V connection.**" Two transformers so connected will give approximately balanced

three-phase secondary voltages, 120° apart in phase; but their capacity is only 58 per cent of that of the three transformers, not $\frac{2}{3}$, or 66.6 per cent, as might be assumed.

In the delta-delta connection, the current in each transformer equals $\frac{\text{line current}}{\sqrt{3}}$, while in the V-connection, the current per transformer must be equal to the line current. Since the rating of the transformers is proportional to the **current**, the load with the V-connection must be reduced in the ratio of $\frac{1}{\sqrt{3}}$, or 58 per cent of that supplied by the three transformers in delta.

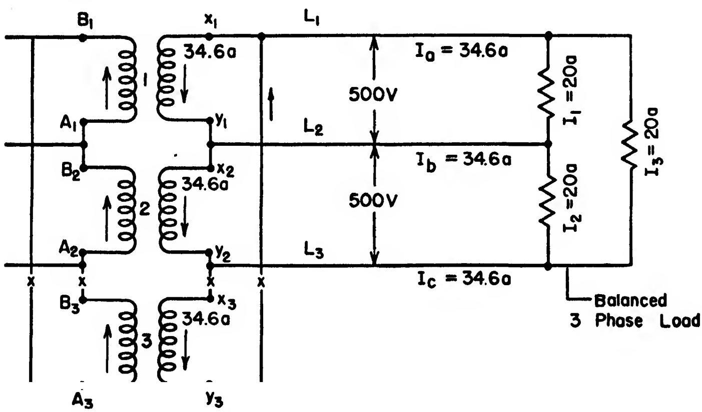


FIG. 47–8. The open-delta connection. Note that transformer 3 is disconnected so that the other two are carrying the three-phase load.

Thus, if a balanced unity-power-factor load of 30 kva at 500 volts is supplied by the three delta-connected transformers of Fig. 47–8, the rating of each must be 10 kva. The current per phase in the load and in each transformer secondary is $\frac{10{,}000}{500}$, or 20 amperes, while the line current is $\sqrt{3} \times 20$, or 34.6 amperes. Now, if one transformer is disconnected, as indicated in the figure, the secondaries of the other two must each carry the line current, or 34.6 amperes; and each must supply $\frac{500 \times 34.6}{1000}$, or 17.3 kva. Therefore, 2×17.3 kva, or 34.6 kva of transformer capacity is required to supply the 30 kva load.

When one transformer is removed, the load must be reduced in

the ratio of $\frac{20 \text{ amperes}}{34.6 \text{ amperes}}$, or 58 per cent of its former value. That is, the load above must be reduced from 30 kva to .58 × 30, or to 17.3 kva. Also the transformers operate at only 86.6 per cent of their rated capacity.

The reason the required transformer capacity in kva is greater than that of the load is explained by the fact that, with the open

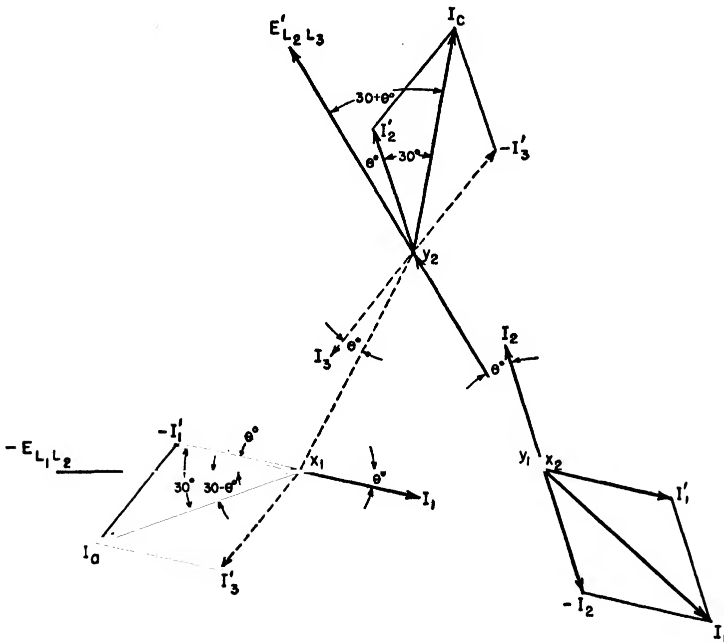


FIG. 48–8. Vector diagram for open-delta connection.

delta connection, the transformers operate at reduced power factor. On a balanced load, at unity power factor, as in the condition above, the current in each transformer coil is displaced 30° from the voltage. Each transformer, therefore, supplies 15 kw $\left(\frac{500 \times 34.6 \times \cos 30°}{1000}\right)$, or **half the power** to the 30 kw load; and also 17.3 kva, as shown.

When the power factor of the balanced load is less than one, or cos $\theta°$, the two transformers, while carrying the **same current,** do **not** supply the same power. This is shown in the vector diagrams of Fig. 48–8, in which I_1, I_2 and I_3 represent the current in the

three phases of the balanced load, lagging $\theta°$ behind their respective voltages. I_a, I_b and I_c are the resulting currents in the three lines. Applying this diagram to the two transformers, the missing transformer, or opening in the delta, is indicated by the broken line. The current I_3, normally flowing in transformer No. 3, must now flow through transformers 1 and 2. The current I_a in transformer No. 1 is the vector sum of I_3 and I_1 reversed, drawn from X_1 and shown as I_3' and $-I_1'$, respectively. Thus, I_a makes an angle of $(30 - \theta)°$ with the voltage, $-E_{L_1L_2}$, across the terminals of transformer No. 1, as shown.

Similarly, the current I_c in transformer No. 2 is the vector sum of I_3 reversed and I_2, drawn from Y_2 and shown, respectively, as $-I_3'$ and I_2'. Thus I_c makes an angle of $(30 + \theta)°$ with the voltage $E_{L_2L_3}$ across the terminals of transformer No. 2.

Therefore, the power supplied by,

$$\text{Transformer No. 1} = EI_a \cos(30 - \theta)°.$$

$$\text{Transformer No. 2} = EI_c \cos(30 + \theta)°.$$

The open-delta connection is usually employed only where the voltage for a comparatively small amount of power is to be transformed. (See D in Fig. 1–1.) This connection is first installed, and the third transformer added, when the load or the circuit demands it. By this means, a 50 per cent increase in transformer investment adds 73 per cent to the capacity of the circuit. Thus, in the illustration above, by adding one more 10 kva transformer, the load can be increased from 17.3 kva to 30 kva, an increase of 30 − 17.3, or 12.7 kva; this is an increase of $\dfrac{12.7}{17.3}$, or 73 per cent.

In the following problems, neglect losses and voltage drop in the windings.

Prob. 69–8. A balanced three-phase 150-kva, 240-volt load at unity power factor is to be supplied from a 2400-volt line by means of an open delta connection.

(*a*) What should be the voltage, current and kva rating, of each transformer?

(*b*) What power does each transformer supply?

Prob. 70–8. Repeat Prob. 69–8 when the power factor of the load is 0.7.

Prob. 71–8. (*a*) What should be the kva capacity of each transformer in an open delta to carry a balanced 230-volt, three-phase, 120-kva load at 0.8 power factor? (*b*) What power will each supply?

Prob. 72–8. Two 15-kva transformers are connected in open delta.
(*a*) What maximum balanced load in kilowatts, at 230 volts and 0.9 power factor, can they supply without overloading?
(*b*) What power is supplied by each transformer?

Prob. 73–8. Using the transformers in Prob. 72–8, on the system of Fig. 48–8, the loads are as follows: 3 kva in phase I_3 at 0.9 power factor, 9 kva in I_2 at 0.9 power factor. How many kva at 0.9 power factor may be taken by I_3, without overloading the transformers?

35–8. Scott, or T-Connection. By means of this connection, often called the "Teaser" connection, two like transformers can be used to transform from three-phase to three-phase, and also

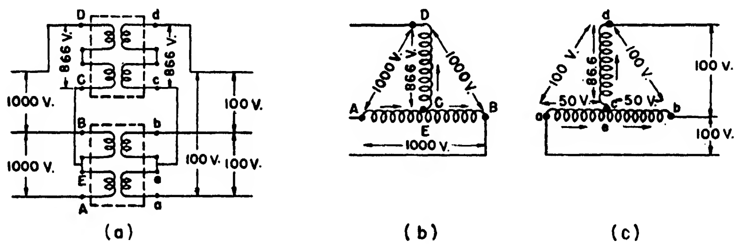


FIG. 49–8. (*a*) T-connection of transformer. (*b*) and (*c*) Relation of voltages in the windings.

from two-phase to three-phase, or from three-phase to two-phase. Figure 49–8 shows the connections and primary and secondary voltages, for three-phase transformation from 1000 to 100 volts; and Fig. 50–8, the connections and secondary coil voltages, for a similar two-phase to three-phase transformation.

In Fig. 49–8, the midpoint of both windings, E and e of transformer AB, called the main transformer, are brought out and connected to terminals C and c of the other transformer, called the "teaser." The secondary windings in Fig. 50–8 are similarly connected and the teaser winding is tapped so that only 86.6 per cent of its turns are in circuit, as shown.

Note here, that in both Figs. 49–8 and 50–8, the voltage on the teaser transformer is only 86.6 per cent of the three-phase line voltage.

If it be assumed that the positive directions of emf, in the secondaries of Figs. 49–8 (*b*) and 50–8 (*b*), are from a to e, e to b and

c to d, as shown by the arrows, the vector diagram of coil voltages can be drawn, as in Fig. 51–8(a). The secondary line voltage from a to b, internally, is thus the arithmetical sum of E_{ae} plus E_{eb},

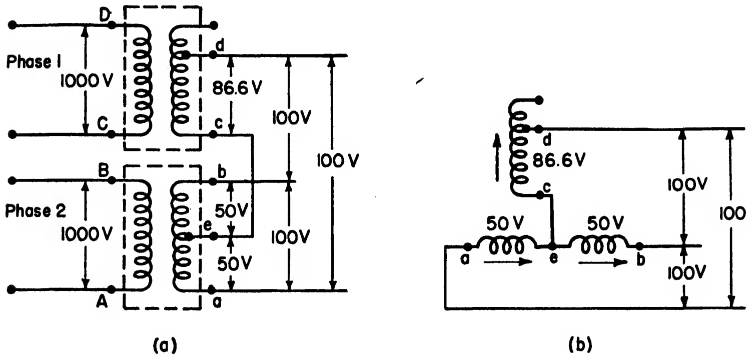


FIG. 50–8. (a) Connection of two transformers to change 2-phase to 3-phase — the Scott connection. (b) Relation of voltage in windings in the three-phase circuit.

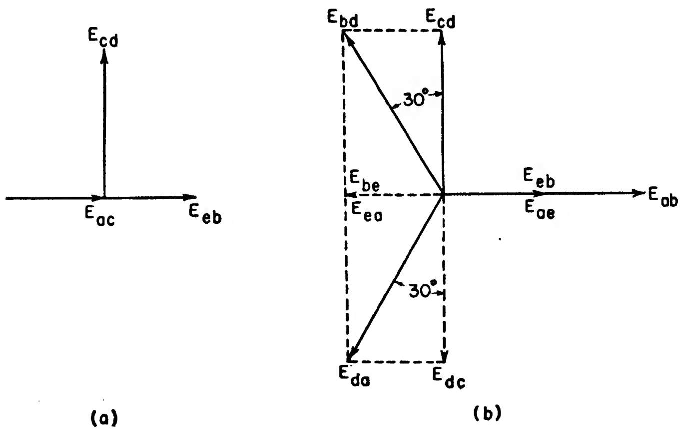


FIG. 51–8. (a) Vector diagram of voltages in the windings of T-connected transformers. (b) Diagram of resulting three-phase voltages.

or E_{ab}, as shown in Fig. 51–8(b); the voltage from b to d, internally, is the vector sum of E_{be} (E_{eb} reversed) and E_{cd} direct, or E_{bd}; and the voltage from d to a, internally, is the vector sum of E_{dc} and E_{ea} (E_{cd} and E_{ae} both reversed), or E_{da}. Each of these connections therefore gives three equal secondary voltages 120° apart in phase, or a symmetrical three-phase system.

On non-inductive load, it can be shown that the current and voltage (86.6 per cent of normal) in the teaser transformer are in phase; while the current in one half the main transformer lags the voltage by 30° (cos = .866), and in the other half, leads the voltage by 30°. Therefore, two T-connected transformers have a three-phase capacity only 86.6 per cent of their normal rating.

For example, assume the transformers, in Fig. 49–8 or Fig. 50–8 above, at full-load current, supply a balanced three-phase 30 kva load, at 100 volts and unity power factor. Full-load current per line and in each transformer equals $\frac{30{,}000}{\sqrt{3} \times 100}$, or 173 amperes. The normal voltage rating for each transformer secondary is 100 volts, and the kva rating for the two transformers is $2 \times 100 \times 173$, or 34.6 kva. The transformers, therefore, operate at $\frac{30}{34.6}$, or 86.6 per cent of their rated capacity.

Note also that 34.6 kva of installed transformer capacity is required to supply a 30 kva load, exactly as in the case of the open-delta connection of Art. 34.

Each transformer delivers half the total power, or

Power in the teaser transformer $= 86.6 \times 173$ 15.0 kw.

Power in one half the main transformer

$= 50 \times 173 \times .866$ or 7.5 kw.

Power in other half the main transformer

$= 50 \times 173 \times .866$ or 7.5 kw.

Total power = 30.0 kw.

When the power factor of the load is less than unity, or $\cos \theta$, the current in the teaser transformer differs in any phase from the voltage by $\theta°$; while in one half the main transformer, it differs in phase by $(30 + \theta)°$, and in the other half by $(30 - \theta)°$. Each transformer now delivers half the total power, but the load on the main transformer is unbalanced in the windings on the two sides of the teaser connection.

It should be noted that two T-connected auto-transformers, with windings properly tapped, can also be used in many combinations to obtain the voltages and phase transformations discussed above.

Three-phase to two-phase transformation is seldom used today, except where a formerly installed two-phase system is to be fed from a modern three-phase line.

In the following problems neglect the losses.

Prob. 74–8. A 125-kva, 230-volt three-phase load at unity power factor is to be supplied from a 2300-volt, three-phase line, by means of T-connected transformers.

(*a*) What are the voltage and current in each coil of the transformers?
(*b*) What must be the kva rating of each transformer?
(*c*) What power will the teaser transformer supply?
(*d*) The main transformer?

Prob. 75–8. Repeat Prob. 74–8, if the power factor of the load is 0.8 lagging.

Prob. 76–8. A 50-kva, 2200-volt, 2-phase balanced load at unity power factor is to be supplied from a 2200-volt, 3-phase line by Scott-connected transformers. For each transformer, compute:

(*a*) Current in primary and secondary coils.
(*b*) Power input. (*c*) Kva rating.

Prob. 77–8. A 30-kva, 100-volt three-phase load at unity power factor is supplied by means of Scott-connected transformers from a two-phase, four-wire, 1000-volt line. For each transformer, compute:

(*a*) The primary and secondary current. (*b*) The power input. (*c*) The kva rating. (*d*) Explain the apparent discrepancy in the ratio of primary to secondary current in each of the two transformers.

Prob. 78–8. Repeat Prob. 77–8, if the load is two-phase, 100-volts, supplied from a three-phase 1000-volt line.

36–8. Three-Phase Transformers. For three-phase transformation, in place of a bank of three separate single-phase transformers, a single three-phase transformer is commonly used. Three-phase transformers have considerably less core weight (about 16 per cent less) and occupy less floor space than three separate transformers of the same kva capacity. Also, where the high voltage coils are connected inside the case, only three high-voltage bushings, or terminals, need be brought out. The disadvantage lies in the fact that, when one winding is disabled, the whole unit must be taken out of service; while with three separate transformers, only one transformer must be replaced. Three-phase transformers are either core-type or shell-type. Each of the primary windings sets up its own flux, which combines with that from the other two windings in certain parts of magnetic circuit. Each leg of the core carries both the primary and secondary windings of a single phase.

In the following figures, only the primary windings are indicated.

Figure 52–8 represents three single-phase, core type, transformers grouped together to form one central core. The coils are

wound on the outside legs and connected in Y to a three-phase line. The currents I_1, I_2 and I_3, displaced 120° in phase, set up the three fluxes, ϕ_1, ϕ_2 and ϕ_3, respectively, also displaced 120°. These

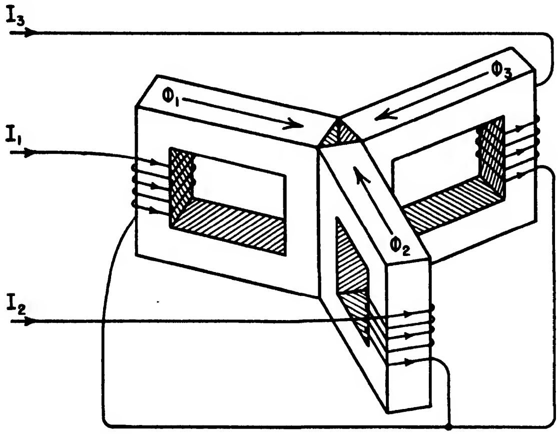


FIG. 52–8. Three single-phase, core-type transformers combined to take the place of a three-phase transformer.

fluxes combine in the central, or common, leg of the core. The resultant flux is therefore zero in this leg, and it can be eliminated.

To simplify construction and reduce the amount of iron, the commercial three-phase, core-type transformer is built in the form indicated in Fig. 53–8. Here again the vector sum of $\phi_1 + \phi_2 + \phi_3$ is zero; so that, at every instant, the flux downward (negative) in one core is exactly equal to the arithmetical sum of the fluxes upward (positive) in the other two cores. Or, with zero flux in one core, the downward (negative) flux in one of the other cores is exactly equal to the upward (positive) flux in the third core. The fluxes in the yokes (horizontal sections) are also equal to the fluxes ϕ_1, ϕ_2 and ϕ_3 in the cores; so that, for the same flux density, the cross section area of the yokes should be equal to that of the cores. With this construction, Fig. 54–8, the re-

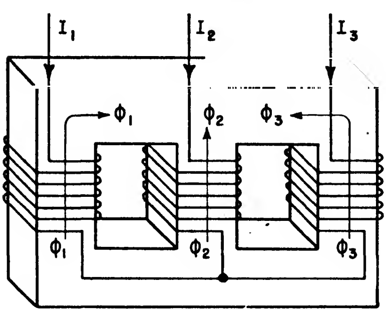


FIG. 53–8. Form of the commercial three-phase core-type transformer.

luctance of the magnetic path for the center coil is somewhat less than that for the other two; therefore, its magnetizing current at zero load is less, but the difference is slight and disappears when the transformer is loaded.

Figure 54–8(a) represents a single-phase shell-type transformer. It is apparent that the core flux divides equally in the two parallel magnetic circuits, so that, for the same flux density through, the cross section area of the yokes and outside legs must be just one half that of the center core.

If three of these single-phase transformers are stacked together, as indicated in Fig. 54–8(b), they form a three-phase, shell-type

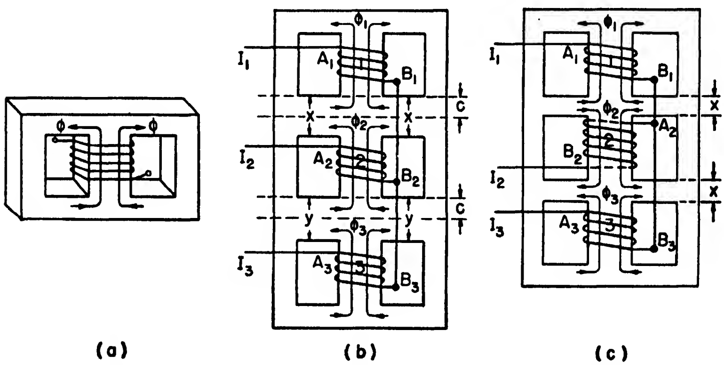


Fig. 54–8. (a) Single-phase, shell-type transformer. (b) Three similar single-phase transformers stacked to form a three-phase shell-type transformer. Coil terminals are connected in the same sequence. (c) By reversing the connections to coil 2 the "c" sections in (b) can be eliminated without increasing the flux density in the core.

transformer, with three separate magnetic circuits. When the primary coils are symmetrically connected to a three-phase line, as shown, each of the combined yoke sections, marked x, which now have the same cross-section area as the center cores, carry **half** the fluxes ϕ_1 and ϕ_2 at 120°, but **opposed;** so the flux in these sections equals half the center core flux times $\sqrt{3}$. The same is true for the sections marked y. Thus the area of these yoke sections can be reduced. **If the connections to coil 2 be reversed,** as shown in Fig. 54–8(c), the flux, ϕ_2, set up by this coil, is also reversed and the fluxes in the x and y yokes are still at 120°, but in the **same** direction. The flux in sections x and y is now equal to **half** the flux in the center cores, as in the single-phase transformer of Fig. 54–8(a). Consequently, the sections of the core c, indicated by the broken

lines, in Fig. 54–8(*b*), can be removed, and the volume and weight of the core structure still further reduced, as indicated in Fig. 54–8(*c*). All parts of the core are now marked at the same flux density.

The reluctance of the magnetic path for all coils is the same in the shell-type transformer, so that there is no unbalance in the magnetizing currents.

In large three-phase transformers, which are to be Y-Y connected, an auxiliary coil is usually installed on the same core with each main winding. These three coils, called the "tertiary" winding are delta connected, and carry the triple frequency component of the magnetizing current (see Art. 16–8), which is suppressed in a Y-Y connection. This prevents distortion of the induced emf waves already mentioned.

37–8. Parallel-Connected Transformer Banks. When two or more three-phase transformers, or banks of three separate transformers are connected in parallel to the same three-phase system, the following conditions must be satisfied. Otherwise, circulating current will flow in the secondaries or a short circuit will occur.

First: The ratio of primary and secondary terminal voltages must be the same in each bank.

Second: The voltages between any two corresponding secondary terminals in each bank must be in phase with each other.

Third: The phase sequence of secondary terminal voltage in each bank must be the same.

The connections which meet the above requirements are given in the following rules:

(*a*) With Y-Y on one bank, the other bank must be Y-Y or delta-delta.

(*b*) With delta-delta on one bank, the other banks must be delta-delta or Y-Y.

(*c*) With Y-delta on one bank, the other banks must be Y-delta, or delta-Y.

(*d*) With delta-Y on one bank, the other banks must be delta-Y, or Y-delta.

The voltage relation, under rules (*a*) and (*b*) above, are indicated in Fig. 55–8. A 2 : 1 three-phase voltage transformer is assumed.

Figure 55–8(*a*) shows the vector diagram of primary and secondary voltages of each of three transformers 1, 2, and 3, or of a

three-phase transformer, connected Y-Y, together with the primary and secondary line voltages. When two similar banks, so connected, are joined in parallel, it is apparent that the respective voltages in the primary coils, and also in the secondary coils, of each bank are respectively the same and in phase. The corresponding secondary line voltages, L_3 to L_1, L_1 to L_2 and L_2 to L_3, in each bank are, therefore, in phase and must be equal in value.

Figure 55–8(b) shows the vector diagram for a delta-delta connection. It is apparent also that, when two such banks are

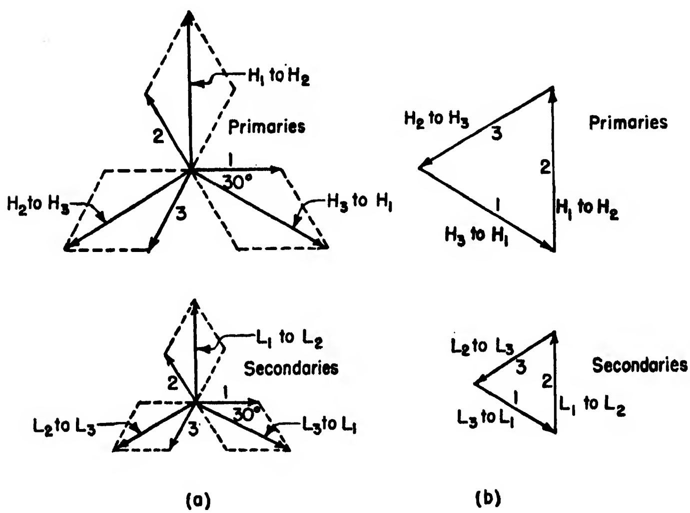


FIG. 55–8. Vector diagram of voltages. (a) In a Y-Y connected 3-phase transformer bank. (b) In a Δ-Δ connected 3-phase transformer bank.

connected in parallel, the same conditions described above are obtained.

When a Y-Y bank is connected in parallel with a delta-delta bank, as indicated by both diagrams in Figs. 55–8(a) and (b), it is apparent that the voltage rating of the primary and secondary coils in the two banks are not the same; neither are their respective voltages in phase. If the transformers, however, have the proper voltage-rating and ratio, the secondary line voltages, L_3 to L_1, L_1 to L_3, etc., in each bank are of the same value and in phase, as shown.

If the primary and secondary voltage ratios and phase sequence are properly adjusted, similar voltage diagrams for parallel con-

nected Y-delta and delta-Y banks, (rule (*c*) and (*d*)) will show that the secondary terminal voltages of both banks will be equal and in phase.

It is important to note that, even with the proper phase sequence and the same three-phase secondary terminal voltages, a Y-Y or a delta-delta connected bank **cannot be parallel** with any Y-delta combination, since their secondary voltages are not in phase. This is illustrated in Fig. 56–8, in which Fig. 56–8(*a*) is a diagram of voltages in a Y-Y bank, and Fig. 56–8(*b*) those in a Y-delta bank, connected to the same high-side line. While the secondary

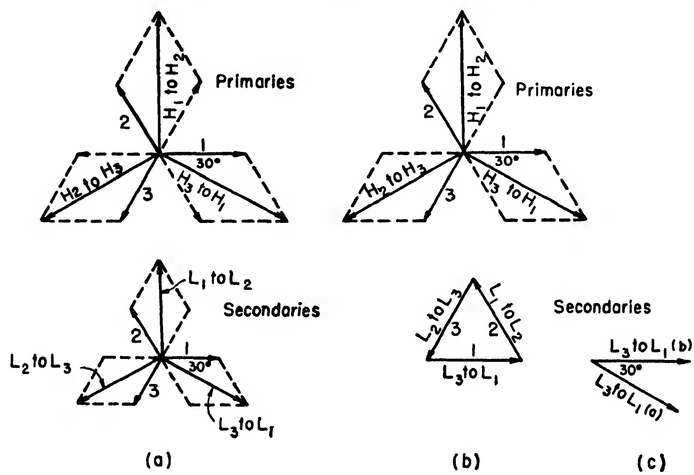


FIG. 56–8. Vector diagram of voltages. (*a*) In a Y-Y connected 3-phase bank. (*b*) In a Y-Δ connected 3-phase bank. (*c*) Note that L_3 to L_1 in (*b*) lead L_3 to L_1 in (*a*) by 30°.

line voltages, L_3 to L_1, etc., in both banks are the same in value, it will be noted that these voltages differ by 30° in phase, as indicated in Fig. 56–8(*c*).

38–8. Tap-Changing Under Load. When a transformer supplies a distribution system, it frequently becomes necessary to change the voltage ratio, in order to compensate for line drop and hold the secondary voltage constant. Also a change in voltage ratio is often necessary when two different power systems are tied together through a transformer.

Such a transformer has taps brought out, either from the primary or secondary winding, so that the number of turns in the coil can be changed. In modern power systems, it is desirable that changes in tap connections be made while the transformer is in

operation under changing load. Tap-changing under load is accomplished by a parallel winding, or by a single winding and an auxiliary auto-transformer.

Figure 57–8 shows a simple diagram of the single-winding method. *AB* represents the primary winding of the transformer with suitable taps brought out through circuit breakers, or switches, 2, 3, 4, 5, and 6, which connect to the terminals of an auto-transformer, *cd*. The midpoint of the auto-transformer, *e*, is con-

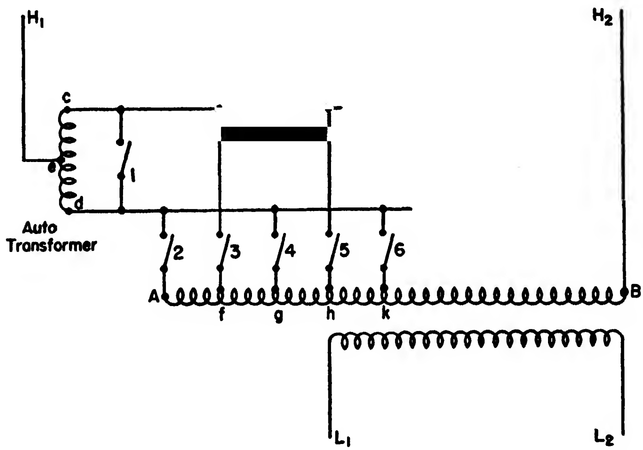


FIG. 57–8. Transformer connections for tap changing under load.

nected to H_1, one side of the supply line, while the terminal *B* is connected to the other side, H_2.

Circuit breaker 1 is connected across the terminals of the auto-transformer.

Assume the transformer is operating at its normal voltage ratio, with all primary turns in circuit. Circuit breakers 1 and 2 are normally closed and all others are open. Current now flows from terminal *A*, and divides equally through *c* and *d* to *e*, and out to the line. Note that, at any instant, the currents in *ce* and *de* are in opposite directions; therefore, the flux and induced voltage in the auto-transformer are zero, and the voltage drop is negligible.

Now assume the supply voltage drops. To keep the secondary voltage L_1L_2 constant, the voltage ratio must be changed without dropping the load. Breaker 1 is first opened and then 3 is closed. The turns Af of the transformer primary are, thus, short circuited through the auto-transformer, which prevents excessive circulating

current through these turns. Breaker 2 is now opened and 1 closed. The transformer now operates at reduced voltage ratio with the turns Af out of circuit, thus raising the voltage of the secondary circuit L_1L_2. Additional turns may be cut out, or in, by similar operations, as the conditions of the load demand. These operations may be automatically controlled by relays, operated by a change of voltage on the transformer.

The circuit breakers and control mechanism are enclosed in moisture proof iron cases, attached to the transformer tank, and may be almost as large as the transformer itself, as indicated in Fig. 19–8. Tap changing equipment is expensive, and is generally used only with transformers of large or moderate capacity.

39–8. Induction Regulator. It has been shown that alternator terminal voltage in a power station can be held constant by voltage regulators (see Ch. VII, Art. 15); and that the secondary terminal voltage of power transformers (in a substation) can be controlled by tap changing (Art. 38). From these constant-voltage sources, scattered distribution centers are supplied by individual feeders of different lengths and load characteristics. The voltages at these different centers of distribution may vary widely under changing load, due to resistance and reactance drops in the line. It is often necessary, therefore, to regulate the voltage of each of these feeders by a separate "feeder voltage regulator," usually called an **Induction Regulator.** The regulator may be placed at the station end of the line, or at the distribution center, in a manhole, or on a pole.

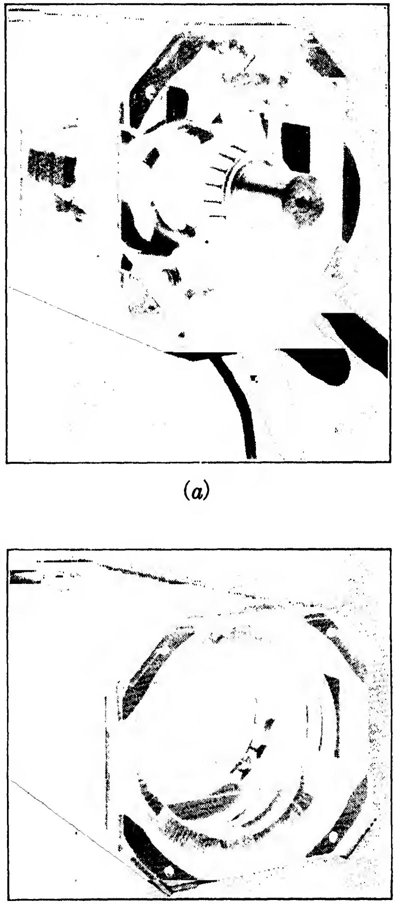
The induction regulator, illustrated in Fig. 58–8, is actually an auto-transformer in which the primary is connected across the line and the secondary in series with the load, as indicated in the simplified diagram of Fig. 59–8. The magnetic circuit is so arranged that one of the windings can be rotated to change its position with respect to the other. This changes the induced secondary voltage, E_s, and causes it to add, or subtract, from the feeder voltage, and so controls the voltage on the load. Induction regulators are built either for single-phase or three-phase circuits.

The construction of the single-phase regulator is represented in Fig. 60–8. The secondary winding, *SS*, is wound in slots, partially distributed, in the stationary member, which is somewhat similar to the stator of a revolving field alternator. The primary winding, ***PP***, is wound in slots in the rotating member, which may be

operated by hand, or controlled automatically. This member is usually stationary and never turns through much more than 90° in either direction. A short circuited winding, *SC*, is also placed in slots in the rotor at 90° to the primary windings.

In position (*a*), most of the flux set up by the primary exciting current links the secondary, since the planes of the two coils are parallel. In this position, the induced voltage in the secondary, and voltage on the load, are a maximum. No flux links the short-circuited coils, *SC*, and it is inactive.

When the rotor is turned clockwise through 90°, as in position (*b*), the plane of the primary coil, *PP*, is now 90° to that of the secondary, *SS*. Consequently, no primary flux links the secondary turns, and no secondary voltage is thereby induced. Since the secondary coil, however, carries the load current, it sets up a flux of its own, which would cause a reactance drop in the load circuit, were it not for the short-circuited coil, *SC*. This coil is now in such a position that it is linked by this flux, due to the load current. It thus becomes a short-circuited secondary, and sets up an opposing mmf which, neglecting leakage, reduces the flux and reactance drop in *ss* to zero. In position (*b*), the regulator, therefore, has no effect on the load voltage.



(*a*)

(*b*)

Fig. 58-8. An induction voltage regulator. (*a*) Top view of rotor and stator. (*b*) View of stator. (*General Electric Co.*)

In any intermediate position of the rotor between (*a*) and (*b*),

the flux linkages and voltage, induced in the secondary coil, are proportional to the cosine of the angle between the planes of the two coils. If the rotor is turned in clockwise direction beyond position (*b*), the primary flux links the secondary coil in reversed sense, so that the secondary voltage "bucks" the feeder voltage,

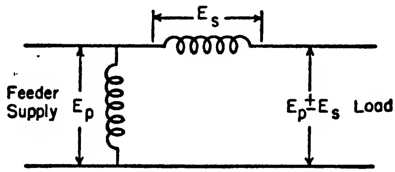


FIG. 59–8. Simplified diagram of a feeder voltage regulator in a line.

and reduces the voltage on the load. The regulator can thus raise or lower the voltage of the feeder, depending upon whether the load on it is heavy or light.

The voltage of a three-phase feeder can be controlled by three single-phase regulators, or by a single three-phase regulator. The

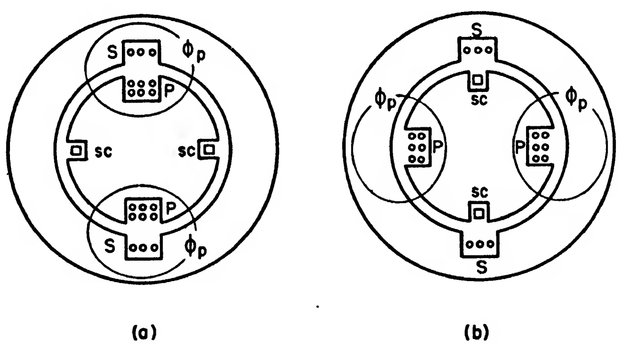


FIG. 60–8. Cross section of the coil arrangement in an induction regulator. (*a*) Position of maximum voltage in the secondary winding. (*b*) Position of zero voltage in secondary winding.

three-phase regulator is similar to that for single-phase feeders, except that three primary and secondary windings spaced 120° apart are necessary. It is essentially an induction motor, the rotor of which can be locked in any position. Its operation can be more readily understood after a study of the polyphase induction motor in Chap. IX.

Voltage regulators are generally designed to raise or lower the feeder voltage through a range approximately 10 per cent. When

automatically controlled, a "voltage-regulating relay" closes a control circuit, if the voltage on the load side of the regulator reaches a value a fraction of 1 per cent above, or below, the value which it is adjusted to maintain. The control circuit operates a small motor in such a direction as to move the rotor and restore the load voltage to the predetermined value. See Arts. 7 and 8, Chap. XII.

Prob. 79–8. The rating in volt-amperes of an induction regulator is generally computed on the current it can deliver, times the amount by which it can raise or lower the feeder voltage above, or below, its normal value. Assuming a 10 per cent range above and below normal voltage, what would be the kva rating of a single-phase induction regulator for a 100-kva, 2300-volt feeder?

Prob. 80–8. Under the conditions of Prob. 79–8, what would be the kva rating of a three-phase induction regulator for a three-phase feeder, delivering 100 amperes per line, at 2300 volts between line wires?

Prob. 81–8. Under the conditions of Prob. 79–8, what would be the limiting values of load voltage and current, supplied through a single-phase regulator, rated at 2.3 kva, for a 2300-volt circuit?

Prob. 82–8. Would it be preferable to use a three-phase regulator or three single-phase regulators under each of the following conditions?

(*a*) Three-phase balanced circuit feeding, principally, three-phase motors.

(*b*) Three-phase unbalanced circuit feeding, principally, a lighting load.

40–8. The Constant-Current Transformer. Electrical energy for power and light is almost universally distributed over a multiple, or parallel circuit at a constant, or approximately constant voltage, in which an increase in load requires an increase in current. Such a circuit is a **constant-voltage, variable-current circuit,** and is served by the constant potential transformer previously considered.

The principal exception to the use of the parallel circuit is that for street lighting. Street lights are usually connected in series for the following reason. The route of a street-lighting circuit, comprising a considerable number of lamps, may be several miles in length. If a multiple circuit is used, the wires must be large enough to carry the combined current of the lamps in parallel, with an allowable small line drop and power loss, in order that the voltage on all lamps may be approximately the same. Also two wires must cover the entire route.

By use of a high-voltage series circuit, a large saving in copper results, since the wire need be only large enough to carry the current of a single lamp, with an allowable power loss; and only one wire need cover the route. For instance, the circuit from the sub-station might be out on one street and return on another. Line drop in this case is not a serious consideration.

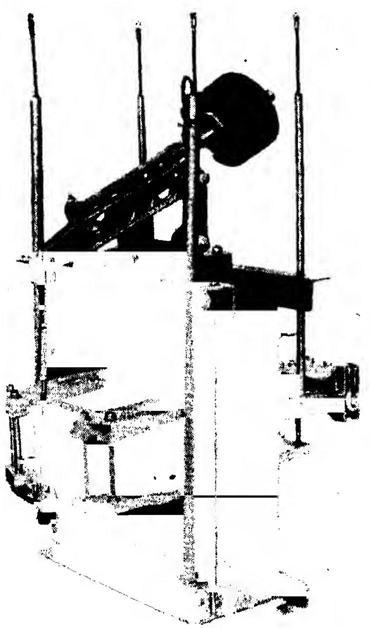


FIG. 61–8. A constant-current transformer. (*General Electric Co.*)

To operate efficiently, series street lamps require a constant current. If, for any reason, one or more of these lamps burn out, an automatic device, in each lamp, short circuits it, in order that the remaining lamps may continue to operate. This reduces the impedance in the circuit and would increase the current, tending to burn out more lamps, were the voltage to remain constant. To operate satisfactorily, therefore, the voltage must vary automatically with the number of lamps in circuit. This is accomplished by means of the **constant current or " Tub " Transformer,** the primary of which is supplied from **constant potential mains** at the sub station; while the secondary delivers a **constant alternating current** at a variable voltage, depending upon the number of lamps in circuit.

The constant-current transformer is so constructed that the primary and secondary windings are free to move with respect to each other. That is, either the primary or the secondary winding may be fixed and the other movable. Both coils are wound on the center leg of a shell-type core. In the more common type of transformer, shown in Fig. 61–8, the lower coil is the primary and fixed in position. The upper coil, or secondary, is free to move, and is suspended by a lever which is not quite counterbalanced by a weight. When the secondary is on open circuit, it rests at the

bottom of its travel, on the fixed coil. When the secondary supplies a load, the currents in the two windings flow in opposite directions. Thus there is **repulsion between the coils,** which acts with the weight, causing the secondary to " float " in a position above the fixed coil. The coils, therefore, are more or less loosely coupled, and the leakage flux, which is objectionable in the constant-potential transformer, is large. The variable voltage characteristic is obtained by a variation in this leakage flux, due to

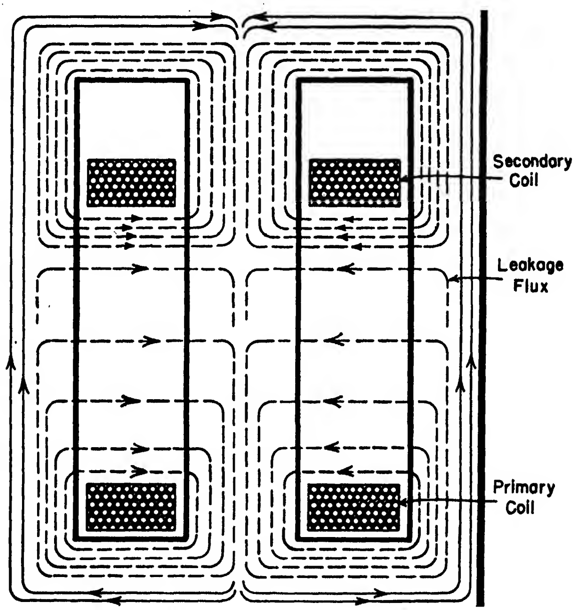


FIG. 62–8. Diagram of the leakage flux in a constant-current transformer.

a change in position of the movable coil. This is indicated in Fig. 62–8. As the distance between the coils increases, the leakage flux and reactance drops in both coils, also increase. Therefore, the voltage, induced in the secondary, decreases as this coil moves away from the primary.

When the transformer is loaded, the secondary (by adjustment of the counterweight) can be made to float in that position which will deliver the proper, or normal, current to the lamps. If the load changes, the position of the floating coil changes. For instance, if a lamp is short circuited, thereby reducing the impedance in the circuit, the current, in both primary and secondary

windings, increases. The repelling force between the coils increases, forcing the secondary to move further away from the primary. This action continues until the secondary current drops to the normal value. A dash-pot is attached to the lower mechanism to dampen the fluctuation of the movable coil.

At light loads, the secondary operates near the upper end of its travel; the leakage flux is high compared to the mutual flux, and the power factor is low. At, or near full load (rated number of lamps in circuit), the secondary operates near the lower end of its travel, close to the fixed coil, the leakage flux is reduced and the power factor is correspondingly improved.

41–8. Instrument Transformers. In alternating-current systems of high and moderate voltage, meters, relays and other instruments are not usually connected directly to the power circuit. They are **insulated** from such circuits by placing them in the secondary circuit of "**instrument transformers,**" known as "**potential transformers**" and "**current transformers.**" Otherwise, it would be dangerous for any one to come in contact with meters or other switchboard equipment. Furthermore, it is impracticable to construct and insulate commercial meters to measure directly the high voltages and large currents employed today. By means of these transformers, ordinary instruments with 150-volts potential and 5-amperes current coils can be used to indicate accurately the voltage, current and power, etc., in such circuits, regardless of the line voltage or of the current they carry. Also low energy relays can be employed to operate protective and control apparatus.

Potential Transformers. Potential transformers are used, on circuits of 300 or 400 volts and above, to step down the voltage to a convenient and safe value. They do not differ greatly from the constant-potential power transformer, except that their rated output is small — generally from 25 to 200 watts. For voltages below 4000 or 5000 volts, they are usually of the dry type, or air cooled. Above this voltage, they are generally oil cooled, mainly because of the insulating properties of the oil. The primary winding is connected directly across the power circuit. The secondary is usually wound for 110 or 115 volts, so that the voltage ratio depends upon the rated primary voltage.

Figure 63–8 represents a potential transformer connected to a 2300-volt line. Its voltage rating is 2300 to 115 volts, or a ratio of 20 : 1. Thus, the 150-voltmeter will indicate 115 volts, or 1/20th

of the line voltage. Or, the voltmeter scale can be multiplied by the transformer ratio, and graduated from 0–3000 volts. The 150-voltmeter would thus indicate the line voltage or 2300 volts directly. While the transformer insulates the voltmeter from the high voltage line, **the secondary circuit should always be grounded, as shown,** to eliminate static on the instrument and as a precaution against transformer failure.

The potential coils or other instruments, supplied by the transformer, should be connected in **parallel** with the voltmeter. And the total secondary load, called the "burden" should never be greater than the transformer rating. The turn ratio of the primary and secondary coils may be slightly less than the voltage ratio to compensate for the transformer impedance drop under

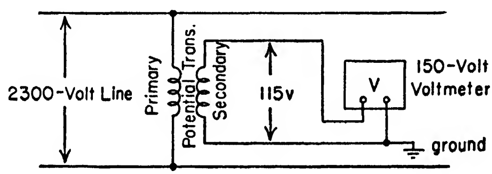


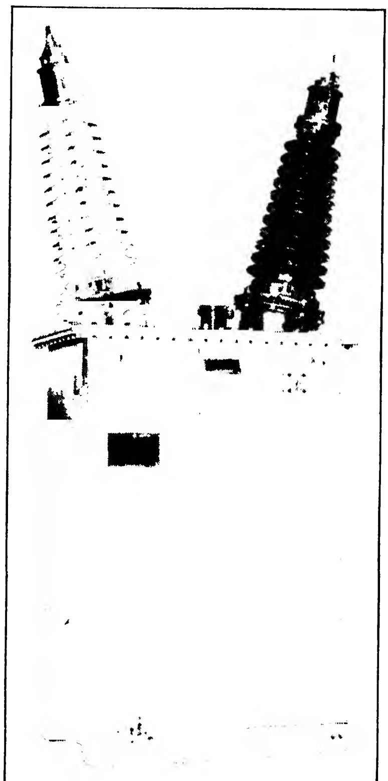
FIG. 63–8. Connection of a potential transformer and voltmeter to measure voltage of a 2300-volt line.

load. Figure 64–8(*a*) is a 2300-volt, dry type potential transformer, for mounting on switchboard supports; and Fig. 64–8(*b*), an oil-filled transformer for a 6600-volt circuit.

Current Transformers. Current or "series," transformers are used in power circuits to step down the current in a known ratio for measurement. Since the current in the windings of a "series" transformer is large, it usually has a primary of only a few turns (one or more), connected in **series** with the power line. The secondary, consisting of a larger number of turns, is connected to the terminals of a low-reading ammeter. The ammeter is thus entirely insulated from the line. The secondaries of practically all current transformers are wound for 5 amperes; therefore, the current, or transformation ratio, is determined by the current rating of the power circuit.

Figure 65–8 represents a current transformer, connected in a 2300-volt circuit. Assume there are 2 turns in the primary, and 300 in the secondary, and neglect for the moment the slight difference between the "turn ratio" and "current ratio" of the transformer. The ratio of the transformer is thus 300 : 2, or 150 : 1.

(a)



(b)

FIG. 64–8. (a) Dry-type instrument potential transformer. (b) Oil-filled instrument potential transformer for high-voltage circuit. (*General Electric Co.*)

If the line current is 600 amperes, as indicated in the figure, the 5-ampere meter will indicate 1/150th of 500, or 4 amperes. Here again, the scale of the instrument may be multiplied by the transformation ratio and graduated from 0–750 amperes, so that

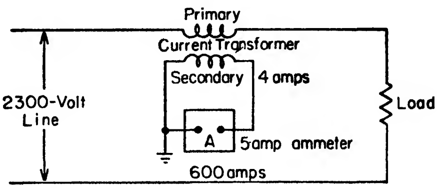


FIG. 65–8. Connection of current transformer and low-range ammeter to measure line current.

the meter indicates the line current directly. The current coils of wattmeters and other instruments are placed in **series** with the ammeter in the secondary circuit. The secondary circuit of the current transformer should also be grounded, as shown.

Figure 66–8 is a current transformer, which can be attached to switchboard supports. And Fig. 67–8 is a portable transformer

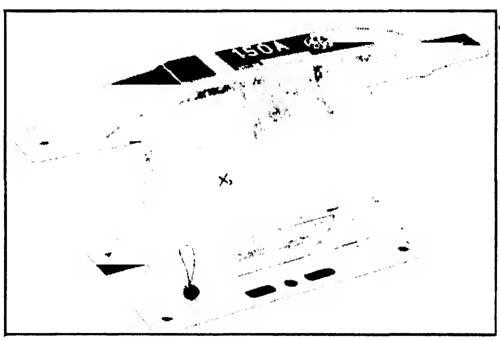


FIG. 66–8. Current transformer arranged for mounting on switchboard supports. (*General Electric Co.*)

for a 2500-volt circuit, having several primary coils, which can be combined to give current ratios of 2 : 1, 2½ : 1, 5 : 1, 10 : 1 and 20 : 1. These coils, with the secondary, are wound on the hollow iron core. The transformer may also be used as a " through " transformer for measuring larger currents. When one conductor of the power circuit is passed **once** through the opening in the core, it is equivalent to **one** primary turn; **two** turns, when passed

twice through the opening, etc. This transformer, with one turn, has a current ratio of 160 : 1; with 2 turns, 80 : 1, with 4 turns 40 : 1, and with 5 turns, 32 : 1. The transformer can thus be used to measure currents from 160 × 5, or 800 amperes, to any fraction thereof.

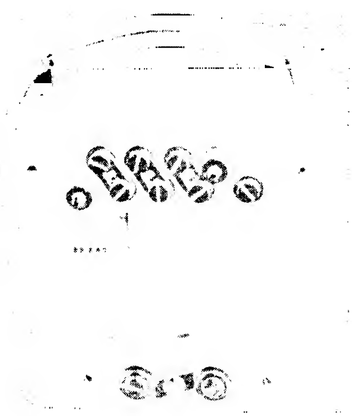


FIG. 67–8. A portable-type "universal" current transformer having several primary coils. This is also a "through-type" transformer. By passing the primary line once through the opening in the core the current ratio is 160 : 1 and the transformer can be used to measure a current of 800 amperes. (*The Esterline-Angers Co.*)

On first thought, it would seem that connecting the secondary to the terminals of an ammeter must result in a short-circuit, and damage to the windings and the instrument, but such is not the case. The current transformer differs from the constant-potential transformer in that its primary current is determined by the current in the power circuit, and not by the load on its own secondary. The primary ampere-turns, therefore, are definitely controlled, or fixed, by the current in the power circuit. Since the secondary is short circuited through a circuit of low impedance, the opposing secondary ampere-turns approach closely in value to those in the primary; and the core flux, which is due to their **difference,** is very small. Thus, as the current in the power circuit and primary ampere-turns is increased or decreased, the secondary ampere-turns and current increase or decrease proportionately.

If the secondary circuit is opened, there are no opposing secondary ampere turns, and the primary ampere-turns act alone to magnetize the core. The flux is thus greatly increased. This causes excessive core losses and heating, a noticeable impedance drop in the power line, and also a high and probably dangerous voltage across the secondary terminals, due to the ratio of turns. Therefore, the **secondary circuit of a current transformer should *never* be opened.**

When instrument transformers are used to measure voltage and

current only, the accuracy depends principally upon their **ratio.** When they are used to actuate wattmeters and measured power, the **phase difference** between primary and secondary circuits becomes an important factor. This applies particularly to the current transformer.

Figure 68–8 is a vector diagram of relations in a 2 : 1 ratio current-transformer. OI_p is the total primary current, or the current in the power circuit, and remains fixed for a given load. OI_p is the vector sum of two component currents. The first is the exciting current OI_E, which supplies the core losses and sets up the core flux or mmf to produce the secondary emf, OE'_S, and current, OI_S. The other component of OI_p is the primary **load current of the transformer,** OI'_P, which supplies the primary ampere-turns to balance the opposing secondary ampere-turns. The ratio of $OI'_P : OI_S$, which always differ in phase by 180°, is the "turn ratio" of the transformer, while the ratio of $OI_p : OI_S$ is the "**current ratio.**" It is therefore seen that the turn ratio is always somewhat less than the current ratio. Note particularly that OI_P and OI_S differ in phase from the ideal 180° position by the small angle θ. This causes the phase angle between the voltage and current in the wattmeter coils to differ from that in the power circuit, and results in a corresponding error in the wattmeter indications. The angle θ, as well as the current ratio, is largely determined by the value and phase position of the exciting current OI_E, which, in turn, is determined by the flux density. OI_E can be reduced to a minimum by liberally designing the cross section of the core so that the flux density is always kept well below the knee of the saturation curve.

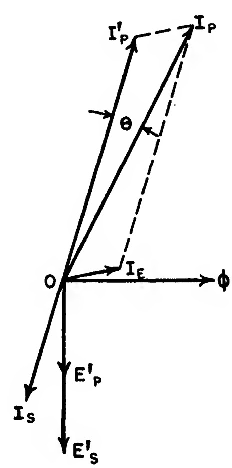


FIG. 68–8. Vector diagram of current, voltage and flux relations in a 2 : 1 ratio current transformer.

It is important to note that **the vector sum of OI'_P and OI_E regardless of their value and phase positions, must always be equal to OI_P.** Thus, any change either in the value or phase position of OI_E, changes the relative value, or phase position of OI_P and also of OI_S with respect to OI_P. This changes correspondingly both the current ratio and the phase angle θ.

Other diagrams, similar to Fig. 68–8 can be drawn to show that

the flux, the phase angle and value of the exciting current, OI_E, and thus the phase angle and ratio of the transformer, are affected by any change in the load on the power circuit; or any change in the impedance or burden of the secondary circuit, such as the

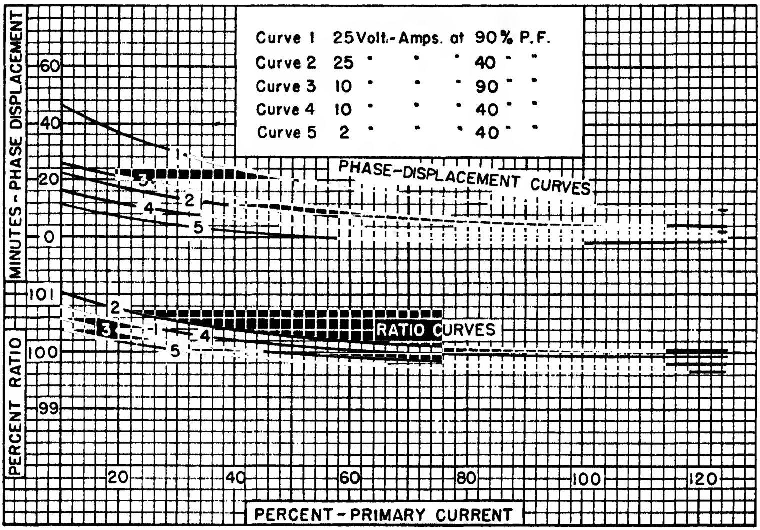


FIG. 69–8. Characteristic curves of a current transformer. The power-factor here referred to is that of the load or burden on the transformer itself. (*Westinghouse Electric Corp.*)

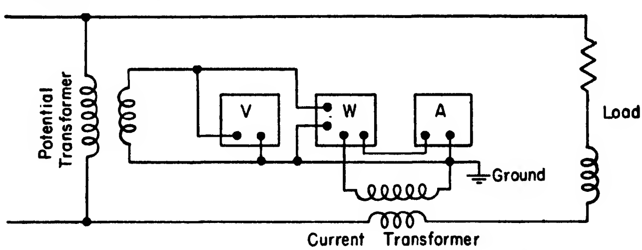


FIG. 70–8. Use of instrument transformers for measuring voltage, current and power in a single-phase circuit.

addition of more instruments; or by any change in the power factor of the burden.

The calibration curves of Fig. 69–8, for the type of current transformer in Fig. 66–8, show that variation from the ideal condition of constant ratio and zero (180°) phase displacement, though small, are quite appreciable. Current transformers are often

designed to give accurate current ratio and practically 180° displacement between primary and secondary at about 65 per cent of their rated capacity.

Instrument Connections. Figure 70-8 illustrates the method of connecting transformers and instruments to measure voltage, cur-

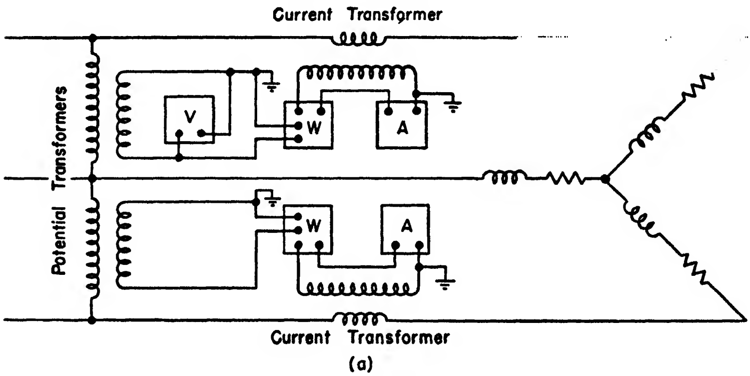


FIG. 71-8(*a*). Arrangement of instruments for measuring voltage current and power in a three-phase circuit by means of two potential and two current transformers.

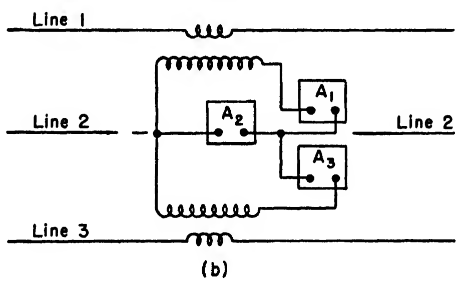


FIG. 71-8(*b*). Connections of three ammeters in the secondaries of two current transformers for measuring current in the three lines of a three-phase circuit.

rent and power in a high-voltage, single-phase circuit. Note that the voltmeter and potential coil of the wattmeter are connected in **parallel** to the potential transformer, while the ammeter and current coil of the wattmeter are connected in **series** in the current transformer secondary. The secondary circuits of both transformers are grounded.

Figure 71-8(*a*) shows the arrangement for measuring voltage, current and power by the two-wattmeter method in a three-phase circuit.

Using only two current transformers, the current in all three lines can be determined by the arrangement of ammeters in Fig. 71–8(*b*). Ammeters A_1 and A_3 indicate the currents in lines 1 and 3, respectively. Since the current in the third line of a three-phase circuit is equal to the vector sum of the currents in the other two, ammeter A_2 is placed to measure the **sum** of the currents in the two transformer secondaries; and, therefore, indicates the current in line 2. In this case, the transformer secondaries must be phased.

A single ammeter may be connected to indicate the current in each of the three lines in turn, by use of a special "jack" switch. This switch connects the ammeter in any one of the three secondary circuits, while the other two are closed.

Prob. 83–8. A current transformer, like that shown in Fig. 68–8, has a ratio of 400 to 5 amperes, when the primary circuit passes once through the opening in the core. Approximately what will be the reading on an ammeter with 5-ampere range, connected to the secondary terminals, when a primary circuit carrying 80 amperes passes 4 times through the core?

Prob. 84–8. How many times should the primary circuit be passed through the core, in the transformer of Prob. 83–8, in order that an ammeter of 5 amperes range may indicate in middle of the range, when used to measure a current of 100 amperes?

Prob. 85–8. The current, voltage and power in a 2200-volt single-phase feeder, carrying 8-amperes, at 0.8 power factor, is to be measured by meters with standard current and voltage ranges of 5 amperes and 150 volts, respectively. The voltmeter has a 750-watt scale. The ratio of both the potential and the current transformer is 20 : 1.

(*a*) Make a wiring diagram, showing the connections of the transformers and meters.

(*b*) Neglecting any phase displacement in the transformers, what would each meter indicate?

(*c*) What power in the primary circuit will the indication of the wattmeter in (*b*) represent?

Prob. 86–8. What maximum voltage, current and power can be measured by the instruments connected as in Prob. 85–8?

Prob. 87–8. A polyphase wattmeter, consisting of two current coils and two potential coils, on a single moving system in one case, is connected through instrument transformers to a 6600-volt, three-wire, three-phase line, according to the two-wattmeter method of measuring power. If the two potential coils and the two current coils are each rated at 150 volts and 5 amperes, respectively, what should be the ratio (nearest multiple of 5) of each potential transformer and each current transformer to measure 200 kw at 0.9 power factor?

Prob. 88–8. (*a*) What current would flow in each current coil of the wattmeter in Prob. 87–8?

(*b*) What would be the pressure across each potential coil?

Prob. 89–8. Make a wiring diagram showing the connections of the transformers and wattmeter in Prob. 87–8, and include in the diagram the arrangement of three ammeters to indicate the current in each primary line.

SUMMARY OF CHAPTER VIII

TRANSFORMERS are machines for transferring energy from one a-c circuit to another a-c circuit. They consist of stationary coils, linked together on a stationary iron core. By means of an alternating flux in the core, they change low voltage to high voltage and vice versa. They are the simplest, most rugged, most efficient and least expensive in first cost and maintenance of any electrical power machine.

The coils, or windings, which receive energy, are called the PRIMARY, and those which deliver energy, the SECONDARY.

The RATIO OF THE TRANSFORMER is the ratio of the number of turns in the high-voltage coils to that in the low-voltage coils,

or
$$\frac{E_p}{E_s} = \frac{N_p}{N_s}.$$

The EQUATION FOR INDUCED EMF (effective values) **is expressed as,**

$$E_{eff} = \frac{4.44fNB_MA}{10^8}.$$

The INDUCED EMF in the coils has a phase difference of 180° to the impressed voltage. The induced emfs lag 90° behind the flux which is set up in the core. The impressed voltage, therefore, leads the flux by 90°.

ON ZERO LOAD, when no current is taken from the secondary, the counter, or induced, emf in the primary prevents the impressed voltage from sending more than an EXCITING current through primary coils. The EXCITING CURRENT is the vector sum of two components at 90°: the MAGNETIZING CURRENT, I_M, which sets up and is in phase with the flux; and the CORE LOSS CURRENT, I_H, in phase with the impressed voltage, and which supplies the iron losses in the core.

Therefore,

$$I_E = \sqrt{I_M^2 + I_H^2}.$$

Thus the Power Factor $= \dfrac{I_H}{I_E} = \dfrac{I_H}{\sqrt{I_M^2 + I_H^2}}.$

WHEN LOADED, the current, taken from the secondary, sets up a counter mmf in the core, which reduces the flux and counter emf in the primary. The impressed voltage can now send enough more

current through the primary to balance this counter mmf, due to the secondary current, and restore the flux to its original value.

THE RATIO OF PRIMARY to SECONDARY CURRENT at full load is approximately the inverse ratio of the number of turns, or $\frac{N_P}{N_S} = \frac{I_S}{N_P}$.

TRANSFORMER CORES are CORE TYPE or SHELL TYPE, or modifications of these types, depending upon the arrangement of core laminations with respect to the coils. In the CORE TYPE, the copper windings surround the core. In the SHELL TYPE, the iron core surrounds the windings.

COOLING OF TRANSFORMERS.

(1) OIL-FILLED transformers are set in a tank filled with oil, which carries the heat from the coils to the case, from which it is dissipated to the surrounding air. The cooling surfaces in large transformers are increased by means of external vertical pipes or radiators welded or bolted to the tank.

(2) AIR-COOLED transformers, built only in small sizes, have no tank and heat is dissipated directly to the surrounding air.

(3) AIR-BLAST transformers are cooled by forcing air through ducts in the coils by means of a blower.

(4) OIL-INSULATED, WATER-COOLED transformers have a coil of copper tubing set in the top of the tank and below the oil level. Water is circulated through this tubing, thereby carrying away heat from the hottest oil, as it rises through the windings.

LARGE OIL-COOLED TRANSFORMERS are generally equipped with an expansion tank, or CONSERVATOR, partly filled with oil, placed above the transformer and connected by pipe to the tank; or with a BREATHER, to prevent moisture from entering the transformer as the oil expands and contracts with temperature change. Moisture and oxygen reduce the dielectric strength of the oil and form a thick sludge, which clogs the oil ducts in the coils and increases the heating.

POWER TRANSFORMERS are installed in power stations and on transmission lines for transforming relatively large amounts of power.

DISTRIBUTION TRANSFORMERS, usually less than 100 kva capacity, are used on secondary lines for distribution of power to local centers.

LIGHTING TRANSFORMERS, usually less than 50 kva, are used to supply relatively small amounts of power to the consumer. Their high-side rating is seldom more than 2300 volts approximately.

If THE MAXIMUM FLUX DENSITY is carried much above the "knee" of the saturation curve, the exciting current rises rapidly, and at very high flux densities may equal or exceed the full-load current of the transformer. The iron losses and heating also are greatly increased. The relation of impressed voltage, frequency, number of turns in the primary, etc., to the maximum flux density, B_m, is expressed as,

$$B_m = \frac{E \times 10^8}{4.44 f N A}.$$

The flux is directly proportional to the impressed voltage and inversely proportional to the frequency and number of turns.

The SHAPE OF THE EXCITING CURRENT WAVE is not a sine curve, but is distorted due to hysteresis in the core. Besides the normal frequency curve, it contains higher frequency components, most marked of which is that of triple frequency.

POLARITY and PHASING. The terminals of the coils in the primary side (and also in the secondary side) in a transformer must be so connected that all the coils in that side set up mmfs in the SAME direction in the core. The coils in each side, therefore, must be joined in additive polarity in their respective electric circuits. The process of determining coil terminals of like POLARITY is called PHASING.

THE LOSSES in a transformer consist of CORE, OR IRON, LOSSES and COPPER LOSSES.

IRON LOSSES are constant at all loads, if the frequency and flux density are constant, and are composed of two separate losses.

(1) HYSTERESIS LOSS, the equation for which is,

$$P_H = WK_H f B_m^{1.6}.$$

(2) EDDY CURRENT LOSS, the equation for which is,

$$P_E = WK_E f^2 B_m^2 t^2.$$

THE COPPER LOSS is the I^2R loss in both primary and secondary windings, and is approximately equally divided between the two.

CORE LOSSES ARE DETERMINED by operating the transformer at zero load, or with the secondary circuit open, at rated impressed voltage and frequency, and measuring the power input. The I^2R losses are negligible in this case and the wattmeter indicates the core loss.

COPPER LOSSES may be DETERMINED from the ohmic resistance of the windings; but are generally determined by short circuiting one side (usually the low-side) and impressing sufficient voltage at normal frequency on the other side to send full load current through both windings. Less than 5 per cent of rated voltage is generally required, so the flux and iron losses are negligible. The power input, indicated by a wattmeter, is the copper loss.

ALL DAY EFFICIENCY of a transformer is the RATIO OF THE WATTHOURS OUTPUT TO THE WATTHOUR INPUT, during a specified time; which may be a 24-hour period, a month, or a year.

LEAKAGE REACTANCE. When loaded the opposing mmf, set up by the secondary NI, forces some of the primary flux out of the core, so that it completes its circuit through the air, without linking the secondary winding. The secondary mmf, since it opposes the core flux, also forces a secondary flux out of the core, completing its circuit through the air, without linking the turns of the primary winding. These leakage fluxes cause reactance, known as primary and secondary leakage reactance respectively, and a voltage drop in each winding. Thus the induced emf in the primary is less than the impressed voltage; and the secondary terminal voltage is less than the secondary induced

voltage. The reactance drop in the two windings, together with the IR drops, cause the ratio of the primary and secondary voltages to differ somewhat from the turn ratio of the coils.

The relations of these voltage drops are all shown in the practical vector diagram of the loaded transformer.

EQUIVALENT PRIMARY RESISTANCE AND REACTANCE. The resistance and leakage reactance of the secondary winding is assumed to be zero, and enough extra resistance and reactance, added to that of the primary, to cause the secondary terminal voltage to be equal to that of the actual transformer for any given load. That is, all the voltage drops are assumed to be in the primary and none in the secondary. The sum of the primary resistance plus the extra resistance, is called the EQUIVALENT PRIMARY RESISTANCE; and the sum of primary reactance plus the extra reactance is called the EQUIVALENT PRIMARY REACTANCE.

The Equivalent Primary Resistance, $R_{ep} = R_p + R_s\left(\frac{N_p}{N_s}\right)^2$.

The Equivalent Primary Reactance, $X_{ep} = X_p + X_s\left(\frac{N_p}{N_s}\right)^2$.

These values can be similarly expressed in terms of the secondary winding as:

The Equivalent Secondary Resistance, $R_{es} = R_s + R_p\left(\frac{N_s}{N_p}\right)^2$.

The Equivalent Secondary Reactance, $X_{es} = X_s + X_p\left(\frac{N_s}{N_p}\right)^2$.

THE IMPEDANCE OF A TRANSFORMER is the equivalent primary impedance. It is expressed in ohms, or in volts to overcome the impedance drop at rated load. It is found by measuring the primary impressed voltage (impedance volts), necessary to overcome the impedance drop, $E = IZ_{ep}$, at rated current on short-circuit test. The watts input in this test (" impedance watts ") divided by the rated current, equals the equivalent primary resistance drop, IR_{ep}. Equivalent primary reactance drop

$$IX_{ep} = \sqrt{\overline{IZ}^2_{ep} - \overline{IR}^2_{ep}}, \quad \text{or} \quad IX_{ep} = \sqrt{E^2 - \left(\frac{W}{I}\right)^2}.$$

THE REGULATION OF A CONSTANT-POTENTIAL TRANSFORMER may be computed from the values of IX_{ep} and IR_{ep} expressed either in volts or in per cent of the rated voltage.

TO OPERATE IN PARALLEL, transformers must have the same rated primary and secondary voltages, that is, the same turn ratio. Also, they must have the same per cent resistance and reactance, in order to divide the load proportionately. Before connecting the secondary winding in parallel they must be phased for polarity.

AN AUTOTRANSFORMER has the primary and secondary windings joined electrically in series, and part of the winding is common to both primary and secondary circuits. Part of the power from the

supply mains is transferred directly to the secondary circuit, and part is transformed. The current in that part of the winding, common to both circuits, is the DIFFERENCE between the primary and secondary currents.

The advantage of the autotransformer lies in reduced copper losses, greater efficiency and lower cost for the same kva capacity. These advantages increase as the voltage ratio approaches unity.

The chief objection to autotransformers on systems of even moderate voltage is that a dangerous potential exists on the low-side winding, because it is electrically connected to the high side. Autotransformers used for motor starters are called "compensators."

TRANSFORMERS ON POLYPHASE SYSTEMS can be used to transform power:

(1) From 3-phase at one voltage, to 3-phase at a different voltage.
(2) From 2-phase at one voltage, to 2-phase at a different voltage.
(3) From 3-phase at one voltage, to 2-phase at a different voltage.
(4) From 2-phase at one voltage, to 3-phase at a different voltage.

THREE-PHASE TRANSFORMATION is generally accomplished by a "bank" of three separate and identical transformers, or by a single three-phase transformer. There are four possible connections, as follows:

(1) Primaries in Y-secondaries in Y;
(2) Primaries in Δ-secondaries in Δ;
(3) Primaries in Y-secondaries in Δ;
(4) Primaries in Δ-secondaries in Y.

Three-phase transformation can be obtained by means of two similar transformers in an OPEN-DELTA or V-V CONNECTION. The two transformers, however, have only 58 per cent of the capacity of three transformers, connected inclosed delta.

SCOTT or T-CONNECTION. THREE-PHASE to THREE-PHASE and THREE-PHASE to TWO-PHASE TRANSFORMATIONS are possible by means of two similar transformers. One terminal of both primary and secondary windings of one transformer, called the "teaser," is tapped to the midpoints of the corresponding windings of the other, or "main" transformer.

When a 3-phase supply is impressed across the three remaining primary terminals, a balanced 3-phase voltage is obtained across the three corresponding secondary terminals.

While the primary windings only are so connected, and only 86.6 per cent of the turns in the teaser winding are used, a 2-phase voltage is obtained across the two pairs of secondary terminals.

The two transformers, connected in this manner, operate at only 86.6 per cent of their rated capacity.

THREE-PHASE TRANSFORMERS are constructed by combining parts of the magnetic circuits of three single-phase transformers to form a single core structure. They occupy less floor space, weigh slightly less and have slightly higher efficiency than three single-phase

transformers of the same total kva capacity. There are two types — the core type and shell type.

If THREE-PHASE BANKS OF TRANSFORMERS are CONNECTED IN PARALLEL to the same system on the primary sides, then to connect the secondary sides in parallel, the connections must be such that the voltage between any two lines on this side must be in phase in each bank. From these relations result the following rules:

(a) With YY on one bank, the other bank must be YY, or ΔΔ.
(b) With ΔΔ on one bank, the other bank must be ΔΔ, or YY.
(c) With ΔY on one bank, the other bank must be ΔY, or YΔ.

It is not possible to connect any ΔY combination with a YY or a ΔΔ combination.

In addition to the above relations, the three phases in each bank, must be connected in the proper sequence, or a short circuit results.

TAP CHANGING UNDER LOAD. To compensate for line drop in primary lines, or when two systems are tied together through a transformer, the voltage ratio of large transformers may be adjusted by bringing out taps from one winding, and changing the number of turns in circuit. Parts of the winding can be inserted or taken out, while under load, and without short circuit, by means of an auxiliary autotransformer, which can be connected in turn across any two adjacent taps. This may be accomplished manually or automatically.

THE INDUCTION REGULATOR is used to keep the voltage of secondary lines, or feeders, constant under changing load. It is essentially an autotransformer in which the line current sets up a flux in a movable core, upon which the primary coil is mounted. This core can be rotated automatically into such a position that it sets up an induced emf in the secondary, which is in series with the line. This emf can be made to " boost " or " buck " the line voltage sufficiently to keep the feeder voltage constant.

THE CONSTANT-CURRENT TRANSFORMER takes power from constant-potential mains and automatically delivers, from its secondary, a constant a-c current to a series lighting system, the impedance and voltage of which vary with the number of lights in circuit. One winding is movable with respect to the other, and the leakage flux is large. The variable voltage characteristic is obtained by a variation in this leakage flux. Any increase in secondary current increases the repulsion between the windings, and they move farther apart, the leakage flux increases and the secondary voltage drops, restoring the current, practically, to its former value.

INSTRUMENT TRANSFORMERS are small transformers for stepping down the voltage or the current from the power circuit. Meters of standard range can thus be used in the secondary circuits to measure the power, voltage, and current in the power circuit. Low-energy control devices can also be used in these circuits. These transformers also insulate the instruments from the high voltage line and insure against dangerous voltage on switchboard apparatus.

A POTENTIAL TRANSFORMER is similar to a constant-potential

power transformer and is used to step down the voltage in a known ratio, which varies slightly from the turn ratio. Potential coils of instruments are connected in PARALLEL across the secondary terminals.

A CURRENT TRANSFORMER is used to step down the current in a known ratio. This is the ratio of the primary of the secondary currents, which differ slightly from the turn ratio, and varies somewhat with the load and with the "burden" on the transformer. The secondary current differs slightly in phase from the primary current — the amount depending on the load. These variations affect principally the accuracy of the wattmeter indications. The current coils of instruments are always connected in series in the secondary circuit.

CAUTION. NEVER OPEN THE SECONDARY CIRCUIT OF A CURRENT TRANSFORMER. IF IT IS NECESSARY TO CHANGE CONNECTIONS, FIRST SHORT-CIRCUIT THE SECONDARY TERMINALS.

PROBLEMS ON CHAPTER VIII

Prob. 90–8. The high-side winding of a 20-kva, 2400-500-volt, 25-cycle transformer consists of 750 turns and is to operate at 70,000 lines per square inch.

(*a*) What must be the net area of the core? (*b*) How many turns in the low-side winding?

Prob. 91–8. The core of the transformer in Prob. 90–8 is to be rewound for the same voltage at 60 cycles and operated at 60,000 lines per square inch.

(*a*) What should be the number of turns in the high-side winding?
(*b*) In the low-side winding?

Prob. 92–8. (*a*) If the current density in the windings of the transformer in Prob. 90–8 is to be approximately 1200 circular mils per ampere, what size wire should be used for the high-side winding? (*b*) For the low side?

Prob. 93–8. A 200-kva, 11,000-2200-volt, 60-cycle transformer operates at a core flux which induces 8 volts per turn in the windings. The net cross section of the core is 46 square inches. Determine:

(*a*) The turns in the high-side winding.
(*b*) In the low-side winding. (*c*) The maximum flux in the core.
(*d*) The maximum flux density.

Prob. 94–8. A 30-kva, 2300-460-volt transformer takes 260 watts and 0.4 ampere from 2300-volt, 60-cycle mains when the secondary or low side is on open circuit.

(*a*) What is the magnetizing current? (*b*) The core loss current?
(*c*) The zero load power factor. Show vector diagrams.

Prob. 95–8. If the high-side winding of the transformer in Prob. 94–8 is on open circuit, how many watts and what exciting current would the low side draw from 460-volt, 60-cycle mains? What would

be the magnetizing current? The core-loss current? The zero load power factor?

Prob. 96–8. When the low side of a 500-kva, 13,200-2200-volt, 60-cycle transformer is connected to 2200-volt, 60-cycle mains, with the high side open, the magnetizing current is 7.1 amperes. If the magnetizing current is 95 per cent of the exciting current, compute:

(*a*) The exciting current in amperes and in per cent of full load.

(*b*) The zero load power factor. (*c*) The core loss.

Prob. 97–8. If the transformer in Prob. 96–8 supplies rated current to a line at 13,200 volts and 0.6 lagging power factor, compute:

(*a*) The total primary current. (*b*) The primary power factor.

Prob. 98–8. A 100-kva, 60-cycle transformer has a primary coil rated at 2400 volts, and two secondary coils each rated at 125 volts.

(*a*) If there are 400 turns in the primary coil how many turns are there in each secondary coil?

(*b*) What is the rated current in each coil?

(*c*) What will be the primary current if the secondary coils are connected to an unbalanced three-wire system (see Fig. 24–8(*a*)), and one coil carries 400 amperes and the other 200 amperes, both at unity power factor? Neglect the exciting current in each case.

Prob. 99–8. A 50-kva, 2300-115-volt, 50-cycle transformer normally operates at a flux density of 65,000 lines per square inch. What will be the flux density if it operates at rated voltage on a 60-cycle circuit?

Prob. 100–8. What will be the voltage and kva rating of the transformer in Prob. 99–8 if it operates at normal flux density on 60 cycles?

Prob. 101–8. A 10-kva transformer has two 1100-volt and two 120-volt coils.

(*a*) Show diagrams of all possible connections when used as an ordinary transformer.

(*b*) What are the primary and secondary line voltage ratings for each connection in (*a*).

(*c*) Compute the rated primary and secondary line currents for each case.

Prob. 102–8. The primary winding of a transformer consists of two 230-volt coils which can be connected either in series or in parallel. When connected in series across 460 volts, the power input at zero load is 90 watts and the current is 0.5 ampere.

(*a*) What will be the current and power input when the coils are connected in parallel across 230 volts?

(*b*) When only one coil is connected across 230 volts?

Prob. 103–8. A 15-kva, 2200-220-volt, 600-cycle transformer operates at a normal flux density of 60,000 lines with a hysteresis loss of 140 watts and eddy current loss of 30 watts. If the voltage impressed on the high-side coil is raised to 2500 volts, compute:

(*a*) The flux density. (*b*) The hysteresis loss. (*c*) The eddy current loss.

Prob. 104–8. A 100-kva, 11,000-2200-volt transformer has a core loss of 650 watts and a copper loss of 870 watts at rated full load. Compute the efficiency for $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{4}{4}$ and $\frac{5}{4}$ rated kva at unity power factor. Plot the efficiency curve using kilowatts output as abscissae.

Prob. 105–8. Repeat Prob. 104–8 for loads at 0.8 power factor.

Prob. 106–8. Compute the all-day efficiency of the transformer in Prob. 104–8 if it operates 3 hours a day at full load, 5 hours at half load, and the remainder of the 24-hour period at no load. Unity power factor load in each case.

Prob. 107–8. Repeat Prob. 106–8 under the same conditions, but with the loads at 0.8 power factor.

Prob. 108–8. The primary or high-side resistance of a 100-kva, 1150-230-volt, 50-cycle transformer is 0.086 ohm. That of the low side is 0.00386 ohm.

(*a*) What is the equivalent primary resistance in ohms and in per cent?
(*b*) The equivalent secondary resistance in ohms and in per cent?

Prob. 109–8. A 10-kva, 2300-115-volt, 60-cycle transformer has a full load copper loss of 120 watts. If this load is equally divided between the two windings, compute:

(*a*) The resistance of each winding.
(*b*) The equivalent primary or high-side resistance.
(*c*) The equivalent secondary resistance.

Prob. 110–8. The full load copper loss in a 5-kva, 2300-230-volt, 60-cycle transformer is 80 watts. If the resistance of the low side is 0.074 ohm, compute:

(*a*) The resistance of the high side.
(*b*) The equivalent primary or high-side resistance.
(*c*) The equivalent secondary resistance.

Prob. 111–8. On short circuit or impedance test of a 20-kva, 2200-440-volt, 60-cycle transformer, 92.5 volts impressed on the high side forces full-load current through the winding. The wattmeter in the circuit indicates 300 watts. Compute the equivalent primary or high-side impedance, resistance and leakage reactance in ohms and in volts.

Prob. 112–8. Compute the regulation of the transformer in Prob. 111–8 on the following loads:

(*a*) Unity power factor. (*b*) 0.8 power factor lagging. (*c*) 0.8 power factor leading.

Prob. 113–8. Compute the per cent impedance, resistance and reactance of the transformer in Prob. 111–8.

Prob. 114–8. From the percentage values obtained in Prob. 113–8, compute the regulation of the transformer in Prob. 111–8 for the loads specified in Prob. 112–8.

Prob. 115–8. The following data are obtained from the open-circuit and short-circuit tests on a 10-kva, 2200-220-volt, 60-cycle transformer.

Open-circuit test, instruments in low side,

$E = 220$ volts, $W = 48$ watts, $I = 1.3$ amperes.

Short-circuit test, instruments in high side,

$E = 55$ volts, $W = 160$ watts, $I = 4.54$ amperes.

Compute:

(*a*) The regulation of the transformer at unity power factor load.

(*b*) The efficiency at rated full load, unity power factor.

Prob. 116–8. Repeat Prob. 115–8, for 0.8 power factor lagging load.

Prob. 117–8. Repeat Prob. 115–8, for 0.6 power factor leading load.

Prob. 118–8. The following data are obtained from the open-circuit and short-circuit tests of a 500-kva, 13,200-2200-volt, 60-cycle transformer.

High side open, instruments in low side,

$E = 2200$, $W = 1800$ watts, $I = 8.5$ amperes.

Low side short circuited, instruments in high side,

$E = 820$ volts, $W = 5200$ watts, $I =$ full load current.

Compute:

(*a*) The per cent resistance, reactance and impedance.

(*b*) The regulation at 0.75 power factor lagging load.

(*c*) The efficiency at rated kva and 0.75 power factor.

Prob. 119–8. A 15-kva, 2300-230-volt transformer is connected as an autotransformer so that it boosts the line voltage from 2300 to 2530 volts. Compute:

(*a*) The maximum kilowatt load the transformer can deliver without overloading either of the transformer windings.

(*b*) The power input. (*c*) The power transformer.

(*d*) The power transferred directly from the primary to the secondary circuit. Neglect the exciting current, losses, and voltage drops in the coil.

Prob. 120–8. Repeat Prob. 119–8 when the secondary coil is reversed so that the resulting secondary voltage is 2070 volts.

Prob. 121–8. A 10-kva, 2200-220-volt, 60-cycle transformer similar to that in Prob. 61–8, has two 1100-volt coils A_1B_1 and A_2B_2; also two 110-volt coils, X_1Y_1 and X_2Y_2. The transformer is to be used as an autotransformer, and is connected as follows, H_1 to A_1 and A_2, B_1 and B_2 to X_1, X_2 and L_1, Y_1 and Y_2 to H_2 and L_2. When operated at normal flux density, determine·

(*a*) Volts between high-side mains and H_1 and H_2.

(*b*) Volts between low-side mains L_1 and L_2.

(*c*) Maximum kilowatt load at unity power factor which can be delivered to the mains L_1L_2 without overloading any coil in the transformer. (*d*) Power transformed. (*e*) Power transferred. Neglect losses and voltage drop in the coils.

Prob. 122–8. Repeat Prob. 121–8 when the transformer is connected as follows: H_1 and L_1 to A_1 and A_2, B_1 and B_2 and X_1 and X_2 to L_2, Y_1 and Y_2 to H_2.

Prob. 123–8. When the transformer of Prob. 121–8 is connected, H_1 to A_1, B_1 to A_2, B_2 to X_1, Y_1 to X_2, Y_2 to H_2, and operates at normal flux density, determine all possible secondary voltages. For each case answer the questions (*a*) to (*e*) called for in Prob. 121–8.

Prob. 124–8. A 6600-volt, Y-connected alternator supplies a balanced three-phase load of 15,000 kva at 0.8 power factor which is stepped up to 13,200 volts by means of three Y-connected autotransformers.

(*a*) Show a diagram of the system. (*b*) Compute the current in the primary and in the secondary coil of each transformer. (*c*) The power transformed. (*d*) The power transferred. Neglect losses.

CHAPTER IX

THE POLYPHASE INDUCTION MOTOR

The great majority of electric motors in use today are of the induction type. While the induction motor is inferior to the d-c shunt motor for variable-speed drive, and to the d-c series motor for electric traction, the fact that alternating current is almost universally employed for the distribution of electrical power has led to its wide use. Furthermore, this motor is less expensive than the d-c motor, is reliable, rugged and maintenance costs are low. There is no commutator and, in its simplest form, there are no sliding electrical contacts. It is essentially a constant-speed motor and well adapted for use in shops, mills and factories, particularly in dusty locations and where explosive gases may be present.

The motor usually consists of a stationary primary winding, called the "**stator**," to which the supply mains are connected; and a secondary winding, called the "**rotor**," short-circuited on itself, or through resistance, which is free to rotate. There is no electrical connection between the stator and the rotor, and the fact that currents are **induced** in the rotor gives the motor its name. In this respect, the induction motor is very similar to the transformer. Torque is produced by the reaction between a **rotating magnetic field**, set up by the currents in the primary or stator, and the induced currents in the secondary or rotor.

1–9. Construction. The stator, or primary of the induction motor, consists of a stationary frame and laminated steel core, so punched and bolted together as to form a hollow slotted cylinder, exactly similar to the armature core of a rotating field alternator. This core carries an insulated winding in the slots which is practically the same as that for a polyphase alternator. See Fig. 1–9. Usually in small and medium sized motors, the stator slots are partially closed, as shown in Fig. 2–9. This reduces the reluctance of the magnetic circuit, the flux density and the iron losses in the teeth. In large motors, the slots are generally of the open type, as the comparative reluctance of the flux paths is lower. It is also more difficult to insert conductors of large cross section in partially closed slots.

The rotor or secondary of the motor also consists of laminated steel punchings, bolted together and mounted on a shaft. This core is cylindrical in shape and carries a drum winding in slots or ducts near its outer surface. The adjacent surfaces of stator and

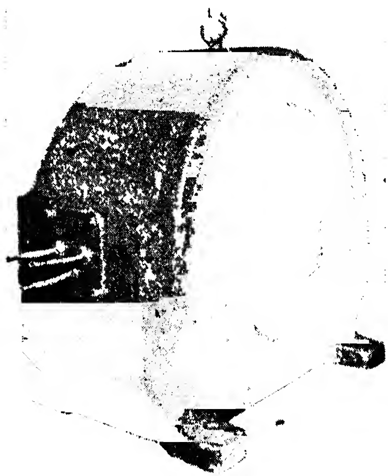


FIG. 1-9. Frame and stator of a polyphase induction motor. (*General Electric Co.*)

rotor are accurately ground or fitted so that the radial length of the air gap is only slightly greater than necessary for mechanical clearance. To prevent deflection or vibration, which would cause

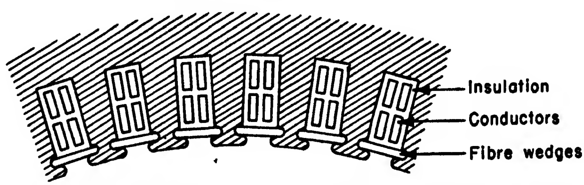


FIG. 2-9. Partially closed slots in the stator core of an induction motor.

the rotor core to rub against the stator, the rotor shaft and bearings are heavier and more rigid than that of other rotating electrical apparatus of comparable size.

There are two types of rotor windings, the " squirrel-cage " and the " wound rotor."

The **squirrel-cage rotor** is the more common and simplest type. This winding consists of bars of copper or alloy metal, laid in the

core slots and short circuited at each end by a ring of the same material to which the bars are bolted, or electrically welded, as shown in Fig. 3–9. In practically all squirrel-cage rotors, the slots are either partially closed, as in Fig. 4–9, or totally enclosed.

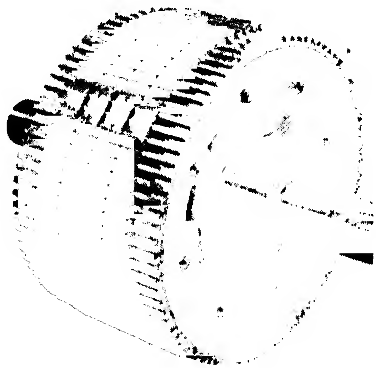


Fig. 3–9. Rotor of squirrel-cage induction motor, showing end ring brazed to the copper bars, which are driven into the core slots. (*Allis-Chalmers Manufacturing Co.*)

The bars are readily driven into these slots from one end. The reluctance of the magnetic circuit is reduced by this type of slot. The bars are provided with little or no insulation, since the emfs induced in this type of winding are low.

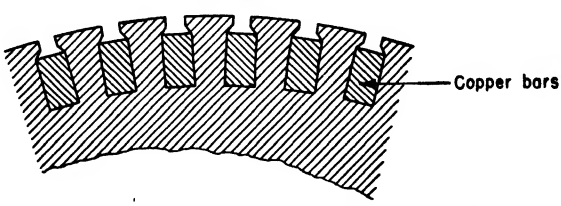


Fig. 4–9. Semi-closed slots in the rotor of the squirrel-cage induction motor.

For small and medium sized motors, it has become common practice to construct the rotor winding of aluminum. This winding, including end rings and ventilating fans, is die cast in one integral piece. In such windings, the slots are generally closed. See Fig. 5–9. Figure 6–9 shows a cast aluminum squirrel-cage winding for a small motor from which the iron core has been eaten away by acid.

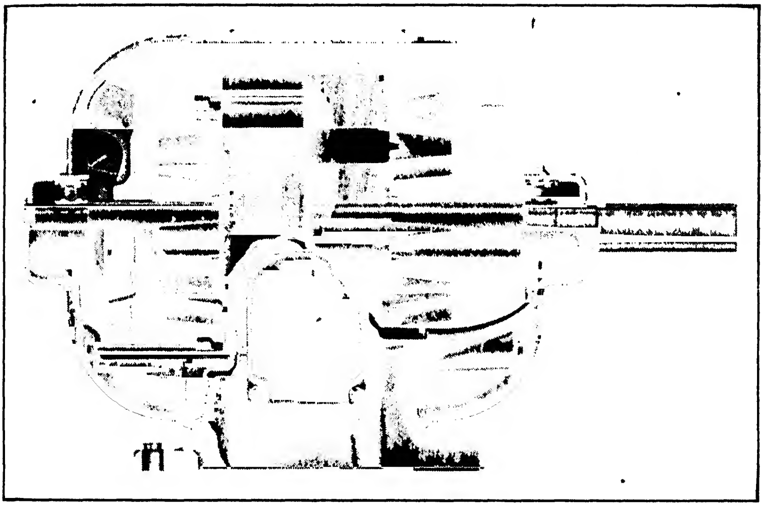


FIG 5-9. Die-cast winding and rotor for a squirrel-cage motor. The rotor slots are entirely closed. (*Westinghouse Electric Corp.*)

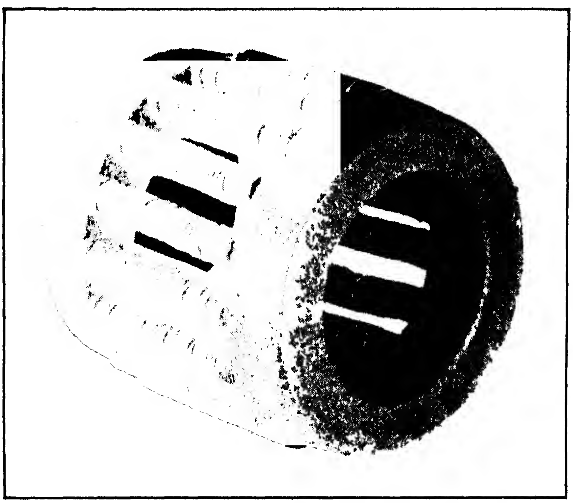


FIG. 6-9. Die-cast aluminum squirrel-cage rotor for a very small motor from which the core has been eaten away by acid. The bars in this winding show the irregularities in the core punchings.

In many rotors of this type, the bars are not placed parallel to the shaft, but are " skewed " by a small angle, as in Fig. 6–9. In general, this gives a more uniform torque at starting and reduces the humming noise when running.

The **wound rotor** has an insulated winding of a considerable number of turns and is often practically the same as that of the stator or very similar to it. When the coils are form wound, the slots are usually of the open type. If the turns in the coils can be inserted separately, partially or semi-closed slots are used to reduce the magnetic reluctance. The terminals of this winding, generally three-phase, Y-connected, are brought out to slip rings

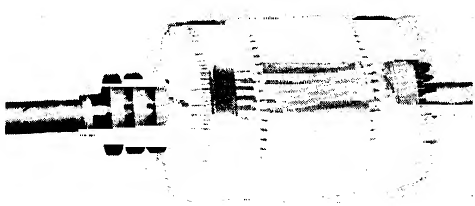


Fig. 7–9. Wound-rotor induction motor. The terminals of the rotor windings are brought out to the slip rings on the left. Brushes on these rings connect the winding to external resistors. (*General Electric Co.*)

on the shaft, as in Fig. 7–9. They are connected through brushes to adjustable resistors. By this means, the speed and starting torque can be controlled, as explained later.

The induction motor will operate equally well if the primary winding is placed on the rotating structure and connected to the power supply through slip rings and brushes. The stationary winding in this case becomes the short-circuited secondary.

2–9. Induction-Motor Action. The induction motor operates on the principle illustrated by the device in Fig. 8–9. This consists of a permanent horse-shoe magnet and a circular copper or aluminum disc, both mounted on a vertical shaft and free to turn independently of each other. If the magnet is rotated by hand, or by other means, in a clockwise direction as shown, the magnetic flux cuts the disc and sets up in it local emfs and current, in accord with Fleming's right-hand rule, as indicated by the broken lines in Fig. 8–9(b). Figure 8–9(c) indicates the direction of these currents under the N pole in the conductor paths in the disc, as

viewed from the position "P" in Figs. 8–9(*a*) and (*b*). The condition is the same as though the N pole were stationary, and

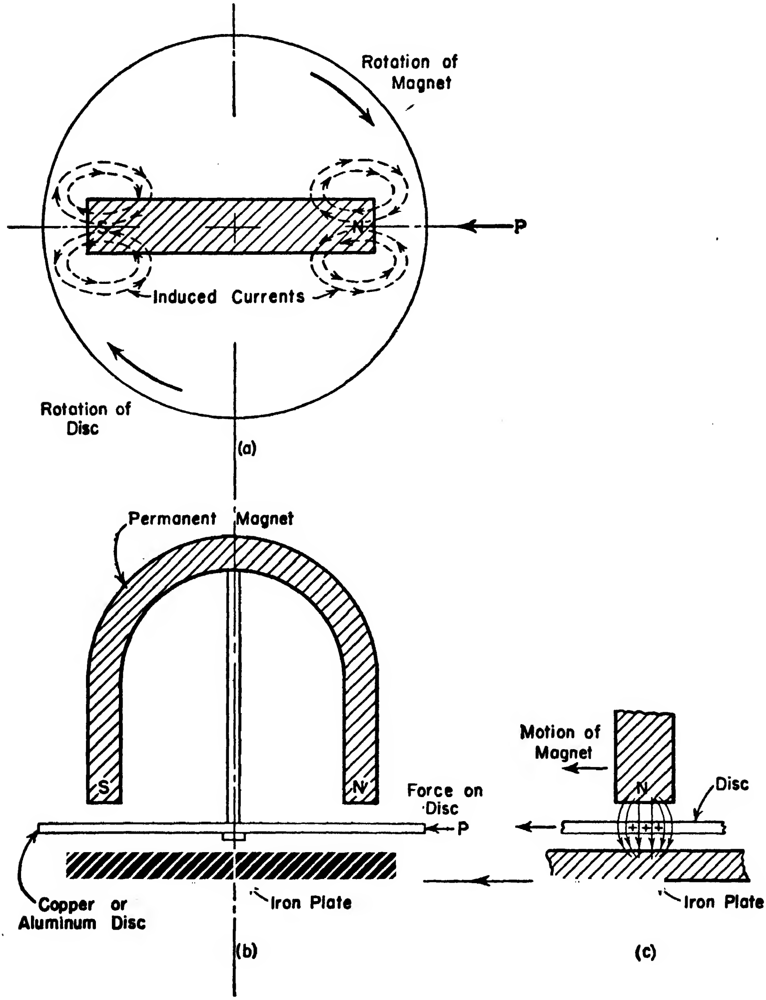


Fig. 8–9. The rotating magnet produces rotation in the copper or aluminum disc.

the disc or conductors, were moving to the right in the opposite or counter-clockwise direction. These emfs and currents are induced in the disc by the generator action of the moving magnet. It is seen that torque is exerted on the disc in such a direction that

it also rotates in the same direction as the permanent magnet, or clockwise.

The disc cannot rotate at the same speed as the magnet or magnetic field, however, for there would be no cutting of the conductor paths in the disc by the magnetic lines. So no current would flow in the disc and no torque would be developed. The disc, therefore, rotates at a lower speed than the magnet. This action is somewhat similar to that of a slipping clutch. The slower the disc revolves with respect to the magnet, the greater is the difference in their relative speeds, and the greater is the emf and current in the disc and the resulting torque. The disc must, therefore, revolve at such a reduced speed that the rate of cutting, or difference in speeds, is sufficient to set up in it enough emf and current to develop the necessary torque. This is the action which takes place in the actual motor. In the commercial motor, the magnetic field rotates at a speed in synchronism with the supply circuit, while the rotor, which is in the form of a cylinder, operates at some speed below synchronism. The induction motor thus falls into that class of electrical equipment known as asynchronous or nonsynchronous apparatus.

3–9. The Rotating Magnetic Field. In Fig. 8–9, the rotation of the magnetic field is produced mechanically. A rotating magnetic field can be obtained electrically by a change in the direction of currents in the windings on a stationary iron core.

Consider Fig. 9–9, which shows the pole pieces of an ordinary four-pole frame. The windings on one pair of oppositely placed poles are connected in series to a d-c supply through the double-pole-double-throw (DPDT) switch *m*. The windings on the other pair of poles are likewise connected through a similar switch, *n*, to the same d-c source. In (*a*), switch *n* is open and *m* is thrown up, and a magnetic field is set up in a direction, according to the right-hand rule for coils, as shown. In (*b*), switch *m* remains closed and *n* is thrown down, closing the circuit on the other pair of poles, and a combined magnetic field is now set up, advanced 45 degrees in space in a clockwise direction from that in (*a*). In (*c*), *m* is opened, while *n* remains closed as before, and the field is seen to have advanced another 45 space degrees in the same direction. In (*d*), *n* remains closed and *m* is thrown down, reversing the current in this circuit from that in (*a*), so that a combined field is again set up in the direction shown. And in (*e*), *n* is opened and the resulting field is reversed from that in (*a*), or the magnetic field

has revolved 180° in space in a clockwise direction. By further manipulation of the switches, it is clear that a rotating magnetic field can be set up in the air gap, although there is no actual movement of the polar structure itself.

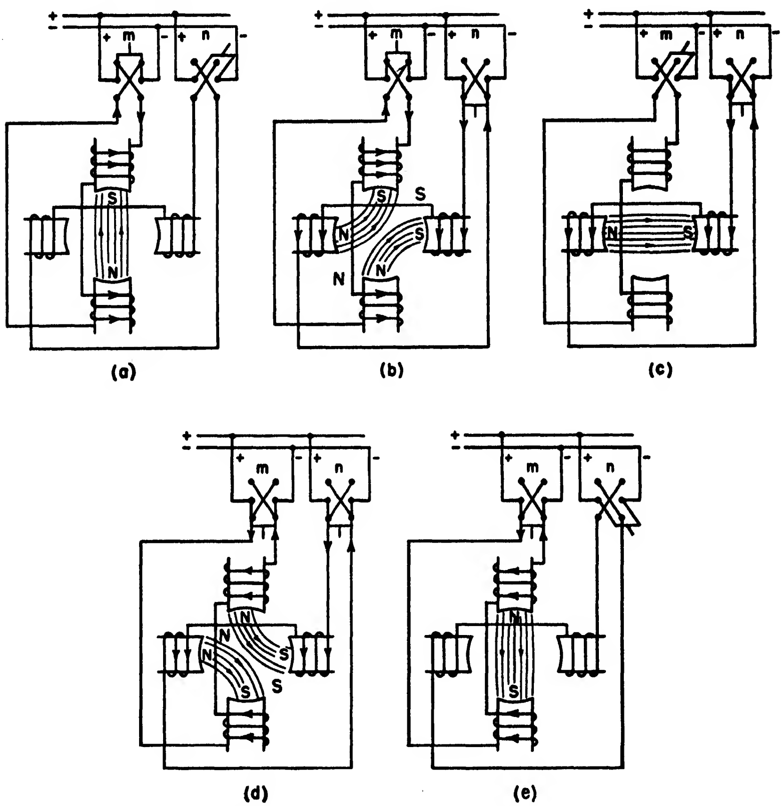


Fig. 9–9. Rotating magnetic field, set up by reversing the direction of a direct current in the windings on a stationary four-pole frame.

Figure 1–9 shows that the induction motor has no salient or definite poles. The revolving field in the uniform air gap is set up by the change in value and direction of polyphase alternating currents in the stator or primary windings. This field of constant value can be set up either by a two-phase or a three-phase winding. Since the two-phase winding lends itself to simpler diagrams, a two-phase motor is represented in the following figures. The field set up by a three-phase winding operates in exactly the same manner.

Figure 10–9 shows the development of a simple two-phase, two-pole single-layer lap winding for an induction motor stator with three slots or conductors per pole per phase. Only one phase is shown connected. Figure 11–9 is a section view of the stator of this motor, taken perpendicular to the shaft. Note that the winding consists of only two coils, placed 90 degrees in space from each other on the stator. The currents in these coils, and therefore the two fluxes, are also displaced 90 time or electrical degrees, as indicated in Fig. 12–9. Figure 11–9(a) corresponds to time t_1 in Fig. 12–9. Current in phase A is a maximum in a positive direction, while that in phase B is zero and N and S poles are set up in the position shown. In Fig. 11–9(b), the current in phase A has decreased in value, while that in B has risen in a positive direction and conditions correspond to time t_2 (Fig. 12–9), 45 time degrees later in the cycle. Note that the position of the magnetic poles has also advanced the same angle in space in a clockwise direction. As the currents in the windings change in value and direction, the position of the field in Figs. 11–9(c), (d) and (e), etc., correspond to time t_3, t_4 and t_5, etc., respectively, in Fig. 12–9. At time t_9, one complete cycle from t_1, the position of the field has reached that of Fig. 11–9(a). Thus in one cycle, the N and S poles make one complete revolution. If the frequency of the supply circuit is 60 cycles, this is 60 rps or 3600 rpm. Since the synchronous speed of an alternator is given by the equation,

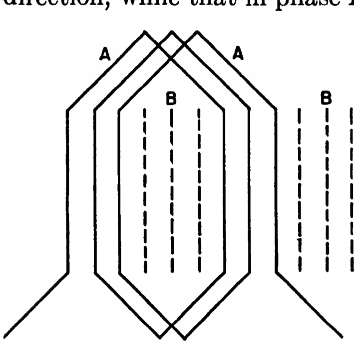


FIG. 10–9. Simple single-layer two-phase lap winding for a stator having two poles.

$$\text{rpm} = \frac{\text{frequency} \times 120}{\text{no. of poles}} \quad \text{(Chap. I, Art. 11)} \tag{1–9}$$

the synchronous speed of a 2-pole, 60-cycle alternator equals $\frac{60 \times 120}{2}$ or 3600 rpm. And the rotating field revolves at synchronous speed. This is true for a motor of any number of poles, as further explained.

Figure 13–9 is a development of the same winding as in Fig. 10–9, except that it is wound for four poles. One phase only is

shown connected. Figure 14–9 again shows a section view of this winding and stator of the motor. In Fig. 14–9(a), the current in

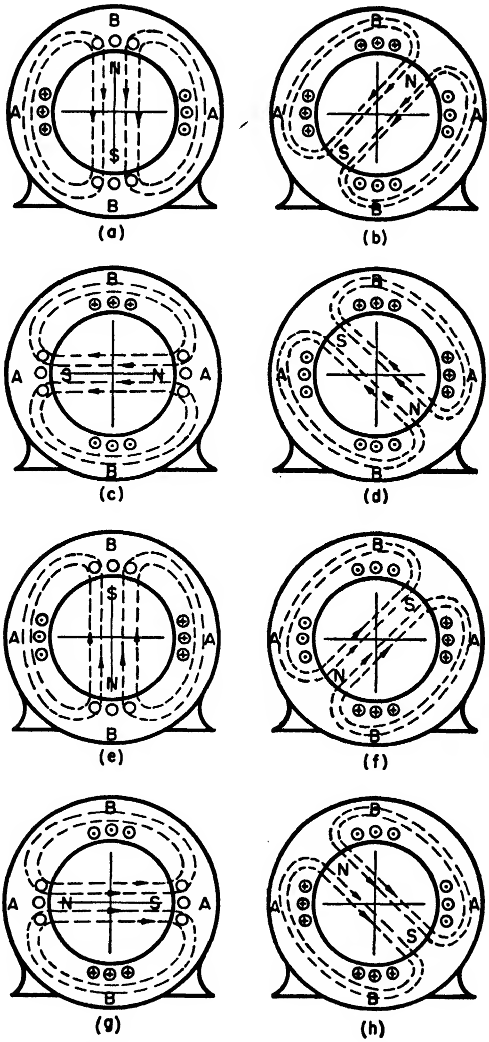


FIG. 11–9. Rotating field, set up by two-phase currents in a two-pole stator winding.

phase A is a maximum in a positive direction, while that in phase B is zero, and corresponds to time t_1 in Fig. 12–9. In Fig. 14–9(b), the currents in both phases are positive and this corresponds to

time t_2 (Fig. 12–9), 45 time or electrical degrees later in the cycle than t_1. Note, however, that the magnetic field has advanced clockwise only 22.5 degrees in space during this time. In (c),

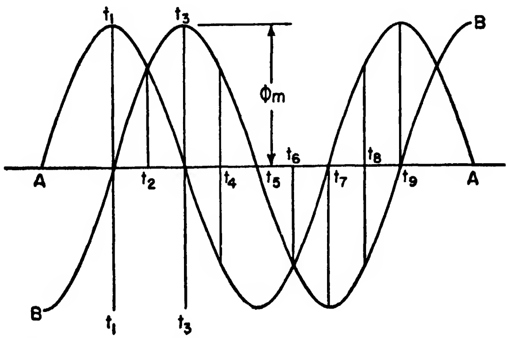


FIG. 12–9. Values of flux produced at various instants by currents in a two-phase winding.

corresponding to time t_3, the current in phase A has dropped to zero and that in B is a positive maximum, 90 electrical degrees later than time t_1. Again note that the field has progressed in a clockwise direction only half the above angle or 45 degrees in

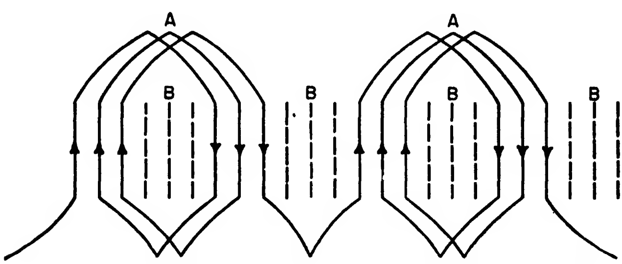


FIG. 13–9. Simple single-layer two-phase lap windings for a stator having four poles.

space during this time. A further consideration of Fig. 14–9 shows that the field moves only one-quarter revolution in a half-cycle, or one-half revolution per cycle, which, on a 60-cycle circuit, is 30 rps or 30 × 60 or 1800 rpm. This is synchronous speed for a four-pole, 60-cycle alternator. Note from the equation,

$$\text{rpm} = \frac{120f}{\text{no. of poles}} = \frac{120 \times 60}{4} = 1800 \text{ rpm}.$$

From the foregoing, it is apparent that the speed of the rotating field is determined by the frequency of the supply circuit and the number of poles for which the motor is wound. The synchronous speed of the field governs to large extent the actual speed of the motor. To meet the speeds called for in common practice, induction motors are wound for different numbers of poles.

In the figures below, a clockwise rotation of the magnetic field has been shown. In a two-phase motor, the direction of rotation

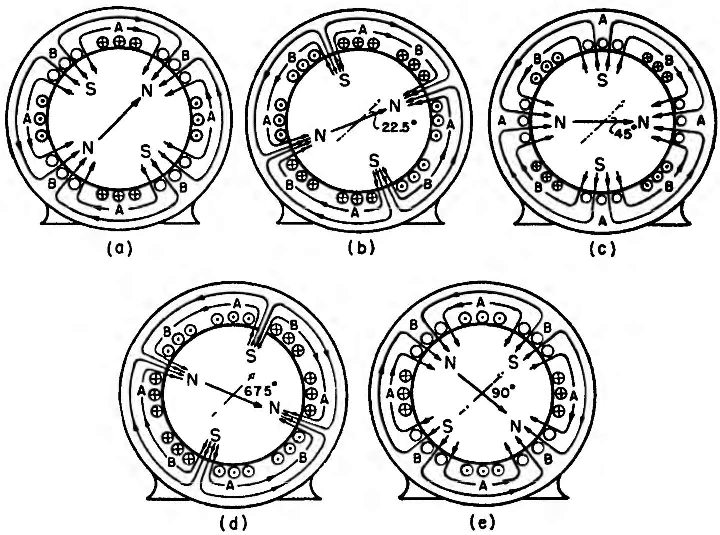


Fig. 14–9. Rotating field set up by two-phase currents in a four-pole stator winding.

of the field, and therefore of the motor, is reversed by reversing the connections of the terminals to one phase. A three-phase motor is reversed by interchanging the connections to any two terminals.

4–9. Slip and Rotor Frequency. The rotating field of the motor sets up emfs and currents in the closed circuit of the rotor in a manner similar to that described in Art. 2. The interaction between these currents and the magnetic field develops torque and the rotor will turn. It has also been stated that the rotor speed must always be less than that of the revolving flux, otherwise there would be no cutting of the rotor conductors by this flux, no voltage would be induced, no current would flow in the rotor winding and no torque would be developed. The rotor must, therefore, revolve at a speed somewhat lower than synchronous speed.

The difference between synchronous speed and rotor speed is called the slip. It is measured in rpm, called **revolutions slip,** or in percentage of synchronous speed. Thus the actual speed of a 60-cycle, 6-pole motor might be 1140 rpm. The synchronous speed of this motor is $\frac{60 \times 120}{6}$ or 1200 rpm. The revolutions slip is 1200 − 1140 or 60 rpm. The per cent slip, s, is computed from the equation,

$$S = \frac{N_1 - N_2}{N_1} \tag{2–9}$$

where N_1 is the synchronous speed and N_2 the rotor speed. Thus the per cent slip in the above motor is

$$S = \frac{1200 - 1140}{1200} = 0.05 \text{ or } 5 \text{ per cent.}$$

From equation (2), at any slip, S, the rotor speed

$$N_2 = N_1 - N_1S$$

or

$$N_2 = N_1(1 - S) \tag{3–9}$$

The speed of a squirrel-cage motor at no load is very nearly the synchronous speed, and the slip may be only a fraction of 1 per cent. At full load, the slip may be as high as 10 or 12 per cent.

The performance of the induction motor is largely controlled by the frequency of the currents in the rotor. This depends upon the **relative** speeds of the stator flux and the rotor conductors — that is, upon the slip. When voltage is impressed on the stator and the rotor is at standstill, the rotating poles of stator flux cut across the rotor conductors at synchronous speed. In a 2-pole, 60-cycle motor, alternate N and S poles would cut across the rotor conductors at 60 rps or 3600 rpm. In a 6-pole 60-cycle motor, the flux cuts across the rotor conductors at 20 rps or 1200 rpm. In either case, the frequency of the currents set up in the rotor is that of the supply, or 60 cycles. The motor simply acts as a transformer in which the stator winding is the primary and that of the rotor, the secondary.

If the rotor in either of the motors above turns at half synchronous speed, or with 50 per cent slip, the **relative speeds** of stator flux and rotor is reduced one half. The stator poles will cut across the rotor conductors at half synchronous speed and the

frequency of the rotor currents will be correspondingly reduced to 30 cycles per second. And if the rotor turns at $\frac{3}{4}$ synchronous speed, or 25 per cent slip, the stator flux will cut the rotor conductor at $\frac{1}{4}$ synchronous speed and rotor frequency will equal $\frac{1}{4}$ of 60 or 15 cycles per second.

The frequency of the currents in the rotor is always equal to the stator frequency multiplied by the per cent slip. This is written

$$f_2 = f_1 s, \tag{4-9}$$

where f_1 and f_2 are stator and rotor frequencies respectively, and s the slip. Thus the frequency of the rotor currents in the 60-cycle, 6-pole motor above with 5 per cent slip is

$$f_2 = 60 \times .05 \text{ or } 3 \text{ cycles per second.}$$

Since the frequency of the emfs and currents in the rotor depends upon the relative speeds of stator flux and rotor conductors, the induction motor may be used as a **frequency changer.** In this case, the stator windings are connected to the normal source of supply and the rotor is driven mechanically at a controlled speed. Current is taken from the rotor (of the wound rotor type) through slip rings and brushes. Part of the power is supplied electrically through the stator and part mechanically through the rotor.

If the rotor of the 60-cycle, 6-pole motor above is driven at 1200 rpm in the **opposite** direction to that of the rotating field, the slip is twice that of synchronous speed or 200 per cent, and $f_2 = 60 \times 2$ or 120 cycles per second. If driven against the rotating field at 600 rpm, or at 150 per cent slip, the rotor frequency $f_2 = 60 \times 1.5$ or 90 cycles per second.

Prob. 1–9. What is the synchronous speed of the rotating flux in rpm in an induction motor, wound for 8 poles and connected to: (*a*) A 60-cycle circuit? (*b*) A 50-cycle circuit? (*c*) A 25-cycle circuit?

Prob. 2–9. If the zero-load speed of a 60-cycle induction motor is 718 rpm, for how many poles must its stator be wound?

Prob. 3–9. For how many poles must a 50-cycle induction motor be wound, if its zero-load speed is 995 rpm?

Prob. 4–9. The full-load speed of the motor in Prob. 2–9 is 698 rpm. What is the slip in rpm and in per cent: (*a*) At zero load? (*b*) At full load?

Prob. 5–9. A certain induction motor driven from 60-cycle mains has a full-load speed of 860 rpm and a zero-load speed of 896 rpm. Calculate: (*a*) the number of poles in the stator; (*b*) the synchronous

speed; (*c*) the per cent slip at zero load; (*d*) per cent slip at full load; (*e*) the per cent regulation.

Prob. 6-9. When the rotor of the motor in Prob. 1-9 is at standstill, what is the frequency of the emf induced in each rotor conductor under conditions (*a*), (*b*), and (*c*)?

Prob. 7-9. What is the frequency of the rotor currents in the motor of Prob. 3-9 at zero load?

Prob. 8-9. What is the frequency of the rotor currents in the motor of Prob. 5-9: (*a*) At zero load? (*b*) At full load?

Prob. 9-9. If the rotor of the motor in Prob. 5-9 is wound with the same number of turns in the same relative positions as the stator (that is, if the rotor winding is a duplicate of the stator winding) and if the voltage per phase of the stator is 134 volts at 60 cycles, calculate the voltage per phase in the rotor winding (*a*) with the rotor blocked (at standstill); (*b*) with the motor running at zero load; (*c*) at full load.

Note: Assume there is no leakage flux, that all the flux produced by the stator winding links also with the rotor winding — a condition impossible to obtain, largely because of the air gap between stator and rotor.

Prob. 10-9. If the circuit of each phase of the rotor winding in Probs. 5-9 and 9-9 above be opened, and the rotor is coupled to an external source of mechanical power, which drives it at synchronous speed in a direction opposite to that of the rotating field, what will be the frequency of the emfs induced between terminals of each phase of the winding?

Prob. 11-9. What will be the voltage induced between terminals of each rotor phase in Prob. 10-9 under the conditions stated for windings and flux in Prob. 9-9?

Prob. 12-9. If the rotor of Prob. 10-9 be driven by an external mechanical force at 1200 rpm in the **same** direction as the rotating field, what will be the frequency of the emfs induced in the rotor?

Prob. 13-9. What will be the induced voltage between terminals of each rotor phase in Prob. 12-9 under the same conditions for windings and flux as in Prob. 10-9?

Prob. 14-9. At what speeds must the motor in Prob. 10-9 be driven in order that the rotor emfs may have a frequency of 25 cycles, when the frequency of the emfs impressed on the stator is 60 cycles?

5-9. Rotor Reactance and Power Factor. All rotor circuits have resistance and inductance. It has just been shown that the frequency of the rotor currents is proportional to slip. At standstill rotor frequency is that of the power supply and the rotor reactance, x_2 is $2\pi f_1 L_2$, where f_1 is line frequency and L_2 the rotor

inductance. The inductance of the rotor may be considered constant if the motor operates at normal voltage and frequency. When the motor is running with a slip s the rotor frequency is $f_1 \times s$ or f_2 and the rotor reactance, x_2, is now $2\pi f_2 L_2$. At no load, the skip (particularly in the squirrel-cage motor) is generally less than 1 per cent; and the rotor frequency and reactance is correspondingly less than 1 per cent of that at standstill. Thus, **rotor reactance increases with the slip.** With constant rotor resistance, r_2, the angle, α, by which the rotor current lags behind its induced emf **increases with the slip** and the rotor power factor decreases. The value of the angle of slip may be found from the equation: tangent $\alpha = \dfrac{x_2}{r_2}\cdot$

Example 1. A 60-cycle induction motor has a slip of 0.6 per cent at no load and 5 per cent at full load. If the resistance of the rotor is 0.005 ohm and the inductance is 0.0004 henry, compute the reactance and the power factor of the rotor: (*a*) at no load; (*b*) at full load; (*c*) at standstill.

Solution:

(*a*) $f_2 = 60 \times .006 = 0.36$

$$x_2 = 2\pi \times 0.36 \times 0.0004 = 0.0009 \text{ ohm.}$$

$$\tan\alpha = \frac{0.0009}{.005} = 0.18 \quad \alpha = 10^\circ\, 10' \quad \cos\alpha = .984$$

(*b*) $f_2 = 60 \times 0.05 = 3$

$$x_2 = 2\pi \times 3 \times 0.0004 = 0.007536 \text{ ohm}$$

$$\tan\alpha = \frac{0.007536}{0.005} = 1.507 \quad \alpha = 56^\circ\, 20' \quad \cos\alpha = 0.554.$$

(*c*) $f_2 = 60$

$$x_2 = 2\pi \times 60 \times 0.0004 = 0.1407 \text{ ohm}$$

$$\tan\alpha = \frac{0.1407}{0.005} = 28.14 \quad \alpha = 88^\circ \text{ (nearly)} \quad \cos\alpha = 0.04.$$

6–9. Torque. In the induction motor, torque is developed in practically the same manner as in the d-c motor. Torque in the d-c motor has been explained, or pictured, as produced by either of two equivalent reactions or methods: (1) the interaction between the field flux and the flux set up by the current in the armature conductors; or (2) the force exerted by the field flux on the current in the armature conductors which lie in this field.

Considering the first method, the current in the armature conductors of the d-c motor sets up magnetic poles at the surface of the armature core, midway between the field poles. (See Vol. I, page 355, Fig. 9–11.) The attraction and repulsion between these two sets of magnetic poles produces torque.

A similar action takes place in the induction motor. When a sine wave of voltage is impressed on the primary winding, the distribution of flux in the air gap under the rotating poles is of sine wave form, except for slight irregularities due to the teeth. This

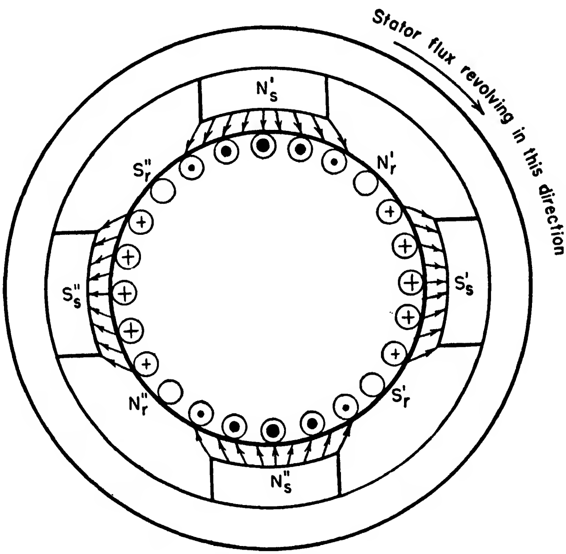


Fig. 15–9. Represents the relative positions of the magnetic poles in the stator and rotor when the slip and the reactance of the rotor are small, the reactance being negligible with respect to the resistance.

flux induces a sine wave of emf in the rotor conductors as it sweeps by them, so that a sine wave of current flows in " belts " of conductors on the rotor and produces another set of magnetic poles in the air gap. The attraction and repulsion between the stator and rotor poles develops torque.

The condition which would be provided at approximately unity power factor in the rotor, that is, when **rotor reactance and slip are equal,** is indicated in Fig. 15–9. This shows a section view of the rotor conductors (small circles) perpendicular to the shaft. For clearness, salient stator poles are shown and are assumed to be rotating clockwise. As the rotor is also moving clockwise at a

speed somewhat less than that of the stator poles, due to the slip, the effect is the same as though the rotor conductors were moving counter-clockwise in a stationary field; and emfs and currents are set up in the direction shown, in accordance with Fleming's right-hand rule. At any instant, the emfs and currents in the belt of rotor conductors in the polar regions of the stator are the greatest at the middle of the poles where the flux is most dense, as shown roughly by the size of the direction symbols in the circles. And conductors midway between the stator poles will have little or no

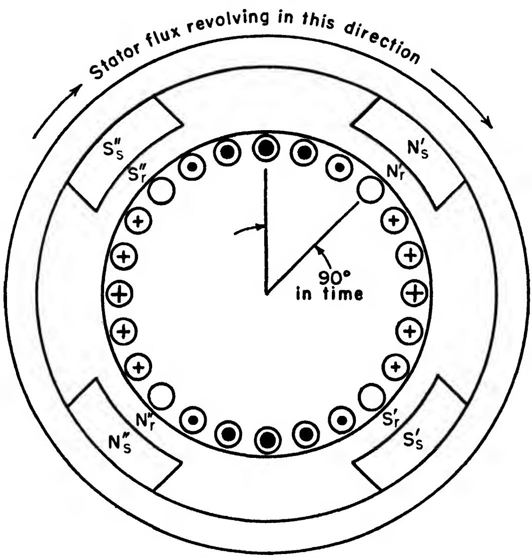


FIG. 16–9. Represents the relative positions of stator and rotor poles when the slip and reactance in the rotor are large, the resistance being negligible with respect to reactance.

current in them. It is readily seen that these rotor currents produce a set of poles N_r', S_r', N_r'', S_r'' on the surface of the rotor, which are **practically midway** between the stator poles N_s', S_s', N_s'', S_s''. As the rotor pole N_r' is pushed by the stator pole N_s' and pulled by S_s', it is evident that torque is produced. This is exactly the effect produced in the d-c motor, as shown in the figure referred to above.

As the stator poles move forward (clockwise), the emfs and belts of current in the rotor progress from conductor to conductor, so that the rotor poles also move clockwise in exact synchronism and in the same relative position with those of the stator. It is im-

portant, however, to visualize the fact that the rotor conductors themselves, due to the slip, do not keep pace with the movement of these poles. Although the currents in both stator and rotor vary with time, the strength of the resulting poles, due to the polyphase currents in the stator windings, is constant in value throughout the cycle and a uniform torque is transmitted to the motor shaft.

Figure 16–9 shows the conditions which would be produced, if the rotor currents had zero power factor — that is, if its **resistance were negligible and its reactance and slip were large.** The nearest approach to this condition in the actual motor would be with the rotor blocked or held stationary. In this figure the belt of current in the rotor conductors lags its induced emf by practically 90 electrical degrees and does not attain its maximum value until the stator poles have advanced $\frac{1}{2}$ pole pitch, or 90 electrical degrees (45° in space or in mechanical degrees in this 4-pole motor). At any instant, the belts of current in the rotor conductors are set up in the positions and directions shown, and progress at synchronous speed with the stator flux. These currents now set up a set of poles in the rotor directly beneath corresponding stator poles, as shown. Again these two sets of poles revolve at synchronous speed in the same relative position to each other. It is apparent that the forces set up between these poles are mainly radial across the air gap and produce no torque.

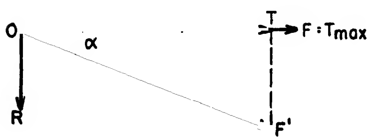


FIG. 17–9. Vector OF represents the total force acting between stator and rotor and is proportional to the maximum torque, if rotor emf and current are in phase. When the same rotor current lags its emf by x degrees, the same force OF is displaced, and produces a reduced torque, OT.

As the rotor circuits in the actual motor contain both resistance and reactance, the rotor power factor is always less than one and greater than zero. Therefore, the rotor poles take an intermediate position with respect to the stator poles, somewhere between the two extremes just described, depending upon the relative values of rotor reactance and resistance, or the amount of slip. In this case, the total forces exerted between the two sets of poles may be considered to consist of two components; one tangential to the rotor, which produces useful torque, and the other radial across the air gap which produces no torque.

This is illustrated in Fig. 17–9 for a given stator flux and rotor

current. The vector, OF, represents the direction and value of the total force acting between the stator and rotor poles. It is proportional to the maximum torque, $T_{m(\text{ideal})}$ developed when rotor reactance and slip are negligible, that is, when the rotor power factor, cos α, is one. Vector OF', represents the same total force acting between the poles when rotor reactance and slip are such that the rotor currents lag their emfs by α degrees. It is now displaced by the same angle α (in electrical degrees) and is composed of the two components, OT, the developed torque; and OR, that component which supplies no turning effort. The effective torque in this case is reduced and may be written as:

$$T = T_{m(\text{ideal})} \cos \alpha \tag{5-9}$$

where $T_{m(\text{ideal})}$ is the torque, developed when rotor currents are in phase with their emfs, and cos α is the actual angle of phase displacement of rotor emfs and currents.

Now consider the second method noted at the beginning of this section. It has been shown in Vol. I that torque in the d-c motor is directly proportional to the density of the field flux and the current in the armature conductors. This is true in the induction motor only for instantaneous values of flux and rotor current.

The torque in lb. ft., developed per conductor in the d-c motor, is expressed as,

$$T = \frac{8.85B\ell I_a \times r}{10^8} \qquad \text{(See Vol. I, page 352.)}$$

Torque in the induction motor depends upon the position of the rotor (armature) currents with respect to the wave of stator flux, as already noted, and this depends upon the slip resistance and reactance of the rotor in a somewhat complicated relation. Therefore, torque in pound-feet for the induction motor cannot be directly expressed by a simple equation such as that above for the d-c motor. The relations of stator flux, rotor current and torque, however, can be shown graphically.

Assume first that the rotor current is **practically in phase with its emf, that is, the reactance and slip are small.** As the sine wave of stator flux moves at uniform rate across the rotor conductors, it induces in them a sine wave of emf and current, which, at every instant, is proportional to corresponding instantaneous values of the flux wave, being greatest where the flux density is greatest. Both the emf and current waves are thus **in phase in space** (in the

air gap) with the flux wave. The vector relations are shown in Fig. 18–9(*a*) in which B, E_2 and I_2 represent respectively the flux density under the stator poles, the rotor emf and the current. The torque at any instant over the cycle is the product of the instantaneous value of the flux wave, B, and the rotor current, I_2, as shown in Fig. 18–9(*b*). The torque wave is of double frequency and similar to the power curve in Fig. 2–3 (Chap. III). The average torque is half the maximum value of the torque curve and represents the condition of maximum developed torque. While

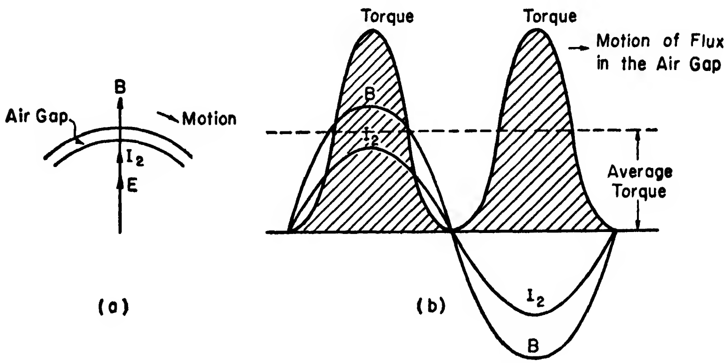


FIG. 18–9. (*a*) Relation of rotor emf and current with respect to the stator flux, B, as it progresses (clockwise) along the air gap; rotor emf and current in phase. (*b*) Torque produced when rotor current and air-gap flux are in phase in space.

the torque is pulsating on the individual conductors, the average torque around the air gap is constant for a given value of flux and rotor current.

When the rotor reactance and slip are such that the rotor currents lag their induced emfs by the angle α, the vector relations of stator flux, rotor emf and current are shown in Fig. 19–9(*a*). The emf again is in phase **in space** with the flux wave but the current does not reach its maximum value until the maximum value of the flux wave has progressed α electrical degrees in space. The torque developed at any instant is shown in Fig. 19–9(*b*). Note that there are negative loops in this curve so that the average torque (for the same stator flux and rotor current) is reduced from that in Fig. 18–9(*b*). This is so because some rotor conductors in the several current belts, due to the lagging current, are thrown into the reverse field of the following stator poles.

The **average or developed torque** is thus again shown to be equal

to the **maximum torque,** multiplied by the **cosine of the phase angle** by which the rotor current lags its emf.

It should again be noted that T_{max}, in equation (5) above, expresses only that maximum torque which would be developed at a given value of flux and rotor current, if that current were in phase with its emf. This value can never be attained in the actual motor, for as load is applied, the slip increases, as shown later; the rotor reactance increases and its power factor decreases. It can be shown, however, that T_m, **the true maximum torque is**

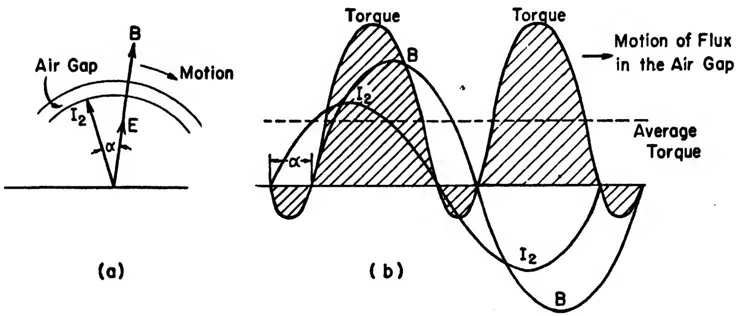


FIG. 19–9. (*a*) Relation of rotor emf E and current I_2 with respect to the stator flux, B, as it progresses (clockwise) along the air gap; rotor current lagging its emf by α degrees. (*b*) Torque produced when rotor current lags the air-gap flux by α degrees in space.

developed at the pulley where rotor reactance (X_2) is approximately equal to its resistance (R_2).

Prob. 15–9. In a two-pole motor, at what angle in mechanical, or space, degrees would the rotor poles be formed, with respect to the stator poles, under the following conditions:

(*a*) Reactance of the rotor in ohms (x_2) is equal to its resistance (r_2).

(*b*) $\dfrac{x_2}{r_2} = \dfrac{3}{4}$. (*c*) $\dfrac{x_2}{r_2} = \dfrac{4}{3}$.

Illustrate with diagrams similar to Figs. 15–9 and 16–9.

Prob. 16–9. Repeat Prob. 15–9 for a four-pole motor.

Prob. 17–9. In Probs. 15 and 16–9, what are the power factors of the rotor circuits in (*a*), (*b*), and (*c*)?

Prob. 18–9. In Probs. 15 and 16–9, on the basis of the same flux and rotor current in each case, the torque, developed by the motors in (*a*), (*b*) and (*c*), is what per cent of the torque which would be developed, if the rotor currents were in phase with their emfs?

7-9. Operating Characteristics of the Squirrel-Cage Motor. It has been shown that the rotor of the induction motor can never run at synchronous speed — that is, there must always be some slip. At zero load, the slip is very small and the rotor current is just sufficient to develop the necessary torque to overcome the no load losses. As a load is applied to the motor, more current must flow in the rotor to develop more torque to carry the load. The rotating stator flux must, therefore, cut the rotor conductors at a greater rate to induce in them a greater emf. Thus, as the load is increased, the slip increases and the speed drops. The resistance of the squirrel-cage rotor is very low; so that throughout the range of ordinary loads only a comparatively small increase in slip is necessary to set up sufficient emf and current to produce the required torque. The slip in large squirrel-cage motor may be as low as **1 per cent at full load.*** The speed therefore falls off very gradually as load is applied. In this respect, the performance of the induction motor is very similar to that of the d-c shunt motor.

The physical relations in the motor can be further illustrated as follows: When a constant alternating emf is applied to the stator, the counter-emf in the windings is practically constant from no load to full load. The flux, or strength of the rotating field, therefore, can also be considered as practically constant through this load range. If we consider a nearly ideal motor in which neither winding has reactance and only the rotor has resistance, the rotor poles are in the position of Fig. 15-9 with respect to those of the stator ($\cos \alpha = 1$). In this case, the torque is directly proportional to the rotor current which is directly proportional to the slip. That is, as the load is increased, the strength of the rotor poles is correspondingly increased. They react with correspondingly greater force against the constant flux of the stator poles in direct proportion to the rotor current. The speed-torque curve of such a motor would thus be a straight line; the speed, dropping off in direct proportion to the load, or required torque, as shown by curve *A* in Fig. 20-9. This would continue from no load to 100 per cent slip, when the motor is stalled.

The speed-torque curve of the commercial motor differs from this straight-line relation. The slip increases and the speed drops at a slightly greater rate throughout the range of ordinary loads,

* In an induction motor, the ratio of slip to the power crossing the air gap (the power input to the rotor) is proportional to the I^2R loss in the rotor.

but at a much greater rate at heavy overloads, until the torque that can be developed by the motor reaches a definite maximum, as shown by curve *B*. If the motor is loaded beyond this point, both the torque and speed decrease rapidly and the motor comes to a standstill. This point of maximum torque in the induction

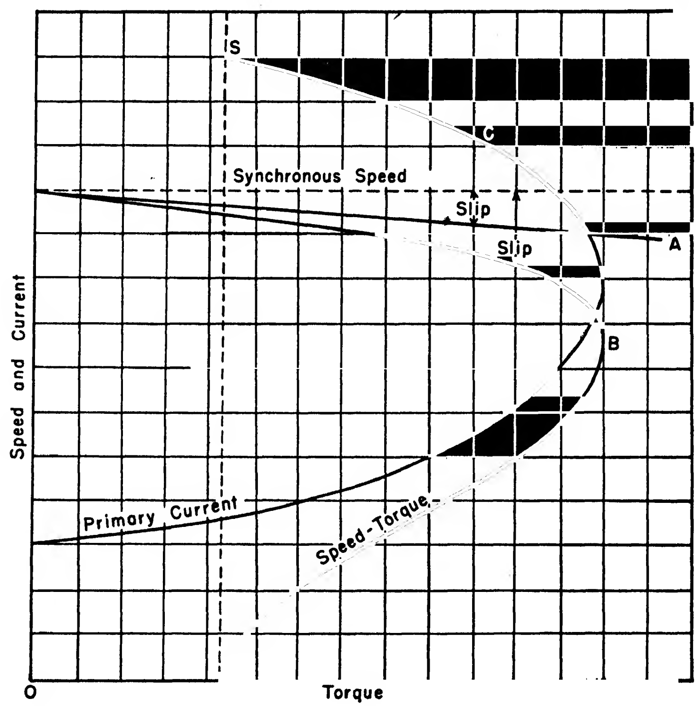


FIG. 20–9. Speed-torque and current curves. *A*. Speed-torque curves for a nearly ideal motor, having no reactance in either winding, and only the rotor has resistance. *B*. Typical speed-torque curve for a squirrel-cage motor. *C*. Primary current curve.

motor is called the **pull-out torque,** or break-down point, and generally occurs at about 200 to 300 per cent of the full-load torque.

First, at heavy overloads, the magnetic leakage increases; thereby reducing the effective or mutual flux across the air gap, so that a greater slip is required to induce the necessary rotor emf and current to develop the required torque.

Second, as the slip increases, the rotor frequency and reactance increase; thereby increasing rotor impedance. This requires

additional slip to induce a greater emf in the rotor to produce the necessary rotor current and torque.

Third, as the reactance increases the power factor of the rotor circuit, cos α, decreases, which in itself reduces the torque so that a still greater slip and rotor current are required to develop the necessary torque.

And finally, as additional load is applied, due to the three effects above, a point is reached where an increase in slip and rotor current develop no more torque. If this load is exceeded, the torque actually decreases and the motor stalls. All these effects are very similar to armature reaction in the d-c machine.

The point of pull-out torque occurs when the rotor reactance becomes equal to its resistance, as noted in the previous article. The rotor resistance is practically constant in a given motor. Thus, **the slip at which the maximum or pull-out torque is developed depends upon the resistance of the rotor circuit.** It is, therefore, also true that **the slip, corresponding to any given torque, is proportional to the rotor resistance.** Note that this does not mean that the **value** of the **pull-out torque** can be increased by increasing the **rotor resistance.*** It will be shown that when a wound-rotor type induction motor is **starting,** the introduction of resistance into the rotor circuit **will increase** the torque, because of the low starting speed, the reactance X_2 is much greater than the constant resistance R_2. This added starting resistance must be cut out as speed increases enough to reduce the value of X_2.

The action described above can be visualized as follows: As load is applied to the motor, the effective stator flux decreases (very slightly up to and above full load), the slip and rotor reactance gradually increase and the rotor poles gradually fall back from their ideal position in Fig. 15–9. As the load is steadily increased, the stator poles continue to weaken slightly, and the rotor poles, while increasing in strength, continue to fall back with respect to those of the stator, due to decrease in rotor power-factor. Thus the rotor poles gradually **approach** the position shown in Fig. 16–9 at which the torque decreases and the motor stalls.

Figure 21–9 shows a vector diagram of the relations of impressed voltage, flux and current in the stator windings of the motor. It is very similar to that for a transformer primary. At no load, the motor takes an exciting current, I_e, lagging the im-

* It can be shown that the value of pull-out torque is also **decreased, if stator resistance** and **stator and rotor reactances are increased.**

pressed voltage, E, by the angle θ_0. This current contains a small power component, I_h, necessary to overcome the no-load losses, which are small; and a large magnetizing component, I_m, in phase with the flux, ϕ, necessary to overcome the reluctance of the air gap. (This explains the reason for making the induction motor air gap as small as possible.) The angle θ_0 is, therefore, large and **the power factor of the motor, cos θ_0, at no load, is low,** often only 1 or 2 per cent. Since the flux is practically constant from no load to full load, and I_e at no load is mostly magnetizing current, it can also be considered as constant throughout the normal range of load, exactly as in the transformer. When a load is applied to the motor, a larger power component of current, I_L, in phase with the impressed voltage, E, is required to carry the load. This combines with I_m to give the total current, I, for this load. The power factor is now cos θ_L. Within certain limits, if the load on the motor is doubled, a doubled (approximately) power component of current I'_L (equal to $2I_L$) is required to carry the increased load. This current again combines with I_m, to produce the total current I' at the increased load, and at a power factor cos θ'_L. Note from the diagram that the angle θ decreases with increase of load, so that **the power factor of the motor increases with the load.** Beyond a certain load, however, the increase in magnetic leakage and decrease in rotor power factor act to prevent this increase, and the power factor decreases. This action is similar to that of a lagging current in a transformer secondary upon the power factor of the primary. Note from the diagram that, due to the large value of no load or exciting current and to increase in power factor, the total current (I and I') does not increase as rapidly as the load (I_L and I'_L). This is shown by the shape of the lower part of current curve, C, in Fig. 20–9. This relation continues approximately up to and beyond full-load torque. As the torque approaches the pull-out value, the current rises rapidly, and reaches a maximum when the motor stalls.

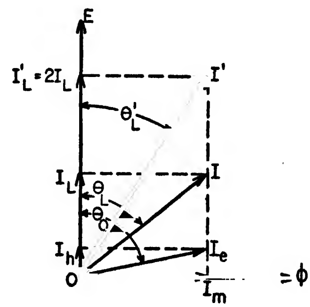


Fig. 21–9. Primary exciting and load currents, showing increase in power factor with increase in load.

Table I gives the performance data for a typical line of squirrel-cage motors.

The performance of the squirrel-cage motor is summarized as follows:

(1) At no load, it operates slightly below synchronous speed, which is determined by the number of poles and the frequency of the supply.

(2) It operates throughout the range of normal load at approximately constant speed, the slip very gradually increasing as load is applied. The speed regulation, due to low rotor resistance, is comparable to that of the d-c shunt motor.

(3) On excessive overloads, the slip increases rapidly and the motor finally reaches a point of maximum developed torque at a value of slip, determined by the resistance of the rotor circuit.

(4) The power factor at no load is very low, but increases, as load is applied, until approximately full load is reached, when it decreases. From this fact, it should be pointed out that a given load driven by an induction motor should not be "over motored." That is, a motor should be chosen of such rating that it must operate at, or near, full load.

8–9. Starting Torque of the Squirrel-Cage Motor. The speed-torque curve, B, in Fig. 20–9, shows that when the motor stalls and comes to standstill, the developed torque OT is greatly reduced from its pull-out value. This is exactly the condition which exists at starting. The motor simply becomes a transformer with the rotor acting as the short-circuited secondary. And if full line-voltage is impressed, the motor takes an excessive current, TS, at a low power factor. This circuit is several times the full load value.

At standstill, or 100 per cent slip, the frequency of the rotor current is equal to that of the supply, or $f_2 = f_1$. Since the rotor reactance increases with frequency, the reactance of the rotor (in this case $x_2 = 2\pi f_1 L_2$) is high compared to its resistance, which is small and constant. Thus the power factor, cos α, of the rotor circuit, and also that of the stator, is low. The rotor poles are in an unfavorable position with respect to the stator poles. Furthermore the magnetic leakage in the stator at this high current also reduces the effective stator flux across the air gap and both the torque and power factor are reduced.

The starting torque, when full-line voltage is impressed, may be approximately only 1.5 to 2.5 times full-load torque. The starting current may be 5 to 10 times the full-load value. The torque produced per ampere of primary current is, therefore, low.

TABLE I

DATA ON TYPICAL LINE OF SQUIRREL-CAGE INDUCTION MOTORS

Three-phase, 60-cycles, 110 to 550 volts, 40° C temperature rise.
Note: Current values are given for 220-volt motors.

Horse-Power	Speed RPM		Efficiency per cent load			Power Factor per cent load			Starting Current Ampere Full Voltage	Torque lb. ft	
	Synchronous	Full Load	Half	Three-Quarters	Full	Half	Three-Quarters	Full		Starting Full Voltage	Pull-out
1	1800	1720	73	77	78.5	57	70	79	27	8	9.5
2	3600	3490	75	79	80	62	75	82.5	47	6	8.3
	1800	1735	77	81	82	63	75	82	47	14.5	19
3	1800	1725	82	83.5	83	71	80	86	60	20.6	25
5	1800	1735	82	84	84	68	80	85	90	30	37
	1200	1150	83	84	84	62	75	82	90	42	60
$7\frac{1}{2}$	1800	1735	83	84.5	84.5	72	82	86.5	120	43	51
	1200	1160	83	85	84.5	68	78	82	120	51	75
10	1800	1740	86	87	86	76	84	86	150	60	61
	1200	1160	85	86	85.5	69	79	83	150	75	95
15	1800	1745	86	87	87	78	84	86	220	95	95
	1200	1165	87	87.5	87	70	80	85	220	91	148
20	1800	1760	87	88.5	88.5	73	83	87	290	90	130
	1200	1170	87.5	88	88	70	82	86	290	121	198
	900	870	87	88	88	62	74	80	290	151	265
30	3600	3550	86	88	89	80	88	90	435	67	98
	1800	1760	89	90	90	75	86	88	435	134	200
	1200	1170	88	89.5	89.5	77	85	88	435	181	295
40	3600	3550	88	89.5	90	85	89	90	580	80	130
	1800	1765	89	90	90	78	86	88.5	580	179	265
	1200	1175	88	89.5	89.5	75	85	88	580	243	390
50	1800	1765	90	90.5	90.5	78	86	89	725	222	325
	1200	1175	88	90	90	75	85	88	725	302	490
60	1800	1765	89.5	90.5	90.5	78	86	89	870	267	390
	1200	1175	90	91	91	75	85	88	870	362	550
	900	875	89	90.5	90.5	75	84	88	870	448	770
75	1800	1765	90	91	91	78	86	89	1085	333	490
	1200	1175	91	92	92	75	85	88	1085	452	700
100	3600	3560	89	91	92	75	86	88	1450	148	325
	1800	1765	91	91.5	91.5	83	88	89	1450	447	660
125	1800	1765	91.5	92	92	83	88	89	1810	560	815
150	3600	3560	89	91.5	92.5	80	88	90	2200	221	650

The squirrel-cage motor thus has the undesirable characteristics of **low starting torque with excessive current at low power-factor.** It should be noted that the excessive current and low power-factor on starting cause fluctuations in the voltage of the ordinary supply circuit. This is particularly objectionable, if the circuit also supplies a lighting load.

9–9. Efficiency. As in other types of electrical machinery, the losses in the induction motor consist of constant or fixed losses and variable or load losses. The constant losses are core losses, friction and windage which are independent of the load. The variable losses are the I^2R losses which vary almost with the square of the load. Due to the low resistance of stator and rotor windings in the squirrel-cage motor, these losses are comparatively small. The efficiency is low at light loads, since the constant losses are large with respect to the input. As load is applied, the efficiency rises rapidly at first and then more slowly until the I^2R losses are equal to the constant losses — the point of maximum efficiency. At loads greater than this, the I^2R losses increase rapidly and the efficiency drops. From the data in Table I, Art. 8–9, the efficiency from half to full load is shown to be comparatively high, and equal to that of the d-c motor of comparable rating.

Prob. 19–9. Assuming a 220-volt rating for the 20-hp, 1200 rpm motor in Table I, compute for full load:
(*a*) The per cent slip.
(*b*) The frequency of the rotor currents.
(*c*) The torque in lb ft.
(*d*) The current input per terminal.

Prob. 20–9. For the motor in Prob. 19–9, make the additional computations:
(*a*) The starting torque in per cent of full load torque.
(*b*) The pull-out torque in per cent of full load torque.
(*c*) The full load torque in lb ft per ampere of line current.
(*d*) The starting torque in lb ft per ampere of starting current of full voltage.

Prob. 21–9. Repeat Prob. 19–9 for the 5-hp, 1800 rpm motor of Table I, assuming it to be wound for 220 volts.

Prob. 22–9. Repeat Prob. 20–9 for the motor in Prob. 21–9.

Prob. 23–9. Repeat Prob. 19–9 for the 100 hp, 3600 rpm motor in Table I, assuming it to be wound for 220-volts.

Prob. 24–9. Repeat Prob. 20–9 for the motor in Prob. 23–9.

Prob. 25–9. (*a*) What is the full load current in amperes per terminal and torque in lb ft for the 30-hp, 1200 rpm motor in Table I, assuming it to be wound for 220 volts?

(*b*) Compute the full load current and torque for the motor in (*a*), if it is wound for 440 volts.

10–9. Relation of Torque to Impressed Voltage. At constant frequency, the flux in the stator of the motor is directly proportional to the impressed voltage, just as in the transformer. If the impressed voltage is halved, the flux of the rotating field is also

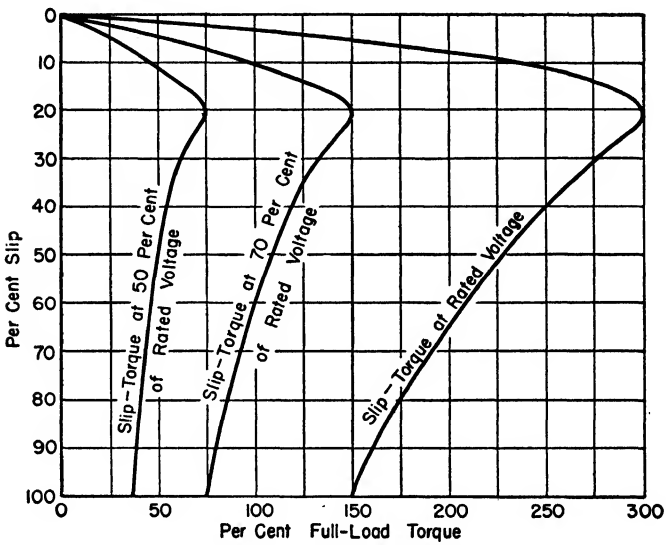


FIG. 22–9. Variation in the slip-torque curve with different values of impressed voltage.

halved (neglecting the resistance and reactance drops in the stator). At normal impressed voltage, the motor develops a definite torque at **a certain slip.** With halved impressed voltage, the halved stator flux sets up a halved emf and current in the rotor at **the same slip.** The rotor poles of half strength now react against the halved stator poles to produce approximately **one quarter** the former torque. It can, therefore, be said that **for a given slip the torque is proportional to the square of the impressed voltage.** This is true for any slip, as shown by the curves in Fig. 22–9.

These curves are similar to the speed torque curve, *B*, in Fig. 20–9, except that they are plotted between percentage slip and

percentage torque and are therefore called "slip torque" curves. The curves are shown for a motor having 300 per cent pull-out torque at 25 per cent slip, and a starting torque of 150 per cent at normal impressed voltage. Accordingly, at half voltage, the pull-out torque is $(\frac{1}{2})^2$ or $\frac{1}{4}$ that at normal voltage and equal to $\frac{1}{4}$ of 300 or 75 per cent. The starting torque is also $\frac{1}{4}$ of 150 per cent or 37.5 per cent. At 70 per cent of normal impressed voltage, both pull-out and starting torque are therefore 0.70^2 or practically 0.5 of normal, giving a pull-out torque of 0.5×300, or 150 per cent, and a starting torque of 0.50×150, or 75 per cent of that at normal voltage, as shown.

It also follows that if the required **torque remains constant, while the impressed voltage is halved, the slip is increased four times.** That is, a halved stator flux, with four times the slip, would induce a doubled emf and current in the rotor (neglecting its reactance). This doubled rotor current, with halved stator flux, would produce the same torque as before. The effect of increased rotor reactance at this larger slip will be such as to cause an even greater slip than the above value.

11-9. Methods of Starting Squirrel-Cage Motors. Very small d-c motors are often started by throwing full-line voltage directly across the armature. The usual method is, of course, to reduce the impressed armature voltage, and increase it as the motor comes up to speed. Both of these general methods are used in starting the squirrel-cage motor.

The first method is called "across-the-line starting."

Any squirrel-cage motor, regardless of its size, can be started, without injury, by throwing it directly across the line. The great objection is the large inrush of starting current, which causes fluctuation in line voltage, and disturbance in the operation of other equipment on the line; also, the shock to the machine which it drives may be prohibitive. Nevertheless, where the capacity of the power supply is sufficient, squirrel-cage motors of several hundred horsepower are sometimes started in this manner, because of its simplicity. Such an application might be the starting of driving motors for large pumps in a power station. However, when large motors are so started, they generally have high resistance rotors and good speed regulation is not of prime importance; or they may be of the double squirrel-cage type (see Art. 14 of this chapter).

Small motors, often as large as 5 to 7.5 hp, can be thrown

directly across the line without too great fluctuation in line voltage. Across-the-line starting is usually accomplished by means of a magnetic switch or contactor, which can be controlled by push-buttons at convenient points. Figure 23–9 shows the circuits in such a switch for a three-phase motor. The "start" push-button is normally open and the "stop" button closed. When the "start" push-button is pressed, it closes a control circuit from the live line wire L_1 through the "stop" button, the solenoid, *M*, of the magnetic contactor and a temperature overload relay, *R*, to

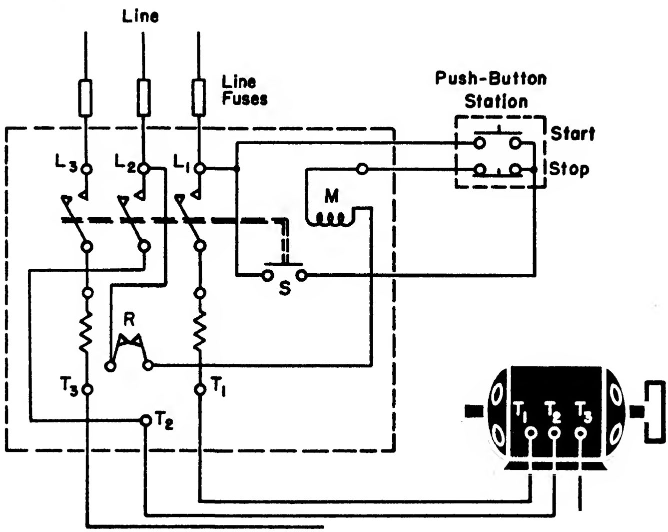


FIG. 23–9. The circuits of an across-the-line magnetic-switch starter. (*General Electric Co.*)

the live line wire L_2. The current in the solenoid, *M*, now closes the main contactors and throws full line voltage on the motor. It also closes the switch, *S*, so that current is maintained in the control circuit through the "stop" push-button, after the "start" push-button is released. The motor can be controlled from several points by using several push-button stations. The "**start**" **push-buttons** are connected in **parallel** and the "**stop**" **push buttons** in **series.** The thermal overload relay is set to protect the motor against continuous overload, but is not affected by the large starting current, nor by sudden momentary overloads. Fuses or circuit breakers in the supply line must carry the starting current and, therefore, must be of such capacity, or set to

carry several times the full-load current. The motors for which data are shown in Table I are designed for across-the-line-voltage.

Standard squirrel-cage motors in sizes above 7.5 hp usually take too much current to start them directly across the ordinary power line, and are started on reduced voltage. This is a definite disadvantage, for the torque decreases with the square of the impressed voltage, as already shown.

The Y-delta connection is one method of starting three-phase motors. The stator windings are normally connected in delta. By means of a triple-pole-double-throw switch, the winding is

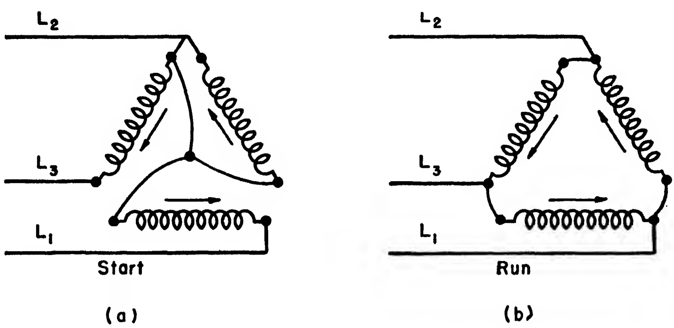


FIG. 24–9. Y-delta method of starting.

connected in Y for starting, which reduces the voltage on the stator windings in the ratio of $\frac{1}{\sqrt{3}}$ or 58 per cent of normal. The starting torque is reduced to only 0.58^2 or 33 per cent of that at normal voltage. The current taken from the line is 58 per cent of that at normal voltage. When the motor reaches sufficient speed the switch is thrown over, thereby connecting the motor in delta across the line. For this connection both terminals of each phase in the stator must be brought out, as indicated in Fig. 24–9.

Another method of starting is by use of resistors in each line which are gradually reduced as the motor comes up to speed. This method, however, is not in general use, due to the large I^2R losses in these resistors.

The usual method of reducing the voltage on the squirrel-cage motor at starting is by means of an auto-transformer and a set of switches called an **auto-starter** or **compensator.**

Figure 25–9 shows the circuits in a typical compensator for a three-phase motor, in which the windings of a three-phase auto-

transformer are connected in Y. In the starting position, the auto-transformers are connected directly across the line and reduced voltage is supplied to the motor from taps on the transformers. Standard motors up to 50 hp have one or two taps on the transformers, and above 50 hp, three or more, depending upon starting conditions. This calls for several positions of the movable

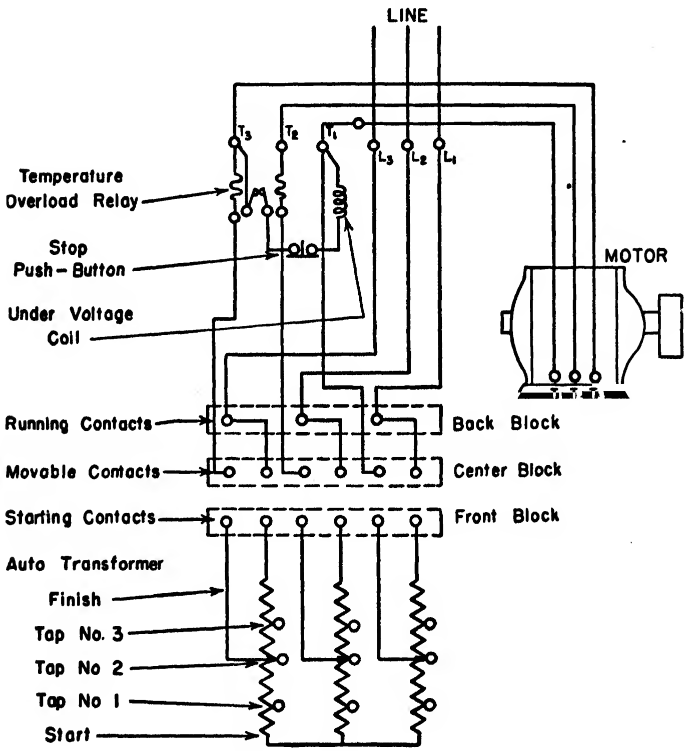


FIG. 25-9. The circuits of an auto-starter or compensator for starting a three-phase squirrel-cage motor.

contacts, or center block, shown in the figure. In the starting position, the motor is protected by fuses, capable of carrying the starting line current. When the switch is thrown to the running position, the auto-transformer is entirely disconnected from the power supply, and the motor terminals are connected directly across the line, as shown by the heavy lines. The motor, in the figure shown, is protected against continued overload by a thermal overload relay.

Instead of a three-coil auto-transformer, some manufacturers make use of two coils, connected in open delta for a three-phase motor. Taps are brought out from these coils and from their junction point so that a symmetrical three-phase voltage at reduced value is impressed upon the motor. Otherwise, the starter operates exactly as that in Fig. 25–9.

The advantage of using auto-transformers rather than resistances to reduce the voltage at starting, is that the line current is

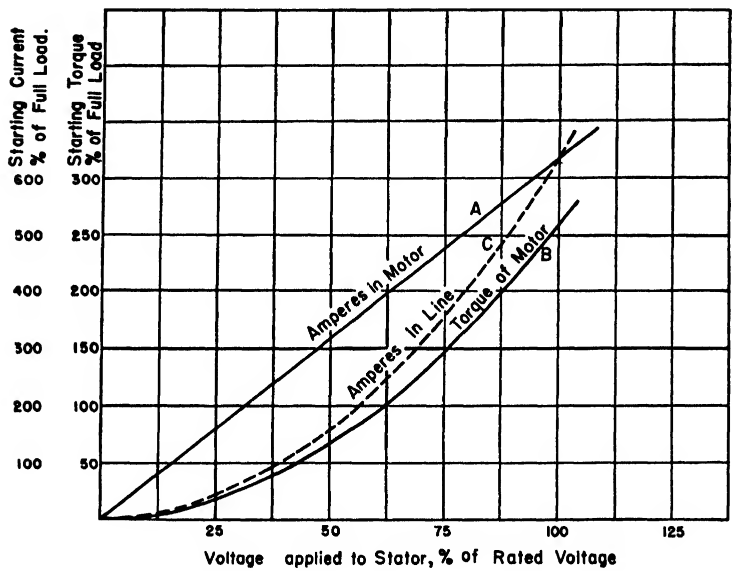


FIG. 26–9. Relation of motor current and torque, at starting, to voltage impressed on the stator of a squirrel-cage induction motor. Curve *A*. Current taken from compensator taps by the stator. Curve *C*. Current supplied to the compensator from the line. Curve *B*. Corresponding values of starting torque. (*Wagner Electric and Manufacturing Co.*)

reduced. The motor at starting is simply a short-circuited transformer in which the current is directly proportional to the applied voltage. At half voltage, the motor current is thus reduced one-half. Also the auto-transformers have a voltage ratio of 2 to 1, so that the line current is also one half the motor current. At half voltage, the motor, therefore, takes $\frac{1}{4}$ the current from the line that it would take were full voltage impressed. That is, if a certain motor takes 200 amperes starting current at full line-voltage, when connected to half-voltage taps on the compensator,

the motor current is 100 amperes, but the current to the compensator from the line is only 50 amperes. Figure 26–9 shows these relations. Note that both line current and torque vary practically with the square of the voltage impressed on the motor, while the motor current at starting is directly proportional to the voltage.

Auto-starters or compensators may be operated by hand, as that in Fig. 25–9, or the switching may be accomplished by means of magnetic contactors with push-button control. Compensator switches for high-voltage motors are usually set in an oil-filled case.

Due to high starting current and low torque, squirrel-cage motors are seldom used where the character of the load requires frequent starts, or where the starting torque is much above full-load value.

Prob. 26–9. Make a wiring diagram of the connections of a triple-pole-double-throw (TPDT) switch to the windings of an induction motor for starting by the Y-delta method. (Note: Connect one terminal of each phase of the stator winding to a center terminal of the switch.)

Prob. 27–9. If the 7.5-hp, 1800-rpm motor in Table I is rated at 220 volts and the stator is normally connected in delta, (*a*) what would be the approximate starting torque in lb ft when started in Y? (*b*) What would be the starting current?

Prob. 28–9. Assuming the 10-hp, 1200-rpm motor in Table I has a normal rating of 440 volts, for what voltage should the taps on a starting compensator be adjusted to start the motor against full load torque?

Prob. 29–9. What current should be taken, by the 220-volt, 15-hp, 1800-rpm motor in Table I, if the applied voltage is just sufficient to start it at 150 per cent of full load torque?

Prob. 30–9. The starting compensator for the 75-hp, 1800-rpm motor in Table I has taps for 40 per cent, 60 per cent, and 80 per cent of rated line voltage. What percentages, respectively, of rated load torque will be obtained when starting on these various taps?

Prob. 31–9. From the curves for a certain standard squirrel-cage motor, shown in Fig. 26–9, what will be the starting line current and stator current in per cent of full-load current: (*a*) When the compensator taps are adjusted to give 50 per cent of normal line voltage applied to the motor? (*b*) The starting torque in per cent of full load torque? (*c*) Repeat (*a*) and (*b*) when the taps are adjusted to give 75 per cent of normal line voltage.

Prob. 32–9. Assuming the inductance of the rotor to be constant, compute the ratio of the rotor reactance at starting (standstill) to the reactance at rated load for the 40-hp, 1200-rpm motor in Table I.

Prob. 33–9. If the power factor of the starting current for a certain motor, when started across the line, is 60 per cent, what will it be under the following altered conditions? (*a*) Stator voltage reduced 50 per cent, frequency unaltered; (*b*) Stator voltage unaltered, frequency reduced 50 per cent; (*c*) Stator voltage and frequency both reduced 50 per cent. Neglect the exciting current.

12–9. Wound Rotor Induction Motor. It has been shown (Art. 6–9) that the slip for any given value of torque is proportional to the rotor resistance. Therefore, if resistance be added to the rotor circuits, the slip for the same value of torque will be increased. This fact is used to control both the speed and starting torque of the induction motor.

The stator flux, as previously stated, is practically constant, since the counter emf is almost constant. The torque is proportional to this flux, the rotor current and rotor power factor, $\cos \alpha$. Assume the resistance, r_2, of the rotor circuit is doubled and that the slip is also doubled. With doubled slip, the stator flux cuts the rotor conductors at twice the former rate and the rotor emf is doubled. At twice the slip, the rotor reactance, x_2, is also doubled. Since both r_2 and x_2 are doubled, the rotor impedance is doubled. A doubled rotor emf will thus send the **same current** through the rotor conductors as before. As r_2 and x_2 are both changed proportionately, the rotor power factor $\cos \alpha$, is unchanged. The torque, therefore, is the same. Thus, if a resistance is added to the rotor circuit, the motor develops the **same torque** with the **same current,** but at a correspondingly **greater slip.**

Figure 27–9 shows the speed-torque and current-torque curves for an induction motor with various values of rotor resistance. Curve *a* represents the speed-torque curve under normal conditions, with low rotor-resistance. Curve *b* shows the performance, with doubled rotor-resistance, and doubled slip; and curve *c*, with quadrupled rotor-resistance, and four times the slip, etc. It will be noted that, as the rotor resistance is increased, full-load torque is obtained at greater slip and lower speed. The maximum, or pull-out torque, is unaffected by the change in rotor resistance except that it occurs at increased rotor slip, at which the rotor resistance is practically equal to its resistance. When sufficient resistance is added to the rotor, the maximum torque is obtained at 100 per cent slip, or at standstill. If rotor resistance is increased above this value, the maximum torque occurs at a negative

speed — that is, when the rotor is driven by an external mechanical source in the opposite direction, while normal voltage is impressed on the stator. As the current for a given torque is constant,

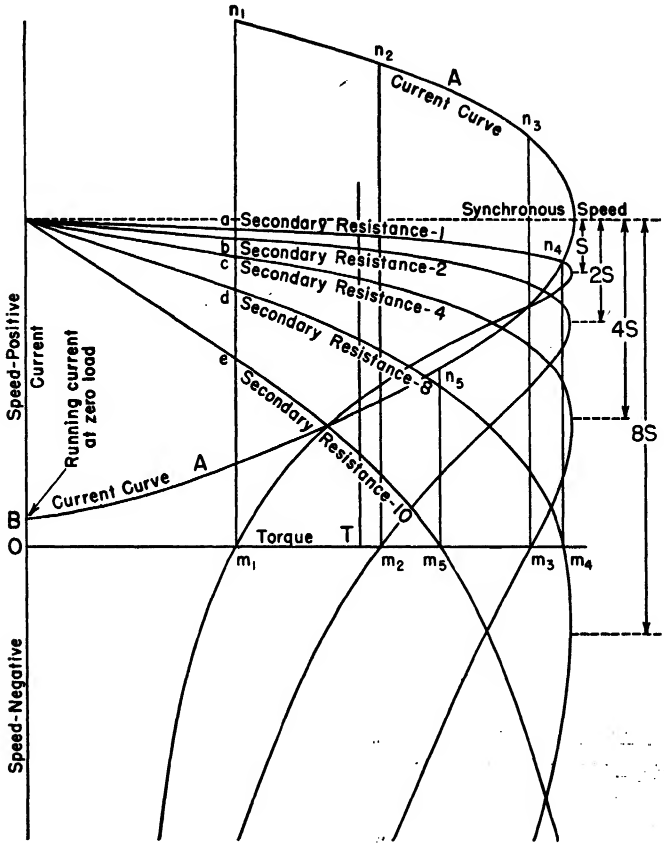


FIG. 27–9. Speed-torque and current-torque curves for a polyphase wound-rotor motor with various values of resistance in the rotor circuit. (*Westinghouse Electric Corp.*)

regardless of the rotor resistance, the current curve, *A*, in Fig. 27–9, applies to all the torque curves, *a*, *b*, *c*, etc.

It has just been shown that added rotor resistance does not affect the torque but does increase the slip and reduces the speed. Since the mechanical power output is proportional to the product of torque and speed, increased rotor resistance reduces the power output and the efficiency of the motor. This is due to increased

I^2R losses in the rotor. With full-load torque at half speed, the rotor I^2R losses are doubled.

It is evident that the speed of the motor can be controlled by introducing resistance in the rotor circuit, but the regulation is poor, as shown by the speed-torque curves of Fig. 27–9. This method of speed control is similar to the armature resistance method of speed control of the d-c motor. (See Vol. I, page 378.)

It is not practical to insert adjustable resistance in a squirrel-cage rotor, so a " wound rotor " is used. In three-phase motors, the rotor winding is either two-phase or three-phase. Such windings are similar to those in the stator, except that the rotor is wound with a different number of slots per pole to prevent locking

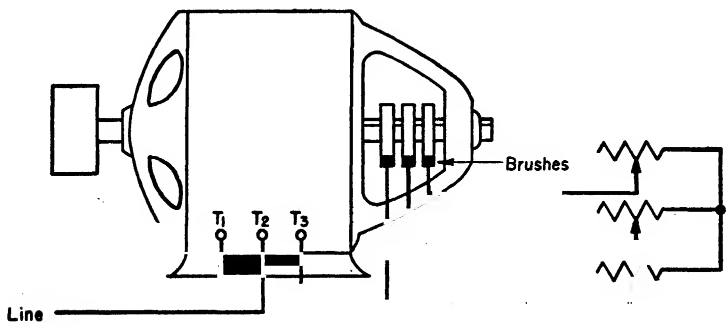


FIG. 28–9. Wound-rotor or slip-ring induction motor with external rotor resistance.

positions of the rotor, at which the motor refuses to start. The three-phase rotor winding may be connected either in delta or in Y. The three terminals are brought out to slip rings, shown in Fig. 7–9. Brushes on these slip rings connect, through a controller, to adjustable external resistors, usually connected in Y, as in Fig. 28–9.

The speed-torque curves in Fig. 27–9 show that the starting torque Om_1, Om_2, Om_3, etc., increases as the rotor resistance increases up to the point of maximum pull-out torque at standstill (where $r_2 = x_2$), and then decreases, if more resistance is added to the rotor circuit. At starting, all the external resistance is in circuit and is gradually cut out as the motor comes up to speed. The resistance of the rotor circuit is high with respect to its reactance, so **the power factor is high on starting.** Even when the motor must develop torque, close to the pull-out value at starting,

the power factor approaches 70 per cent $\left(\tan \alpha = \frac{x_2}{r_2} = 1\right)$. The curves of Fig. 27–9 also show that the starting current is represented by the vertical m_1n_1; on curve b, by m_2n_2, etc. And when the motor starts with a torque equal to Om_5 on curve e, the

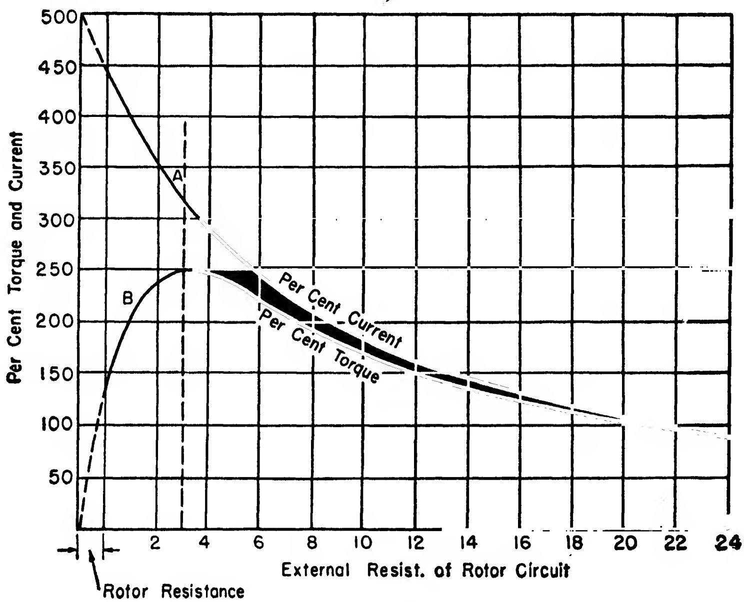


FIG. 29–9. Curve A — relation of starting current to rotor resistance at constant impressed voltage. Curve B — relation of starting torque to rotor resistance at constant impressed voltage. (*Wagner Electric Manufacturing Co.*)

starting current is reduced to m_5n_5. Thus the **wound rotor motor starts with less current.**

The relations between rotor circuit resistance and the starting current and torque for a typical motor, when full line-voltage is applied to the stator, are shown in Fig. 29–9. These curves are plotted between percentage of full-load current and torque, and the relative resistance in the rotor circuits. Curve B shows that the highest torque, developed by this motor, is 250 per cent of that at full load. It is attained when the external rotor resistance is about three times that of the rotor winding. The starting torque is less than this for either higher or lower values of rotor-circuit resistance. Curve A shows the current taken from the supply line for various values of rotor-circuit resistance. The current

taken by the motor at maximum torque is only about 325 per cent of full load value. The curves also show that this motor develops 100 per cent of full-load torque, with approximately 100 per cent of full-load current, when the resistance of the rotor circuit is about 21 times that of the rotor winding.

It is evident that the wound-rotor motor has the advantage of developing large starting torque at high power-factor, and with less starting current than the squirrel-cage motor. Its speed, also, can be controlled, but at the cost of reduced efficiency. The main disadvantage is that it has poorer speed regulation. Even in the running position with all external resistance cut out, the resistance of the rotor, brushes and brush contacts is considerably greater than that of the squirrel-cage motor of comparable rating, so that the slip is greater. The **wound-rotor motor, therefore, has better starting characteristics** and poorer running characteristics than the squirrel-cage motor. The wound-rotor motor, exclusive of the controller and external resistance, also costs more to manufacture due to the expense of winding the rotor coils and installing the slip rings.

Wound-rotor motors are used when the conditions of the load require frequent starts and stops, where good starting torque is required and where close speed-regulation is not important. They are used for cranes, hoists, elevators, air compressors, ice machines, ships, etc. In the electric propulsion of ships, the motors are connected directly to the propeller shafts. Two running speeds are obtained by a switching arrangement which changes the number of poles. Other intermediate speeds are obtained, within limits, by changing the speed of the driving generator and the frequency of the supply. (See Art. 19, Chap. IX.)

Prob. 34–9. (*a*) When the external resistance in the rotor circuit of the wound-rotor motor of Fig. 29–9 is adjusted to such a value that the motor takes 150 per cent of full-load current at starting, what per cent of full load torque is developed? (*b*) When the external resistance is adjusted so that the motor starts with 200 per cent of full-load torque, what per cent of full-load current does it take?

Prob. 35–9. When the resistance in the rotor circuit of the motor of Fig. 29–9 is such as to develop rated load torque at starting, what will be the I^2R losses in the motor in per cent of the I^2R losses at rated load, .en the external resistance is cut out of circuit?

13–9. Starting Wound-Rotor Motors. Consider Fig. 30–9, which shows the starting and running characteristics of a wound-

rotor motor, designed for crane service, and equipped with a controller having 8 steps. That is, 8 different values of external rotor resistance can be connected in the rotor circuits. Curve 1 represents the controller contact on the first step with all the resistance in the rotor circuit. Assume the motor is to start against a rated load torque of approximately 200 lb ft. Setting the controller on steps 1, 2 and 3 does not develop sufficient starting torque, but when the controller is shifted to step 4, a torque of about 235 lb ft is developed and the motor starts. It accelerates

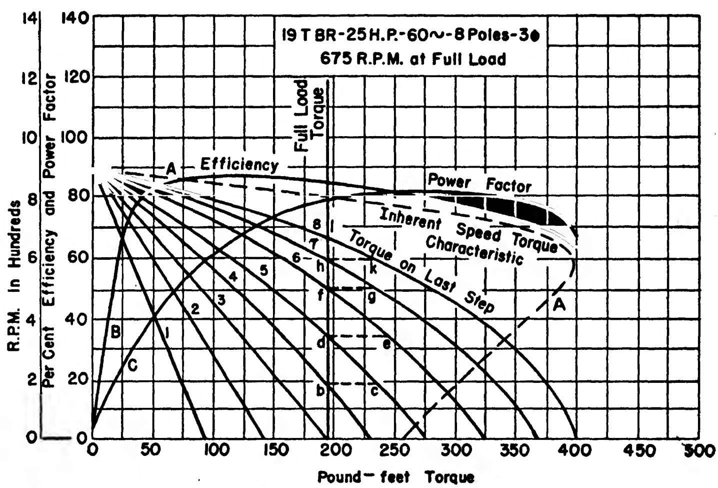


FIG. 30-9. Characteristics of wound-rotor motor designed for crane service. Curve *A* is the speed-torque curve, with all external resistance cut out. Curves 1–8 are the speed-torque curves, with various amounts of external resistance in the rotor circuits. No. 1 is the curve with all external resistance in circuit.

on curve 4 to 200 rpm (point *b* on the curve), where the developed torque is equal to the load torque. The controller is now shifted to curve 5 (point *c*) and the motor accelerates on this curve to *d*, approximately 370 rpm at the required torque. The controller is next shifted to curve 6 at this speed (point *e*) and the motor accelerates to *f* at about 500 rpm. Thus the controller is shifted from one step to the next until, finally on the last step, the motor accelerates, on curve 8, from point *k* to *ℓ*, at 675 rpm.

As the controller is shifting from one step to the next, the current momentarily rises and then decreases as the motor accelerates. It becomes steady when the motor reaches its ultimate speed on

that step. The greater the number of steps in the controller, the less will be the change in current and the smoother will be the acceleration. The use of external rotor resistance thus produces smooth acceleration and the motor draws less current from the line. This method of starting the wound-rotor motor is very similar to that of starting the d-c motor.

In solving the following problems, note that the curves A and B in Fig. 30–9 represent the inherent speed and efficiency respectively, for this motor with no external resistance in the rotor circuit. Also the power factor for any given torque does not change materially with change in rotor resistance; so curve C applies to all the speed curves.

Prob. 36–9. The motor of Fig. 30–9 is rated at 25-hp, 250-volts, 60-cycles, 675-rpm, three-phase, and has 8 poles. Calculate:
(*a*) The useful torque in lb ft at rated load.
(*b*) Synchronous speed in rpm.
(*c*) Slip in per cent at rated load.
(*d*) Slip in per cent at rated load, with no external rotor resistance (Curve A).

Prob. 37–9. From the data of the curves of efficiency, power factor, and speed (Curve A), and Prob. 36–9, calculate for rated load torque:
(*a*) The horse-power output without controller, or external rotor resistance.
(*b*) The watts input. (*c*) Volt-amperes input. (*d*) Current per line.

Prob. 38–9. When the motor of Fig. 30–9 operates at rated load torque on point 8 of the controller:
(*a*) What is the horse-power output? (*b*) The current per line wire?
(*c*) The watts input?
(*d*) Compute the efficiency and compare it with the value on Curve B.

Prob. 39–9. Repeat Prob. 38–9 when the motor of Fig. 30–9 operates at rated load torque on point 6 of the controller. Compare the horsepower and efficiency in Probs. 37–9, 38–9 and 39–9 at rated load torque and explain why they differ.

Prob. 40–9. From the curves of efficiency, power factor and speed (Curve A) for the motor of Fig. 30–9, calculate the line wire current at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{1}{2}$ and 2 times full-load torque. Draw the current-torque curve to scale, using current as ordinates. Compare the shape of this curve with those in Figs. 20 and 27–9.

Prob. 41–9. If the maximum torque at standstill is obtained when rotor, or secondary resistance is equal to its reactance, and if the secondary reactance varies in direct proportion to the slip, calculate:

(*a*) The power factor of the rotor currents, when the motor of Fig. 30–9 operates on curve 8 at rated load with speed of 675 rpm.

(*b*) Compare the value obtained in (*a*) with that observed from the power factor curve of Fig. 30–9 and explain why these values differ.

14–9. Double Squirrel-Cage Motor. It has been shown that the ordinary squirrel-cage motor has excellent running characteristics, but very poor starting characteristics. That is, it develops poor starting-torque with high current at low power-factor. The wound-rotor motor, on the other hand, has excellent starting characteristics, but rather poor running characteristics. It also costs, with the controller and adjustable resistors, considerably more than the squirrel-cage motor. A motor comparable in cost to the standard squirrel-cage motor, with both good starting and running characteristics, is desirable. The rotor of such a motor must have high resistance on starting and low resistance on running. Motors have been constructed with resistors mounted inside the rotor core, which are in circuit with the rotor winding on starting, and are cut out, either by hand or automatically, as the motor comes up to speed. These motors have not been entirely satisfactory. Moreover, they are of the wound-rotor type and correspondingly expensive.

To meet the requirements of both good starting torque and good speed regulation, the "double-deck" or **double-squirrel-cage rotor** has been developed. This may consist of two different windings, one of high and the other of low resistance, placed one above the other in the slots and separated by magnetic shunts, as indicated in Fig. 31–9(*a*). The bars of the low resistance winding occupy the bottom of the slots. Due to the leakage flux across the magnetic shunt, as shown, the self inductance of this winding is high, while that of the high resistance winding at the top of the slot is comparatively low. At starting, the rotor frequency is that of the supply. Thus the reactance ($2\pi f_2 L_2$) and the impedance of the lower winding is much greater than that of the other. Most of the current, therefore, flows in the high resistance winding, which can be so designed that its resistance and reactance at starting are somewhat the same, and the starting torque is high. As the motor comes up to speed, the frequency and reactance of the low-resistance winding decreases and most of the current now flows in this winding, so that the slip is small and the running characteristics are good. The high-resistance bars may be of copper or higher resistance material and may be connected to

separate end-rings; or both sets of bars may be welded to the same end-rings.

Several manufacturing companies have developed an aluminum die-cast double-deck rotor, with bars similar in shape to that in Fig. 31–9(*b*). The lower part of the bar, due to its larger cross section, is of low resistance, but the self-inductance is high, due to the leakage flux across the thin shank, as indicated. That

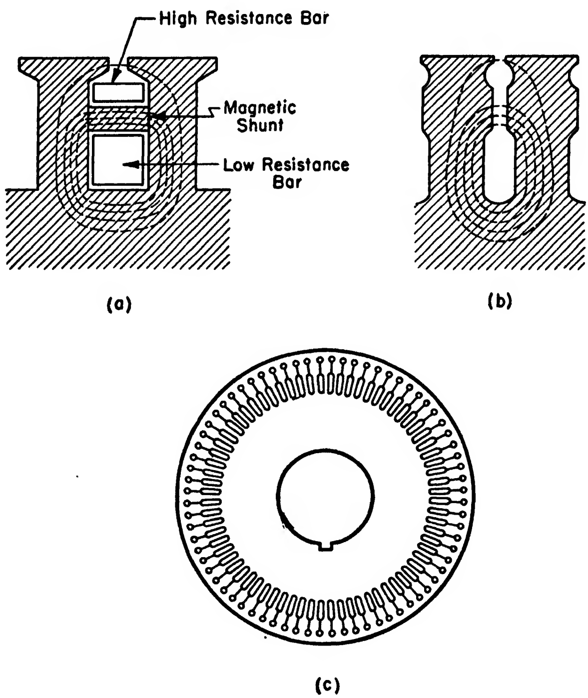


Fig. 31–9. Types of slots and rotor bars in double squirrel-cage rotors.

section of the bar in the upper part of the slot, due to its relative cross section and position, has considerably more resistance and less self-inductance than the bottom section. At starting, most of the current flows in the upper section giving high starting-torque. As the motor accelerates, the rotor frequency and reactance of the lower section decrease, so that the current automatically shifts from the high-resistance section of the conductor to that of low resistance. Figure 31–9(*c*) shows the punching of the Westinghouse double-deck rotor in which the two sets of slots are entirely

enclosed. Connecting these the larger slots are smaller slots for additional bars in the high-resistance winding.

By properly proportioning the double winding, a motor can be built with a starting torque at least equal to the pull-out torque, and with a slip at full load somewhat greater than that of the ordinary squirrel-cage motor. Or the motor can be designed to give somewhat less starting torque with a slip at full load not greater than the standard squirrel-cage motor. Double rotor motors are classed as high-starting-torque, low-starting-current motors, and are especially adapted for across-the-line starting. (See Art. 11, Chap. IX.) These motors are built in sizes up to 200 or 300 h[illegible]r.

15–9. [illegible] **the Air Gap.** It was stated in Art. 1, that the air gap [illegible] luction motor is made as short as mechanical clearan[illegible]. When normal voltage at constant frequency is impres[illegible]e stator, the counter emf is practically constant from no lo[illegible] full load. Since this emf is set up by the flux cutting the [illegible] conductors at constant speed, the flux in the air gap also must [illegible]ractically constant throughout this load range. Therefore, the r[illegible]ed magnetizing current must also be constant. If the length of the air gap in any particular motor is increased, the reluctance of the magnetic circuit is increased. Since the flux is constant, a greater magnetizing current, I_m (Fig. 21, Chap. IX) is required. The core losses with constant flux also remain the same, so that the power component, I_p, does not change. Thus the exciting current I_e is increased and its power factor is decreased; so the power factor of the motor is reduced. **The air gap of the motor is, therefore, made as short as possible to reduce the reluctance of the magnetic circuit, thereby decreasing the exciting current and increasing the power factor.**

The use of totally inclosed slots also reduces the reluctance of the air gap, but increases the leakage flux and the reactance in both stator and rotor. This increases the power factor, but reduces both the starting and pull-out torques. On the other hand, the use of open slots increases the air-gap reluctance and decreases the leakage flux. This decreases the power factor, but increases the torques. Thus high power-factor and high starting and pull-out torques are obtained at the expense of one another. As a compromise between these two extremes, partially closed slots are used. With partially closed slots the reluctance of the air gap is somewhat less than that obtained with open slots and the

power factor is higher. Also, the leakage flux and reactance is somewhat less than that obtained with entirely closed slots and the starting and pull-out torques are higher.

16–9. Effect of Change in Frequency. Neglecting the differential belt factor (see Art. 23–6), the induced or counter emf in the stator of an induction motor can be expressed, as in the transformer,

$$E' = \frac{4.44 f N' \phi_{\max}}{10^8} \tag{6–9}$$

where N' is the number of series conductors per phase.

Then,

$$E' = K f \phi_{\max} \quad \text{or} \quad \phi_{\max} = \frac{E'}{Kf}. \tag{7–9}$$

Thus at constant impressed voltage, a reduced frequency must result in a corresponding increase in stator flux. This requires a large exciting current and increases the core losses and the heating. The synchronous speed of the rotating field is also correspondingly reduced.

If rated voltage at a halved frequency is impressed on the motor, the following relations hold (neglecting the effect of reduced rotor frequency and reactance):

(1) At halved frequency and the same voltage, the flux is doubled.

(2) A doubled flux will develop the same torque with a halved rotor current. Therefore, the rotor emf is reduced one half.

(3) A doubled flux will induce the same rotor emf at one half its normal slip in rpm. Therefore, one-half rotor emf is induced at one-quarter the original slip in rpm.

(4) Since the rpm of stator flux is reduced one half, one-quarter the original slip in rpm results in a slip in per cent, only half or 50 per cent of that at normal frequency.

As the frequency of the rotor emfs above is reduced, however, the rotor reactance is also reduced, and the slip will be somewhat less than that stated above. Thus, the speed will be slightly greater than 50 per cent of that at normal frequency and the pull-out torque will be increased.

Such a large change in frequency as that above is not likely to occur in any motor, unless a 60-cycle motor is connected to a 25-cycle circuit. In which case, the no-load or exciting current would be excessive and great enough to burn out the windings.

By similar reasoning, it can be shown that an increase in frequency, above normal, increases the per cent slip and reduces the pull-out torque.

17–9. Slip Measurement. The slip of a motor in revolutions per minute is relatively very small, compared either to the synchronous speed of the stator field or the actual speed of the rotor. Slight changes in frequency, or errors in measuring rotor speed, cause disproportionate errors in obtaining the slip from the difference of these speeds ($s = N_1 - N_2$).

Figure 32–9 shows a "stroboscope," which is the simplest and most common method of measuring slip. A disc is fastened to the shaft or pulley of the motor. This disc is painted in alternate and

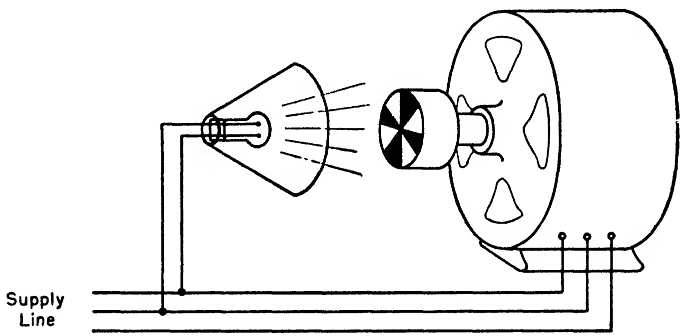
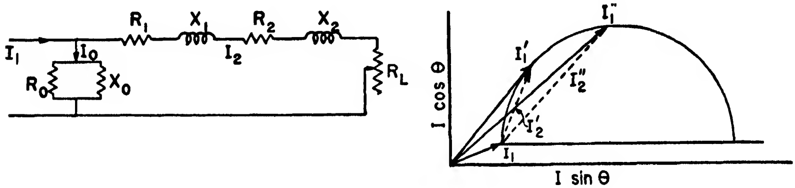


FIG. 32–9. Determination of slip by means of a stroboscope.

equal sectors of black and white, there being as many black and as many white sectors as the number of poles for which the motor is wound. A neon glow lamp, arranged with a reflector to illuminate the disc, is connected to the motor supply line, either directly or through a transformer. The lamp ceases to glow when the instantaneous voltage across its terminal falls to somewhat less than half its effective value. The light, therefore, actually goes out twice each cycle, although the eye fails to detect it; and the disc is illuminated twice each cycle, or every 180° in time. If there were no slip, the sectors would advance one pole (90 degrees in space for the 4-pole motor, shown in the figure), during this time. The black sectors, therefore, at the instant of maximum illumination, would advance and occupy the exact position of the black sector just preceding it. This is also true for the white sectors. During the period of advancement, there is little or no illumination on the disc, and the sectors are not clearly visible.

The sectors would, therefore, **appear** to be standing still in space. Due to the slip, the sectors do not advance one pole each half cycle and **do not reach the position of the preceding sector of the same color** at the instants of maximum illumination on the disc. The sectors will not appear to be standing still in space, but will **seem** to be slowly rotating backward in the opposite direction to that of the motor. By counting the **apparent** number of revolutions of the disc in a given time, the number of revolutions slip per minute is obtained.

18–9. Circle Diagram of the Induction Motor. The performance of small induction motors is readily obtained with sufficient accuracy by loading them. The cost and the complication of loading large polyphase motors make it desirable to predetermine their performance by tests, which require comparatively small



(a) (b)

FIG. 33–9. (*a*) Equivalent circuit of the polyphase induction motor. (*b*) Locus of the currents in (*a*) when R_L is varied.

amounts of power. This performance can be quite accurately obtained by a no-load, or running-light test and a blocked-rotor test, similar to the no-load and short-circuit tests, on the alternator and transformer. These data are used to construct a **circle diagram,** from which the input, output, efficiency, slip, power factor, and torque for any **assumed** stator current may be determined.

The circuits in the induction motor can be approximately represented by a series-parallel arrangement of resistances and reactances, as in Fig. 33–9(*a*). In this figure I_1 represents the total primary current; I_0, the no-load or exciting current, and I_2, the rotor currents in terms of the primary winding (corrected for the ratio between primary and secondary turns). R_L represents the load. This is known as the **equivalent circuit** of the induction motor. In such a circuit, if R_L is varied, while the other values of R and X remain practically constant, the resulting current, I_1,

closely follows the variation of stator current in the motor, as load is applied. It can be shown that, if the total current, I_1, be plotted between values of its power component ($I_1 \cos \theta$) and its reactive component ($I_1 \sin \theta$), the locus of the vector, representing the current for various values of R_L (or load), closely follows that of a circle, as in Fig. 33–9(*b*). These currents are shown as I_1, $I'_1 I''_1$, etc., while the secondary (or rotor) currents, corrected for the ratio of turns, are indicated as I'_2, I''_2, etc.

Data for the Circle Diagram. The three-phase motor may always be assumed to be Y-connected. It is also convenient to use values of voltage, current and power **per phase** in the construction of the diagram.

The necessary data are:

(1) The stator resistance per phase.

(2) The voltage, current and power taken by the motor at rated impressed voltage and frequency, when running light, or at no load.

(3) The same data as in (2), when the rotor is blocked at standstill. stator

The ~~d-c~~ resistance per phase is obtained by measuring the hot resistance of the windings between each pair of terminals in turn, averaging the results and taking one-half this value. The effective, or a-c resistance, is somewhat greater than this (see Ch. VII, Art. 3), and may be taken as 10 to 40 per cent greater than the d-c value.

The current, taken by the stator of a squirrel-cage motor at rated voltage on blocked-rotor test, is excessive and will generally burn out the windings, so that it cannot be measured directly. This current at approximately constant power-factor, varies directly with the impressed voltage in a straight line relation, exactly as in the transformer and alternator on short circuit. For this test, therefore, a low-voltage is impressed on the stator and gradually raised, while instrument readings of voltage current and power input are taken, until the current reaches 40 to 50 per cent above full-load value. From these readings, the current at rated value can be computed. That is, if the line current of a 220-volt motor is 30 amps with 55 volts impressed on blocked-rotor test, the current at rated voltage is $\frac{220}{55} \times 30$ or 120 amperes.

It is to be noted that if power in the two tests above is measured

by the two-voltmeter method, the power factor is such that one meter gives a negative reading, so the total power is the difference of the two readings. (See Ch. V, Art. 6.)

Construction of the Diagram. (Three-phase motor.) In Fig. 34–9, lay off the vertical vector $\frac{E}{\sqrt{3}}$ equal to the rated voltage per phase. From O, lay off the no-load current OI_e per phase, ob-

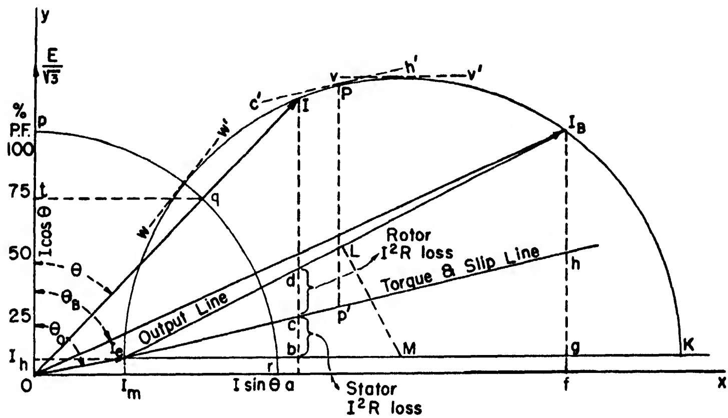


FIG. 34–9. Circle diagram of the polyphase induction motor.

tained from the running-light test, and lagging the voltage by the angle θ_o $\left(\cos\theta = \frac{P_o}{\sqrt{3}EI_e}\right)$. The power component equals OI_h and the magnetizing component OI_m. Now lay off OI_B equal to the blocked rotor current at rated voltage per phase to the same scale as OI_e, and lagging the voltage by the angle θ_B. The coordinates O_y and O_x, thus represent respectively to the same scale, the total power component of stator current, $I\cos\theta$, and the reactive component, $I\sin\theta$, for any load. From I_e, draw I_eK parallel to O_x. Now I_e and I_B are two points on the arc of a circle, the diameter of which is on the line I_eK. The center of the circle, M, is found by drawing the line, I_eI_B, and erecting a perpendicular LM from its middle point, L. With I_eM as a radius and M as a center, the semi-circle, I_eI_BK is drawn.

The line fI_B represents the component of phase current in phase with the phase voltage, $\frac{E}{\sqrt{3}}$, with rated voltage applied on blocked-

rotor test. Its length, therefore, represents and is proportional to the power input per phase. Since the rotor is blocked, the output is zero, and fI_B indicates the losses per phase in the motor, of which fg (equal to I_mI_e) is the no-load loss. gh and position of point h are determined by dividing the stator I^2R loss per phase with rotor blocked (from test data) by the voltage per phase, or $I_B^2R_1 \div \frac{E}{\sqrt{3}} = gh \left(\frac{P}{E} = I_B \cos \theta_o\right)$ where R_1 is the effective resistance per phase. That is, if the blocked rotor current for the 220-volt motor above is 120 amperes at the voltage and the effective resistance is 0.2 ohm, $gh = 120^2 \times 0.2 \div \frac{220}{\sqrt{3}}$ or 22.6 amperes. Lay out this value from q and point h is determined. Also draw the line I_eh.

Take any stator current OI and aI is its power component. Therefore, the power factor $\cos \theta$ is $\frac{aI}{OI}$. On O_y, choose a distance O_p which can be conveniently divided into 100 equal parts. With Op as a radius and O as a center, draw the arc pqr. Where OI intersects the arc, draw qt parallel to Ox. It is now the power factor at this load. Thus for any assumed current, the power factor can be read directly from the diagram. For the current OI, the following values per phase are determined from the diagram.

Total power input, $Pi = aI \times E$ per phase.
Iron and friction loss, $Ph = ab \times E$ per phase.
Stator copper loss, $I^2R_1 = bc \times E$ per phase.
Rotor copper loss, $I_2^2R_2 = cd \times E$ per phase.
Total loss, $P_t = ad \times E$ per phase.
Power output, $p = dI \times E$ per phase.

$$\text{Efficiency} = \frac{dI \times E \text{ per phase}}{aI \times E \text{ per phase}}$$

Since slip is proportional to ratio of rotor I_2^2R loss to power input to the rotor,

$$\text{Slip in per cent of synchronous speed} = \frac{cd}{cI}.$$

Developed torque is proportional to the power input to the rotor.

Thus torque developed in watts per phase at synchronous speed is:

$$T = cI \times E \text{ per phase.}$$

This is known as the torque in "synchronous watts."

The horsepower developed per phase at synchronous speed, is expressed as,

$$\frac{2\pi T N_1}{33{,}000} = \text{HP}, \tag{8-9}$$

where N_1 is synchronous speed in rpm and T, the developed torque in lb ft.

$$\text{HP} = \frac{\text{watts}}{746} = \frac{cI \times E \text{ phase}}{746} \tag{9-9}$$

Substituting in eq. (8)

$$\frac{2\pi T N_1}{33{,}000} = \frac{cI \times E_{\text{phase}}}{746}.$$

$$T = \frac{33{,}000 \times cI \times E_{\text{phase}}}{2\pi \times N_1 \times 746} = \frac{7.04 \times cI \times E_{\text{phase}}}{N_1} \tag{10-9}$$

This is the developed torque in lb ft per phase on the shaft. Multiplying by the number of phases gives the total torque in lb ft developed by the motor.

Thus, any number of currents, or loads, such as OI can be assumed and a set of values, like those above, of input, losses, output, efficiency, torque, etc., determined for each. From these values, curves can be drawn, showing the performance of the motor.

Also, additional information can be obtained from the diagram, as follows: Draw the line $c'h'$ parallel to $I_e h$, and tangent to the circle at P, and the perpendicular pp' is proportional to the maximum or pull-out torque—and can be converted to ft lbs in the manner shown above. A line vv', drawn parallel to Ox, and tangent to the circle gives the maximum input; and the line ww', drawn from O and tangent to the circle, gives the maximum power factor of the motor.

Example 2. Construct the circle diagram for a 5-hp, 3-phase, 110-volt, 60-cycle, 1200-rpm induction motor; full-load current, 28 amperes per terminal. Motor is assumed to be Y-connected. Rated voltage per phase $= \dfrac{110}{\sqrt{3}} = 63.5$ volts.

Test Data	E	W_a	I_a	W_b	I_b
Running Light,	110 volts	890 watts	12.8 amps	−490 watts	12.6 amps.
Blocked Rotor	30 volts	1020 watts	40.5 amps	−70 watts	40.3 amps.

Direct current stator resistance between terminals = 0.165 ohm. Effective resistance = 0.165 × 1.20 = 0.198 ohm. Resistance per phase $= \frac{0.198}{2} = 0.099$ ohm.

Solution: Lay out the coordinates, Oy and Ox on graph paper to as large a scale as possible.

Running-Light Data:

Average current per terminal (phase current) $= \frac{12.8 + 12.6}{2} = 12.7$ amperes.

Power input = 890 − 490 watts = 400 watts.

Power factor, $\cos\theta_o = \frac{w}{\sqrt{3}EI} = \frac{400}{\sqrt{3} \times 110 \times 12.7} = 0.166$. $\theta_o =$ 80° 30′.

Lay out 12.7 amperes (OI_e in Fig. 34–9) to a convenient scale lagging the voltage 80° 30′. I_mI_e $(= fg)$ = 12.7 cos 80° 30′ = 12.7 × 0.166 = 2.1 amperes. Draw the line I_eK.

Blocked Rotor Data:

Average current per phase $= \frac{40.5 + 40.3}{2} = 40.4$ amperes.

Power input = 1020 − 70 = 950 watts.

Power factor, $\cos\theta_B = \frac{950}{\sqrt{3} \times 110 \times 40.4} = 0.453$. $\theta_B = 63°$.

Current per phase at rated voltage, $\frac{I_B}{40.4} = \frac{110}{30}$.

$$I_B = \frac{110}{30} \times 40.4 = 148 \text{ amperes.}$$

Lay out 148 amperes (OI_B in Fig. 34–9) to the same scale as OI_e lagging the voltage by 63°. Through the points I_e and I_B construct the circle. The current fI_B = 148 cos 63° = 148 × 0.453 = 67 amps.

$$gh = \frac{\text{Stator } I^2R \text{ per phase}}{E \text{ per phase}} = \frac{148^2 \times 0.099}{63.5} = 34 \text{ amperes.}$$

The point h is thus found. Draw the line I_eh, completing the diagram.

Prob. 42-9. Construct the circle diagram for the motor of Example 2.

Prob. 43-9. For full load current (28 amperes) in the motor of Example 2, compute the following values from the circle diagram. (*a*) Power input. (*b*) Total losses. (*c*) Power output. (*d*) Efficiency in per cent. (*e*) Power factor. (*f*) Slip in per cent. (*g*) Torque in lb ft.

Prob. 44-9. The following measurements were made on a 50-hp, 3-phase, 440-volt, 25-cycle, 6-pole induction motor.

(*a*) Running light at no load; volts between terminals, 440; amperes per terminal, 16; kilowatts input, 1.14.

(*b*) With blocked rotor (computed from measurements at reduced voltage); volts, 440; amperes, 436; kllowatts input, 173.

(*c*) Effective resistance, hot, between any two stator terminals is 0.118 ohm. Construct the circle diagram using "per phase" values for all current vectors and to as large a scale as possible.

Prob. 45-9. From the diagram of Prob. 44-9, compute, for this motor, the maximum value of:

(*a*) Kilowatts input;
(*b*) Horsepower output;
(*c*) Torque in pound-feet;
(*d*) Per cent power factor.

Prob. 46-9. Assume a sufficient number of stator currents for the motor of Prob. 44-9 from which, by means of the circle diagram, a set of characteristic curves can be drawn. Plot these curves with torque in lb-ft. as abscissa, and as ordinates: (*a*) amperes per terminal; (*b*) kilowatts input; (*c*) horsepower output; (*d*) per cent slip; (*e*) speed in rpm; (*f*) power factor; (*g*) efficiency.

Prob. 47-9. Prove that the total copper loss in a three-phase winding is $\frac{3}{2}I^2R$, where I is the current per line wire and R is the resistance between line wires, regardless of whether the phases are connected in Y or in delta.

19-9. Speed Control of the Induction Motor. The synchronous speed of the induction motor, N_1, is expressed as

$$N_1 = \frac{f \times 120}{p},$$

and the speed of the rotor, N_2, is given in equation (3), Art. 4-9, as,

$$N_2 = N_1(1 - s).$$

Therefore,
$$N_2 = \frac{f \times 120}{p}(1 - s), \tag{11-9}$$

where f is impressed frequency, P the number of stator poles and, s, the percentage slip.

From the equation above, it is seen that the speed of the motor can be controlled by changing the supply frequency, the number of stator poles, or the slip. None of these methods is entirely satisfactory, and the induction motor in this respect is inferior to the d-c shunt motor.

Change of Frequency. The motor will operate satisfactorily on small changes of frequency, but as explained in Art. 16, a decrease in applied frequency, while decreasing the speed and the slip, increases the flux, the core losses and the heating. And an increase in frequency, while increasing the speed, reduces the torque and increases the slip for a given rotor current. Furthermore, induction motors are almost universally operated on distribution systems of constant frequency supplying other loads, so this method is not practical. The principal exception is in the electric propulsion of ships, where the driving motor is the only load on the alternator and turbine. In this case, the speed is changed by changing the turbine speed. With constant alternator excitation, the motor voltage and frequency vary proportionately, so that the stator flux is constant (Eq. 7, Art. 15). This method is objectionable, however, because at other than normal speed, the efficiency of the turbine is reduced.

Change in Number of Poles. By means of a special switch, the connections of the stator coils can be rearranged in such a manner as to change the number of poles. This changes the synchronous speed of the stator field and the speed of the rotor. If the number of poles in a 60-cycle motor are changed from 6 to 4, the synchronous speed is changed from 1200 to 1800 rpm. This does not provide any intermediate speeds. Due to the cost and complication of the switching arrangement, it is not practical to provide more than two arrangements of poles and two normal speeds. Also because of the cost, it is applicable only to large motors. In some cases, instead of the switching arrangement, two separate stator windings are used, the two windings having a different number of poles. This method, together with change in supply frequency, may be applied to the electric propulsion of ships.

Change in Slip. The usual method of changing slip is by means of adjustable resistance in the rotor circuit, and applies only to the wound-rotor motor, discussed in Art. 12, Chap. IX. The objections to this method are the decrease in efficiency, due to the addi-

tional I^2R losses in the rotor circuits and the poor regulation. The main advantage is the high starting torque which can be obtained.

Introduction of emfs into the secondary circuit. Instead of introducing resistance into the rotor or secondary circuit of the motor, the slip can be controlled by introducing emfs into the circuit. These emfs are introduced at rotor, or slip, frequency by a suitable source. There are several methods of accomplishing this. One method is by use of a "frequency converter," direct connected to the motor shaft.

By another method, this emf is supplied in a single machine, known as the "Schrage" motor, or the BTA motor, as manufactured by the General Electric Company. It is also known as the "brush-shift" motor. The primary (ordinarily the stator) in this motor rotates and receives power through slip rings and brushes, Fig. 35–9, while the secondary (ordinarily the rotor) is stationary. The motor has an additional winding, called the speed-adjusting winding, which is similar to a d-c armature, placed in the same slots with the rotating primary and connected to a commutator. Brushes on this commutator are arranged in two groups or sets, each carried by its own yoke. The two yokes are geared together so that when one group is moved in one direction around the commutator, the other group moves an equal distance in the opposite direction. The two groups are so placed that they can be moved past each other on the commutator. The stationary winding is similar to a three-phase wound-rotor induction motor. This winding is not interconnected but each phase, or group of coils, is connected to a pair of the brushes (A and B in the figure), one in each set, there being one pair of brushes per phase per pair of poles. A two-pole motor is shown in Fig. 35–9.

Since the rotating field is set up by the primary (rotor) winding, which is free to turn, and the secondary winding is fixed in position, the force action, set up between the poles in the two windings, causes the rotor to turn in the opposite direction to the rotating field. When the primary rotates at synchronous speed, the flux is fixed in position in space. The frequency of the induced emfs in the stationary winding is that of the slip. That part of the adjusting winding between each pair of brushes is always in the same relative position with the stationary or secondary winding. It is also linked by the same flux. So the emf induced in the adjusting winding must also be of slip frequency.

When the two brushes of each pair are on the same commutator

segments, the adjusting winding is inactive, the various phases, or sections, of the stationary winding are short circuited and the motor acts like an ordinary wound-rotor induction motor.

When the several pairs of brushes are moved apart in one direction, an emf of slip frequency is imposed on the stationary winding in opposition to its emf. This reduces the current in this

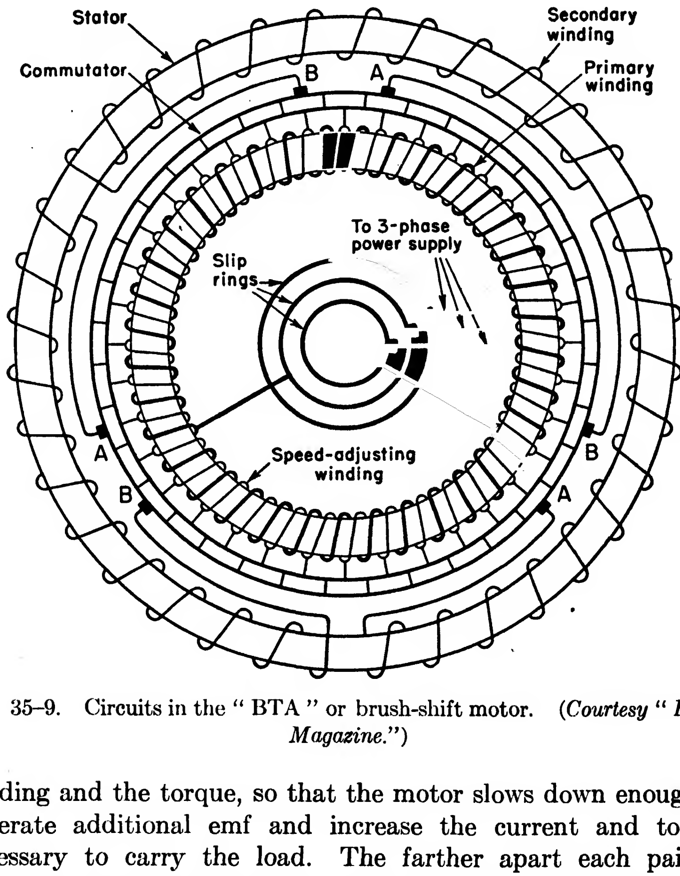


FIG. 35–9. Circuits in the " BTA " or brush-shift motor. (*Courtesy " Power Magazine."*)

winding and the torque, so that the motor slows down enough to generate additional emf and increase the current and torque necessary to carry the load. The farther apart each pair of brushes is moved, the greater is the emf imposed on the secondary and the slower is the speed. This emf imposed on the secondary acts like resistance added to rotor circuit of the ordinary wound-rotor induction motor.

When the brushes are moved apart in the opposite direction, the emf of the adjusting winding is imposed in the opposite direction and adds to the emf of the stationary winding. This increases the

current in the winding and the torque. The motor speeds up to reduce the rate of cutting the flux and reduce the current and torque and to that value necessary to carry the load. The action is like cutting resistance out of the circuit of the wound-rotor motor. In this case, the motor may run **above** synchronous speed. This motor then is a variable speed motor. It ordinarily has a speed range of about 3 to 1.

The control of speed by any method of imposing an emf on the secondary circuit of an induction motor is complicated and expensive. Such methods are occasionally applied to motors of large size for driving steel mills, forced draft, fans, etc.

Concatenation or Tandem Connection. Several operating speeds may be obtained by connecting induction motors in tandem. This is called "concatenation," or "cascade connection," and is explained in the following section.

20–9. Induction Motors in Cascade. This connection requires two motors, the first of which must have a wound rotor. It should also have a one to one voltage ratio. That is, at standstill with the rotor circuit open, the voltage across the slip rings should be equal to that across the stator terminals. The stators of both motors should be wound for the same voltage. The second motor may be of the squirrel-cage type or have a wound rotor with external resistance. The rotor shafts are directly coupled or rigidly connected through gears. Motor No. 1, Fig. 36–9, is connected to the supply line. The stator of motor No. 2 is connected to the rotor of motor No. 1, as shown. Thus part of the power input to No. 1 is converted into mechanical output, and part is transmitted through the rotor, as electrical power input, at slip frequency, to motor No. 2. When electrically connected so that both motors tend to turn in the **same direction,** the resulting speed is dependent upon the **sum** of the number of poles in the two motors. This is called **direct concatenation.** When the motors are so connected that they tend to turn in **opposite directions,** the speed depends upon the **difference** in the number of poles, and this connection is called **differential concatenation.**

Assume the two motors have the same number of poles and are electrically connected so that they both tend to turn in the same direction (direct concatenation). If the slip of motor No. 1 is 50 per cent the frequency and the emf of its rotor is practically half that of the supply line, motor No. 2 thus has impressed upon it half line voltage at half frequency, and operates with normal flux.

Its synchronous speed is reduced one half and, if its rotor is short circuited, runs at practically half speed, or at the same slip with respect to the line frequency as motor No. 1. **The two motors, therefore, operate at half speed.** Speed adjustment between the two motors is such that their combined torque will just carry the load. Thus, the full power of one motor is derived at half speed by the use of two motors. One motor alone or the two in parallel can be used at the normal speed and impressed voltage of each. For

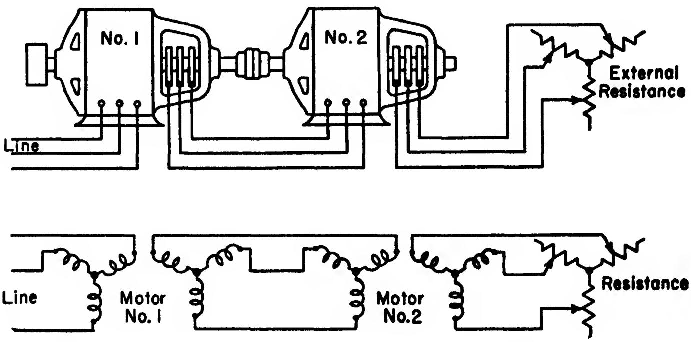


FIG. 36–9. Induction motors connected in concatenation or cascade.

starting and for lower speeds, adjustable resistance in No. 2 rotor may be used. This arrangement has been used for electric traction, particularly in Europe.

The motors may be wound for the same or a different number of poles. In either case, the relations are as follows:

Let f_1 and f_2 be the impressed frequency on motors 1 and 2 respectively; p_1 and p_2 the number of poles; s_1 and s_2 the slip; and N_1 and N_2 the rotor speeds in rpm.

$$\text{Synchronous speed in rpm of No. 1} = \frac{120f_1}{p_1} \qquad \text{(Eq. 1–9)}$$

$$\text{Rotor speed at any slip,} \quad N_1 = \frac{120f_1}{p_1}(1 - s) \qquad \text{(Eq. 11–9)}$$

$$f_2 = f_1 s_1 \qquad \text{(Eq. 4–9)}$$

$$\text{Synchronous speed in rpm of No. 2} = \frac{120f_1 s_1}{p_2}$$

$$\text{Rotor speed of No. 2 at any slip,} \quad N_2 = \frac{120f_1 s_1}{p_2}(1 - s_2) \qquad \text{(12–9)}$$

Since the motor shafts are rigidly connected, $N_1 = N_2$.

$$\text{Therefore,} \quad \frac{120f_1}{p_1}(1 - s_1) = \frac{120f_1s_1}{p_2}(1 - s_2). \tag{13–9}$$

$$\text{Dividing by } 120f_1, \quad \frac{1 - s_1}{p_1} = \frac{s_1 - s_1s_2}{p_2},$$

$$\text{From which,} \quad s_1 = \frac{p_2}{p_1 + p_2} - s_2p_1.$$

When the rotor of motor No. 2 is short circuited, the term s_2p_1 is very small compared to p_1 and p_2 and can be dropped so the slip of the set,

$$s_1 = \frac{p_2}{p_2 + p_1} \tag{14–9}$$

$$\text{The synchronous speed of the set} = \frac{120f_1}{p_1}(1 - s_1).$$

$$= \frac{120f_1}{p_1}\left(1 - \frac{p_2}{p_2 + p_1}\right) = \frac{120f_1}{p_2 + p_1} \text{ rpm} \tag{15–9}$$

This is the synchronous speed when the motors are connected in direct concatenation.

If motor No. 2 is so connected that it tends to turn in a direction opposite to that of No. 1 (in differential concatenation), $N_1 = -N_2$; Equation (13) now becomes

$$\frac{120f_1}{p_1}(1 - s_1) = -\frac{120f_1s_1}{p_2}(1 - s_2);$$

$$\text{from which the slip of the set becomes, } s_1 = \frac{p_2}{p_2 - p_1} \tag{16–9}$$

$$\text{and the synchronous speed of the set} = -\frac{120f_1}{p_2 - p_1} \text{ rpm.} \tag{17–9}$$

Note that the set revolves in the opposite direction. Four speeds are obtained from a set of two motors with a different number of poles.

Example 3. What synchronous speeds can be obtained from a combination of two 60-cycle induction motors, if the first has 12 poles and the second 4 poles?

Solution: $p_1 = 12$ poles. $p_2 = 4$ poles.

$$(a) \text{ Twelve-pole motor alone} = \frac{120 \times 60}{12} = 600 \text{ rpm.}$$

$$(b) \text{ Four-pole motor alone} = \frac{120 \times 60}{4} = 1800 \text{ rpm.}$$

$$(c) \text{ In direct cascade} = \frac{120 \times 60}{12 + 4} = 450 \text{ rpm.}$$

$$(d) \text{ In differential cascade} = -\frac{120 \times 60}{12 - 4} = -900 \text{ rpm.}$$

Prob. 48–9. What synchronous speeds can be obtained when two 25-cycle induction motors, one of 8 poles and the other of 4 poles, are connected in concatenation?

Prob. 49–9. What synchronous speeds are obtainable when two 60-cycle motors, one of 16 poles and the other of 4 poles, are operated in cascade?

Prob. 50–9. Two 100-hp motors, which are operated in direct concatenation, are equally alike in all respects, except that the rotor of No. 2 is short circuited. The rated voltage and frequency of each motor is such that the two may be operated in parallel from the supply line. At rated full load current for each motor, what is the ratio of the power which can be taken from the two motors, when operated in direct concatenation, to that when operated in parallel from the supply line?

21–9. The Induction Generator. If a polyphase induction motor is connected to a constant voltage, constant frequency supply, and its rotor is driven above synchronous speed by an external source of mechanical power (in the same direction it had as a motor), the machine will act as a generator and will deliver electrical power to the line, instead of receiving it.

The machine, so operated, differs from the synchronous alternator in that it is not driven at synchronous speed to deliver power at a given frequency. Its output depends upon the speed at which the rotor is driven, or upon the slip. Therefore, it is not a synchronous machine and is known as an **asynchronous** generator.

Any generator, to operate, must be excited by a magnetic field. In the synchronous alternator, a direct current in a separate field winding is employed to set up N- and S-poles and the flux in the air gap. The induction generator has no such winding and must receive its excitation through the stator winding, from the a-c line to which it is connected. It can, therefore, operate as a generator only when it is connected in parallel with other alter-

nators, of which one at least must be of the usual synchronous type.

When the machine operates as a motor, as already explained, the stator draws current from the line, consisting of two components; an exciting component lagging the impressed voltage by practically 90°, which sets up the rotating field and the air gap flux; and a load component which varies with the load.

When the motor is driven mechanically at synchronous speed, no voltage or current is set up in the rotor, since there is no cutting of the air gap flux by the rotor conductors. No load component of current is therefore required to balance any rotor ampere-turns. However, the stator still draws the same 90° component of exciting current and the stator flux remains unchanged. The machine takes from the line only sufficient power to supply the core losses. The friction losses are supplied mechanically to the rotor. When the rotor is driven **above synchronous speed,** or at a **negative slip,** the stator continues to draw the same exciting current and the air gap flux remains practically the same and unchanged. But the relative direction in which the rotor conductors cut this flux is reversed, so the emfs and currents set up in the rotor are reversed from their normal direction under motor action.*

Neglecting the exciting current, the ampere turns in the stator, due to transformer action, must balance those in the rotor, which are now reversed. Thus the load component of stator current is also reversed. This reversal of current represents power output and the machine becomes a generator.

The relations described above are approximately indicated in Fig. 37–9. When the machine operates as a motor, E_m represents the impressed voltage, I_{ex}, the exciting or magnetizing current, lagging practically 90°, and I_L, the load component of current. The resultant I_m is the total motor current, lagging the voltage by m degrees. When the rotor is driven above synchronous speed, at sufficient negative slip, the machine changes from motor to generator action. The flux and magnetizing current, I_{ex}, remain

* Under motor action, with positive slip, the rotor conductors move more slowly than the rotating flux and the relative direction of the rotor emf is the same as though the air gap flux were stationary and the rotor were turning, at a speed equal to that of the slip, in a direction **opposite to the actual rotation.** When the rotor is driven above synchronous speed, the relative direction of rotor emf is the same as though the air gap flux were stationary and the rotor were turning at a speed equal to that of the negative slip, **in the same direction as the actual rotation.**

This reduces the efficiency due to added I^2R losses and impairs the speed regulation. (d) Slip can be changed by introducing emfs at slip frequency into the rotor circuit instead of resistance. This avoids excessive I^2R losses, but is expensive. This is accomplished in several ways, as with a "frequency converter," direct connected to the motor shaft; or by means of a specially constructed motor, known as the brush-shift motor. (e) By connecting two motors in concatenation or cascade.

CONCATENATION OF MOTORS. The shafts of two motors, one of which having a wound rotor, are direct connected, or rigidly coupled through gearing. The first motor is connected to the supply line and the stator of the second motor is connected to the rotor of the first through the slip rings. If the rotor of the second motor is short circuited, and the motors are electrically connected so that both tend to turn in the same direction (direct concatenation), the synchronous speed of the set is rpm $= \dfrac{120f}{p_2 + p_1}$, where p_1 and p_2 are the number of poles in the first and second motors respectively. If the two motors have the same number of poles, the synchronous speed of the set is half that of either motor alone. The full power of one motor is thus derived at half speed. If the motors are connected, so that they tend to turn in opposite directions (differential concatenation), the synchronous speed of the set is

$$\text{Rpm} = \frac{120f}{p_2 - p_1}.$$

INDUCTION GENERATOR. If a polyphase induction motor is connected to a source of power, and its rotor is driven above synchronous speed by a mechanical source in the same direction it had as a motor, it becomes an alternating-current generator. The a-c source of power must be supplied by a synchronous alternator to excite the stator core at a controlled frequency. The power the machine delivers depends upon the speed at which it is driven, that is, upon the (negative) slip. The machine ceases to generate when short-circuited. This generator action has been made use of for "regenerative braking" on locomotives driven by induction motors.

PROBLEMS ON CHAPTER IX

Prob. 51–9. Compute the synchronous speeds for three-phase 60-cycle induction motors, having 2, 4, 6, 8, 10, 12 and 16 poles. Construct a table showing your data.

Prob. 52–9. Repeat Prob. 51–9 for three-phase 25-cycle motors.

Prob. 53–9. Through how many degrees in space will the rotating flux in a three-phase 60-cycle motor advance in one cycle, if it has 8 poles?

Prob. 54–9. Answer the question for the motor of Prob. 53–9, if it is two-phase.

a high resistance winding for starting and a low resistance winding for running. Current in the rotor bars automatically change from one winding to the other as the motor comes up to speed.

EFFECT OF AIR GAP. (a) An increase in the length of the air gap increases the reluctance of the magnetic circuit and the exciting current, which lowers the power factor. (b) The use of totally enclosed slots reduces the reluctance of the magnetic circuit and improves the power factor, but reduces both the starting and pull-out torques. (c) Open slots have a reverse effect. (d) The use of semi-closed slots is a compromise between (b) and (c).

CHANGE IN FREQUENCY. (a) Synchronous speed varies directly with the frequency. (b) The core loss and exciting current at normal voltage increases, as frequency is lowered; the slip for given torque is decreased and the pull-out torque is increased. (c) An increase in frequency has the opposite effect from that in (b).

The simplest method of MEASURING SLIP is by means of a STROBOSCOPE, consisting of a neon glow lamp in a reflector, connected to the same supply circuit as the motor, and a disc mounted on the motor shaft and illuminated by the lamp. The disc has as many equal black sectors and as many white sectors as the motor has poles. The neon lamp goes out twice each cycle although the eye fails to detect it. The black sectors are plainly visible at the instant of maximum illumination of the disc, and APPEAR to be slowly rotating in the opposite direction to the actual rotation of the disc. By counting the APPARENT revolutions or the black sectors in a given time, the slip in rpm is obtained.

THE CIRCLE DIAGRAM. The locus of the ends of the vectors, representing the current taken by a polyphase induction motor for a series of loads (from no-load to blocked rotor), lie on the circumference of a circle. The diameter of this circle can be determined from the no-load current and power factor, and the blocked rotor current and power factor, both taken, or calculated, at rated voltage. From this circle, the following can be determined for any assumed value of primary current. (a) True or effective power input, reactive power and power factor. (b) Stator copper loss (from measurement of stator resistance). (c) Copper loss in rotor. (d) Core and friction loss. (e) Mechanical output, torque and efficiency. (f) Slip, speed and regulation. It is convenient to use "per phase" values in constructing the diagram.

SPEED CONTROL of the polyphase induction motor is accomplished: (a) By varying the FREQUENCY. Commercially this method can be applied only when the motor is the only load on the driving alternator: used in electric propulsion of ships. (b) By changing the NUMBER OF STATOR POLES. Accomplished either by a special switch which changes the arrangement of coils in the stator winding, or by placing two separate sets of windings on the stator. Because of complication of connections, only two different sets of poles and two speeds are provided. (c) By changing the SLIP. The usual method is to introduce resistance into the rotor circuit of a wound-rotor motor.

The STARTING CURRENT OF THE SQUIRREL-CAGE MOTOR is HIGH, being 3.5 to 10 times the full load values, if started at normal voltage. The motor at starting is simply a transformer with a short-circuited secondary. As the rotor is at standstill, the conductors are cut at synchronous speed by the stator flux and the rotor emf and current are high. The rotor ampere-turns must be balanced by a corresponding number of primary ampere turns so the primary current is high. THE STARTING POWER FACTOR IS LOW in this motor. Since the frequency of the rotor at standstill is that of the supply, rotor reactance is high with respect to its resistance and the power factor, cos α, is low. This reacts on the primary, causing low power factor in this circuit.

THE STARTING TORQUE IS LOW. Since the rotor reactance is high with respect to its resistance, the power factor, cos α, is low; the rotor poles are in an unfavorable position with respect to the stator poles and the torque is therefore low. Starting torque of the squirrel-cage motor is usually from 1.5 to 2.5 times the full load value.

The EFFICIENCY of the squirrel-cage motor compares favorably with that of the d-c motor of corresponding rating.

For a given slip the TORQUE IS PROPORTIONAL TO THE SQUARE OF THE IMPRESSED VOLTAGE. The stator flux is proportional to the impressed voltage. At doubled voltage, the stator flux is doubled. At the same slip, the rotor emf is doubled, the impedance and power factor are unchanged and the rotor current and flux are doubled. A doubed rotor flux reacts against a doubled stator flux to produce four times the torque.

STARTING SQUIRREL-CAGE MOTORS. Small standard motors below 7.5 hp, and motors with high rotor resistance up to 200 or 300 hp may be started, without too much fluctuation of line voltage, by throwing them directly across the line, usually accomplished by means of magnetic contactor, controlled by push-buttons. This is called across-the-line-starting. The stator may be connected in Y for starting and the connections changed to delta for running. This is accomplished by means of a triple-pole-double-throw switch. The usual method is by means of an auto-transformer or compensator, which reduces the voltage at starting and throws full voltage on the stator as it comes up to speed.

THE WOUND-ROTOR MOTOR usually has external resistance, connected in the rotor circuit through the slip rings. By adjusting this resistance, full-load torque is obtained at greater slip and lower speed and with the same current, so the speed can be controlled. The efficiency at reduced speed is decreased, due to additional rotor I^2R losses, and the speed regulation is poor. For a maximum torque, equal to the pull-out value at starting, the resistance in the rotor circuit is so adjusted that it is equal to the reactance. The motor starts with less current and at a higher power factor than the squirrel-cage motor. The external resistance is cut out, as the motor attains full speed.

DOUBLE-SQUIRREL-CAGE MOTORS have two rotor windings;

the "WOUND" type consisting of an insulated winding, similar to that of the stator, laid in slots, and brought out to slip rings on the shaft.

A REVOLVING FLUX or ROTATING FIELD is set up by the alternating currents in the stator winding of a polyphase motor. This revolving flux cuts the rotor conductors and sets up currents in them in such a direction as to produce torque and cause the rotor to revolve in the same direction.

If UNLOADED, the rotor will revolve at nearly the same speed as the stator flux. This is called the SYNCHRONOUS SPEED.

The SYNCHRONOUS SPEED OF THE FLUX in revolutions per minute equals 120 times the frequency, divided by the number of poles, as in any synchronous machine. WHEN LOADED, the rotor does not revolve at synchronous speed, but "slips," so that the rotating flux cuts the rotor conductors at a greater rate and produces the necessary rotor emf, current and torque to carry the load. The SLIP in revolutions per minute is the difference between synchronous speed and rotor speed, or $S = N_1 - N_2$. The percentage slip, $S = \dfrac{N_1 - N_2}{N_1}$, where N_1 and N_2 are the synchronous speed and rotor speed respectively.

The FREQUENCY OF ROTOR CURRENTS is proportional to the slip, or $f_2 = f_1 S$, where f_1 and f_2 are line and rotor frequency, respectively.

ROTOR REACTANCE, $x_2 = 2\pi f_2 L_2$ where L_2 is rotor inductance. It, therefore, increases with rotor frequency and slip.

ROTOR EMF is proportional to the stator flux and the slip.

The POWER FACTOR of the rotor, $\cos\alpha$, decreases with increase in slip. $\operatorname{Tan}\alpha = \dfrac{x_2}{r_2}$, where r_2 equals rotor resistance.

TORQUE developed depends upon the strength of the rotating stator flux and the currents in the rotor conductors. However, due to rotor reactance, the lag of rotor current behind its emf causes the field set up by these currents to be retarded and occupy such a position with respect to the stator flux that the torque is reduced. The torque is expressed as, $T = T_{max} \cos\alpha$, where T_{max} is the torque developed were the rotor currents in phase with their emfs.

OPERATING CHARACTERISTICS OF THE SQUIRREL-CAGE MOTOR. At normal voltage and frequency, the air-gap or stator flux may be considered as practically constant from no load to full load. As load is increased, the speed gradually decreases. The increase in slip causes the stator flux to cut the rotor conductors at a greater rate, increasing rotor emf and current and the torque necessary to carry the increased load. At heavy overloads, the magnetic leakage increases, reducing the effective air-gap flux. The rotor reactance and impedance increase with the slip and rotor power factor decreases, all tending to reduce the torque and the motor finally stalls. The point of maximum torque at which the motor stalls is called the PULL-OUT torque. It generally occurs at $1\frac{1}{2}$ to 3 times full-load torque, and at a slip at which the rotor reactance is equal to its resistance, $x_2 = r_2$.

The SLIP CORRESPONDING TO A GIVEN TORQUE is proportional to the rotor resistance.

connected synchronous alternator. This alternator must also supply the lagging reactive magnetizing current to the induction generator. On lagging loads, the synchronous alternator is, therefore, compelled to supply a heavy current at a low lagging power factor. This is the principal objection to the use of the induction generator.

Its main advantage lies in the fact that it reduces the "short-circuit risk" of a power station. In case of short circuit on the system, the induction generator loses its excitation, generator action stops and it supplies little or no power. Also, it carries no sustained short-circuit current.

The induction generator is inferior to the synchronous generator, so its use has been limited. It has been used in isolated plants, driven by hydraulic turbines and operated in parallel with other synchronous power stations. This principal has also been used for regenerative braking on electric railroads, in which the locomotives are equipped with induction motors, and requires no complicated switching equipment. If the train exceeds the synchronous speed of the rotors on down grades with the motors across the line, they automatically act as generators and return power to the line. This exerts a retarding force on the motors and acts as a brake on the train. However, as soon as the speed of the train reduces to such an extent that the rotor is going at synchronous speed the braking action ceases. Since the normal running speed of a train is usually as high as is safe, anything above this speed is likely to be more or less hazardous.

SUMMARY OF CHAPTER IX

THE POLYPHASE INDUCTION MOTOR usually consists of a stationary or primary winding, called the STATOR, to which the supply lines are connected; and a secondary winding, called the ROTOR, which is free to rotate and is short circuited on itself or through resistance. The motor differs from other types in that the currents are induced in the rotor by the magnetic action of the current in the stator. In its simplest form, there are no moving electrical contacts and it is inherently an approximately constant speed motor.

The STATOR is a polyphase insulated winding laid in slots, and practically identical with the armature of any a-c generator of the stationary armature type, and is wound for any number of poles.

The ROTOR may be of the SQUIRREL-CAGE type, consisting of heavy insulated copper bars, laid in slots in a laminated iron core, and short circuited at both ends by "end rings"; or the bars and end rings may consist of die-cast aluminum in one integral piece. It may be of

practically the same in value and direction. But the load component of current, due to the negative slip, reverses 180°, shown as $-I_L$, and is now in phase with the stator generator voltage, E_g. The current, $-I_L$, now combines with I_{ex} to give the generator current output, I_g.

Frequency and load. It has previously been shown in Art. 6 that the rotor currents in the induction motor, regardless of their frequency or the slip, set up corresponding magnetic poles in the air gap, which revolve at the same frequency as the stator poles. That is, the rotor currents, regardless of the slip, react on the stator at the same frequency as the supply. So, in the induction generator, a change in rotor speed and slip makes no change in the frequency of the output current, which is always equal to that of the rotating field and, therefore is the same as that of the connected supply line. Thus **the frequency of the induction generator is determined by the frequency of the synchronous generator in parallel with it.**

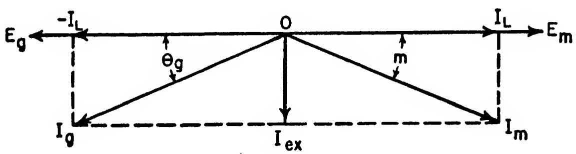


FIG. 37–9. Vector diagrams showing approximate relations in an induction motor and an induction generator.

An increase in speed and slip increases the rate at which the rotor conductors cut the stator flux, which increases the rotor emf and current. This increases the load component of current $-I_L$, in the stator and the generator output. Thus, **the load on the generator is determined by the slip.** A negative slip of possibly 5 per cent above synchronous speed is generally sufficient to place full load on the generator.

Power factor. Note in Fig. 37–9 that the load component of current in the generator, $-I_L$, is in phase with the stator voltage, while the magnetizing component, I_{ex}, **leads** it by practically 90°. And the resultant generator current, I_g, leads the emf by the angle θ_g. This angle is not determined by the load, but by the machine itself. Therefore, **the induction generator always supplies a leading current to the line.** Any lagging or reactive component of current, demanded by the connected load, cannot be supplied by the induction generator, but must be furnished by the parallel

Prob. 55–9. A 150-hp, 50-cycle, three-phase induction motor has 10 poles and a slip of rated load of 1.5 per cent. Compute: (*a*) the synchronous speed in rpm; (*b*) the slip in rpm; (*c*) the speed of the rotor in rpm; (*d*) the frequency of the rotor currents.

Prob. 56–9. What is the frequency of the currents in the rotor of the motor of Prob. 55–9 at starting?

Prob. 57–9. A three-phase motor operating from 40-cycle mains, has a no-load speed of 1197 rpm and a rated load speed of 1120 rpm. (*a*) For how many poles is the motor wound? (*b*) What is the slip in per cent at no load? At rated load? (*c*) What is the frequency of the rotor currents at no load? At rated load?

Prob. 58–9. For a 6-pole induction motor, compute the angle in mechanical- or space-degrees at which the rotor poles would be formed, with respect to the stator poles, if the ratio of rotor reactance to rotor resistance, $\frac{x_2}{r_2} = \frac{5}{6}$. Illustrate with diagrams similar to Figs. 15–9 and 16–9.

Prob. 59–9. A 25-hp, three-phase, 440-volt, 60-cycle induction motor at rated load has a speed of 1175 rpm and an efficiency of 90 per cent. The pull-out torque is 250 per cent of the rated load value and occurs at 25 per cent slip. (*a*) Assuming constant flux, constant stator and rotor inductance and resistance, what will be the rotor power factor at rated load? (*b*) Neglecting the primary exciting current, what is the power factor of the motor and the line current at rated load?

Prob. 60–9. A delta-connected, 10-hp, 240-volt, 60-cycle squirrel-cage motor starts at full line voltage, with 625 per cent of rated current and 230 per cent of rated torque. When started from 240-volt mains, by means of a Y-delta switch, compute in per cent of the corresponding value at rated voltage: (*a*) starting current; (*b*) starting torque.

Prob. 61–9. A $7\frac{1}{2}$-hp, 60-cycle, 6-pole squirrel-cage motor starts at full line voltage with 625 per cent of rated current and 250 per cent of full-load torque. (*a*) For what per cent of normal line voltage must the compensator taps be adjusted to start the motor with a line current equal to the rated load value? (*b*) Under this condition, what per cent of rated-load current will the motor take? (*c*) What will be the starting torque in per cent of rated-load torque?

Prob. 62–9. A 100-hp, 6-pole, 60-cycle, three-phase, 440-volt squirrel-cage motor, at rated load, has an efficiency of 90 per cent and a power factor of 92 per cent, with a slip of 2.5 per cent. (*a*) Compute the rated line current in amperes and the speed in rpm. (*b*) The full load torque in pound-feet.

Prob. 63–9. If the motor of Prob. 62–9 were to be started at full line voltage, it would take, at the instant of closing the switch, 850 per cent of rated-load current, 260 kw from the line, and develop 150 per

cent of rated-load torque. If a compensator, with 40 per cent voltage taps, were used for starting, compute: (*a*) the motor current in amperes; (*b*) the line current; (*c*) the motor power factor; (*d*) the power taken by the motor; (*e*) the starting torque in pound-feet. Neglect the exciting current in the motor and losses in the compensator.

Prob. 64–9. Repeat Prob. 63–9, if the motor is started on 60 per cent voltage taps on the compensator.

Prob. 65–9. What synchronous speeds can be obtained by the use of two 24-cycle induction motors, either singly or connected in cascade, if the first has 4 poles and the second 12 poles?

Prob. 66–9. A 60-cycle, 6-pole, three-phase wound-rotor induction motor with a resistance per phase of 0.072 ohm develops its maximum torque at a speed of 960 rpm. What must be the inductance per phase in the rotor?

Prob. 67–9. Assuming constant rotor inductance, how much resistance must be added per phase to the rotor of the motor, in Prob. 66–9, in order that it may develop the maximum possible torque at starting (standstill)?

Prob. 68–9. A 50-hp, three-phase, 60-cycle motor with external rotor resistance develops rated output and torque at 1700 rpm. Under this condition, the core and friction losses are 3.3 kw; stator copper loss, 2.8 kw and rotor copper loss, 2.1 kw. (*a*) Compute the efficiency and torque in pound-feet. (*b*) If the resistance per phase of the rotor circuit is doubled, at what speed will the motor develop rated load torque? (*c*) Under this condition, what will be each of the three losses above? Neglect the change in rotor core and friction losses. (*d*) The horsepower output? (*e*) The efficiency? (*f*) How will the power factor and the line current be affected?

Prob. 69–9. If the resistance per phase in the rotor circuit of the motor, in Prob. 68–9, is increased to three times its original value, repeat parts (*b*), (*c*), (*d*), (*e*) and (*f*) of that problem.

Prob. 70–9. Under normal conditions, with external rotor resistance short circuited, a 25-hp, 6-pole, three-phase, 240-volt, 60-cycle wound rotor induction motor develops rated-load torque at 1150 rpm with an efficiency of 88 per cent. If the controller is so set that the resistance per phase of the rotor circuit is four times its original (or inherent) value, compute under the new conditions, at rated-load torque: (*a*) the speed; (*b*) horsepower; (*c*) efficiency.

CHAPTER X

SINGLE-PHASE INDUCTION MOTORS
COMMUTATOR TYPE MOTORS

It was shown in the preceding chapter that torque is developed in the induction motor by polyphase currents in the stator windings which set up a rotating magnetic field. After the motor has been started and accelerates to normal speed, however, if one terminal (in a three-phase motor) is disconnected, the motor will continue to run on a single phase, but with very much reduced torque, lower efficiency and lower power factor than before. It will develop no torque at starting, but if given a sufficient impulse by external mechanical means in either direction, it will rotate as a single-phase motor. This is due to the fact that the field set up by a single-phase current in a single stator winding does not rotate. Such a field is pulsating or oscillating in value, and stationary in space, so that no starting torque is developed. But when the motor turns somewhere near synchronous speed, the rotor currents set up additional poles in the stator core. These, together with the main stator poles, result in an imperfect rotating field, somewhat resembling that in the polyphase motor, and torque is developed.

Thus, the single-phase induction motor is not self starting. Many special devices and designs have been developed to produce a single-phase a-c motor with satisfactory starting and running characteristics. This has resulted in the split-phase, capacitor, shaded-pole, repulsion and series or universal motors, etc. There are several designs for each type to meet special starting and running requirements.

The single-phase motor is inferior to the polyphase motor in operating characteristics, weighs more, occupies more space and costs more per rated horsepower. It has very much lower efficiency and, in most cases, a lower power-factor. It is used mainly for motors of fractional horsepower rating of which there are millions in commercial and domestic use. These and motors of larger rating (5 to 10 hp and in some cases 25 hp) are used where polyphase power is not available.

The more important types of these motors are considered in this chapter.

1–10. Physical Theory of the Single-Phase Induction Motor. The construction of the single-phase induction motor is similar to that of the polyphase motor. The stator structure completely surrounds the rotor, which is generally of the squirrel-cage type, and the air gap is uniform. The reactions which take place in the rotor are somewhat complicated, but the simple physical theory can be described as follows.

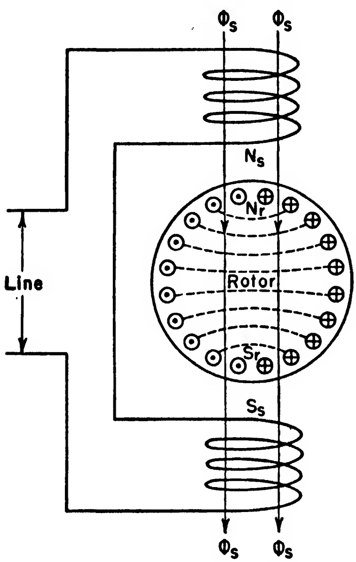


FIG. 1–10. Conventional diagram of a single-phase induction motor. At standstill, the alternating stator flux sets up the rotor poles N_r and S_r by transformer action. These poles are in line with the stator poles N_s and S_s and no torque is developed.

Figure 1–10 is a conventional diagram of a two-pole motor with squirrel-cage rotor in which the stator winding is shown as two oppositely placed coils. The rotor conductors or bars (small circles) are connected through the end rings, so that corresponding bars may be considered to form a single short-circuited turn of a coil, as shown by the dotted lines in the figure. At the instant shown, the direction of the main stator flux, ϕ_s, which is alternating in value, is **increasing** and in a direction **downward**, as indicated. It links the rotor coils and induces in them emfs and currents due to ordinary transformer action. These are called **transformer emfs and currents.** These induced currents, according to Lenz's law, must flow in such a direction as to oppose the flux, ϕ_s. The current at this instant, therefore, must be flowing **inward** on the right-hand side of the rotor and **outward** on the left-hand side; and the poles N_r and S_r are formed in the rotor core. These poles are in line with the stator poles N_s and S_s, and it is evident that the reaction between these two sets of poles can produce no torque or turning effort.

Now assume the rotor is revolving in a clockwise direction. There will be an emf induced in the rotor conductors, which is

entirely due to their rate of cutting the flux, ϕ_s. This is called the **speed emf.** It is alternating in value and a maximum when the flux is a maximum, and, at any instant, in a direction in accordance with Fleming's right-hand rule. Thus in Fig. 2–10, the direction at this instant of the emf in the conductors in the upper half of the rotor is **inward,** and in the lower half it is **outward** as shown. As the rotor bars are short circuited, currents flow in them as a result of this speed emf, and since the frequency of this induced emf is high, the rotor reactance is high with respect to its resistance; so these currents lag the speed emf nearly 90°. The speed currents, therefore, set up a flux, ϕ_r, at right angles to the flux, ϕ_s (Fig. 2–10). This produces the poles N_r' and S_r' in the rotor core. This field, which is called the cross field, in turn sets up the corresponding opposite poles N_s' and S_s' in the stator core, which completely surrounds the rotor.

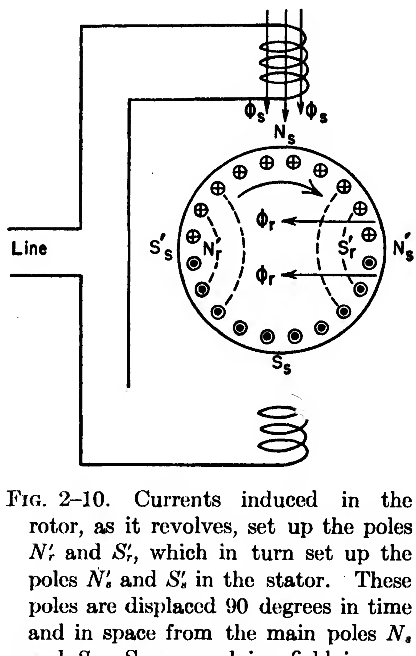


FIG. 2–10. Currents induced in the rotor, as it revolves, set up the poles N_r' and S_r', which in turn set up the poles N_s' and S_s' in the stator. These poles are displaced 90 degrees in time and in space from the main poles N_s and S_s. So a revolving field is produced.

It has just been noted that the speed emf is a maximum when the main field flux, ϕ_s is a maximum. It is, therefore, in time phase with the stator poles N_s and S_s. But the speed currents in the rotor lag practically 90° in time behind the speed emfs. The flux, ϕ_r, and stator poles, N_s' and S_s', do not reach their maximum value, therefore, until one quarter cycle, or 90 electrical degrees after the maximum value of the main field poles N_s and S_s. Thus there are set up in space, two fluxes or two fields at right angles to each other, and differing 90° in time, and a rotating magnetic field is produced.

When the rotor turns near synchronous speed, the flux, ϕ_r, and the poles, N_s' and S_s', are practically equal to those of the main field, and a so-called "circular field" is produced. As the slip increases, the rate of cutting the flux ϕ_s decreases. At reduced

speed, then, two unequal fields at right angles exist, and a so-called "elliptical field" is produced. At standstill, the flux, ϕ_r, is zero, and only a pulsating field results, which, as stated before, produces no torque.

If the rotor in Fig. 2–10 is turned in a counter-clockwise direction, the speed emfs, current and flux ϕ_r are all reversed in direction with respect to ϕ_s of the main field. This reverses the polarity of the poles, N_s' and S_s'. The combined field now rotates in the opposite direction and reverses the torque. Thus the single-phase induction motor will rotate in the direction in which it is started.

Since the flux, ϕ_r, and the poles, N_s' and S_s', are set up by the rotor itself, they cannot contribute power to the rotor. This must

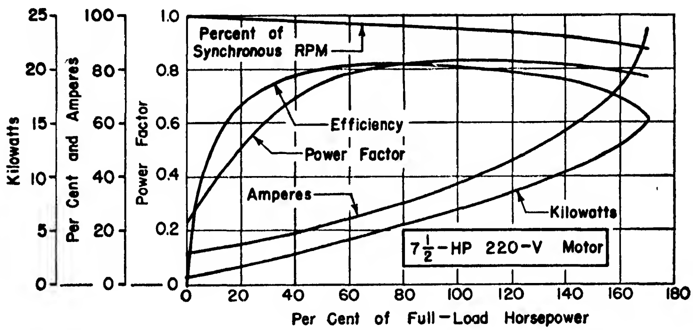


FIG. 3–10. Operating characteristics of a single-phase induction motor.

all come from the single-phase stator circuit. It has been shown that power in a single-phase circuit is pulsating (Chap. III). When the power factor is less than unity, as in the usual motor, the power has actually negative values during some part of each cycle. This means that a single-phase motor, supplying a constant load, must be able to store the excess energy, when the instantaneous power is a maximum and greater than the average, in order to carry the load when the power is zero; and also during the time the power in the circuit is reversed. Thus, the rotor must be heavier to give it the necessary fly-wheel effect. For a given slip the average torque and the pull-out torque are less for the single-phase induction motor than for the polyphase motor. Its operating characteristics are shown by the curves of Fig. 3–10, in which the left-hand column of ordinates represents the power in kilowatts. The right-hand column shows the current in amperes, the per cent

power-factor, the per cent efficiency and the speed as a percentage of synchronous speed. Note that the operating characteristics for the single-phase motor are quite similar to those of the polyphase motor.

2–10. The Split-Phase Motor. The split-phase motor is the most widely used type of single-phase induction motor. It has a squirrel-cage rotor. In order to make it self starting, it is constructed with two stator windings — a main or running winding and an auxiliary, or starting winding. These windings produce a sort of rotating field. The two windings are entirely separate and each consists of as many coil groups as there are poles. In most motors, particularly those of fractional horsepower rating, the windings are generally of the two-layer chain or concentric type (see Art. 27–6). The starting winding is less bulky, occupies less winding space and generally has fewer turns than the main winding. This is to reduce its reactance and allow more current to flow, which tends to increase the flux set up by the winding. Also, the starting winding is wound in place last and occupies the space in the top of the slots. The two windings are placed 90 electrical degrees apart, and are connected in parallel across the line at starting, as indicated in the conventional diagram of Fig. 4–10. A starting switch, mounted on the rotor structure and operated by centrifugal force, is connected in series with the auxiliary starting winding. When the rotor accelerates to about 75 per cent of synchronous speed, the starting switch flies open, and the motor operates on the main winding only.

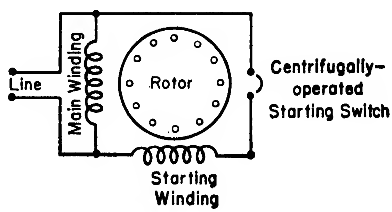


Fig. 4–10. Conventional diagram of the split-phase induction motor. The main and starting windings are drawn at right angles to indicate a displacement of 90 electrical degrees in their positions on the stator.

The two windings set up two fluxes and two sets of stator poles at 90 electrical degrees, as in the two-phase motor. But if the two stator windings of a two-phase motor are connected across the same single-phase line, the current in these windings will be in phase. The two sets of stator poles will be in time phase and reach their maximum values at the same instant. The combined field will be pulsating not rotating, and no torque will be developed. To set up the semblance of a rotating field and produce torque at starting,

there must be a phase displacement of the currents in the two windings. If additional resistance is placed in series with the starting winding of the split-phase motor, it will reduce the cur-

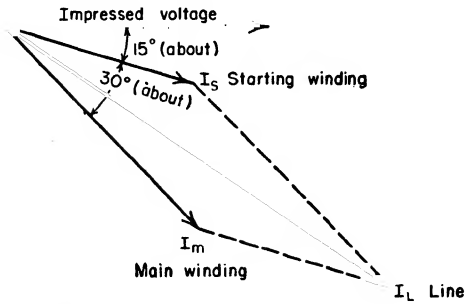


FIG. 5–10. Vector diagram of the starting currents in a split-phase induction motor. The currents in the two windings differ approximately 30° in phase.

rent in that winding, but will bring this current more nearly into phase with the impressed voltage. Additional resistance in the starting winding is generally obtained by using wire of smaller cross section than in the main winding, or by using wire of higher specific resistance than copper. In the commercial motor, the phase difference of the currents in the two windings is in the neighborhood of 30°, as indicated in the vector diagram of Fig. 5–10. In the diagram, I_s is the current in the starting winding, I_m the current in the main winding, and I_L the total motor current. The motor thus starts as an imperfect two-phase motor.

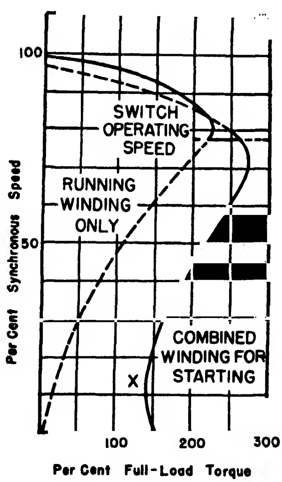


FIG. 6–10. Typical speed-torque curves for the split-phase motor.

Figure 6–10 shows the typical speed-torque curves for a split-phase motor. Note that the starting torque, developed by the main winding alone, is zero, while that developed by the combined windings, in this case, is about 150 per cent of full-load value. Also note that the two curves cross at about 85 per cent of synchronous speed, called the "cross-over point," and above this, the main winding alone develops more torque, for a given slip,

than both windings acting together. The motor accelerates on the two windings to approximately that speed at which the main winding torque is a maximum. At this speed, the centrifugal switch opens, cutting out the starting winding. The torque developed by the main winding when the switch opens is called the "pull-in torque."

Note that, as the motor accelerates, the torque reaches a minimum value, indicated by point x, on the combined curve. This is called the "pull-up torque."

The direction of rotation of the motor is reversed by reversing the connections of the **starting winding** to the line.

In many small split-phase motors, the connections to the coils are so arranged that the motor may be operated on either 110 or 220 volts. These are called **dual** or **two-voltage** motors. The coils of the main windings are all connected in series for operation on 220 volts, and are connected in two equal parallel groups for 110 volts. The starting winding may or may not be likewise arranged. In the latter case, this winding is designed for 220 volts and on 110 volts, the starting torque of the motor is reduced.

Another form of split-phase motor is that in which a **reactor** is placed in series with the **main winding** only while the motor accelerates. This is known as the reactor-start split-phase motor.

3–10. The Capacitor Type Motor. A form of split-phase motor, called the **capacitor motor,** has come into wide use during the past few years, particularly in fractional horsepower sizes. This has been largely due to the development of the electrolytic condenser. This condenser is manufactured cheaply today, is reliable and more compact than the older types.

There are three general types of capacitor motor which are classified by motor manufacturers as the "**Capacitor-Start Motor,**" the "**Capacitor Motor,**" or "Permanent-Split Capacitor Motor," and the "**Two-Value Capacitor Motor.**" These motors, almost universally, have squirrel-cage rotors.

The Capacitor-Start Motor is very similar to the ordinary split-phase type in that it has a main winding and an auxiliary winding, spaced 90 electrical degrees apart, as indicated in the conventional diagram of Fig. 7–10(a). The electrolytic condenser, or capacitor, is connected in series with the auxiliary winding and a centrifugally operated starting switch. This switch, as in the ordinary split-phase motor, is also set to open when the rotor reaches about 75 per cent of synchronous speed. The capacitor may be cylindrical

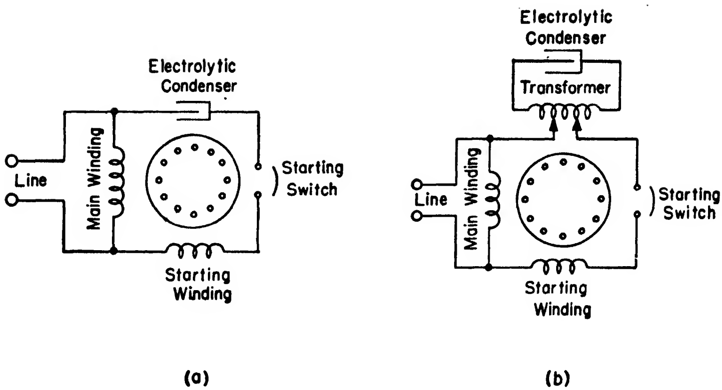


FIG. 7–10. Conventional diagram of the capacitor-start motor. (*a*) Capacitor connected directly in series with the starting circuit. (*b*) Capacitor connected into the starting circuit through an auto-transformer.

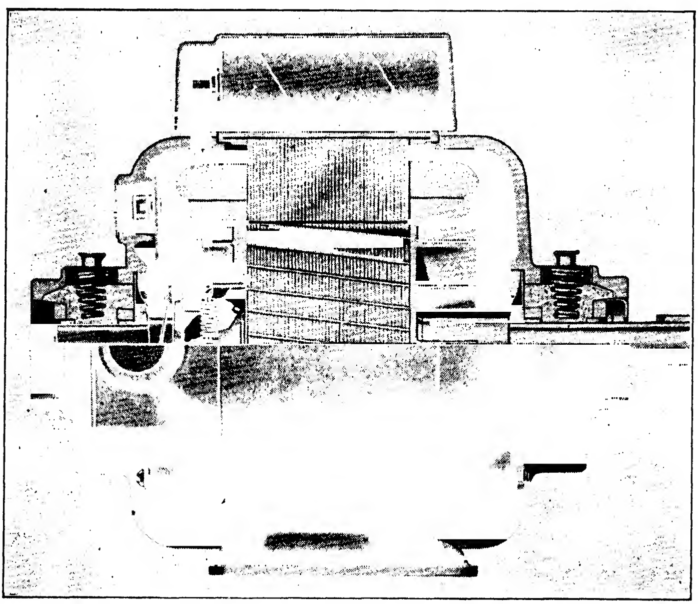


FIG. 8–10. A capacitor-start motor. (*General Electric Co.*)

in form and mounted on the motor frame, as in Fig. 8–10, or it may be ring shaped and mounted inside the bearing bracket, or end bell. Condensers for capacitor-start motors vary from about 80 μf for $\frac{1}{8}$ horsepower motors to 400 μf for one-horsepower motors.

The windings in the capacitor-start motor differ somewhat from those in the ordinary split-phase motor. The starting winding, while consisting of smaller wire, has more turns than the main winding. The increased reactance, due to the larger number of

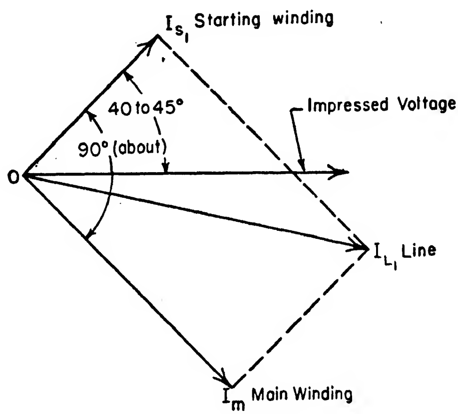


FIG. 9–10. Vector diagram of the starting currents in a capacitor-start motor. The currents in the two windings differ almost 90° in phase.

turns, is counteracted by that of the capacitor; so the current and flux of this winding is increased.

The effect of the capacitor causes the current in the starting winding to lead the impressed voltage, as indicated in Fig. 9–10. The reactance of this winding and the capacitance are so proportioned that the current in this circuit leads the voltage by close to 40 or 45 degrees. This causes a phase displacement between currents in the two windings approaching 90°, as shown in the diagram. The motor thus starts almost under the conditions of a two-phase motor.

It can be shown that the starting torque in a single-phase induction motor, with two windings spaced 90 electrical degrees apart, is proportional to the sine of the angle of phase displacement between the two currents in these windings. Thus the capacitor-start motor, with practically 90° displacement of the currents, has

much greater starting torque than the split-phase motor, with only 30° displacement (Fig. 5–10).

The vector diagrams of Fig. 5–10 and Fig. 9–10 are assumed to be for motors of the same rating. For approximately the same currents in the corresponding windings of the two motors, a comparison of two diagrams shows that the capacitor-start motor takes less current from the line at a higher power factor than the split-phase motor. Or conversely, for the same line current at starting, more current can be allowed in either or both the two windings

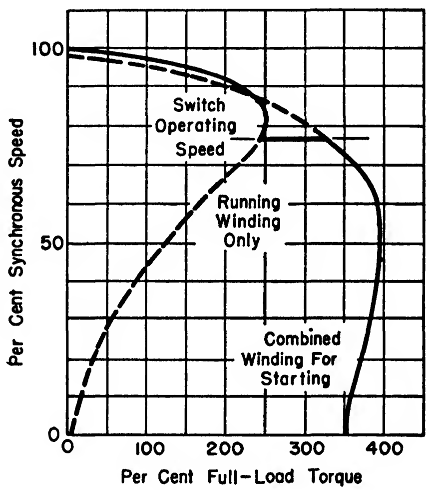


FIG. 10–10. Typical speed torque curves for the capacitor-start motor.

than in the split-phase motor, thereby producing more torque. The starting current in the capacitor-start motor may be made close to unity power factor.

The condenser is sometimes connected into the starting circuit through a small auto-transformer, as indicated in Fig. 7–10(*b*). Doubling the voltage on a condenser doubles the current, so the volt amperes vary with the square of the voltage. This gives the effect of four times the condenser capacity. The transformer thus permits the use of a higher-voltage condenser of less capacitance.

Figure 10–10 shows the typical speed-torque curves for the motor. Note the increase in starting torque compared to that in Fig. 6–10.

Below is a comparison of the minimum torque requirements of the National Electrical Manufacturers' Association for split-

phase and capacitor-start motors, rated at $\frac{1}{8}$ hp, 1725 rpm, 60-cycles. The values are given in per cent of full-load values.

Motor	Starting Torque	Pull-up Torque	Pull-out Torque
Split-phase	150	150	200
Capacitor-Start	350	200	200

It is evident that the capacitor-start motor is particularly adapted for service requiring high starting torque.

Permanent-Split Capacitor Motor. The windings in this motor are the same as for the capacitor-start motor. It both starts and runs with a fixed value of capacitance in series with the auxiliary winding, and the starting switch is omitted, as shown in Fig. 11–10. In place of the electrolytic condenser, which deteriorates and breaks down under continuous duty, an oil-filled condenser is used. Due to its increased bulk, weight and cost, it is impractical to use oil-filled condensers of as high a capacitance as that obtained with the electrolytic type. The capacitance of these condensers varies from about 2 or 3 μf for the smallest motors to around 20 μf for motors of $\frac{3}{4}$ horsepower. Less capacitance, however, is necessary to produce satisfactory running characteristics.

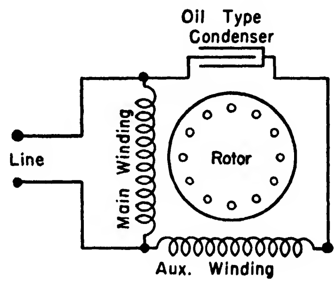


Fig. 11–10. Diagram of circuits in a permanent-split capacitor motor.

Because of less capacitance in the auxiliary circuit, this motor has lower starting torque than the capacitor-start motor. But under running conditions, the lower capacitance causes sufficient phase displacement in this circuit to produce the cross field, explained in Art. 1. Thus the motor both starts and runs as an imperfect two-phase machine. Due to this fact, it operates more quietly, at better efficiency, higher power factor, greater pull-out torque, and with less pulsating effect in the torque than the single-phase motors previously described. It is much used for oil burners, fans, etc., where high starting-torque is not required, and where silent operation is desirable.

The motor can be arranged to operate either on 110- or 220-volt circuits. One method is indicated in Fig. 12–10 in which a 220-

volt, 2 to 1 ratio auto-transformer is employed. The transformer is tapped at its midpoint as shown. The main winding is designed for 110 volts and connected across the half-voltage taps. The auxiliary winding with its condenser is designed for 220 volts, and connected across the 220-volt taps. For operation on 110 volts, the line is connected to transformer taps *a* and *b* in the figure. This puts the normal voltage on the main winding and the transformer steps up the voltage to 220 volts for the auxiliary circuit. For operation on 220 volts, the line is connected to taps *a* and *c*. This puts 220 volts on the auxiliary circuit and steps the voltage down to 110 volts for the main winding. The motor thus operates under normal conditions on either voltage, and is called a dual-voltage motor.

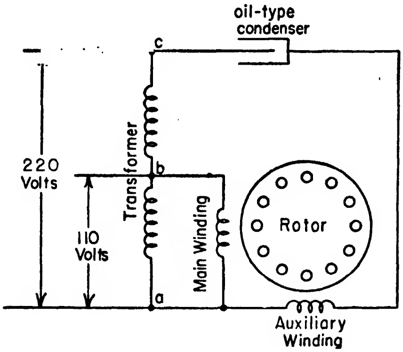


FIG. 12-10. Circuits in dual-voltage permanent-split capacitor motor.

Two-value Capacitor Motor. The windings in the two-value capacitor motor are the same as in the other types of capacitor motors. It differs from other types, however, in that it starts with one value of capacitance in series with the auxiliary winding and runs with another of smaller value. There are two general designs.

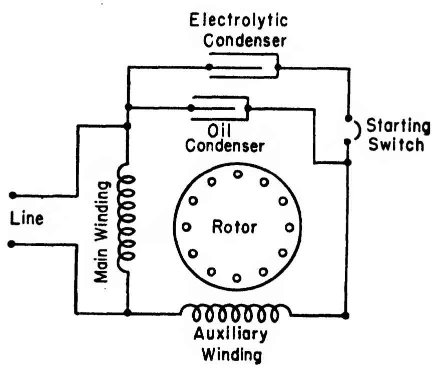


FIG. 13-10. Two-value capacitor motor with two parallel connected condensers for starting.

One design uses two parallel connected condensers in series with the auxiliary winding, as in Fig. 13-10. One condenser, of the oil type, is connected permanently in circuit for running. The other, an electrolytic condenser of much higher capacitance, is used for starting only, and is cut out automatically

by a centrifugal switch, when the motor reaches about 75 per cent of synchronous speed.

In the other design, an oil-filled condenser is connected into the auxiliary circuit through a small auto-transformer, as indicated in Fig. 14–10. A centrifugal switch connects the transformer in circuit through the taps *a* and *b*. This steps up the voltage on the condenser; thereby producing the effect of increased capacitance on starting. When the motor accelerates nearly to the cross-over speed, the switch closes the circuit through transformer tap *c*, giving sufficient capacitance for running. The main objection to this arrangement is the weight and the size of the container holding the transformer and condenser; as used with fractional horsepower motors it may be almost as large as the motor itself.

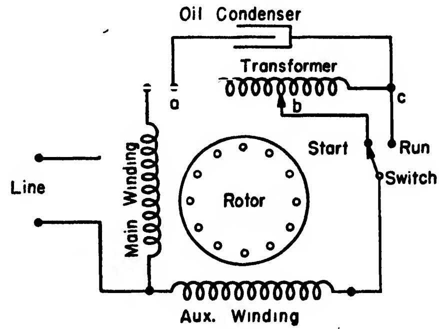


FIG. 14–10. Two-value capacitor motor with a single condenser, connected into the auxiliary circuit through an auto-transformer.

The two-value capacitor motor operates, therefore, as a permanent-split motor with its desirable running characteristics and, in addition, has the high starting-torque of the capacitor-start motor.

4–10. Speed Control of the Single-Phase Induction Motor. The normal speed-characteristics of the motors, so far considered in this chapter, are very similar to that of the polyphase induction motor. They are approximately constant speed machines, the slip gradually increasing with the load. As in the polyphase motor, the speed depends primarily on the frequency of the supply circuit and the number of poles for which the motor is wound. Attempts to obtain multiple or variable speed are met with the same difficulties that exist in the polyphase motor.

" Multispeed " motors may be designed for two or three speeds by constructing them with two or three sets of stator windings, each set, consisting of a main and an auxiliary winding, wound for a different number of poles. The two windings of each set are connected in parallel and the speed is changed by switching from one set of windings to another. The disadvantage of this scheme,

as in the polyphase motor, is that no intermediate speeds are provided. The motor, also, must be larger to accommodate the additional windings. Figure 15–10 is a diagram of the circuit in a two-speed capacitor-start motor, using a single condenser. A double-pole-double-throw switch changes the line connections from the low-speed to the high-speed windings. An objection to this arrangement is that the centrifugal switch must be set to operate near the cross-over point of the low-speed winding. It will, therefore, open at a low point on the high-speed torque-curve, which reduces the torque when accelerating on this winding.

It is common practice to control the speed, particularly of fan and blower motors, by changing the slip. The use of a high

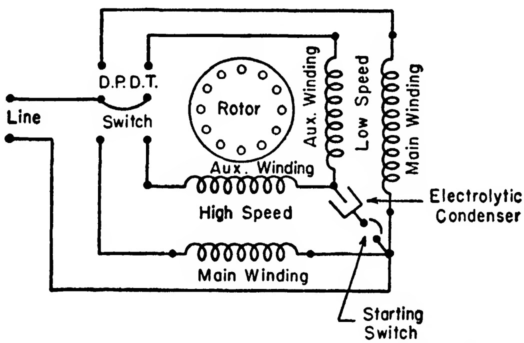


FIG. 15–10. Two-speed capacitor-start motor, using a single condenser and two sets of stator windings.

resistance rotor increases the slip, but the motor has only one speed at full load. It is applicable and commercially used where a motor with normal slip does not give the desired full-load speed. For instance, a standard 4-pole, 60-cycle motor may have a full-load speed between 1700 and 1750 rpm and a 6-pole motor a speed between 1100 and 1150 rpm. If the desired speed is about 1400 rpm, this can be obtained by use of a 4-pole motor with a higher-resistance rotor. The additional rotor I^2R losses decrease the efficiency, but this is not serious in a small motor.

The usual method of varying the slip in small motors is by changing the value of the flux. This is done by changing the impressed voltage on both windings or by varying the value of the voltage on either winding with respect to the other. It has been shown (Art. 10, Ch. 9) that the torque, developed for a given slip, is proportional to the square of the impressed voltage; or, for any

given torque, the slip is increased by reducing the applied voltage. Figure 22–9 also shows that the slip-torque curve changes with change in applied voltage. The voltage applied to the motor is usually varied by means of an auto-transformer. This is generally, but not necessarily, mounted on the motor frame. Figure 16–10 shows one arrangement for varying the voltage on a permanent-split capacitor motor. Speed is varied by connecting the motor to the various transformer taps. Starting torque is reduced on the low-speed taps.

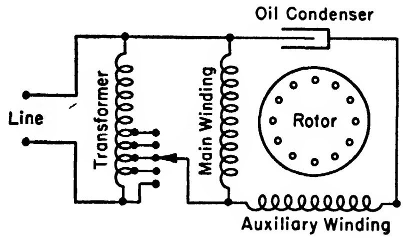


FIG. 16–10. An auto-transformer with adjustable voltage taps for controlling the speed.

5–10. The Shaded-Pole Motor. This motor is a form of single-phase induction motor and is generally built in very small sizes up to $\frac{1}{20}$ horsepower. It has a squirrel-cage rotor, but the stator is constructed with definite or salient poles. The main stator-winding is concentrated and wound on these poles, or on the

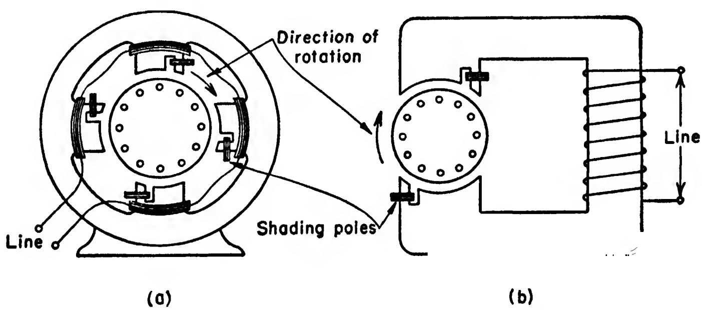


FIG. 17–10. (*a*) Shaded-pole motor with four poles. (*b*) The more common type of cheaply constructed shaded-pole motor.

magnetic circuit, and is similar to that for a d-c motor. A short-circuited copper strap, or coil, of low resistance is placed around a section of the main pole, as indicated in Fig. 17–10. This is known as a "shading-coil," and that portion of the main pole encircled by this coil is called a "shaded- or shading-pole." The poles and frame are, of course, laminated.

When the pole flux is increasing, a part of this flux attempts to

pass through the shading pole. This sets up an emf and current in the shading coil which, according to Lenz's law, sets up a counter mmf, opposing the flux entering this shaded section. Thus, at first, the greater part of the flux passes through the unshaded section of the poles. As the main flux reaches its maximum value, where the rate of change is zero, the emf and current in the shaded coil drop to zero, and the flux is more evenly distributed over the entire pole face. When the main flux decreases, the emf and current in the shaded coil reverse and set up an mmf tending to oppose this decrease. Thus the flux in the shaded pole is displaced in time from that in the main pole. The total flux, therefore, first reaches its maximum value in the main pole and later in the shaded pole. The combined effect is that of a sweeping action of flux across the pole face, resulting in a rather poor rotating field. This action pushes the rotor along with it and the rotation is in a clockwise direction; or from the main section toward the shaded section of the pole, as indicated in the figure.

The operating characteristics of the shaded-pole motor are very similar to those of the ordinary split-phase motor. Among its advantages are its simple construction and low cost, due to the type of stator winding. It is reliable, rugged, has relatively high power-factor and no sliding electrical contacts or automatic switches. However, it is noisy, has low starting-torque and low efficiency. It is used for driving small fans, thermostatic control of heaters, and other small devices.

6–10. Commutator Type Motors. Because of its almost constant-speed, the single-phase induction-motor is said to have shunt motor characteristics. A single-phase motor is desirable for many applications in which a drop in speed is accompanied by a very large increase in torque; that is, an a-c motor having d-c series motor characteristics. So far, the only a-c motors with such characteristics are of the commutator type. These motors are built in fractional horsepower sizes and in larger ratings, notably for hoists and a-c traction.

7–10. Series A-C Motor — Universal Motor. If the current in both field and armature of a d-c series motor is reversed, the direction of the torque remains unchanged. That is, reversing the polarity of the applied voltage does not reverse the torque nor the rotation. Thus, if an a-c voltage is impressed on a d-c series motor, it develops torque and rotation in one direction only. The field flux is practically in phase with field current, so that both field

flux and armature current are substantially in phase, and reverse at practically the same instant. A pulsating torque is developed, varying from zero to a maximum. The average torque for the same current is approximately equal, therefore, to that obtained when operating on direct current. The question naturally arises as to the operation of the d-c shunt motor on alternating current. The reactance of the shunt winding, however, precludes the development of torque in this motor. It is further considered in Art. 11.

The standard d-c series motor does not operate satisfactorily on alternating current. The efficiency and power factor are low, and severe sparking occurs at the brushes. This is largely due to reactance in the motor circuit. To overcome these defects, so far as possible, the construction of the a-c series motor differs from the d-c machine in the following respects and for the following reasons.

First. An alternating flux sets up large eddy currents in the poles and solid frame of the d-c motor, causing excessive heating and iron losses, and lower efficiency. Therefore, the entire field structure, as well as the armature core, is laminated.

Second. In the d-c motor, the field coils are wound with a comparatively large number of turns and the flux density at full load is high. This winding, thus, has high inductance and the reactance drop with alternating current is high. This causes low power-factor and relatively little output for a given current.

To improve the output and produce a satisfactory a-c motor, the reactance, and, therefore, the inductance of the field winding must be reduced to a minimum. Since the inductance is proportional to the product of flux and turns (Vol. I, page 238, Art. 7), the necessary flux to develop the required torque must be obtained with a minimum number of field turns per pole. That is, the field ampere-turns per pole must be as low as possible. Therefore, the reluctance of the magnetic circuit must be as low as possible. This is obtained by a very short air gap and by working the iron at low flux density, and therefore, at high permeability.

The motor is constructed either with salient poles, similar to the d-c motor, as in Fig. 18–10(*a*); or with a field structure similar to the stator of an induction motor, as in Fig. 18–10(*b*). When constructed, as in Fig. 18–10(*a*), the poles are short, due to the few turns in the field coils, and of large area in order that the flux density may be low. When constructed, as in Fig. 18–10(*b*), the distributed field winding, shown by the full lines, occupies only

part of the stator slots, since it is not efficient to fully distribute a single-phase winding.

Third. The armature is similar to that of the d-c motor, except that it is wound with an unusually large number of conductors. Developed torque is proportional to the product of the flux density of the field, the current per conductor and the number of conductors (Vol. I, page 350). Since the poles are worked at much lower flux density than in the d-c machine, the number of armature conductors must be increased to produce the necessary torque with a weaker field. The armature ampere-turns are, therefore, large with respect to the field ampere-turns. They set

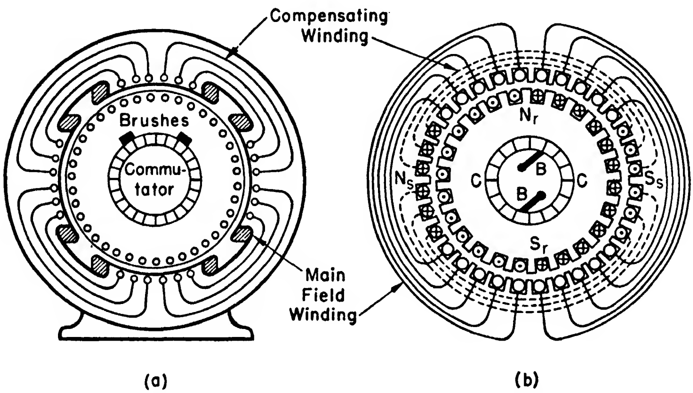


FIG. 18–10. The windings in a series a-c motor. (*a*) Salient-pole motor. (*b*) Two-pole motor with uniform air gap.

up an mmf and a flux at 90 electrical degrees to the field. This corresponds exactly to armature reaction in the d-c motor, except that the effect is greatly increased in the a-c motor. The relations are indicated in Fig. 18–10(*b*). The poles N_s, S_s are set up in the stator by the field, and the poles N_r, S_r, are set up in the armature core by the current in the armature. The latter poles are much stronger, with respect to N_s, S_s, than in the d-c machine. They cause a greater distortion of the field flux and a greater shift in the position of the commutating neutral point, which causes sparking at the brushes. Furthermore, the alternating armature flux links the armature conductors and causes reactance, and a reactance drop in the armature, which also lowers the power factor.

To reduce armature reactance drop and improve commutation, the motor is equipped with a compensating winding, similar to

that in some d-c machines (see Vol. I, page 336). This winding, at 90 electrical degrees to the main field winding, sets up an opposing mmf which completely nullifies the armature mmf, prevents distortion of the main field flux, and reduces armature reactance. The leakage flux around the individual armature conductor in the slots cannot be compensated for, however, and armature reactance cannot be entirely eliminated. In the motor of the type shown in Fig. 18–10(*a*), the compensating winding is set in slots in the pole faces. In Fig. 18–10(*b*), this winding occupies those stator slots not used by the main field winding, as indicated by the dotted lines.

The compensating winding may be connected in series with the armature and main field winding, as indicated in the conventional

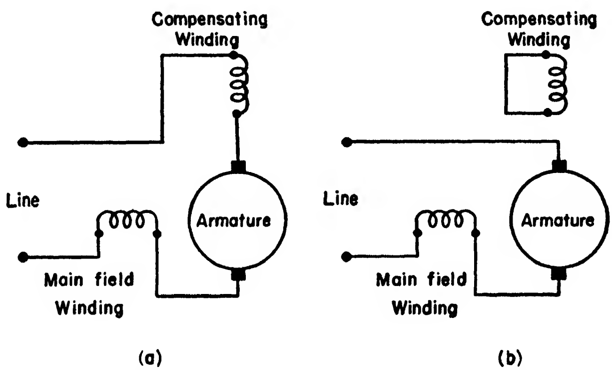


FIG. 19–10. Conventional diagram of circuits in a series motor. (*a*) Conductive compensation. (*b*) Inductive compensation.

diagram of Fig. 19–10(*a*). This is called "**conductive compensation.**" Or the winding may be used short circuited on itself as in Fig. 19–10(*b*), and current induced in it by the alternating armature flux. It acts like the short-circuited secondary of a transformer, of which the armature winding is the primary. This is called "inductive compensation." Reactance cannot be entirely eliminated here because of the leakage flux between the winding and the armature.

Fourth. An additional problem exists in the a-c motor which is not present in the d-c motor. In Fig. 20–10(*a*) (a ring armature being shown for clearness), when the coils c_1 and c_2 are undergoing commutation, they are short circuited by the brushes B_1 and B_2, respectively, and are linked by the main field flux, ϕ. As this flux is alternating, a high voltage and large current is induced in the

short circuited coils by transformer action. This also causes severe sparking at the brushes. The emf induced per turn in the short-circuited coils is proportional to the flux per pole. To reduce the voltage between adjacent commutator segments to the lowest possible value, the armature coils are wound of a **single turn,** and the flux per pole reduced to a minimum. The number of poles must be increased, therefore, to obtain the necessary total flux to develop the required torque. As the number of armature conductors is unusually large, the number of commutator segments and the size of the commutator is greatly increased. Thus the series **a-c motor** has an unusually large commutator, composed of

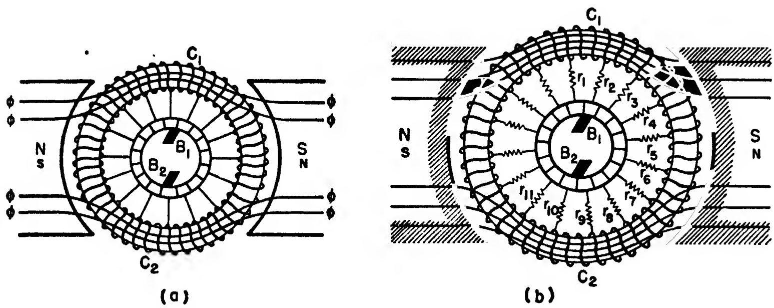


FIG. 20–10. (*a*) The alternating flux, in the series motor, links the short-circuited coils C_1 and C_2, and induces in them heavy currents by transformer action. (*b*) Resistance leads r_1, r_2, etc., connected between the commutator segments and the coils, limit these short-circuit currents.

very thin commutator bars, and a larger number of field poles of greater cross section area than the d-c motor of comparable rating. All this limits the voltage rating of such motors to about 250 volts.

To reduce the current in the short-circuited coils to the lowest possible value, resistance- or "preventive-leads" have been inserted in the leads between the coils and commutator segments, as in Fig. 20–10(*b*). This increases the impedance of these coils on short circuit. The resistors r_1 and r_2 thus act in series to limit the current in any short-circuited coil, but act in parallel to the main armature current. These resistors carry current only at the instants when they are connected to segments which are short circuited by the brushes. While this increases the I^2R loss in the motor, it is not excessive. The main objection is the cost of installing these resistors and the fact that they may burn out if the motor stalls or takes too long in starting. In motors of the type

shown in Fig. 18–10(*a*), the current in the short-circuited coils may be reduced by installing interpoles, as in the d-c motor.

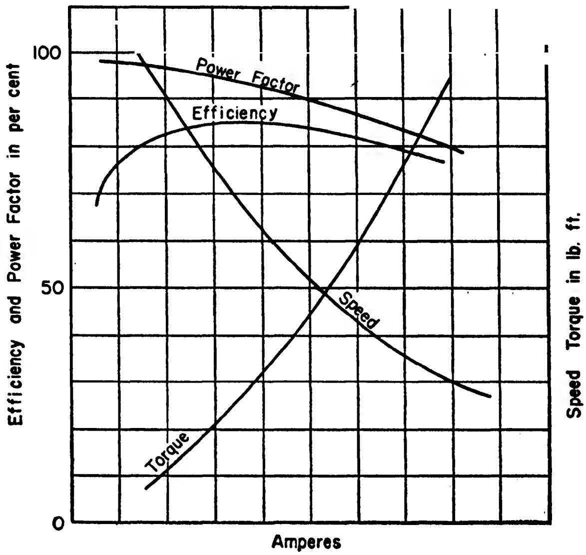


FIG. 21–10. Typical characteristic curves for the series a-c motor.

Operating Characteristics. The operating characteristics of the series a-c motor are similar to those for the d-c motor, as shown in Fig. 21–10. Note that the torque and the speed vary almost with the square and the inverse square of the current respectively, as in

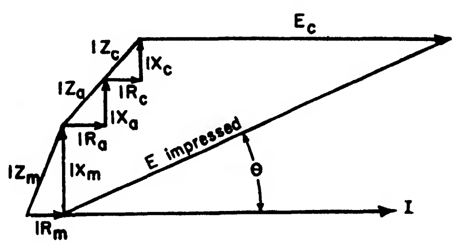


FIG. 22–10. Vector diagram of the series a-c motor.

the d-c motor. (See Vol. I, Fig. 30–11, page 392.) The motor is suitable, therefore, and is used to some extent in railway applications.

Figure 22–10 shows the vector diagram for this motor. The resistance drops in the main field, the armature and the com-

pensating winding are shown as IR_m, IR_a and IR_c respectively. The corresponding reactance drops IX_m, IX_a and IX_c lead the current by 90 degrees. Note that the reactance drops in the armature and compensating winding are small with respect to that in field, since their respective mmfs tend to neutralize each other. It is impossible to compensate for reactance in the main field. The counter emf, E_c, is due to the rate of cutting the main field flux and is in phase with the current, since it is a maximum when the flux is a maximum, and zero when the flux is zero. The impressed voltage is the vector sum of the counter emf and the several impedance drops. It leads the current by the angle, θ, cosine θ being the power factor of the motor. As load on the motor is increased, the current and flux increase and the speed falls. The impedance drops increase and E_c decreases, thus increasing the angle θ. The power factor is higher at light loads, therefore, and decreases with increases in load, as shown by this curve in Fig. 21–10. The efficiency is somewhat lower than for the corresponding d-c machine.

It has been shown that reactance in the motor causes brush sparking and lowers the power factor. Since reactance varies with the frequency ($x = 2\pi fL$), the motor operates best on circuits of low frequency. Except in fractional horsepower sizes, it is rarely used in the United States on circuits above 25 cycles. In Europe, lower frequencies are used. In fact, the series a-c motor gives better performance on direct current. Commutation and efficiency are improved and it may run at higher speeds, all due to decreased reactance in the circuit. If it is to be operated both on direct and alternating current, the motor must be compensated conductively, as in Fig. 19–10(a).

An outstanding example of the application of the a-c motor for electric traction on both alternating and direct current is the electrification of the New York, New Haven and Hartford Railroad. From New Haven to Harlem, the locomotives take power from an overhead trolley at 11,000 volts and 25 cycles. The voltage is stepped down by an auto-transformer on the locomotive to 250 volts on the motors, all connected in parallel. From Harlem to Grand Central Station, the tracks of the New York Central are used, and the motors, two in series, take power at 600 volts d-c from a third rail.

The Universal Motor. Series a-c motors of fractional horsepower rating, designed for use on both 60-cycle alternating- and

direct-current circuits, are known as **universal motors.** They are widely used in many domestic and commercial applications, for vacuum cleaners, sewing machines, food mixers, portable drills, office machines, etc. The relations involved and operating characteristics discussed above apply also to this motor. It is usually built in sizes not exceeding one horsepower. In very small sizes, where cheapness is a controlling factor, it may be uncompensated. But most universal motors are conductively compensated, as the performance is much improved. This motor is often equipped with an automatic centrifugally operated switch or governor. When a certain predetermined speed is reached, the governor contacts open, inserting a resistance in series with the motor circuit, as in Fig. 23–10. Below this speed, the contacts

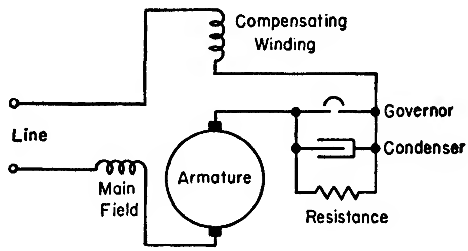


FIG. 23–10. Diagram of circuits in a compensated universal motor, equipped with a governor to limit the speed.

remain closed. To prevent arcing and burning of the contacts, they are shunted by a condenser, as shown. The speed characteristics are thus changed and the motor operates throughout a restricted load range at somewhere near a constant speed. Rated speeds of the universal motor are high, ranging from about 3600 to possibly 10,000 or 15,000 rpm.

8–10. The Repulsion Motor. The repulsion motor in many respects is similar to the series a-c motor. It consists of an armature and commutator like that in the d-c motor and, in its simplest form, a single stator winding on a laminated core, similar to that in the single-phase induction motor. A single-phase alternating magnetic field is set up, and **if the brushes are short circuited,** torque is developed.

First, consider Fig. 24–10(*a*), which is a conventional diagram of the stator or field winding for a two-pole motor connected to a single-phase line. A ring armature is shown for clearness in tracing the circuits. The short-circuited brushes are here placed

along the axis of the poles. The alternating stator flux, which is practically in phase with the line current, passes half through each side of the ring, as shown. The armature is thus divided into two paths and is equivalent to two separate transformer secondaries, each short-circuited through the brushes. And each has an emf and a current induced in it by transformer action. At the instant shown, the field or primary flux is increasing and sets up the stator poles N_s and S_s. According to Lenz's law, the mmf set up by the current in each armature, or secondary circuit, opposes that in the primary. At this instant, the emfs and currents in the two halves

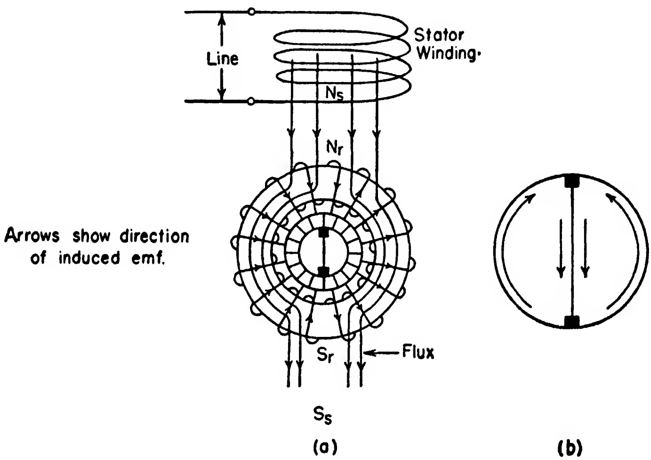


Fig. 24–10. (*a*) Currents and emfs in the armature of a simple repulsion motor, when the short-circuited brushes are placed along the axis of the stator poles. (*b*) General direction of the currents in the armature and through the short-circuited brushes.

of the armature, are, therefore, increasing also, and in a direction shown by the arrows on the coils. The diagram of Fig. 24–10(*b*) shows the general direction of these currents, which is up through the armature and down through the brushes. And according to the right-hand rule for coils, the poles N_r and S_r are set up in the armature core. These poles are in line, and practically in time phase, with the stator poles N_s and S_s. It is evident that although current flows in the armature, no torque is developed.

Now consider Fig. 25–10(*a*), in which the brush axis is shifted 90 electrical degrees (90 space degrees in a two-pole motor) from the polar axis — that is, to the mechanical or geometrical neutral. An emf is induced in both the armature paths, as before; but no

current can flow between the brushes, since they are connected between points of equal potential in the armature. Equal emfs also oppose each other in the closed circuit of the armature, as indicated in the diagram of Fig. 25–10(*b*). Thus the armature current is zero and no torque is developed.

Note that when the brushes are placed along the polar axis, as in Fig. 24–10, they are in the most favorable position to produce current in the armature, but are in an unfavorable position to develop torque. And when the brushes are placed on the mechanical neutral, as in Fig. 25–10, ordinarily the most favorable

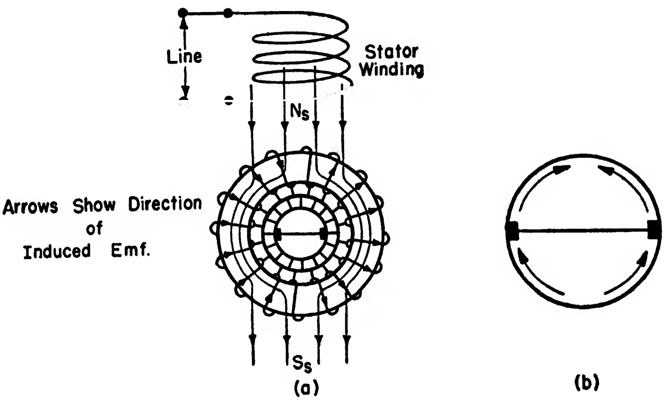


FIG. 25–10. (*a*) Emfs in the armature, when the brushes are shifted 90 degrees and set on the mechanical neutral. (*b*) General direction of opposing emfs in the closed-circuit paths in the armature.

position to develop torque in the d-c motor, no armature current is produced. In either case, the motor develops no torque.

However, if the brushes are placed in an intermediate position between the two shown above, they will short circuit points on the armature between which a difference of potential exists. This is shown in Fig. 26–10(*a*), which represents the conditions at the same instant as in the preceding figures. Currents will now flow in the armature and establish the poles N_r and S_r. Repulsion between these poles and the corresponding stator poles N_s and S_s develops torque and produces rotation. In this case it is clockwise. The general direction of the currents in the armature paths and through the brushes, at this instant, is indicated by the full line arrows in the diagram of Fig. 26–10(*b*).

Note, in the current path *c d e c*, Fig. 26–10(*b*), the emf *d* to *c*,

shown by the broken arrows, is opposed in the circuit to a greater emf from d to e. Also in the path $c\,f\,e\,c$, the emf from e to f is opposed to the greater emf from c to f. The current flow is in the direction of the greater emf, as shown in Fig. 26–10(b). Thus, while the resultant emfs in each path are equal, they are less than when the brushes are set in line with the stator poles, and the armature current is less. Therefore, as the brushes are shifted from the position in Fig. 24–10 toward that in Fig. 25–10, the torque gradually increases, while the resultant armature emf and current decreases. A point is reached where the torque for a

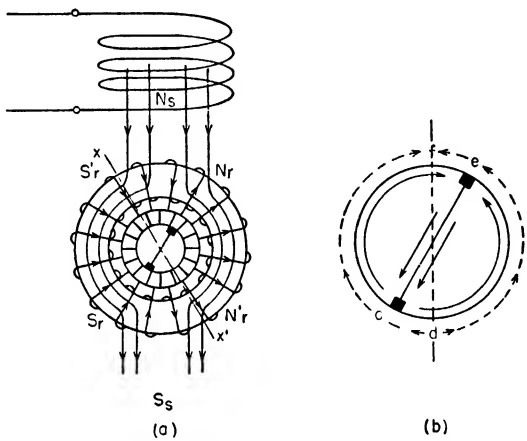


FIG. 26–10. (a) Brushes are shifted out of line with the polar axis to a position which gives both armature current and torque. (b) General direction of emfs (broken arrows) and currents in the closed armature paths.

given field flux is a maximum. If the brush axis is shifted beyond this point, the torque decreases. In the commercial motor, the brush axis is shifted 20 to 30 electrical degrees from the polar axis. If the brush axis is shifted an equal angle in the opposite direction, to the line xx_1, Fig. 26–10(a), the poles $N_r'S_r'$ are set up and the direction of both torque and rotation is reversed.

A four-pole motor would have two pairs of short-circuited brushes, a six-pole motor three pairs, etc., placed 90 electrical degrees to each other and displaced from the polar axis, as in the two-pole motor.

Instead of a single stator winding, the motor usually has two such windings, as shown in the conventional diagram of Fig. 27–10(a). The brushes are placed along the axis of one winding,

called the **transformer,** or **inducing** field, which produces current in the armature by transformer action. It also performs the function of the **compensating** winding in the series a-c motor. The other winding, placed at 90 electrical degrees to the first, develops torque as in the d-c motor. This is called the **main,** or torque field. The two fields at 90° create a resultant field, as in Fig. 27–10(*b*), and produce the same effect as the brush shift in Fig. 26–10. The power factor of the motor is improved and becomes unity at synchronous speed.

By comparing Fig. 19–10(*b*) (Art. 7–10), for the series motor with Fig. 27–10(*a*), it will be noted that the circuit diagrams are the same, except that the armature and compensating or transformer fields are interchanged. Since the armature currents in the

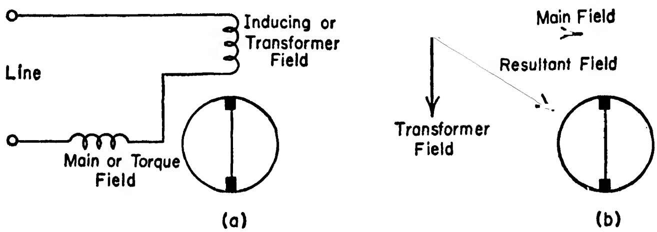


FIG. 27–10. (*a*) Conventional diagram of a repulsion motor with two stator windings — a transformer winding and a main, or torque, winding. The brushes are set along the axis of the transformer winding. (*b*) The combined field produces the same effect as shifting the brushes in Fig. 26–10.

repulsion motor are induced by the transformer action of the inducing field, they are proportional to the currents in this field. As the inducing field is in series with the main field winding, an increase of current in this main winding is accompanied by a corresponding increase in armature current. This is exactly the action which takes place in the series motor. Thus, **the repulsion motor has series motor characteristics, with correspondingly high starting torque.** There are several types of this motor.

The internal reactions which take place in the repulsion motor are very much involved, largely due to the speed emf induced in the armature by rotation, and to the fact that the resultant transformer field linking the armature decreases with increase in speed. These can be analyzed only by a complicated vector diagram. This is mainly of interest only to those persons engaged in the design of such motors and is not considered here.

The repulsion motor has been widely used for driving fans and blowers, etc., but has been practically displaced by the capacitor motor. This principle is used, however, in starting single-phase induction motors.

9–10. Repulsion-Start Induction Motor. This was one of the first types of single-phase motors to be developed commercially. As the name implies, it starts as a repulsion motor with high starting torque, but runs as a single-phase induction motor at approximately constant speed. It has the same type of a-c armature, a distributed stator field, and the circuits are the same as in the simple repulsion motor of Fig. 26–10, (Art. 8–10). It differs from the latter motor, however, in that it is constructed with a mechanical device operated by centrifugal force, which short-circuits the commutator segments at a predetermined speed. This device is called a "governor," or "short-circuiter," and converts the armature into a short-circuited induction motor rotor. There are many types of governors, which differ in mechanical construction. They are universally mounted inside the armature core, or between the core and the commutator, and short circuit the commutator segments on inner surfaces provided for that purpose.

Figure 28–10 is a typical speed-torque curve for this motor. Note that it accelerates as a repulsion motor approximately to the crossover point of the two curves, near 75 per cent of synchronous speed. At this speed, the governor short circuits the armature and the motor operates with induction motor characteristics. At starting, it usually develops around 400 per cent of full-load torque.

The commutator may be of the usual axial or cylindrical type, with brushes bearing on the outside surface, as in the d-c machine. In which case the brushes are usually held permanently in contact with the commutator. This is called a "brush riding" motor. Or, the commutator may be of the radial type, in which the bearing surface is perpendicular to the motor shaft. On this surface, the commutator segments are wedge shaped. In motors with commutators of this type, the brushes are usually lifted from the bearing surface by the governor, when the armature is short circuited.

The direction of rotation is reversed by reversing the angle of brush shift, as explained in the preceding article.

The main objection to this motor is the complication of the governor and increased cost of manufacture and maintenance.

To large extent, it has been superseded by the capacitor-start motor, previously described.

The repulsion-start induction motor is often misnamed a repulsion induction motor. The latter motor is considered in the following article.

10–10. Repulsion-Induction Motor. The repulsion-induction motor has a squirrel-cage rotor winding in addition to the repulsion

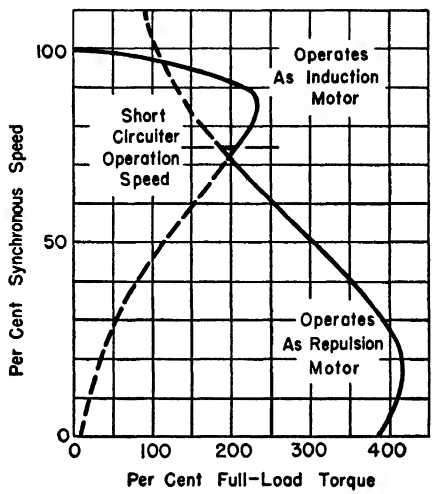


FIG. 28–10. Typical speed-torque curve for the repulsion-start motor.

motor armature winding. Otherwise the circuits are the same as in the simple repulsion motor of Fig. 26–10 (Art. 8–10). The motor has no governor nor brush lifting device and the two rotor windings are not connected in any way. Both windings may be placed in the same slots and separated by magnetic shunts, similar to the arrangement in Fig. 31–9(*a*) (Art. 14–9); or the slots may be sectionalized, as indicated in Fig. 29–10. The squirrel-cage winding is placed in the bottom section of the slots, and the principle upon which the motor operates is similar, in some respects, to that of the double-squirrel-cage motor (Art. 14–9). The leakage reactance of the squirrel-cage winding embedded in the core, is high at starting and at low speeds. The current and developed torque, therefore, are negligible. So the motor starts as a repulsion motor with high torque. As the motor accelerates and attains operating speed, the current in the squirrel-cage winding increases, and it develops a large proportion of the torque required

by the load. Since the repulsion winding tends to produce series motor characteristics, if the load is sufficiently reduced, the motor will run at synchronous speed and no current will flow in the squirrel cage. And if the load is further reduced, the series motor action of the repulsion winding tends to drive the motor at still higher speed. But the squirrel-cage winding now develops generator action, as in the induction generator (Art. 21–9), and opposes any excessive increase in speed. On very light loads, the motor runs slightly above synchronous speed. Otherwise its performance is similar to that of the induction motor, except that the drop in speed with increase in load is somewhat greater. A typical speed-torque curve is shown in Fig. 30–10. The squirrel-cage winding may be of copper, die-cast aluminum, or of higher resistance material, depending on the speed characteristics desired.

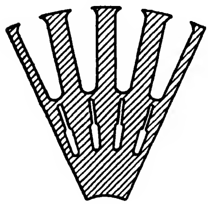


FIG. 29–10. One form of rotor slot in the repulsion-induction motor.

The resultant torque, developed by this motor, is the combined torque of the two rotor windings. Except for the reaction of one winding on the other, its performance may be compared to the combined action of two motors, a simple repulsion motor and a single-phase induction motor, whose shafts are coupled together.

This motor is manufactured both in fractional horsepower and in larger sizes.

11–10. Adjustable-Speed Commutator Motor. As stated in Art. 7–10, it is not possible to operate a d-c shunt motor from a single-phase alternating-current circuit. The relations which exist when such a motor is connected to a single pair of a-c terminals are as follows:

The field current is mainly a magnetizing current and lags practically 90 degrees behind the applied voltage. The flux set up by this current also lags by approximately the same angle, while the armature current is more nearly in phase with the applied voltage. Thus the field, or air-gap flux, and the armature current are nearly 90 degrees out of phase; so that, when the armature current is a maximum, the flux is zero. Also the reactance of the field winding is high and the resultant field current is low. The motor, therefore, develops little or no torque. Furthermore, the armature counter emf, induced by rotation and the cutting of this flux, must be at 90 degrees with the impressed voltage. So

the armature current is determined, not by the load and counter emf, as in the d-c shunt motor, but by the reactance of the armature circuit. Thus the armature current will be high, even for light loads, and the power factor low. So if the torque were sufficient to cause rotation, the motor would not have shunt motor characteristics.

In order to obtain a satisfactory torque of high power factor, armature impressed voltage, armature current and air-gap flux

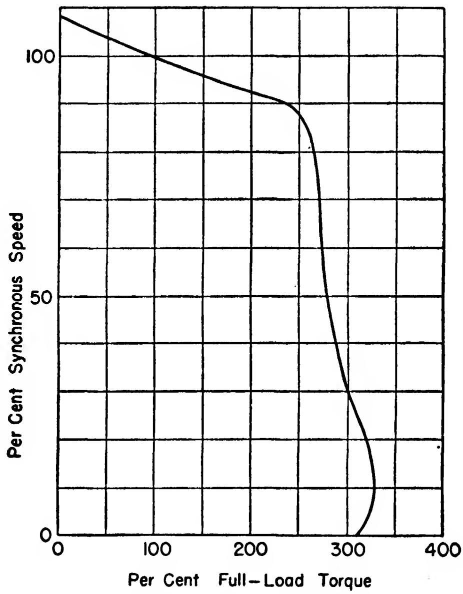


Fig. 30–10. Typical speed-torque curve for the repulsion-induction motor.

must all be substantially in time phase with each other. This cannot be obtained when both field and armature are connected to the same pair of a-c terminals. However, the motor can be adapted for use on a polyphase circuit, and will operate at adjustable speeds with characteristics very similar to those of the d-c shunt motor.

Such a motor has been developed by Messrs. Conrad, Smith and Ordung, and is described in *Electrical Engineering* (Sept. 1943). This motor has a uniform air gap, a distributed stator winding, similar to that in a single-phase induction motor, and an armature like that in the repulsion motor. In order to reduce armature reactance, a compensating winding is placed at 90 electrical degrees

to the main stator winding, and may be connected in series with the armature or short circuited on itself, producing inductive compensation.

When the field, or stator, winding is connected to one phase of a two-phase circuit and the armature to the other phase, the voltage applied to the field is at 90 degrees to that applied to the armature. Since the field current lags its voltage by nearly 90 degrees, the field, or air-gap flux, will be nearly in phase with the armature voltage and torque is developed. The counter or back emf, due to rotation, will oppose the applied armature voltage. The armature current is thus determined by the value of the

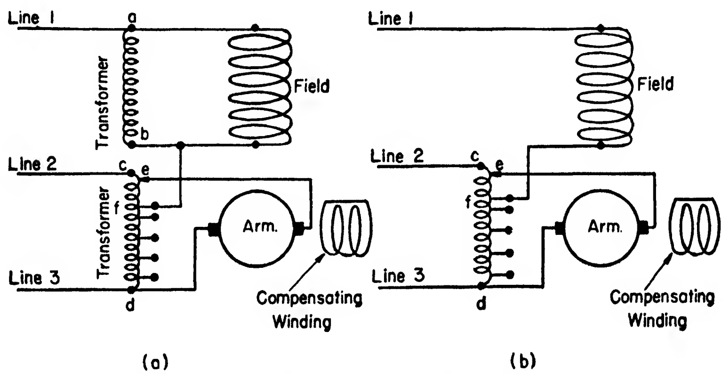


FIG. 31–10. (*a*) Circuit diagram of an adjustable-speed commutator motor, using Scott-connected transformers to produce voltages on the armature and field, displaced 90 degrees. (*b*) A connection in which the field winding replaces one transformer. Armature voltage and speed are adjusted by connecting the contact, *e*, to various taps on the transformer, *cd*.

counter emf and the motor will operate very much as a d-c shunt motor.

The motor can be supplied from a three-phase line by means of a modification of the Scott-connection for transforming from three-phase to two-phase (Art. 35–8). In Fig. 31–10(*a*), the field is connected across the transformer *ab* and the armature across the transformer *cd*. Note that the point *f* is not connected to the mid-point of the transformer winding *cd*, as in the ordinary Scott-connection. This shifts the phase position of the voltage on transformer *ab* (and also on the field) with respect to the voltage on transformer *cd*, such that the field flux lags slightly behind the armature voltage. The air-gap flux thus is thrown more nearly in

time phase with the armature current and improves the power factor.

Instead of two transformers, the stator field winding itself can be used in place of one transformer, as in Fig. 31–10(*b*). The armature terminals are connected to the movable contact *e* and point *d* on transformer *cd*, called the adjusting transformer. The voltage on the armature, and therefore the speed of the motor, can be adjusted by shifting the position of the movable contact *e*. Moving this contact toward point *d*, reduces the armature voltage and the speed. This corresponds exactly to a similar method of speed control of the shunt motor. At starting, the movable contact *e* is simply set to impress a low voltage on the armature, and its position is gradually shifted, as the motor accelerates. Thus no additional starting equipment is necessary and the motor accelerates smoothly.

For a given voltage applied to the armature, the performance is similar to that of the d-c shunt motor, except that the regulation is somewhat poorer. The power factor varies from about 50 per cent leading at light loads to approximately unity near full load and decreases with lagging values at heavier loads. The efficiency is lower than for the corresponding d-c motor, but compares favorably with that of the single-phase motor.

The motor takes unbalanced line currents from the three-phase mains. Line 1 (Fig. 31–10(*b*)) supplies the field winding with a current which is practically constant at all loads, while lines 2 and 3 supply the armature current, which varies with the load. This unbalancing is undesirable in large motors, but is not objectionable in small motors. The application is limited, therefore, to motors of small horsepower rating.

SUMMARY OF CHAPTER X

THE SINGLE-PHASE MOTOR is inferior to the polyphase motor, since power obtained from a single pair of terminals is not uniform but is pulsating. It is larger than the polyphase motor of comparable rating, has much lower efficiency, poorer operating characteristics, and, in most cases, lower power factor. This motor is used mainly in fractional horsepower sizes and where polyphase power is not available.

THE SINGLE-PHASE INDUCTION MOTOR has only a single stator winding. This produces no rotating field, but only a pulsating or oscillating field which develops no torque. However, if the motor is brought up to speed in either direction, by external means, it will run.

The emfs and currents induced in the squirrel cage by rotation, set up a secondary set of poles in the stator, which differ in time phase with the main stator poles. This produces a sort of rotating field and a resulting torque. These poles, at synchronous speed, are substantially as strong as the main poles, but decrease in value as the speed decreases and disappear when the motor is at standstill.

THE SPLIT-PHASE MOTOR is a single-phase induction motor with an additional stator winding connected in parallel, and placed at 90 electrical degrees to the main winding. This auxiliary winding is used for starting only. It is connected in series with a switch operated by centrifugal force, which opens when the motor accelerates to about 75 per cent of synchronous speed. The starting winding is so designed that its current is about 30° out of time phase with the current in the main winding. This produces an imperfect rotating field and the motor starts with from 150 to 250 per cent of full-load torque. Reversing the connections of the terminals of the starting winding REVERSES THE DIRECTION OF ROTATION.

THE CAPACITOR-START MOTOR has come into use the past few years, due to the development of the electrolytic condenser or capacitor. This condenser is much less bulky and has higher capacitance than the older types. The motor is similar to the split-phase motor with an auxiliary winding placed at 90 electrical degrees to the main winding. The electrolytic condenser is connected in series with the starting winding and a centrifugally operated switch, which is also set to open at about 75 per cent of synchronous speed. Due to the condenser, the current in the starting winding leads the impressed voltage by nearly 40 to 45 degrees. This causes a phase displacement of currents in the two windings close to 90 degrees. The motor thus starts almost under the conditions of a two-phase motor. It develops much greater starting torque with less line current than the ordinary split-phase motor, with only 30 degrees displacement of current in the two windings. The starting current may be close to unity power factor, and the starting torque from 300 to 400 per cent of full-load value.

The condenser is sometimes connected into the starting circuit through a small auto-transformer. By raising the voltage on the condenser, the capacitance effect is increased.

This motor is applicable for service requiring high starting torque.

THE PERMANENT-SPLIT CAPACITOR MOTOR is similar to the capacitor-start motor, except that it BOTH STARTS AND RUNS with a fixed value of capacitance in series with the auxiliary winding. The starting switch is omitted. An oil-filled condenser is used, since the electrolytic condenser breaks down under continuous duty. Due to its increased bulk, weight and cost, it is impractical to use such a condenser of as high a capacitance as that obtained in the electrolytic type. Because of lower capacitance in the auxiliary circuit, this motor has LESS STARTING TORQUE. But under running conditions, the phase displacement of current in this circuit is sufficient to produce a semblance of a rotating field. The motor thus starts and runs as an imperfect two-phase motor. It, therefore, operates more quietly, with

better efficiency, at higher power factor and with more uniform torque than other single-phase induction motors. It is much used where high starting torque is not required and where silent operation is desirable.

THE TWO-VALUE CAPACITANCE MOTOR has the same winding as other types of capacitor motors. It starts with one value of capacitance in series with the auxiliary winding and runs with a different and smaller value. There are two methods.

One method uses two parallel connected condensers in series with the auxiliary circuit. One condenser, of the oil type, is connected permanently in the circuit for running. The other, an electrolytic condenser of much higher capacitance, is used only for starting. It is connected in series with the centrifugally operated switch, which opens when the motor attains about 75 per cent of synchronous speed.

The other method uses a single oil-type condenser, connected into the auxiliary circuit through a small auto-transformer with two voltage taps. On starting, a centrifugal or magnetic switch closes the circuit through one transformer tap, putting increased voltage on the condenser. This gives the effect of higher capacitance. When the motor attains speed, the switch opens the circuit to the first tap and closes it through the lower voltage tap. This produces lower capacitance for running.

This motor, thus, has both high starting torque and the desirable running characteristics of the single-value permanent split capacitor motor.

" MULTISPEED " MOTORS are obtained by placing two or three separate sets of stator windings on the motor, each set wound for a different number of poles. They give two or three different rated speeds but no intermediate speeds.

THE SLIP AND SPEED of small induction motors is changed by use of an auto-transformer to vary the voltage on either or both stator windings. This changes the flux and the shape of the slip-torque curve.

THE SHADED POLE MOTOR is a single-phase induction motor with salient field poles. The faces of these poles are divided and a short-circuited " shading coil " of low resistance is wound on the trailing sections. The changing pole sets up currents and mmfs in these coils which oppose and retard the flux in these " shaded " sections of the poles. This causes a sweeping action of the flux across the entire pole face, which is somewhat similar to the movement of a rotating field. These motors are built only in very small sizes, seldom exceeding $\frac{1}{20}$ hp. The starting torque is low.

SERIES A-C MOTOR. When alternating current is impressed on the terminals of a d-c series motor, the torque is in one direction only. However, a SERIES MOTOR TO OPERATE SATISFACTORILY ON ALTERNATING CURRENT must be altered in design from that of the d-c motor. On alternating current its increased reactance causes severe sparking at the brushes, lowers the power factor and reduces the output.

TO REDUCE THE REACTANCE AND IMPROVE COMMUTATION:

(a) The field coils are wound of fewer turns, the air gaps are shortened and the iron is worked at lower flux density. To obtain the necessary total flux to develop the required torque, the cross section and the number of poles is increased.

(b) The number of armature conductors is greatly increased to provide the necessary torque with a weaker field. This increases armature reactance and lowers the power factor. So a COMPENSATING WINDING is placed on the stator at 90 degrees to the main field. Current in this winding nullifies the armature ampere-turns. The compensating winding is placed in series in the circuit producing CONDUCTIVE COMPENSATION; or is short circuited on itself, producing INDUCTIVE COMPENSATION by current induced in it by the alternating armature flux.

(c) Currents, induced by transformer action of the alternating flux in the armature coils, short circuited by the brushes, cause sparking. This is reduced by winding the armature with coils of a single turn. So the number of commutator segments in the motor is unusually large.

(d) To further reduce this short-circuit current, resistance leads are inserted between the coils and commutator segments.

(e) To reduce eddy current losses and heating, the entire field structure, as well as the armature core, are laminated.

Since reactance varies with frequency, large motors for hoists and traction are designed for frequencies not above 25 cycles. This motor has operating characteristics similar to the d-c series motor, and when conductively compensated performs best on direct current.

Series motors of fractional horsepower rating are designed for service on both 60-cycle and direct-current circuits. They are known as UNIVERSAL MOTORS and, if compensated, conductive compensation must be used.

THE REPULSION MOTOR has a rotor similar to a d-c armature and, in its simplest form, has single stator winding on a laminated core with uniform air gap, similar to the single-phase induction motor. The brushes bearing on the commutator are short circuited, and currents are induced in the armature by transformer action, which sets up poles in the rotor core. When the brushes are shifted slightly out of line (about 30 electrical degrees) with the stator poles, repulsion between the two sets of poles produces torque and rotation.

Generally the motor has two stator windings connected in series. The short-circuited brushes are placed in line with the field of one winding, called the transformer field, which sets up currents in the armature by transformer action. The other winding, at 90 electrical degrees, is called the main or torque field and furnishes the flux which produces torque. Armature current varies with, and is proportional to, the flux of the stator windings. This produces series motor characteristics with correspondingly high starting torque.

THE REPULSION-START MOTOR is a simple repulsion motor equipped with a mechanical device, operated by centrifugal force.

an alternator, therefore, will operate as a synchronous motor, just as a d-c generator will operate equally well as a motor.

The design of the synchronous motor, however, differs in some respects from that of the alternator. When used as a motor it usually has a squirrel-cage winding placed in the faces of the poles. It also has better operating characteristics when it has a higher percentage of armature reactance than that of the alternator of the

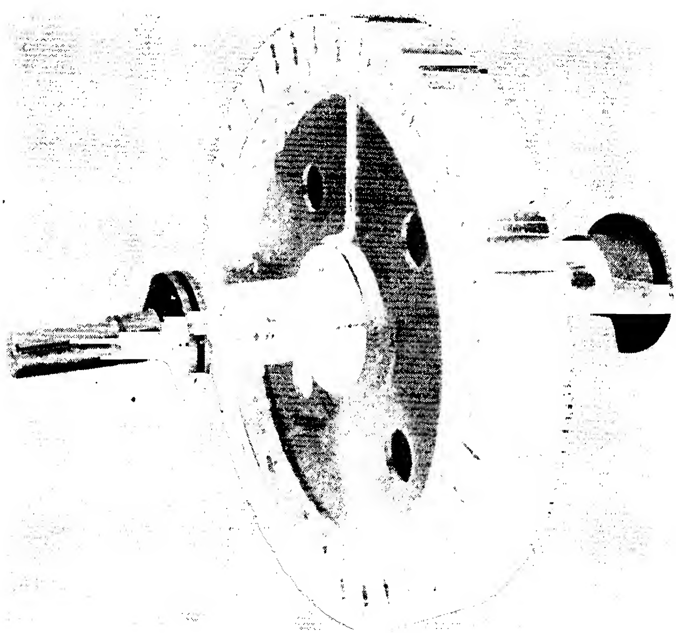


FIG. 1–11(*b*). The rotor field of the motor of Fig. 1–11(*a*).
(*Allis-Chalmers Manufacturing Co.*)

same rating. Small motors may be constructed with a rotating armature but larger sizes almost always have a rotating field structure with salient poles, as indicated in Fig. 1–11(*b*). The field winding is excited by direct current from a separate source, as an exciter, through collector rings on the shaft. A non-excited type of synchronous motor is also built in fractional-horsepower sizes (Art. 19–11).

Synchronous motors are generally rated either at 80 or 100 per cent power factor, which means they are designed to deliver rated power output at their respective power factors.

1–11. Synchronous Motor Action. The principle upon which the synchronous motor operates is indicated in Fig. 2–11, which represents a motor with a rotating armature. Only one armature coil is shown and the position and polarity of the field poles are fixed. In Fig. 2–11(*a*) the current in the left-hand coil-side is *out* and that in the right-hand side is *in*. According to Fleming's left-hand rule for motors, the force action or torque on both conductors is in a clockwise direction as shown by the arrows. If alternating current flows in the coil, the current in both conductors will be reversed during the next half cycle, as in Fig. 2–11(*b*), and the force action is reversed, producing torque in the opposite, or counter-clockwise direction, as shown by the arrows.

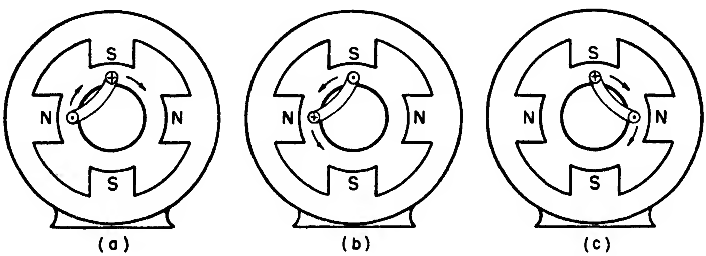


FIG. 2–11. (*a*) A single armature coil carrying a current as marked will tend to rotate clockwise. (*b*) A coil in the same position as in (*a*), but carrying a current in the reverse direction will tend to rotate in the reverse direction. (*c*) When the current is caused to reverse as it reaches a pole of opposite polarity, the coil tends to continue to rotate in the same direction as in (*a*).

So the resulting torque over a cycle is zero. The synchronous motor of itself, therefore, produces no starting torque. However, if the armature and coil can by some means be rotated or advanced clockwise one pole pitch during the time the current in the coil is reversed, as in Fig. 2–11(*c*) the conductors are brought under poles of opposite polarity and the torque will still be in a clockwise direction. This will produce rotation.

Consider Fig. 3–11 also in which the motor has a stationary armature and a rotating-field structure. During one-half cycle, if the current in the armature conductors is in the direction shown in Fig. 3–11(*a*), the force acting on both conductors, according to the left-hand rule, is to the left on in a counter-clockwise direction. But the conductors are embedded in a stationary frame and cannot move. Therefore the force action on the poles is in a clockwise direction, as shown by the arrows. During the next half cycle, the

current in the armature coils is reversed, as in Fig. 3–11 (*b*) and the force acting on the poles is also reversed. So the torque over the cycle again is shown to be zero. But if the poles can be advanced clockwise one pole pitch during the time the current is reversed, as in Fig. 3–11 (*c*) the force action on poles of opposite polarity still produces torque in clockwise direction.

In order that a synchronous motor may operate and produce torque, the rotating member, either armature or field structure, must therefore rotate or advance a distance equal to one pole pitch each half cycle. It will be noted in the four-pole motors in the figures above the rotating structure must advance one quarter revolution in a half cycle, or one revolution every two cycles. In a

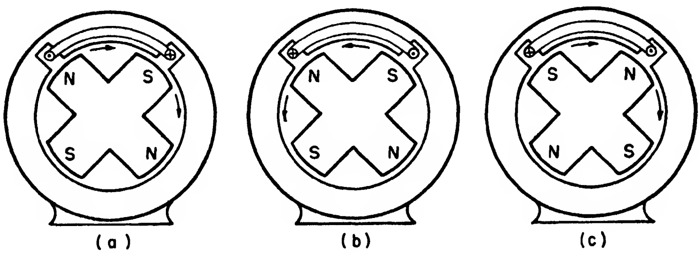


FIG. 3–11. (*a*) The poles tend to rotate as in Figs 2–11 (*a*) because the relation between the current and the magnetic field is the same as in Fig. 2–11 (*a*). (*b*) When the relation between the current direction and the polarity of the poles is reversed from that of (*a*) the poles tend to rotate in the opposite direction (*c*) The relation between the pole field and the coil current is the same as that in (*a*); therefore the pole tends to rotate in the same direction as in (*a*).

60-cycle motor this is 30 revolutions per second or 1800 rpm, which is exactly the synchronous speed of a four-pole 60-cycle alternator. At the same circuit frequency a synchronous motor must therefore operate at the same speed as an alternator with the same number of poles, or at synchronous speed. This gives the motor its value.

The speed of any synchronous motor can be computed by the same equation as for the synchronous alternator, as follows:

$$\text{Rpm} = \frac{2 \times 60 \times f}{P} = \frac{120f}{P} \qquad (1\text{–}11)$$

where f = frequency in cycles per second
P = number of poles.

The synchronous motor on a constant frequency circuit thus

operates exactly at a constant speed. Its speed may momentarily fluctuate slightly on change of load but it quickly settles down to a speed which is always exactly equal to the synchronous speed.

The polyphase synchronous motor can also be compared to the polyphase induction motor. Since the armature winding of the one is exactly similar to the stator winding of the other, both have a rotating magnetic field setting up polyphase currents in a stationary distributed winding. The two motors differ in the following respects. In the induction motor, poles of flux are set up in the rotor, which react upon the rotating fluxes of the stator to produce

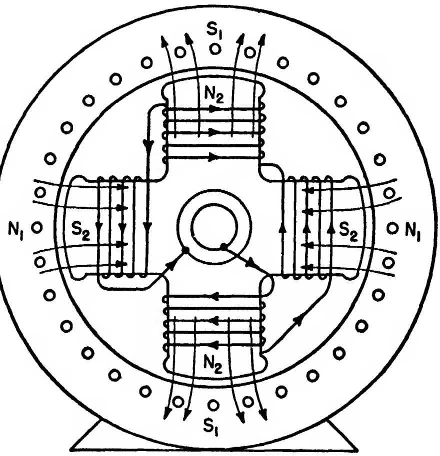


FIG. 4–11. Diagram of a synchronous motor. The field structure rotates in synchronism with the rotating flux of the stator poles, N and S.

torque. But these rotor poles themselves depend upon the **slip** of the rotor for their existence; while in the synchronous motor the flux set up in the poles of the rotating field structure is supplied by direct current in windings on these poles. The rotating structure itself thus locks in step and remains in synchronism with the rotating field of the stator poles. This is indicated in Fig. 4–11 in which N_1, S_1, etc., are the rotating stator poles set up by the alternating currents in the stator, and N_2, S_2 are the poles set up by direct current in the windings on the rotating structure. In fact, if the load on the motor is sufficiently reduced, this interlocking action continues even when the direct-current field circuit is opened. The poles in the field structure are magnetized by the action of the alternating currents in the armature. Under this condition the machine is said to operate as an **hysteresis** motor.

Prob. 1–11. At what speed will a 12-pole, 3-phase, 25-cycle synchronous motor operate?

Prob. 2–11. How many poles must a synchronous motor have in order to operate at 514 rpm on a 60-cycle line?

Prob. 3–11. At what speed will the motor of Prob. 2 operate on a 25-cycle line?

2–11. Effect of Load: Counter EMF, Armature Current and Synchronous Position. When any motor is in motion, a counter emf is set up in the armature of any motor. Also, when a load is applied to the shaft of any motor operating on a constant potential line, increased current must be drawn from the line to supply the additional torque and power to carry the increased load.

When a load is applied to a d-c shunt motor, the speed decreases slightly, thereby decreasing the back or counter emf, E_c, which is directly opposed to that of the line. The line voltage E_L, must, therefore, be sufficient to overcome the counter emf and also supply a voltage necessary to overcome the I_aR_a drop in the armature, that is,

$$E_L = E_c + I_aR_a$$

or

$$I_a = \frac{E_L - E_c}{R_a}$$

where E_L is the line voltage, E_c is the counter emf, I_a is the armature current, and R_a is the resistance of the armature. Thus, as additional load is applied, the speed and counter emf both decrease and the motor armature draws more current from the line.

The a-c voltage impressed on the armature of the synchronous motor is approximately of sine wave form and the induced counter emf has approximately the same form. The pole pieces are so shaped and the armature conductors are so distributed that this form of wave is produced. When a load is applied to the motor its counter emf cannot decrease by change in speed, for the synchronous motor must always operate at a **constant speed equal to the synchronous speed.** It cannot, therefore, draw additional current from the line by a decrease in counter emf. Also, the motor is separately excited and its counter emf is controlled solely by the value of the direct current in its field windings. Under normal conditions the field current is usually so adjusted that the counter emf is practically equal to the impressed or line voltage. This may be called the **normal excitation** of the machine.

Even under no-load conditions this counter emf cannot reach its maximum value at exactly the same instant as the line voltage, for it would be directly opposed to the latter voltage so no current would be drawn from the line and the motor would stop. Therefore, as load is applied to the synchronous motor, its rotor momentarily slows down due to the resisting torque of the load and drops back in phase position. It may oscillate a little here, but it still rotates at synchronous speed because it must maintain synchronous speed in order to rotate at all. This action is indicated in Fig. 5–11 for a motor with a rotating field structure. In Fig. 5–11 (*a*) when little torque is required to drive the motor, the flux

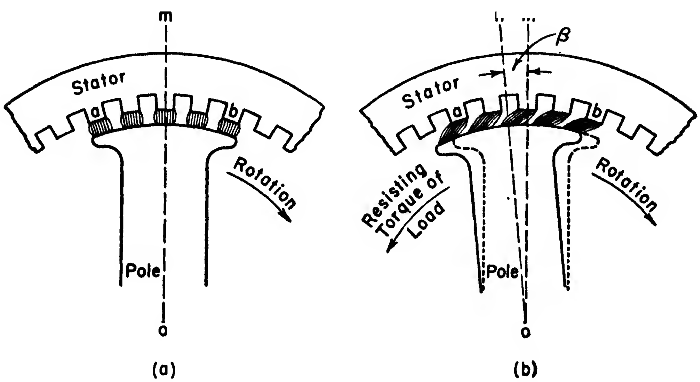


FIG. 5–11. A synchronous motor under load. The rotating field pole shown in (*b*) has dropped back in position by an angle β with respect to the rotating stator pole, but is still rotating at synchronous speed.

lines can be considered as groups of rubber bands stretched radially across the air gap, from fixed positions on the pole structure to succeeding positions on the stator, as the armature flux moves around the stator core. The center of the pole flux, the armature mmf and the maximum point of the induced or counter emf wave are in the position indicated by the line *mo*. The counter emf under this condition is practically 180° from the impressed voltage. When load is applied, the resisting torque may be considered as stretching these rubber bands, as indicated in Fig. 5–11 (*b*), and the poles and center line of the pole flux shift backward to the position shown as *no*. The armature mmf and counter emf are now displaced and the latter lags the line voltage by the angle β. But the rotor still maintains synchronous speed.

This action can also be represented by the use of vectors, as in Fig. 6–11. Assume that little or no current is required to drive the motor at no load and that the counter emf, E_c, is practically equal to the line voltage, E_L, which is constant. The induced or counter emf reaches its maximum at the same instant as the line voltage. The vector E_c can, therefore, be drawn 180° out of phase with E_L and equal to it. Thus, no vector difference exists. When load is applied to the motor, more current must be drawn from the line and the resisting torque of the load causes the vector E_c to shift backward in phase by the small angle β to the position E_c'. A vector difference between E_c' and E_L now exists which produces the resultant voltage E_R necessary to overcome the synchronous I_aZ_a drop in the armature and draw the required current from the line. The reactance of a synchronous motor is generally very high with respect to its resistance and the armature current

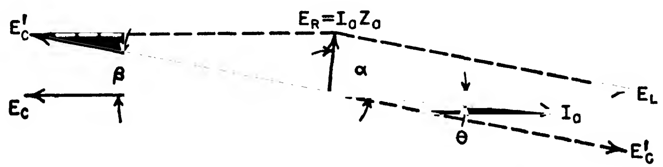


FIG. 6–11. Vector diagram of the current and voltage relations in a synchronous motor under load conditions.

lags the resultant voltage, E_R, nearly 90°, shown as angle α in the figure. This current, I_a, is nearly in phase, therefore, with the line voltage E_L and each phase of the motor draws a power component of current $I_a \cos \theta$ and thus is taking increased power from the line. Thus, **as load is applied, the rotor of the synchronous motor, by shifting backward in phase position, draws a power component of current from the line, which supplies the additional power necessary to carry the increase in load.**

The angle by which the rotor shifts backward at any load is called its **synchronous position,** and the angle by which it shifts backward at full load is called the **torque angle.** A motor which has a small torque angle is said to be "hard coupled," while one which has a large torque angle is "soft coupled."

If the vector E_c' in Fig. 6–11 be reversed and drawn as $-E_c'$ in Fig. 7–11, it represents the component of the line voltage necessary to overcome the counter emf. The vector sum of this voltage and the armature I_aZ_a drop is equal to the impressed or line

voltage. That is,

$$E_L = E_c' \oplus I_aZ_a, \text{ or } I_a = \frac{E_L \ominus E_c'}{Z_a} \text{ and } E_c' = E_L \ominus I_aZ_a \qquad (2\text{–}11)$$

This is similar to the relations which exist in the d-c shunt motor, except that these voltages are vector quantities. It should be noted that the resultant voltage, E_R, or the armature I_aZ_a drop, always takes such a position with respect to the counter emf that the vector sum of the two is equal to the line voltage E_L.

Example 1. A 1500-hp, 6600-volt, Y-connected, 3-phase, 60-cycle, 12-pole, 80 per cent power factor synchronous motor has an efficiency of 93.5 per cent, exclusive of field loss. It is operating with its counter emf practically equal to the line voltage and with a load such that the

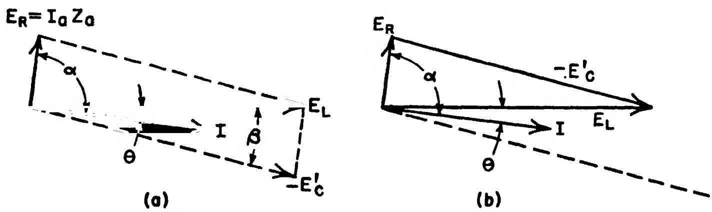


FIG. 7–11. The vector $-E_c'$ is the negative of the counter emf E_c' of Fig. 6–11. Note the line voltage E_L must equal the vector sum of the armature impedance drop (I_aZ_a) plus the counter emf ($-E_c'$).

rotor is shifted backward 5 electrical degrees in synchronous or phase position.

(*a*) What resultant emf, E_R, is produced in each phase of the armature?

(*b*) If the synchronous reactance per phase is 12 ohms and the effective resistance per phase is 0.4 ohm, what current will be drawn from the line?

(*c*) What will be the phase position of this current with respect to the resultant emf, E_R? With respect to line voltage?

(*d*) What total power does the motor draw from the line?

Solution:

$$E_L \text{ and } E_c' \text{ per phase} = \frac{6600}{\sqrt{3}} = 3810 \text{ volts}$$

From Fig. 8–11:

(*a*) $E_R = \sqrt{(3810 - 3810 \cos 5°)^2 + (3810 \sin 5°)^2}$

$= \sqrt{15^2 + 331^2} = \mathbf{332\ volts}$

(*b*) Synchronous Impedance $= \sqrt{0.4^2 = 12^2} = 12$ ohms (about).

$$I_a = \frac{332}{12} = \mathbf{27.6\ amperes}$$

(c) $\tan \alpha = \frac{x}{R} = \frac{12}{0.4} = 30.$ Angle $\alpha = 88.08°$.

$$\text{Angle } \theta = 88.08° - \frac{175°}{2} = 0.58°$$

(d) P per phase $= 3810 \times 27.6 \cos 0.58° = 3810 \times 27.6 \times 0.999$

$= 105{,}000$ watts or 105 kw.

Total power taken by the motor $= 3 \times 105 =$ **315 kw.**

Prob. 4–11. How many mechanical degrees in space is the rotor of the motor in Example 1 shifted backward or retarded?

Prob. 5–11. (a) What must be the phase or synchronous position of he rotor of the motor in Example 1, if it draws 80 amperes from the line? (b) Compute the power factor. (c) What power will the motor take from the line?

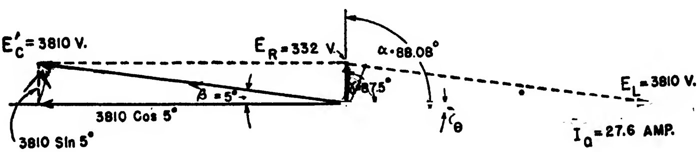


FIG. 8–11. Vector diagram of a synchronous motor operating with the rotor shifted 5 electrical degrees behind its synchronous position.

Prob. 6–11. For the motor in Example 1, assuming the counter emf at no load to be adjusted equal to the line voltage, compute: (a) The full-load armature current per terminal. (b) The resultant emf, E_R, produced per phase causing the motor to draw the full-load current in (a). (c) The synchronous position or backward shift of the counter emf in electrical degrees. (d) The angle in (c) in mechanical space degrees. (e) The power factor of the motor under this condition. (f) The electrical power input exclusive of the field loss.

Prob. 7–11. Compute the speed of the motor in Prob. 6–11.

3–11. Maximum Load for a Synchronous Motor. Pull-out Torque (Constant Fixed Excitation). It has been shown in the preceding article that, in order to carry an increase in load and draw additional current and power from the line, the rotor of the synchronous motor shifts backward in phase position with respect to the stator. As more and more load is applied, the rotor continues to shift backward by an increasing angle. But it must not be assumed that this backward shift can proceed indefinitely and the motor carry an indefinitely large load. For every synchronous motor reaches a load limit above which an increase in

backward shift of the rotor actually **reduces** the power input and the motor pulls out of synchronism and stops, although the current input increases. This can be visualized by considering Fig. 5–11(*b*). If the motor is loaded beyond a certain point, the flux lines (likened to stretched rubber bands) reach their elastic limit and break. The interlocking action between stator and rotor ceases and the motor pulls out of step and stops.

To determine the load limit of the 1500-hp, 3-phase, 6600-volt motor of Example 1, assume the field strength is such that the counter emf per phase is equal to the line voltage per phase, as in that example. Construct diagrams similar to Figs. 6–11 and 8–11. The armature current and power input per phase can be computed for successive positions of angle β as the rotor drops further backward. The mechanical power developed can then be found by subtracting the armature I^2R loss. The results for several values of angle β are given in Table A. Note that the power developed by the motor continues to increase as the rotor shifts backward until there is a phase difference of 88.058° between the induced and counter emfs. At this position (and load) the armature current is 442 amperes and the power input 1249 kw per phase. The armature I^2R loss is 78 kw and the mechanical power developed per phase equals 1249 − 78 or 1171 kw.

Any further increase in load, with an accompanying increase in the angle β, causes the armature current to increase, but the increase in armature I^2R loss and the decrease in power factor causes the mechanical power developed to decrease. Thus, if a load is applied requiring a power input greater than 1249 kw per phase, the rotor would shift backward more than 88.058° in an attempt to carry it. This reduces the power intake and causes the motor to drop out of synchronism. Furthermore, an armature current of 442 amperes per phase is over three times the full load current of 131 amperes (computed in Prob. 6–11). The motor would be unable to carry this load for more than a very few minutes without burning out.

It can be shown and is clearly indicated in Table A that the maximum or pull-out load for any synchronous motor occurs when the rotor shifts backward in phase position by an angle the tangent of which is equal to $\frac{X}{R}\left(\tan\beta = \frac{X}{R}\right)$, where x is the synchronous reactance per phase and R the effective reactance per phase of the stator. Thus in the 1500-hp motor above, the

synchronous reactance is 12 ohms and the effective resistance 0.4 ohm. Tangent $\beta = \dfrac{12}{0.4} = \tan^{-1} 30 = 88.058°$, approximately.

The maximum load above which the synchronous motor pulls out of synchronism is generally given in terms of the torque in pound-feet developed at this load and is called the "pull-out" torque. This is usually expressed in percentage of full-load torque.

TABLE A

Synchronous position of rotor angle β	Resultant voltage developed E_R	Current per phase I	Angle between impressed voltage and current θ	Power factor $\cos\theta$	Power delivered to motor per phase kw	Armature I^2R loss per phase kw	Total mechanical power developed per phase kw
0°	0	0	0°				
*5°	332	27.6	0.58°	0.999	105	.76	104.24
10°	683	56.8	3.058°	0.998	216	1.29	214.7
14.33°	960	80	5 25°	0.9957	303.4	2.56	300.84
20°	1377	114.8	8.058°	0 990	432.5	5.3	427.2
30°	1970	164.	13 058°	0.974	617	10.75	606.25
60°	3810	317.5	28.058°	0.882	1067	40.3	1026.7
80°	4890	407.5	38.058°	0.787	1222	66.4	1155.6
88.058°	5300	442.	42.058°	0.742	1249	78.	1171.
90°	5385	448	43.058°	0.731	1240	80.	1166
100°	5833	486	48.058°	0.668	1235	94.	1141
120°	6600	550	58.058°	0.528	1106	121	985
140°	7160	597	68.058°	0.373	848	143	705

* From Example 1.

The pull-out torque of the synchronous motor may vary from about 150 to 300 per cent of the normal full-load value. It can also be shown that the pull-out torque varies directly with the impressed voltage and the value of the field excitation, or counter emf. In general, a strong field increases the pull-out torque while a weak field may greatly reduce it, because of unstable conditions which arise. The effect of changing the field strength, and thereby changing the induced or counter emf, is explained in the following article.

Example 2. (*a*) What is the full-load torque in pound-feet of the 1500-hp, 60-cycle, 12-pole motor of Example 1? (*b*) Assuming an efficiency of 92 per cent for the maximum possible load on the motor, compute the pull-out torque in pound-feet. (*c*) The pull-out torque is what per cent of the torque at full load?

Solution:

$$(a)\ \text{Speed} = \frac{120f}{P} = \frac{120 \times 60}{12} = \mathbf{600\ rpm.}$$

$$\text{Hp} = \frac{2\pi T \times \text{Rpm}}{33{,}000}, \quad \text{or} \quad 1500 = \frac{2\pi T \times 600}{33{,}000}$$

$$T = \frac{1500 \times 33{,}000}{6.28 \times 600} = 13{,}137 \text{ lb-ft.}$$

where f = frequency, P, the number of poles, and T, the torque in lb-ft.

(*b*) From Table A, maximum power delivered to the motor per phase is 1249 kw per phase.

Horsepower output at this load = 3 × 1249 × 0.746 × 0.92 = 2572

$$2572 = \frac{2\pi T 600}{33{,}000} \quad T = \frac{2572 \times 33{,}000}{6.28 \times 600} = \mathbf{22{,}525\ lb\text{-}ft.}$$

$$(c)\ \frac{22{,}525}{13{,}137} \times 100 = \mathbf{171\ per\ cent.}$$

Prob. 8–11. (*a*) What is the full-load current and power in kilowatts taken from the line by a 330-hp, 2300-volt, 3-phase, Y-connected, 60-cycle, 10-pole, 100 per cent power factor synchronous motor, if the efficiency is 90 per cent exclusive of field loss? (*b*) What is the full-load torque in lb-ft.?

Prob. 9–11. (*a*) What would be the synchronous position of the rotor of the motor in Prob. 8–11 at its maximum or pull-out load if its synchronous reactance is 10 ohms per phase and effective armature resistance is 0.43 ohm? Armature counter emf is equal to the line voltage per phase. (*b*) What is the resultant voltage E_R, or IZ drop in the armature at this load? (*c*) Compute the line current, power factor, and total power in kilowatts drawn from the line. Construct the vector diagram.

Prob. 10–11. Compute the pull-out torque in lb-ft. for the motor in Prob. 9–11, assuming an efficiency at this load of 87 per cent exclusive of field loss.

Prob. 11–11. What would be the maximum or pull-out load (a-c input) for the motor of Probs. 8–11 and 9–11, if the field strength were of such value that the counter emf per phase is reduced to 1000 volts per phase (the motor is operated from a circuit of rated voltage)? (*b*) If the efficiency at this load is 85 per cent exclusive of field loss, what is the pull-out torque in pound-feet? Compare this value with that found in Prob. 10–11.

4–11. Effect of Varying the Field Current. Power Factor. It has been shown that the rotor of a synchronous motor shifts backward in phase position with respect to its stator as load is applied, but continues to run at a constant speed in synchronism with the

line voltage. Another characteristic of this motor is its ability to operate at **unity power-factor** or at either a **leading** or a **lagging** power factor. The power factor can be controlled by increasing or decreasing the d-c field current. This action in the synchronous motor is very different from that in the d-c shunt motor in which the **speed** is changed by varying the field current. (1) When the field current in the synchronous motor is such that its power factor is close to unity it can be said to be **normally excited.** (2) **When the field current in the motor is increased,** it cannot slow down, as does the d-c shunt motor, but must run at a constant speed. Since the speed is constant, the counter emf is controlled solely by the field excitation, and is increased when the field current is increased. The counter emf is independent of the load, therefore, and the motor will operate even when its counter emf is greater than the line voltage. In the direct-current motor an induced or counter emf greater than the line voltage would produce generator action and the machine would cease to operate as a motor. The synchronous motor continues to operate, however, but takes a **leading** current from the line. Under this condition the motor is said to be **overexcited.** The reason for this may be explained as follows.

When an alternator operates as a synchronous motor, the armature current, in relation to the induced or counter emf, must be considered as reversed, or 180° from its former relation under generator action. That is, as an alternator, the armature is supplying current and power to the line; while as a motor, it is receiving current and power from the line. Thus a leading current in a synchronous motor has the same effect on the air gap flux as a lagging current in the same machine used as an alternator. It has been shown in Ch. VII, Fig. 12–7, that a **lagging** current in an **alternator** demagnetizes or weakens the field. **Thus a leading current in a synchronous motor produces a demagnetizing action which reduces the air gap flux to a value just sufficient to set up the necessary induced or counter emf in the armature.**

The effect of increasing the field strength on the power factor can be shown by a vector diagram as in the example below.

Example 3. At rated impressed voltage, assume the field current in the Y-connected motor of Example 1 is increased sufficiently to raise the no-load counter emf to 4300 volts per phase. If the necessary load is now applied to the motor under this condition to shift the synchronous position of the rotor backward 5° (Angle $\beta = 5°$), as in that example, compute for each phase: (*a*) The resultant voltage, E_R, developed in

the armature. (*b*) The armature current. (*c*) The angle θ and power factor. (*d*) The power drawn from the line.

Solution: From Fig. 9-11:

$$(a)\ E_R = \sqrt{(4300 \cos 5^\circ - 3810)^2 + (4300 \sin 5^\circ)^2}$$

$$= \sqrt{(4280 - 3810)^2 + (374)^2} = \sqrt{470^2 + 374^2} = 600 \text{ volts}$$

(*b*) Synchronous impedance, $z = 12$ ohms. $I = \frac{600}{12} = \mathbf{50}$ **amperes**

(*c*) cos angle $\phi = \frac{470}{600} = 0.783$. Angle $\phi = 38.5^\circ$. Angle $\alpha = 180 - 38.5^\circ = 141.5^\circ$; angle β $\left(\tan^{-1} \frac{x}{R}\right) = 88.085^\circ$, $\theta = 141.5^\circ - 88.085^\circ =$ **53.42° leading.** P.F. $(\cos \theta) = 0.596$

(*d*) Power input per phase $= 3810 \times 50 \times 0.596 =$ **113.5 kw.**
Total power input $= 3 \times 113.5 =$ **340.5 kw.**

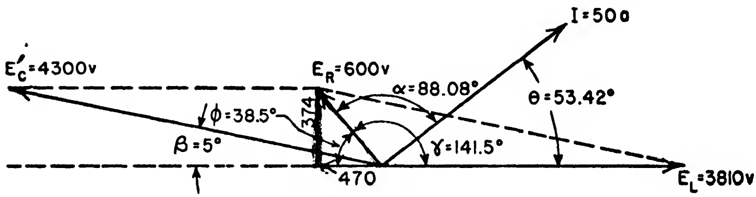


FIG. 9-11. Vector diagram of the motor of Fig. 8-11 when the field current has increased sufficiently to raise the counter emf E_c' to 4300 volts.

Note, in the example above, that the motor, with increased field and counter emf, now takes a current leading the voltage by 53.42° and this current has risen from 27.6 amperes in Example 1 to 50 amperes. Also the power input for practically the same load has increased from 105 kw to 113.5 kw per phase. This may be accounted for by increased iron and armature I^2R losses. The efficiency of course has also been reduced.

5-11. Effect of Decreasing the Field Current. When the field current in the synchronous motor is decreased, the counter emf is reduced, but again the motor cannot speed up as does the d-c shunt motor, for it must run at a constant speed. The armature current is increased, but the motor now takes a **lagging** current from the line and is said to be **underexcited.**

In accord with the reasoning above, armature reaction, produced by a lagging current in a synchronous motor, acts in the same manner as a leading current when the machine is used as an alternator. It has been shown in Ch. VII, Fig. 13-7, that a **leading current** in an **alternator** produces a magnetizing action which

increases the air-gap flux and the induced voltage. **Thus a lagging current in the synchronous motor produces a magnetizing action which increases the air gap flux and raises the counter emf to the necessary value.**

The effect of decreasing the field current on the power factor of the synchronous motor is shown in the example below.

Example 4. At rated impressed voltage, assume the field of the Y-connected motor of Examples 1 and 3 is weakened sufficiently to reduce the no-load counter emf to 3300 volts per phase. If the necessary load is now applied to the motor to shift the synchronous position of its rotor

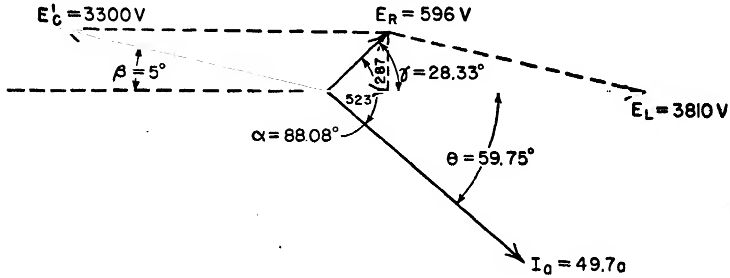


FIG. 10–11. Vector diagram showing the effect of reducing the field current below its normal value.

backward 5° (β = 5°), as in those examples, compute for each phase: (*a*) The resulting voltage, E_R, produced in the armature. (*b*) The armature current. (*c*) The angle θ and power factor. (*d*) The power drawn from the line.

Solution. From Fig. 10–11:

(*a*) $E_R = \sqrt{(3810 - 3300 \cos 5°)^2 + (3300 \sin 5°)^2}$

$= \sqrt{(3810 - 3287)^2 + (287)^2} = \sqrt{523^2 + 287^2} = 596$ volts.

(*b*) Synchronous impedance = 12 ohm. $I = \frac{596}{12} = 49.7$ amperes.

(*c*) $\tan \gamma = \frac{287}{523} = 0.5394. \quad \gamma = 28.33°$

$\theta = 88.08° - 28.33° = 59.75°$ Power factor, $\cos \theta = 0.504$

(*d*) Power input per phase = $3810 \times 49.7 \times 0.504 = 95.4$ kw.

Note, from the example above, that the current has increased from 27.6 amperes, as in Example 1, to 49.7 amperes and now lags the phase voltage by 59.75°. The power input per phase has decreased from 105 to 95.4 kw per phase, partially due to decreased iron losses.

The examples above show that a weakened field causes the

synchronous motor to take a lagging current from the line, while a strong field causes it to take a leading current. This fact makes it possible to use the synchronous motor to correct the power factor of a system.

6–11. Excitation in a Synchronous Motor on a Constant-Voltage Circuit. In Chs. VIII and IX, it has been shown that under normal load conditions the flux in both the transformer and the induction motor is practically constant when operated from a constant potential circuit. These machines obtain all their excitation from the a-c line.

The synchronous motor, however, may obtain its excitation partly from its d-c field and partly from the a-c line, or entirely from either source. When this motor is underexcited, a deficiency in air-gap flux exists and it draws a lagging component of current from the line. This produces a magnetizing action, as shown in the previous article, which increases the excitation and brings the flux up to normal. Incidentally, this lagging line current tends to reduce the excitation and induced emf in the driving alternator, so that additional current and flux must be supplied from its d-c field.

Also, as pointed out in Art. 1–11, if the d-c field circuit is opened and the load is sufficiently light, the synchronous motor will continue to operate. It now receives all its excitation from the a-c line by means of a lagging current at a very low power factor. But if the d-c field is of such value as to supply normal excitation and flux, the motor draws a unity-power-factor current and receives no excitation from the a-c line.

In an over-excited synchronous motor, the d-c field supplies an excess of normal flux requirements. The motor draws a leading current from the line which produces a demagnetizing action that reduces the air-gap flux to normal. This leading current reacts upon the driving alternator tending to increase its excitation and less has to be supplied by its d-c field. Or if the over-excited motor operates in parallel with induction motors, its leading component of current supplies excitation to these machines. Thus the over-excited synchronous motor, by means of its leading current, may supply excitation to other apparatus connected to the same line.

On a line of constant voltage, therefore, the synchronous motor, like the transformer and induction motor, operates at practically a constant excitation regardless of its power factor.

7–11. Calculation of Induced or Counter E.M.F. in the Synchronous Motor. In Art. 4–11, it was shown that an increase or decrease in d-c field current changes the power factor of the synchronous motor. Since the induced emf is directly proportional to the field excitation, it is desirable to determine the induced emf required for any specified load and power factor. It has also been shown that the resultant voltage, E_R, produced in the armature is equivalent to an IZ drop; and that the induced emf, E_c, per phase is equal to the vector difference of the line voltage, E_L, per phase, and the IZ drop per phase (Eq. 2–11).

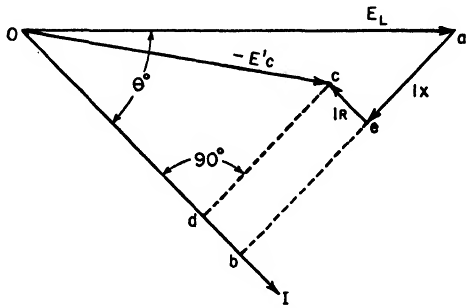


FIG. 11–11. Vector diagram of the voltage and current relations in a synchronous motor with lagging armature current.

The induced emf due to field excitation can be determined by means of the clock diagrams shown in previous figures, but is more readily computed in a manner similar to that for the alternator (Figs. 5–7 and 6–7). Thus in Fig. 11–11, **for a lagging current OI and power factor, cos θ,** the synchronous reactance drop, ae and effective resistance drop, ec, per phase are laid off from E_L, the vector ae lagging OI 90° and the vector ec at 180° to the current OI. The construction lines ab and cd are drawn perpendicular to OI. The line voltage, IR and IX drops are thus projected on the current vector and Odc is a right triangle of which the induced emf, E_c', is the hypotenuse, and $-E_c' = \sqrt{(Od)^2 + (dc)^2}$. But $Od = Ob - bd = E_L \cos\theta - IR$; and $dc = ba - ae = E_L \sin\theta - IX$.

Therefore $-E_c' = \sqrt{(E_L \cos\theta - IR)^2 + (E_L \sin\theta - IX)^2}$ (3–11)

For **unity power factor** the equation for counter emf becomes

$$-E_c' = \sqrt{(E_L - IR)^2 + (IX)^2} \qquad (4\text{–}11)$$

For a leading current, OI, and power factor cos θ (Fig. 12–11), the IR and IX drops are drawn from E_L as before and projected on the current vector. Odc again is a right triangle of which $-E_c$ is the hypotenuse and $-E_c = \sqrt{(Od)^2 + (dc)^2}$

$$Od = Ob - bd = E_L \cos\theta - IR; \; dc = be + ea = E_L \sin\theta + IX$$

Thus $$-E_c = \sqrt{(E_L \cos\theta - IR)^2 + (E_L \sin\theta + IX)^2} \quad (5\text{–}11)$$

Example 5. A 100-hp, 600-volt, Y-connected, 0.8 power factor synchronous motor has an effective resistance of 0.08 ohm and a synchronous reactance of 0.9 ohm per phase. Assuming an efficiency, exclusive of field losses, of 88 per cent at rated load and 0.8 lagging

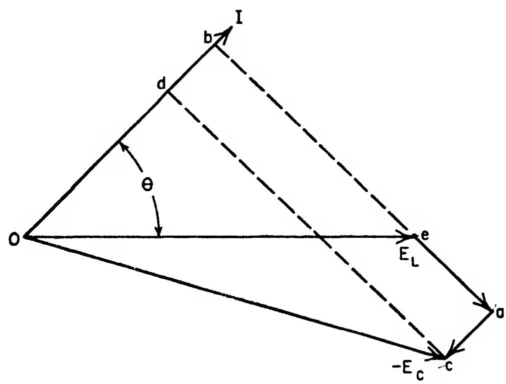


FIG. 12–11. Vector diagram when the armature current is leading.

power factor, compute: (a) the full load current per phase; (b) the induced or counter emf per phase (produced by field excitation) necessary to carry full load at the above power factor.

Solution:

$$(a) \text{ Power Input} = \frac{100 \times 746}{0.88} = 84{,}770 \text{ watts}$$

$$\text{Full load } I = \frac{84{,}770}{\sqrt{3} \times 600 \times 0.8} = 102 \text{ amperes}$$

$$(b) \; E_{\text{phase}} = \frac{600}{\sqrt{3}} = 346 \text{ volts.} \quad \theta = \cos^{-1} 0.8 = 36.83°$$

From Eq. 3–11 and Fig. 13–11:

$$E_c = \sqrt{(346 \times 0.8 - 102 \times 0.08)^2 + (346 \times 0.6 - 102 \times 0.9)^2} = 292.5 \text{ volts}$$

Example 6. Assuming an efficiency of 90 per cent for the motor in Example 5–11 at rated load and **unity** power factor, compute the full load current and counter emf under these conditions.

Solution:

$$\text{Input} = \frac{100 \times 746}{0.90} = 83{,}000 \text{ watts}$$

$$I_{\text{per phase}} = \frac{83{,}000}{\sqrt{3} \times 600} = 79.8 \text{ amperes}$$

From Eq. 4 and Fig. 14–11:

$$E_c = \sqrt{(346 - 79.8 \times 0.08)^2 + (79.8 \times 0.9)^2} = 347.1 \text{ volts.}$$

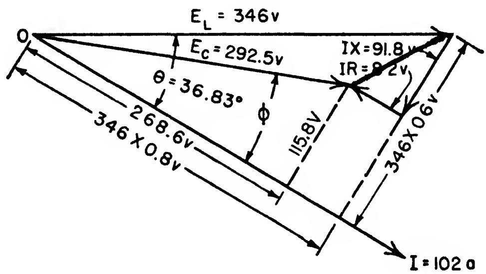


Fig. 13–11. Vector diagram when the armature current is lagging.

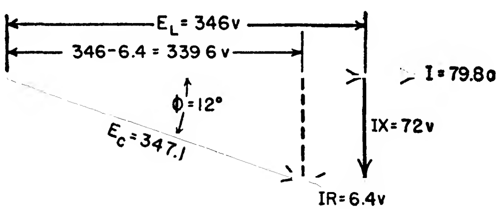


Fig. 14–11. Vector diagram of motor of Fig. 13 when operating at unity power factor.

Example 7. Assuming the same efficiency as in Example 5–11, compute the counter emf in the motor of Examples 5 and 6 at rated load but for an 0.8 **leading** power factor.

Solution:

$$\text{As before, input} = \frac{100 \times 746}{0.88} = 84{,}770 \text{ watts.}$$

$$I = \frac{84{,}770}{\sqrt{3} \times 600 \times 0.8} = 102 \text{ amperes}$$

From Eq. 5 and Fig. 15–11:

$$E_c = \sqrt{(346 \times 0.8 - 102 \times 0.08)^2 + (346 \times 0.6 + 102 \times 0.9)^2} =$$
402 volts.

Thus, for a given load on the motor, the examples above show that the value of the counter emf with a leading power-factor is much greater than with a lagging power factor.

The mechanical power, P, developed in the motor per phase, also, can be computed from the diagrams above, if the angle, ϕ, between the counter emf, E_c, and the armature current, I, is determined, that is,

$$P = E_c I \cos \phi.$$

Example 8. Compute the mechanical power per phase developed in the motor, (*a*) in Example 5; (*b*) in Example 6; (*c*) in Example 7.

Solution: (*a*) From Fig. 13–11, $\tan \phi = \dfrac{115.8}{268.6} = 0.430. \quad \phi = 23.33°$

$$P = 292.5 \times 102 \cos 23.33° = 295.5 \times 102 \times 0.918 = 27.4 \text{ kw.}$$

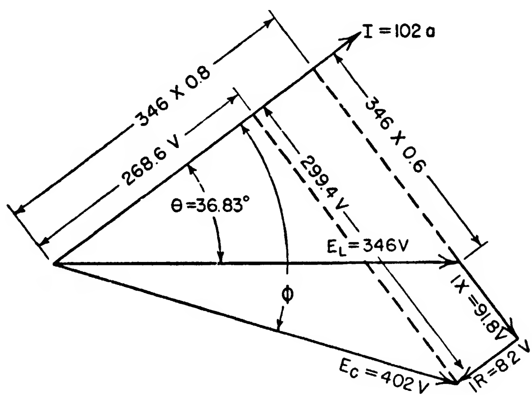


FIG. 15–11. Vector diagram when motor of Fig. 13 is operating with a leading power factor. Note that counter emf E_c under this condition is 402 volts, but in Fig. 13 is only 292.5 volts with a power factor of same value, but lagging.

(*b*) From Fig. 14–11, $\tan \phi = \dfrac{72}{339.6} = 0.212 \quad \phi = 12°$

$$P = 347.1 \times 79.8 \cos 12° = 347.1 \times 79.8 \times 0.978 = 27.1 \text{ kw.}$$

(*c*) From Fig. 15–11, $\tan \phi = \dfrac{299.4}{268.6} = 1.115. \quad \phi = 48.17°$

$$P = 402 \times 102 \cos 48.17° = 402 \times 102 \times 0.667 = 27.35 \text{ kw.}$$

It was indicated in Art. 3 and in Table A that the mechanical power developed also is equal to the power input less the armature I^2R loss. Thus

$$P = 346 \times 102 \times 0.8 - 102^2 \times 0.08 = 27.4 \text{ kw per phase.}$$

This checks with the values computed in Example 7.

Note that the mechanical power determined above includes the rotational or stray power losses, so that the power actually delivered to the motor shaft is somewhat less than the computed values.

Prob. 12–11. Assume the efficiency of the motor in Examples 5 and 6 at rated load and unity power factor is 90 per cent, exclusive of field loss, and compute under these conditions, (*a*) the counter emf per phase; (*b*) the angle ϕ between counter emf and current; (*c*) the mechanical power per phase developed in the motor.

Prob. 13–11. A 200-hp, Y-connected, 2300-volt, 0.8 power factor synchronous motor has an effective resistance of 0.5 ohm and a reactance of 0.4 ohm per phase. At rated load and 0.6 lagging power factor, compute for each phase, (*a*) the current for rated load at this power factor; (*b*) the counter emf; (*c*) the angle ϕ between counter emf and current; (*d*) the mechanical power developed. Assume an efficiency of 90 per cent exclusive of field loss.

Prob. 14–11. Repeat Prob. 13–11 for rated load on the motor at 0.6 leading power factor, assuming the same efficiency.

8–11. Synchronous Motor Characteristics of "V" Curves. If the load on a synchronous motor is held constant the power input must also be practically constant. For a three-phase motor, this is expressed as,

$$P = \sqrt{3}E_L I_L \cos\theta$$

where P is power input, E_L the line voltage, I_L the line current and $\cos\theta$ the power factor. When both P and E_L in this equation are constant, the line current must increase if the power factor is decreased and decrease if the power factor is increased. The **power component** of line current, however, remains the same (except for slight change due to variation in the losses). It has been shown that an underexcited synchronous motor draws a lagging current from the line, while an overexcited motor draws a leading current. Thus in Fig. 16–11, if the field current or counter emf is such as to produce a lagging power factor, $\cos\theta$, the line current is I_1. If the field current and counter emf is increased the power factor rises to $\cos\theta_2$ and the current decreases to I_2. If the field current is sufficiently increased to give normal excitation, the power factor rises to unity and the current decreases to I_3, the minimum value at this load. A further increase in field current and counter emf produces a leading power factor and increases the current to I_4, etc. This action is shown at rated load for the motor in Examples 5, 6, and 7. At 0.8 lagging power factor, the motor draws 102 amperes from the line. At unity

power factor, the current drops to 79.8 amperes. And at 0.8 leading power factor the current rises again to 102 amperes.

Since the power components of the line current, $I_1 \cos \theta_1$, $I_2 \cos \theta_2$, etc., are constant for a given load, the current vectors in Fig. 16–11 all terminate on the line **mn**, drawn perpendicular to the voltage E_L. However, the leading and lagging reactive components of these currents, I_3I_4, I_3I_5, I_3I_2, etc., increase as the power factor decreases. By means of field adjustment, the leading or lagging reactive component of current taken by the syn-

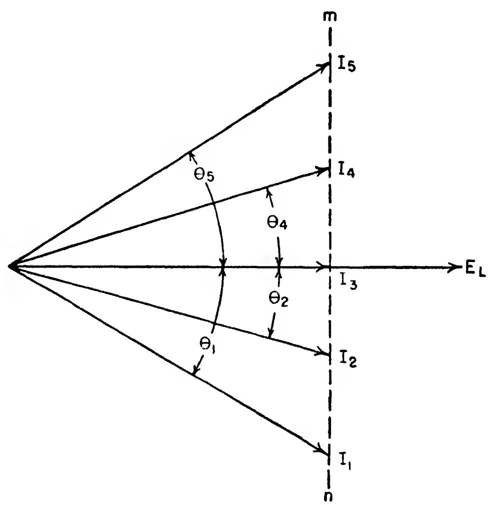


Fig. 16–11. The current I decreases from lagging I_1 to I_3 as the lagging power factor ($\cos \theta$) increases, and increases from I_3 to I_5 as the leading power factor decreases.

chronous motor can be so increased that, even at no-load, the motor will draw a line current equal to, or greater than the full-load value.

If the values of line current I_1, I_2, I_3, etc., for a given load in Fig. 16–11 are plotted against field current or counter emf, a "V-curve," so called from its shape, results. This is shown in Fig. 17–11(a). Note that for a very low value of field current and counter emf, the line current (even for a very light load) is large and lagging. As the field current is increased the line current decreases until it reaches a minimum value at unity power factor. A further increase in field current causes the line current to increase with leading power factor. Since the line current,

for a given load, varies inversely with the power factor, an inverted V-curve, Fig. 17–11(*b*), can also be plotted between power factor and field current or counter emf. This curve shows that for very

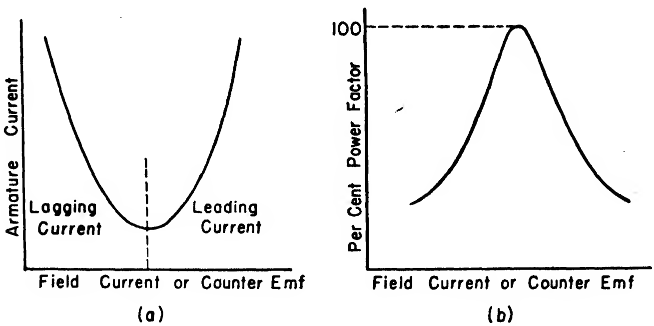


FIG. 17–11. Curves showing relation of armature current to field current in a synchronous motor with constant load.

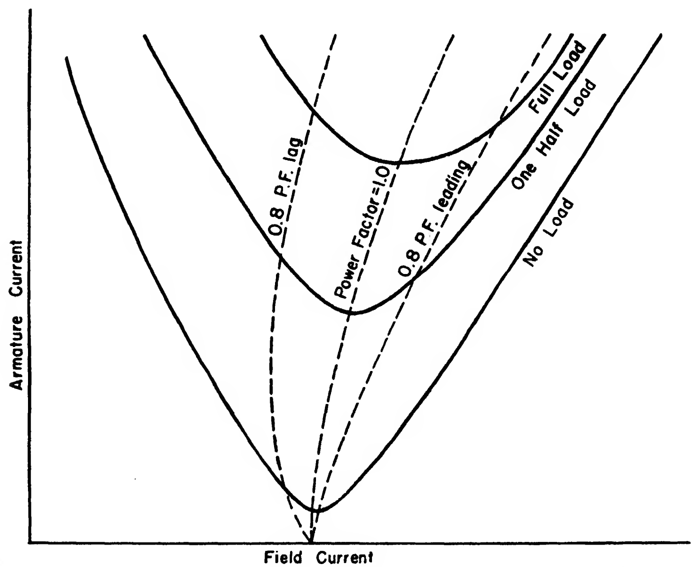


FIG. 18–11. Family " V-curves " showing relation between armature current and field current for different loads.

low values of field current (underexcitation) the power factor is low and lagging. As the field current is increased, the power factor rises to a maximum or unity, and then decreases and becomes leading on overexcitation. Note that the power factor is unity

and the line current a minimum at that field current which produces normal excitation.

A set of V-curves of current can be obtained, each for a different load on the motor. Figure 18-11 shows a set of such curves similar to those obtained from an actual motor, at no-load, one-half-load, and full-load, plotted between line current and field current. Note that these curves are not symmetrical in shape and the minimum value of line current for the several loads occurs at a different value of field current. Thus normal field excitation varies with the load on the motor. The dash lines in the figure are drawn through points of equal power factor for the several loads and are known as "compounding curves." These curves

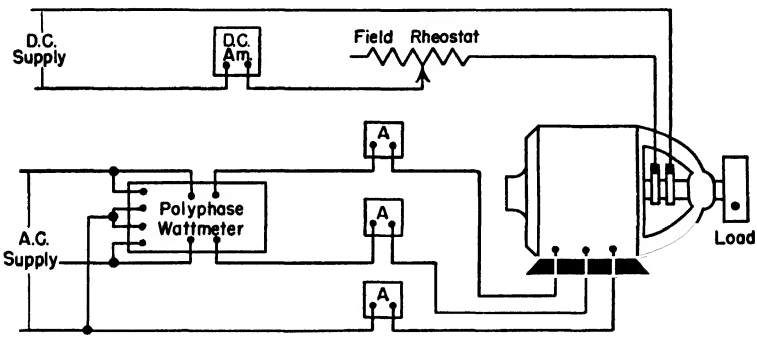


FIG. 19-11. Diagrams of connections for obtaining the V-curves of Fig. 18-11.

all show the characteristics of the synchronous motor as to its ability to produce sufficient reactive current to correct the power factor of a distribution or transmission line.

9-11. Determination of the V-Curves for a Synchronous Motor. Figure 19-11 shows the connections for obtaining the V-curves for a three-phase synchronous motor. A direct-current ammeter and a field rheostat are connected in series with the field winding and the alternating-current instruments are inserted in the line, as shown. Note that a polyphase wattmeter is used. This avoids the necessity of adding or subtracting the indications of two single-phase wattmeters to obtain the power input through a wide range in power factor (see Art. 3-11). The power input is held constant by adjustment of the mechanical load on the motor. The impressed or line voltage should be constant for this test. By means of the rheostat the field current is varied in steps from the lowest to the highest possible value. For each setting of the

rheostat, the value of the field current, power input and line currents are obtained. The same data are taken through a range of different loads on the motor, and a set of curves similar to those in Fig. 18–11 plotted.

It may be found that the indications of the instruments, especially on the no-load test, show that the computed power factor is less than unity for the minimum value of line current. This may be due to the fact that either, or both, the line voltage and counter emf waves are not of true sine-wave form (that is, contain higher-frequency harmonics) rather than due to errors in the instruments.

10–11. Hunting. Amortisseur or Damper Windings. The synchronous motor is very sensitive to any sudden change in load. When it is operating under steady load conditions the rotor, and counter-emf vector, take a certain synchronous position at a definite torque-angle β. If an additional load is suddenly applied, the torque already developed is less than that required for the added load and the rotor starts to slow down, or shift backward in synchronous position to a new torque-angle β'. Due to the momentum of the rotor, it cannot immediately stop this slowing down, or decelerating action, at exactly the proper synchronous position, but slips past it to a larger torque angle than required for the new load. The motor now develops more torque than necessary and the rotor tends to increase in speed and surges forward. Due to its momentum it advances past its proper synchronous position, and the torque developed is insufficient to carry the load and so the same cycle of events is repeated. Thus, while the motor maintains an average speed equal to the synchronous speed, when the load is suddenly increased or decreased, the rotor oscillates about its synchronous position with respect to the stator field. These oscillations of the rotor cause momentary inrushes of current and power, shown by the ammeters and wattmeters in the line. This action is called **hunting,** and is similar to that in the alternator (Art. 25–7).

Since the flux lines in the air gap have been likened to stretched rubber bands (Fig. 5–11), the rotor can be considered as locked in position with the stator flux through a spring coupling. The rotor, therefore, has a natural period of oscillation, depending upon its radius and its mass or weight. If changes in the load occur at intervals of the same frequency as this natural period of oscillation, the swing of the rotor above or below synchronism may be so increased that the motor pulls out of step.

Hunting is effectively eliminated by placing a short-circuited winding in the faces of the salient poles. In the more common form, the winding consists of heavy insulated copper bars placed in slots in the pole faces and connected by end rings, as shown in Fig. 1–11. It is similar to the squirrel-cage winding in the induction motor and is known as an **amortisseur or damper winding.** When hunting occurs, the armature flux shifts across the faces of the poles. In such a closed winding this induces emf and currents in the conductors in the pole faces. According to Lenz's law these currents act to prevent any change in the relative positions of the stator and rotor flux. That is, when the rotor tends to swing backward at lower than synchronous speed, motor action is produced in the damper winding tending to force the rotor ahead. And, when it swings forward, generator action is produced similar to that in the induction generator, tending to prevent the swing of the rotor above synchronous speed. The winding thus acts as a damper to prevent any tendency of the rotor to oscillate. When the motor is running at synchronous speed under a steady load, there is no relative motion between stator and rotor flux, and no voltage or current is induced in the damper winding and it is inoperative.

The lower the resistance of the damper winding the more effective it is in eliminating hunting. If the motor is to be started by induction motor action, the winding also acts as a starting winding. But a low resistance in this winding, as in the squirrel-cage induction motor, is not productive of high starting torque (see Art. 8–9). If only moderate starting torque is required the damper winding is of heavy copper as described above. Where larger starting torque is required the winding may consist of higher resistance material. And where maximum starting torque is required, the damper winding may be phase wound and insulated with the terminals brought out through collector rings to external resistors as in the wound-rotor induction motor.

11–11. Starting the Synchronous Motor. It has been shown that the synchronous motor of itself has no starting torque. When the proper line voltage is impressed on a small motor, however, the eddy current and hysteresis effect in the pole faces may produce a torque sufficient to start it. But before a large motor will start, it first must be brought up approximately to synchronous speed. It may be started mechanically from an external source, or electrically from the a-c line.

Starting from an External Source. A small induction motor direct-connected or geared to the shaft may be used to bring the rotor up to speed. The field is then excited and the motor synchronized with the line in the same manner as the alternator. If direct-connected, the number of poles in the starting motor must be such that its speed, allowing for slip, is equal to, or slightly greater, than the normal speed of the synchronous motor.

If the synchronous motor has a direct-connected exciter and sufficient direct-current power is available, the exciter may be used as a starting motor to bring the rotor up to speed. The field is then excited and the motor synchronized with the line as before. The exciter is now disconnected from the d-c line and its field strengthened, converting it from a motor to a generator. Or if the synchronous motor is used to drive a d-c generator, the latter may be used in the same manner as a starting motor. The above methods are applicable only to starting without load and are seldom used, as most synchronous motor applications require starting under load.

Induction Motor Starting. Practically all synchronous motors today are started as induction motors by means of the short-circuited-damper-or-starting winding described in the previous article. In starting the standard or conventional motor, the field circuit first is opened, next a polyphase a-c. voltage is impressed on the armature or stator. To prevent too large a current and kva inrush at starting, a compensator (or series reactance) similar to that employed in starting the induction motor is used. This in itself reduces the starting torque. The rotating field, set up by the polyphase currents in the stator, produce currents in the squirrel-cage damper winding and torque is developed. If this is high enough to start the load, or " break away," the rotor accelerates until close to synchronous speed. The field circuit is now closed just before the compensator is thrown to full running position and the rotor pulls into synchronism, the rotor poles locking in step with the rotating poles in the stator. The torque developed at that speed (usually about 95 per cent of synchronous speed) at which the motor changes from induction motor action and pulls into synchronism with the supply line is called the " pull-in torque."

Slipping a Pole. When the field circuit is closed, the field poles may be excited to the same polarity as the corresponding rotating poles in the stator, as in Fig. 20–11. The air-gap flux and the

torque are reduced and the rotor drops back one pole, or is said to "slip a pole" to the position shown in Fig. 4–11. This may cause a momentary rush of current and disturbance in the a-c system. For this reason, the field circuit is closed before the compensator is thrown to the full running position. Also, if the motor is loaded, closing the field circuit at this time increases the pull-in torque.

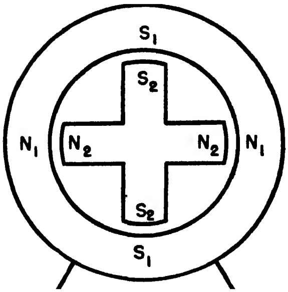


FIG. 20–11. If the rotating field comes up to a synchronous speed with like poles facing each other, as shown in this figure, it is necessary for the rotor to slip back one pole in order to be in normal synchronism.

Hysteresis Effect. In some cases the motor, if lightly loaded, may pull into synchronism before the field circuit is closed. This is due to hysteresis, or the residual magnetism which remains in the field poles after the magnetizing force is removed. As the rotor approaches synchronous speed, the rotating stator flux, as it passes the salient field poles, shows a decreasing tendency to leave them. If this hysteresis effect is strong enough, the rotor will pull into synchronism without the aid of the field current.

Resistance in the Starting or Damper Winding. Both the starting and pull-in torques in the synchronous motor, with its short-circuited starting-winding, are induction motor torques. If the resistance of this winding is low, its reactance on starting is relatively high and produces low starting-torque, exactly as in the squirrel cage induction motor (Art. 8–9). If the motor is loaded, however, the slip at the last point of the starting compensator is correspondingly low and the rotor approaches more closely to synchronous speed. This increases the pull-in torque. On the other hand, if the resistance of the starting winding is high, its reactance is relatively low and the starting torque is increased. But the slip is increased, the rotor does not accelerate so closely to synchronous speed and the pull-in torque is decreased. Thus in general, a starting winding of low resistance reduces the starting and increases the pull-in torque, while a high-resistance winding increases the starting torque but decreases the pull-in torque. Because of these opposing effects, the design of the damper winding in the standard motor must necessarily be a compromise.

It should be noted that the synchronous motor, on starting, is a distorted form of induction motor. The damper winding does not form a perfect squirrel-cage winding because of the gap between the salient poles. Also, in order to reduce leakage reactance and produce better synchronous motor characteristics, the air gaps are longer than in the induction motor. This increases the magnetizing current, reduces the power factor and increases the current and kva inrush at starting.

Emf Induced in the Field Circuit. When voltage is first applied to the armature or stator of the synchronous motor on starting, the rotating stator flux cuts the turns of the field coils at synchronous speed. These coils are spool wound and contain a large number of turns with respect to those in the stator. The stator winding thus becomes the primary of a step-up transformer and a very high voltage is induced in the field circuit. The field coils must, therefore, be more heavily insulated than is required for the normal d-c excitation voltage (generally from 125 to 250 volts). As the motor accelerates, the relative speeds of the rotating stator flux and the field poles decrease and the induced emf in the field circuit also decreases and finally disappears, when the rotor attains synchronous speed. It is desirable to reduce this transformer emf on starting and the following methods can be used.

(1) The field circuit may be divided into sections by a field break-up switch mounted on the rotor. This switch is open on starting and is automatically closed by centrifugal action when the rotor attains sufficient speed to reduce the transformer emf to a safe value.

(2) On starting, the field circuit may be short-circuited on itself, or through a comparatively low resistance. This produces transformer currents in the field circuit. Since its reactance is high with respect to its resistance, the induced field emf is largely consumed in the reactance drop in the several coils. But the field circuit now acts in parallel with the damper winding, reduces the effective resistance of the rotor and decreases the starting torque. The pull-in torque, however, is increased.

(3) The direct current for the field may be supplied at a low voltage, using a 30 to 40 volt exciter. The number of turns in the field coils and the transformer emf is likewise reduced proportionally. To provide the necessary exciting field ampere-turns the field current also must be correspondingly increased. However,

a low-voltage exciter to supply the very much larger field current is more expensive than one designed for a standard voltage.

Another reason for starting with opened field circuit, noted above, is also due to this transformer action. If this circuit with high reactance and low resistance is closed, it draws a low power-factor current from the a-c line, increases the kva inrush and reduces the starting torque.

12–11. Relation of Starting Current and Developed Torques to Impressed Voltage. The starting current in the synchronous motor, like that in the induction motor, varies directly with the impressed voltage. With both impressed voltage and starting current doubled, the kva input is increased four times. Thus the kva inrush at starting varies with the square of the impressed voltage.

Both the starting and pull-in torque vary practically with the square of the impressed voltage. For if this is doubled the rotating flux set up in the stator is doubled and the currents in the damper winding and flux so set up are also doubled. Thus the torque is practically increased four times.

The pull-out torque, however, varies directly with the impressed voltage. At synchronous speed, the flux in the salient field poles is largely determined by the constant direct-current excitation, which is independent of the load. So, if the impressed voltage is doubled, the air-gap flux and also the pull out is doubled. This gives the synchronous motor an advantage which enables it to carry its load through a temporary drop in line voltage that will cause the induction motor (in which the break-down torque varies with the square of the impressed voltage) to pull out.

13–11. Multiple Winding Synchronous Motor. To reduce the starting current without the use of a compensator, the armature or stator may be parallel (or multiple) wound. That is, each phase may consist of two complete sets of windings, usually placed in alternate slots and connected in parallel for normal operation. When so connected these windings have normal reactance and resistance. One set of windings is connected through a three-pole short-circuiting switch, as indicated in Fig. 21–11. If this switch is opened on starting, the reactance and resistance of the armature is approximately doubled and the motor starts with less current and kva inrush. The starting torque, however, is reduced. On approaching synchronous speed, the short-circuiting switch is closed, connecting both windings in Y in parallel. The motor

now pulls into synchronism in the same manner and with the same pull-in torque as the standard motor.

14–11. Synchronous Motor with Phase-Wound Starting or Damper Winding.* For high starting torque developed with the lowest possible inrush of line current, the wound-rotor induction motor is superior to all other alternating-current motors. The motor which most closely approaches it, and to which it is similar, is a synchronous motor with a polyphase damper or starting winding set in the rotor field. The winding is three-phase with the terminals brought out to three collector rings which are connected

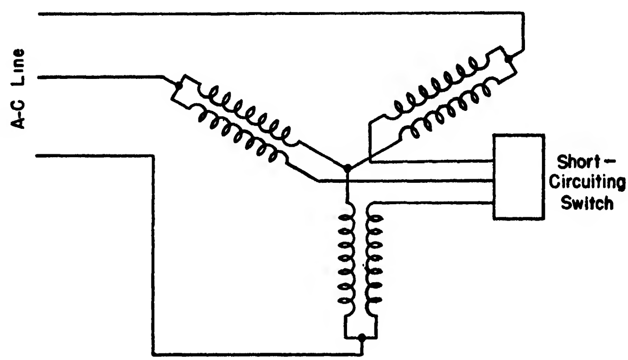


FIG. 21–11. When the switch is open, one of the parallel windings is cut out. Closing the switch puts the two windings in parallel again.

through a controller to external resistors. The Westinghouse Simplex Rotor, Fig. 22–11, is of this type. Note in the figure that two of the five collector rings are for direct-current supply to the field coils. The motor is started with the field circuit open and all external resistance in the damper circuit. As the motor accelerates this resistance is gradually cut out, the field is excited, making definite pole areas on the rotor, and the motor pulls into synchronism in the same manner as the standard synchronous motor. The starting performance of this motor is close to that of the wound-rotor induction motor.

In slow speed motors of this type the damper winding generally consists of a single heavy insulated copper bar in each pole-face slot. In high speed motors the current in the damper winding tends to become excessive. To reduce this current, the damper

* See "High torque synchronous motor," by M. R. Losy in The *Electric Journal,* May, 1937.

winding is generally made of form-wound insulated coils with several conductors in each slot. This reduces the current through the collector rings and controller circuit, while the total current per pole-face slot remains about the same as in the bar winding.

Such motors produce a starting torque close to 200 per cent of normal full-load value, a pull-in torque near 125 per cent and a pull-out torque 175 to 200 per cent of normal. They are more expensive than the standard synchronous motor due to the extra cost of the phase-wound damper winding, the controller, switches,

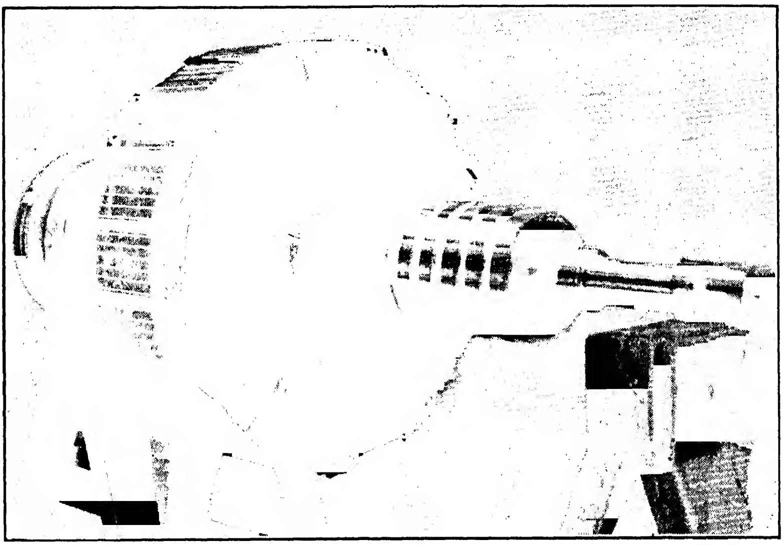


FIG. 22–11. Simplex Rotor. (*Westinghouse Electric Corp.*)

relays, etc. Its principal advantage is its high torque and lower circuit-and-kva inrush at starting. It is adapted for starting heavy equipment such as grinding machines, crushers, conveyors, etc., and is probably the most widely used type of high-torque synchronous motor.

15–11. The Synchronous-Induction Motor. This motor is somewhat similar to the motor with polyphase damper winding, described above, except that it has no salient poles, but a round rotor and a uniform air gap. It also has no extra winding for direct-current excitation. It is fundamentally a wound-rotor slip-ring induction motor. The air gap is longer and the rotor slots are fewer and larger than is common in the induction motor.

The motor is started with resistance in the damper circuit, which is gradually cut out as the motor accelerates, exactly like the wound rotor machine. When the resistance has been all cut out and the rotor has approached synchronous speed, two terminals of the three-phase damper winding are connected to a direct-current source. This produces fixed poles in the rotor and the motor pulls into synchronism. One disadvantage of this motor is that, since the damper winding itself has low resistance and comparatively few turns, the d-c excitation must be low, about 25 volts, and the exciting current correspondingly high. More power is required for excitation than in other types of synchronous motor and the overall efficiency is lower. This motor has high starting and pull-in torques comparable to the phase-wound motor in the previous article, but the pull-out torque is lower. This motor is expensive and due to its lack of space for the field winding it is not adapted for over excitation and leading power factor.

16–11. The Super-Synchronous Motor. A special type of motor designed to give a starting torque equal to its pull-out torque, called the " super-synchronous motor " has been developed by the General Electric Company. The stator or armature, which is stationary in the usual type of synchronous motor, is free to rotate about the revolving field structure, but may be restrained or locked by a band brake. The field structure or rotor is direct connected to the load. In starting the brake is released and a low three-phase voltage is applied to the stator or armature through collector rings. As the rotor is connected to the load it remains stationary and the stator accelerates, due to the induction motor torque produced by the squirrel cage damper winding on the rotor, but in a direction **opposite** to that required by the load. When the stator approaches close to synchronous speed, the field switch is closed, full voltage is applied to the stator and the motor pulls into synchronism. The brake is now applied and the stator gradually brought to standstill. Since the stator and rotor are now operating in synchronism, their relative speeds must be equal to synchronous speed. As the speed of the stator is gradually reduced by the brake, that of the field structure and the load must, therefore, gradually increase. And when the stator is brought to standstill the field must turn at synchronous speed. The motor thus starts under no-load conditions of torque and current inrush, but on starting the load develops a torque which is always equal to the pull-out value.

Due to its complicated mechanical features of design this motor is expensive to build and is not in common use.

17–11. The Synchronous Condenser. The fact that the armature current taken by a synchronous motor may be made to lead or lag the impressed voltage simply by overexciting or underexciting the field makes it very useful for improving the power factor. In preceding chapters it has been shown that low power factor results in greater line current for a given kilowatt load, poorer generator regulation, increased line losses, and reduced efficiency, besides requiring greater generator capacity.

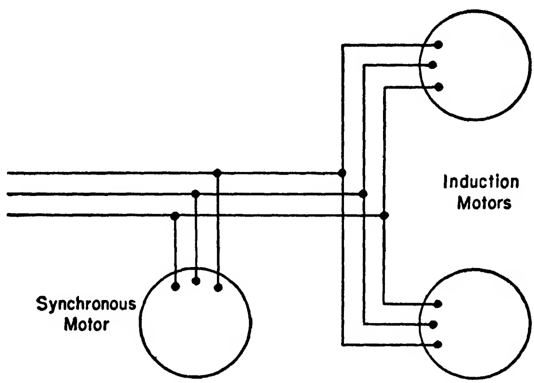


Fig. 23–11. The overexcited synchronous motor operated in parallel with induction motors improves the power factor of the load.

When the load on a line consists largely of induction motors, as that in a mill or factory, it may have a lagging power factor as low as 50 or 60 per cent. If an overexcited synchronous motor is connected in parallel with such a load, as in Fig. 23–11, it acts as a capacitor and draws a leading reactive component of current or leading kilovolt-amperes. This improves the power factor and reduces the current in the line. It is important to note that the synchronous motor must be placed at the load end of the line rather than at the generator end. The overexcited motor may be connected to a mechanical load and serve the double function of a motor drive and a capacitor on the line. Or it may be run light without load. In which case its sole function is to improve the power factor and it is called a "synchronous condenser."

If the synchronous motor is of sufficient rating it may be adjusted by field control to draw the necessary leading reactive current to raise the power factor of the line to any desired value.

If this leading reactive component of current can be made equal to the lagging reactive current taken by the induction motor load, the line power factor is raised to unity. Thus for a three-phase system, as in Fig. 23–11, the rating of the synchronous condenser necessary to raise the power factor of the line to unity can be determined as follows.

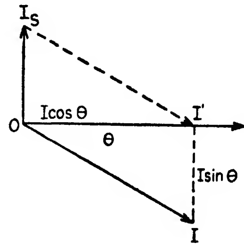


FIG. 24–11. The synchronous motor adds the leading current I_s to the lagging reactive current of the induction motor to produce the resultant current I', which is in phase with the voltage and produces unity power factor.

Using phase values in Fig. 24–11, let E be the impressed voltage and I the current taken by the induction motor load at a lagging power factor $\cos\theta$. The current I is resolved into two components, a power component $I\cos\theta$ $(=I')$ in phase with the voltage and a reactive component $I\sin\theta$ $(=I'I)$ lagging the voltage by 90°. When the synchronous motor is run light, as a condenser, it draws only a small power component of current necessary to supply its own losses, which can be neglected. The necessary condenser current I_s is thus almost all reactive and is drawn at 90° to the voltage and leading. It is, therefore, practically equal to $I'I$, the lagging reactive current taken by the inductive motor load. And the volt-ampere rating of the condenser per phase is EI_s. The line current has been reduced in value from I to I', or $I \oplus I_s = I'$. Actually the resulting line current is slightly greater than I' due to the small power current required to supply the losses in the condenser.

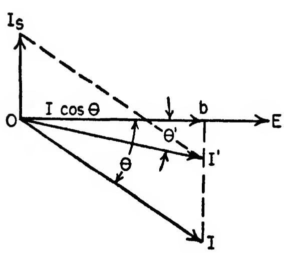


FIG. 25–11. In this case the synchronous motor reduces the total current from I to I' and greatly increases the power factor $(\cos\theta)$.

In practice it seldom pays to raise the power factor of a line above 90 per cent, for above this value little is gained in the reduction of current and considerably greater condenser rating is required. The rating of a synchronous condenser necessary to raise the line power factor from $\cos\theta$ to $\cos\theta'$ and reduce the current from a value I to that of I' is indicated in Fig. 25–11. The current I is resolved into a power component, $I\cos\theta$ $(=I'\cos\theta',\ =ob)$, and

reactive component $I \sin \theta$ $(=bI)$. At power factor $\cos \theta'$, total reactive component to the line equals bI'. The leading reactive component of current which must be supplied by the condenser equals $bI - bI'$ or $I'I$. This is laid off as I_s 90° from E and leading. The rating of the synchronous condenser is thus EI_s volt-amperes. Note that $I \oplus I_s = I'$.

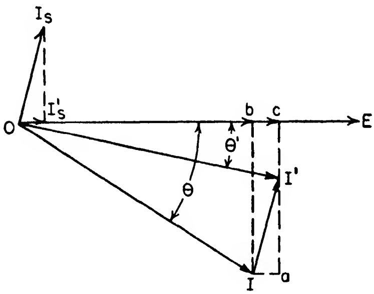


FIG. 26–11. The synchronous motor carries its own load and still supplies a leading I_sI_s' which improves the power factor of the line.

When the synchronous motor is used **both** to correct power factor and supply a mechanical load it must draw a power component of current from the line. In Fig. 26–11 let I be the current per phase taken by the induction motor load at a lagging power factor $\cos \theta$. It is desired to determine the rating of a synchronous motor necessary to raise the line power factor to $\cos \theta'$ and at the same time carry a load requiring a power current I_s' amperes per phase. Current I is resolved into its two components $I \cos \theta$ $(=ob)$ and $I \sin \theta$ $(=bI)$. Vectors bc and Ia are drawn equal and parallel to I_s', and ca equals bI.

Total power component of current per phase taken by the combined load $= ob + bc = oc$.

At power factor $\cos \theta'$:

Total current per phase taken by the combined load equals

$$\frac{oc}{\cos \theta'} = I'$$

Total reactive current per phase equals $oc \tan \theta' = cI'$

$ca - cI' = aI'$, leading reactive current which must be provided by the synchronous motor.

Total synchronous motor current $= \sqrt{(Ia)^2 + (aI')^2} = II'$ or I_s, at a power factor $\cos \theta'$.

The rating of the synchronous motor in this case is now EI_s volt-amperes per phase. Note again that $I \oplus I_s = I'$.

It was pointed out in Art. 5–11 that the leading reactive component of current taken by an overexcited synchronous motor produces a demagnetizing action which brings the air gap flux

back to normal. And this reacts upon the driving alternator, producing in it a magnetizing effect. It was also shown in Art. 7–11 that the induction motor is magnetized by a lagging reactive component of current taken by this machine. When an overexcited synchronous motor operates in parallel with an induction motor load, however, its leading reactive current does not flow in the line and alternator circuit, but circulates between the synchronous and induction motors. The action is similar to that in a parallel connection of condenser and induction coil. Thus the leading reactive current in the overexcited synchronous motor becomes a lagging reactive magnetizing current in the induction motor circuit.

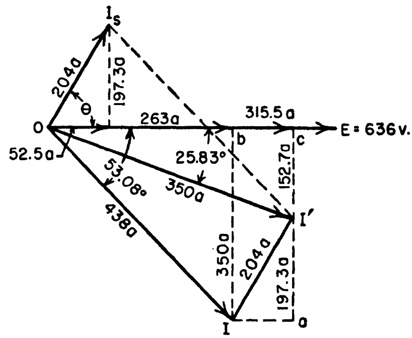


Fig. 27–11. The synchronous motor can deliver 197.3 leading amperes to improve the power factor and still supply (52.5 × 636) watts of power.

Example 9. An induction motor load on a factory takes 500 kw from a 1100-volt, 3-phase line at 0.6 lagging power factor. A synchronous motor is to be installed to take an additional load of 100 kw and at the same time raise the line power factor to 0.9. What must be the rating of the synchronous motor?

Solution: The system is assumed to be Y-connected and phase values are used. See Fig. 27–11.

$$\text{Voltage to neutral, } E = \frac{1100}{\sqrt{3}} = 636 \text{ volts.}$$

$$\text{Current to induction motor load} = \frac{500{,}000}{\sqrt{3} \times 1100 \times 0.6}$$

$$= 438 \text{ amperes.}$$

$$\text{Power current to load, } I \cos\theta = ob = 438 \times 0.6$$
$$= 263 \text{ amperes.}$$

$$\text{Reactive current to load, } I \sin\theta = Ib = ca = 438 \times 0.8$$
$$= 350 \text{ amperes.}$$

$$\text{Power current to synchronous motor, } I's = \frac{100{,}000}{\sqrt{3} \times 1100}$$

$$= 52.5 \text{ amperes}$$

$$\text{Total power current to combined load, } oc = 263 + 52.5$$
$$= 315.5 \text{ amperes}$$

Power factor to combined load $= 0.9$. $\cos^{-1} 0.9 = 25.83°$

$$\text{Total reactive current to combined load, } cI' = oc \tan 25.83°$$
$$= 315 \times 0.484$$
$$= 152.7 \text{ amperes}$$

Reactive current to synchronous motor, $(ca - cI')$

$$= aI' = 350 - 152.7 = 197.3 \text{ amperes}$$

Total synchronous motor current, $\sqrt{(Ia)^2 + (aI')^2}$

$$= \sqrt{52.5^2 + 197.3^2} = 204 \text{ amperes}$$

$$\text{Rating of synchronous motor per phase} = 636 \times 204$$
$$= 129.7 \text{ kva}$$

Or a total rating of $3 \times 129.7 =$ **389 kva** **Ans.**

$$\text{Power factor of synchronous motor, } \cos \theta_s = \frac{52.5}{204} = 0.257.$$

$$\theta_s = 75.08°$$

$$\text{Total current taken by combined load} = \sqrt{315.5^2 + 152.7^2}$$
$$= 350 \text{ amperes}$$

Thus the synchronous motor in the example above takes an additional load of 100 kw from the line, raises the power factor from 0.6 to 0.9 and reduces the line current from 438 to 350 amperes.

Since the voltage is constant, Example 9 can be computed by the power method using kilowatts and kilovars (Ex. 10–III). This simplifies the calculations as in Example 10 below.

Example 10. (Data from Example 9. In Fig. 28–11 the 500 and 100 kw loads taken by the induction and synchronous motors are both laid off on the impressed voltage vector, E (1100 volts).)

$$\text{Kva to induction motor load, } od = \frac{500}{0.6} = 833$$

$$\text{Kvars to induction motor load, } bd\ (= ca) = od \sin 53.16°$$
$$= 833 \times 0.8 = 666.4$$

Total power to combined load, $oc = 500 + 100 = 600$ kw.

$\cos^{-1} 0.9 = 25.83°$. Tan $25.83° = 0.484$

At 0.9 power factor, kilovars to combined load,

$$ce = oc \tan 25.83°$$
$$= 600 \times 0.484 = 290.4$$

Kilovars taken by synchronous motor, ae ($= ca - ae$)

$$= 666.4 - 290.4 = 376.$$

Rating of synchronous motor, de ($= of$) $= \sqrt{100^2 + 376^2}$

$$= \mathbf{389\ kva.}\quad \textbf{Ans.}$$

Power factor of the synchronous motor $\cos \theta_s = \dfrac{100}{389} = 0.257$

$$\theta_s = 75.08°$$

This checks with the values in Example 9.

Kva taken by the combined load $= \sqrt{600^2 + 290.4^2} = 737.6$

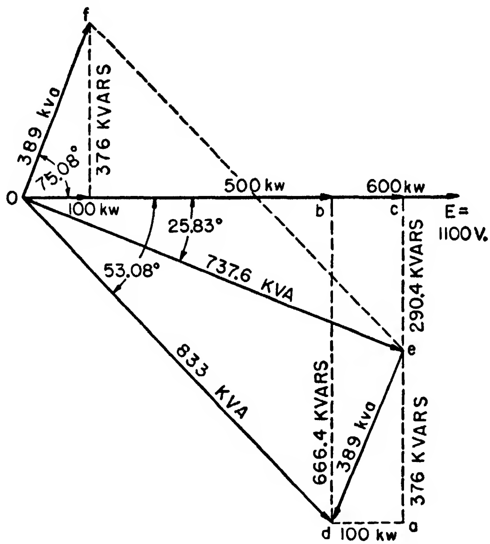


Fig. 28–11. The diagram of Fig. 27–11 constructed with power and reactive-power vectors.

Thus, by use of the synchronous motor, the power load is increased and the total kva taken from the line is reduced 833 to 737.6 kva.

Synchronous Reactor. Long, high-voltage transmission lines at light loads generally have a leading power factor due to their charging current. (See Art. 5–13.) A large synchronous motor installed across such a line is operated without load and generally is underexcited to take lagging kilovolt amperes. The motor in this case actually operates as a reactor on the line and is called a "synchronous reactor." The power factor and receiving voltage vary with the load on the line, both of which may be controlled

by automatic adjustment of the motor field. Such motors are operated without mechanical load, are often entirely enclosed, sealed against atmospheric pressure and hydrogen cooled. (See Art. 31–7.) They are usually installed in the open without protection against the weather (Fig. 29–11) and may on occasion be operated either as a condenser or a reactor.

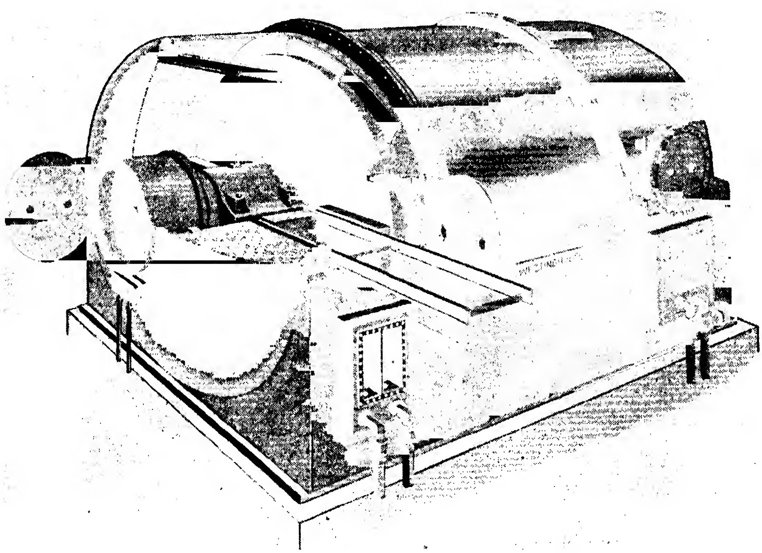


FIG. 29–11. Cutaway view of a hydrogen-cooled synchronous condenser. (*Westinghouse Electric Corp.*)

Prob. 15–11. The 60-cycle induction motors in a certain plant draw 1200 kw at 80% power factor from the power line. (*a*) What must be the kva output of a synchronous condenser which will raise the power-factor of the plant to unity.

Prob. 16–11. Assume that the power losses in synchronous condensers decrease regularly from 8% of the kva output rating of those having an output of 200 kva to 3% for those having an output rating of 2,000 kva.

(*a*) What would be the total kilowatt load in the plant in Prob. 15 before the condenser was installed?

(*b*) After the condenser was installed?

(*c*) What would be the total kilovolt-ampere load before the condenser was installed?

(*d*) After the condenser was installed?

(*e*) At what power-factor would the synchronous condenser be operating?

Prob. 17–11. If it had been desired to raise the power-factor of the plant in Prob. 15 to 90 per cent only, what kilovolt-ampere synchronous condenser would have been needed?

Prob. 18–11. Apply the questions in Prob. 16 to the conditions of Prob. 17.

Prob. 19–11. If it is desired to maintain the same kilovolt-ampere load at 90 per cent power-factor in Prob. 17 as when the induction motors only were running, the additional load being in the form of a mechanical load on the synchronous condenser, what size (kilovolt-ampere) condenser should be installed?

Prob. 20–11. (*a*) What effective power would be available as output from the synchronous motor in Prob. 19?

(*b*) At what power-factor would the synchronous motor operate?

Prob. 21–11. What kilovolt-ampere synchronous condenser should be installed in Prob. 16 so that the power plant would be delivering the maximum effective power and still not increase the total kilovolt-ampere output, the additional load being in the form of a mechanical load on the synchronous condenser?

Prob. 22–11. (*a*) At what power-factor would the synchronous motor in Prob. 21 be operating?

(*b*) How much would the effective power output of the plant be increased?

18–11. Synchronous Motors Without D-C Field Excitation.* The standard synchronous motor, with its salient field poles, collector rings and direct-current excitation, is seldom built in sizes smaller than 5 horsepower because of the necessity of a d-c exciter, or other source of direct-current supply. To meet the need for a small a-c motor with more constant speed characteristics than those obtained in the induction motor, and which will develop greater torque than that obtained in the very small clock-motor described in the next article, the nonexcited synchronous motor has been developed. It has no collector rings and requires no d-c field excitation.

In construction, the motor is almost identical with that of the small induction motor and is usually built from induction motor parts. The polyphase motor of this type has the standard three-phase induction motor stator and squirrel cage rotor, except that 35 to 40 per cent of the teeth are cut out of the rotor punchings, Fig. 30–11, forming the same number of physical poles, or projec-

* See "Small Synchronous Motors without Exciters," by P. H. Trickey in *Electrical Journal*, April 1933.

tions in the rotor as are set up in the stator winding. The squirrel-cage end-rings and all the rotor bars are left, even in the spaces where the teeth have been removed. This gives the motor better starting characteristics. Also the number of teeth left for each rotor pole need not be the same, for poles of nonuniform width have a tendency to develop a more uniform torque. That is, they tend to prevent "cogging" (a variation of starting torque with rotor position). Skewing of the rotor slots also tends to prevent cogging.

The operation of the nonexcited synchronous motor is similar to that of the standard synchronous motor with its damper wind-

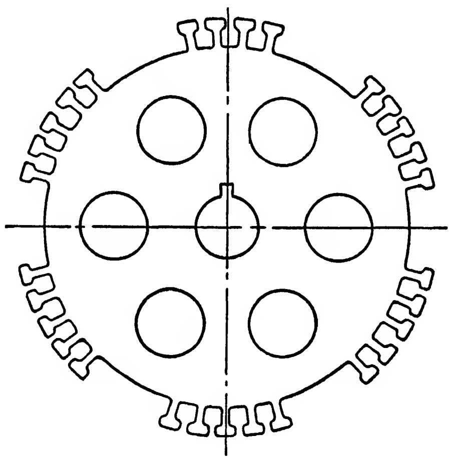


FIG. 30-11. A squirrel-cage rotor with 35 to 40% of the teeth cut out of rotor punchings for easy starting purposes.

ing, but with the d-c field excitation reduced to zero. The non-excited motor also starts as an induction motor in which the average torque is little affected by cutting out part of the rotor teeth. As the rotor accelerates and approaches synchronous speed, the stator flux, in sweeping by rotor poles, has a decreasing tendency to leave them, due to the hysteresis effect. If this action is strong enough, the rotor pulls into synchronism. A high resistance rotor or damper winding, as in the standard synchronous motor, produces high starting torque, but reduces the pull-in torque; while a low resistance winding produces a lower starting torque, but increases the pull-in torque. After the motor has pulled into synchronism rotor resistance has no effect on the torque.

Figure 31–11 shows a typical speed torque curve for a nonexcited synchronous motor. Note that the synchronous pull-out torque is only about one-third the maximum induction motor torque. This means that the frame size of the nonexcited synchronous motor must be three times larger than that of the comparable induction motor of the same horse-power rating. In other words, a one-horse-power nonexcited synchronous motor must be built from the parts of a three-horse-power induction motor frame. Furthermore, the power factor and efficiency of

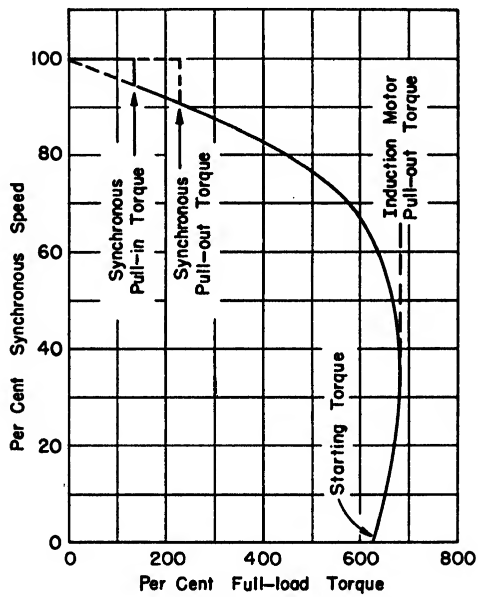


FIG. 31–11. A typical speed-torque curve of a nonexcited synchronous motor.

the nonexcited motor is low, being approximately that of the comparable induction motor at one-third to one-half load. This is not too serious in a small motor.

A single-phase, nonexcited synchronous motor is also built, in which the rotor is the same as in the polyphase motor. The stator windings are exactly the same as in the split-phase or several types of capacitor induction motor. (See Ch. X.) In cases where the centrifugal starting switch is usually used, it opens at the same speed as in the induction motor and before the rotor pulls into synchronism. The rating is also about one-third that of the comparable induction motor.

It is interesting to note that a small induction motor may be converted into a synchronous motor by milling equally spaced gaps in the rotor punchings, there being as many gaps as there are poles in the stator. To accomplish this, part of the end rings and rotor bars in the gaps may necessarily be cut away, but this will not effect its performance materially. The rating of the machine as a synchronous motor will, of course, be about one-third of its original rating as an induction motor.

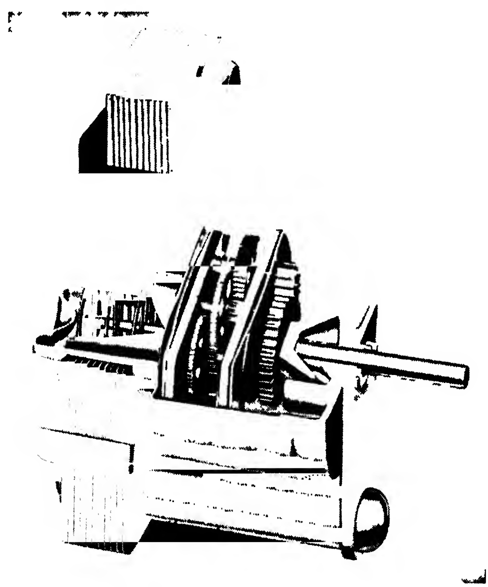


FIG. 32–11(*a*). Cutaway view showing shading coil and rotor of Telechron clock motor. (*General Electric Co.*)

19–11. Very Small Clock-or-Timing Motors. The fact that the rotor of a small motor will run at a constant or synchronous speed without d-c field excitation has lead to the development of very small synchronous motors for clocks and industrial timing devices, etc. These motors have very low power-factor but this is unimportant in such small motors. There are several types.

One of the more common types is that invented by H. E. Warren and manufactured by Telechron Inc. This is a shading-pole motor (Art. 5–10) and consists primarily of a two-pole laminated frame or stator with a single a-c exciting coil. The shading coils, each a heavy copper ring, are arranged on one half of the

divided salient poles. The rotor consists of two or more hardened steel rings with cross bars, as shown in Fig. 32–11 (*a* and *b*), mounted on a light shaft. The rotor starts due to the rotating action of the flux set up by the exciting and shading coils. Since the hysteresis loss in hardened steel is high, considerable energy must be consumed in the rotor in a reversing or rotating field and the starting torque is high.*

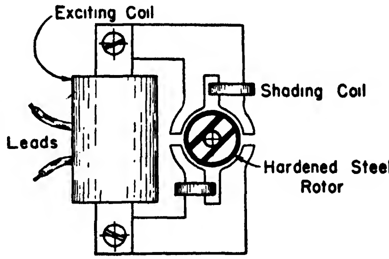


FIG. 32–11(*b*). Diagram of a Telechron clock motor.

As the rotor approaches synchronism, the flux tends to take the path of least reluctance through the cross bars, the rotor becomes permanently magnetized and pulls into synchronism. The normal speed of the rotor on 60 cycles is 3600 rpm so the rotor is connected to its load through a worm on its shaft and reducing gear. The motor is lubricated by capillary action from a reservoir at the bottom of the gear case, the bearing surfaces are covered at all times with a thin film of oil and friction is reduced to a minimum. Due to the lightness of construction the rotor shaft assembly is said to "float in the rotating magnetic field." The lighter types of this motor require approximately only 2 watts to operate, while heavier duty motors require from 6 to 12 watts.

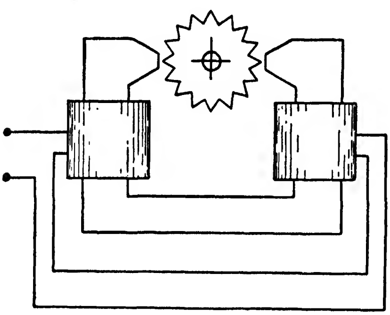


FIG. 33–11. Small salient poles on rotor; two laminated-iron main poles.

Another type of motor, shown in Fig. 33–11, consists of a two-pole laminated stator and a heavy iron rotor with a large, and even, number of projections or salient poles. This motor is not self-starting but must first be brought up to synchronism, gener-

* It was shown in Ch. IX that increased rotor resistance in the induction motor, with corresponding increase in rotor losses, increases the starting torque.

ally by spinning it by hand. Its action when operating at synchronous speed, is as follows. As the field flux is increasing, diametrically opposite rotor poles are attracted to the field poles. As the field flux is decreasing and passing through zero, the rotor continues to rotate, due to its inertia. And, as the field flux increases in the opposite direction, the following pair of rotor poles is attracted to the field poles. To increase the inertia effect, the rotor assembly may include a flywheel on its shaft. The ordinary synchronous motor has a synchronous speed determined by the number of field poles (and the frequency) but this motor operates at a speed determined by the number of salient poles on the **rotor.** For instance, such a motor, with a rotor of 20 poles on a 60-cycle circuit, has a speed equal to $\frac{120 \times 60}{20}$ or 360 rpm. Because this motor operates at a speed lower than that determined by the field poles it is called a **subsynchronous** motor. Such motors may be difficult to start and may lock into step at half their synchronous speed.

20–11. The Selsyn Motor and Selsyn System of Remote Control.* The Selsyn motor, so called, is a small self synchronizing device used in the electrical transmission of angular motion, or as a position indicator, between two or more remote appliances which cannot readily be interconnected mechanically. The distance of transmission may be only a few feet or several miles. In construction the machine is somewhat similar to a small rotating-field, three-phase alternator, except that the rotor is in the form of a two-pole shuttle, as indicated in Fig. 34–11. The stator is wound with a three-circuit, distributed, Y-connected winding, and the rotor has a concentrated winding connected through collector rings to a single-phase source, Fig. 35–11. Such a machine is variously termed a "Selsyn," a

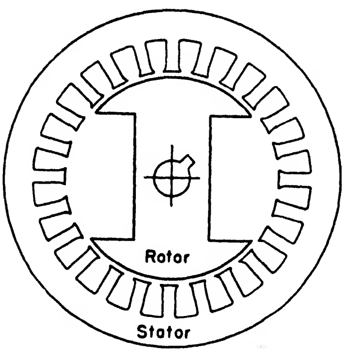


FIG. 34–11. Rotor and stator of a Selsyn motor.

* See "Principles of Selsyn Equipments and their Operation," by L. F. Holder, in *General Electric Review*, Sept. 1930.

"Teletorque," an "Autosyn," or a "Synchro," but is commonly known as a "Synchro-Mechanism."

The simplest form of Selsyn system consists of two identical machines, as indicated in Fig. 36–11. The rotor windings of the two machines are connected to the same single-phase circuit, or

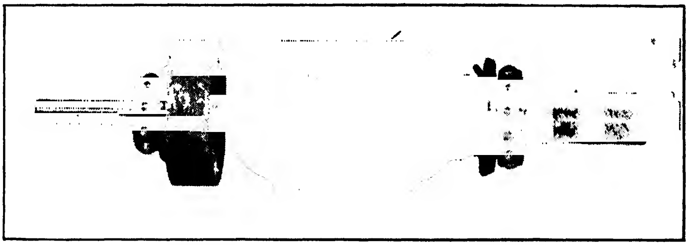


FIG. 35–11. Selsyn rotor with damper. (*General Electric Co.*)

to the same phase of a three-phase circuit and are permanently excited. The stator windings of both machines are interconnected in the same sequence, s_1 to s_1, s_2 to s_2, and s_3 to s_3. The rotor of one machine is connected mechanically, generally through gearing, to the appliance whose position is to be transmitted. This is

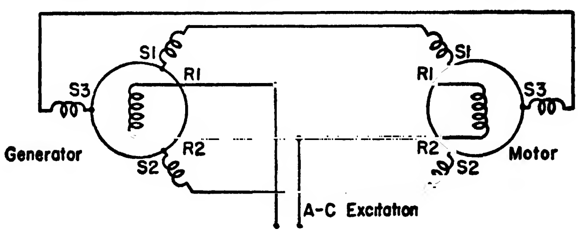


FIG. 36–11. The simplest form of a Selsyn system.

known as the generator or transmitter, while the other machine is known as the motor, or receiver, or indicator.

The principle upon which the system operates is as follows: In any given position, the rotor of the generator, by transformer action, sets up emfs of different magnitude in the three legs of the stator winding. These emfs are all in phase but differ in value with change in the position of the rotor. They produce unequal currents in the stator windings of the motor and a resultant field corresponding in direction to that set up in the generator stator. If the rotor of the motor is displaced in position with respect to its stator field, torque is developed, since the current in the rotor

winding produces a field which tends to align itself parallel with that of its stator field. The rotor, if free to turn, therefore assumes the same position as that of the rotor in the generator. In this position, if the currents in the two rotors are in phase, the rotor of the motor, by transformer action, induces emfs in the three legs of its stator, equal and opposite to those induced in the stator windings of the generator. No current then flows in the stator windings of the machines and the system is said to be in equilibrium. Thus, as the position of the generator rotor is changed by the motion of the appliance to which it is mechanically connected, the position of its resultant stator field, and also that in the motor,

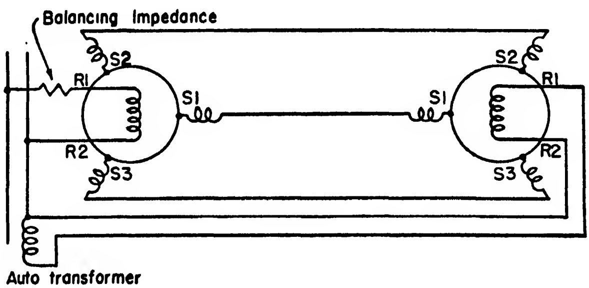


FIG. 37–11. Auto transformer for distant Selsyn.

likewise change in position and the rotor of the motor moves synchronously to a corresponding position.

While the generator and the motor, as previously stated, are identical machines, they actually differ mechanically in that the motor is equipped with a flywheel mounted through a friction plate to a sleeve attached to the shaft. Any excessive oscillation of the rotor rotates the sleeve which the flywheel cannot immediately follow because of its inertia. Thus any oscillating motion of the rotor is quickly stopped. See Fig. 35–11.

It should be noted that although the Selsyn transmitter is customarily called a generator and the receiver a motor, this terminology is not strictly correct. For the one does not generate electrical energy, nor does the other transform it into mechanical motion. In the Selsyn system, neglecting friction losses, the mechanical power input to the transmitter is exactly equal to the mechanical power output of the receiver.

The system can be arranged for multiple operation in which several identical Selsyn motors or receivers, placed in parallel at

several locations, may be operated by a single Selsyn generator or transmitter. In this case the transmitter is built with a larger frame and of sufficient capacity to supply normal torque to each of the receivers.

To prevent dangerous circulating currents in the stator circuits of the system, the currents in all rotor circuits should be in phase and the rotor impressed voltages should have the same value. (The torque output of the system is proportional to the square of the excitation voltage on the rotors.) As previously stated, the rotors of the Selsyn units are all connected to the same a-c source. When the units are separated by considerable distance, long lines must be run to the rotor of the distant Selsyn unit and the *IR* and *IX* drops reduce the rotor voltage and cause a phase displacement in the current. To compensate for the *IR* drop, an auto transformer is placed in the circuit to the distant rotor (Fig. 37–11), and correction for phase displacement is obtained by placing a balancing impedance in the rotor circuit of the near Selsyn unit.

The Differential Selsyn. A series type of Selsyn systems is obtained by use of one or more **differential** Selsyn units. This unit differs from the standard unit only in the shape of the rotor punchings which are similar to those in the induction motor. The rotor carries a distributed, three-circuit, Y-connected winding, with the same number of turns per circuit as the stator, brought out to three collector rings. The entire unit is similar to a three-phase wound-rotor induction motor, except that three-phase voltages and currents do not exist. It is essentially a one-to-one ratio, single-phase transformer in which one set of windings is free to rotate.

The series system, in its simplest form, consists of a single differential unit connected between a Selsyn generator and a Selsyn motor, as in Fig. 38–11. The stator of the generator unit is connected in the same sequence, s_1 to s_1, s_2 to s_2, and s_3 to s_3, to the stator of the differential, and the rotor of the differential is connected, also in the same sequence, to the stator of the motor unit. From the figure it is apparent that the windings of the differential unit are excited from the other two Selsyn units rather than directly from the outside source. The voltages impressed on, and the currents flowing in the differential stator windings, and the direction of the resultant field, must be the same as that in the generator unit. Similarly, the voltages impressed on the

differential rotor windings and their resultant field must be the same as that in the motor unit. When the two fields or fluxes in the differential unit are in alignment, both the impressed and induced voltages in the differential rotor are of the same value, but in opposite directions. No current flows in the windings of the differential unit, therefore, no torque is developed and the system is in equilibrium. If the relative directions of these fields in the differential is changed, a circulating current is set up in the system developing torque. This is produced by moving the rotor of any one of the three Selsyn units. If the position of any one is fixed and a second is turned through a given angle, the third, being free to turn, will rotate through the same angle. Therefore,

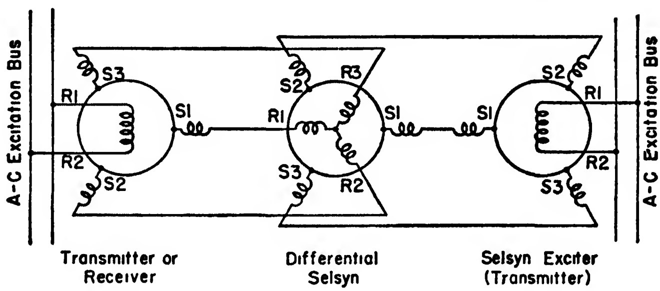


FIG. 38–11. Series differential Selsyn system.

in the system of Fig. 38–11, the generator and motor are electrically connected through the differential Selsyn exactly as the drive wheels of a motor vehicle are mechanically connected through differential gearing. The differential unit thus provides a means of controlling the position of a third Selsyn unit from either of the other two.

One of the advantages of the Selsyn system over other methods of transmitting mechanical motion is that, after an interruption of power, the system automatically resets itself. That is, it is self synchronous. In recent years the Selsyn has assumed a position of growing importance in a wide variety of applications as a means of remote signalling, remote control and the transmission of synchronous power. In the latter case the units are rotated continuously at various speeds, their size depending upon the amount of energy to be transmitted. These systems may be operated manually or automatically.

The first notable application of the Selsyn system was in the operation of the Panama Canal locks. These are operated from

a central station, Selsyn units being used to indicate the positions of the lock gates, fender chains, intake and outlet valves and the water level, etc. The system is used for signalling from the bridge to the engine of a ship; for navigation in conjunction with the gyroscope compass, etc. It has recently been used by the U. S. Army and Navy in range finding. The position of the target may be located by radar or other means, which is connected through Selsyn units to guns, searchlights, etc., and are thus automatically trained on the target.

SUMMARY OF CHAPTER XI

THE SYNCHRONOUS MOTOR has the same construction as an alternating-current generator, and is usually of the revolving field type. If the rotor of such a machine is revolving at synchronous speed, and the alternating current is maintained in the armature, a torque is exerted by the interaction between the rotor fields and the revolving fields produced by the armature current, which keeps the speed up to synchronism until the machine is greatly overloaded.

THE ARMATURE CURRENT is proportional to the resultant of the impressed emf and the counter emf induced in the armature windings by the revolving field.

THE COUNTER EMF depends in value upon the field excitation, since the speed of the field poles is constant at all loads. It does not, therefore, change in value as the load on the motor is increased. It merely comes more nearly into phase with the impressed emf so that the resultant voltage is increased and a greater armature current results.

THE ROTOR FALLS BACK as the load is increased and produces the necessary change in phase relation between the counter and the impressed emf in order to increase the resultant emf. The position which the field poles occupy in space with regard to the revolving poles set up by the armature current is called the synchronous position.

THE MAXIMUM OR "PULL-OUT" LOAD occurs when the rotor has dropped back through an angle the tangent of which is $\frac{X}{R}$. In this expression, X is the synchronous reactance of the armature and R the effective resistance. In modern machines the value of $\frac{X}{R}$ is usually between 50 and 100 and thus the pull-out angle is approximately 90°. The normal load of such machines is usually about $\frac{1}{2}$ or $\frac{1}{3}$ of the maximum load, the machine being unstable in the neighborhood of maximum load.

VARYING THE FIELD EXCITATION changes the phase relation between the impressed voltage and the armature current, — a weak field causing the armature current to lag the impressed voltage and a strong current to lead it. For a given load, the field current which produces unity power factor results in the lowest armature current.

Any reactive component in the armature is approximately proportional to the amount of under- or over-excitation.

"V"-CURVES result when, for various loads, the line currents, are plotted against field currents or counter emfs.

HUNTING is the term applied to an oscillating motion of the rotor, back and forth across the synchronous position. It may be caused by a sudden increase of load on the rotor or by the pulsation of the prime movers, which produce an uneven frequency. Hunting may become great enough to pull the motor out of step, if it is not stopped or damped. Hunting is effectively prevented by damping windings inserted in the pole faces of the motor.

A LARGE SYNCHRONOUS MOTOR HAS OF ITSELF NO STARTING TORQUE. It may be brought up to synchronous speed by another motor and then thrown on the line, or it may be started as an induction motor by means of the short-circuited damper windings in the pole faces. In this case the field is not excited until the speed is almost at the "pull-in" point. Modification of the field and armature windings, may be incorporated into the design, which will make it possible to start the motor, especially under no load or a light load.

THE SYNCHRONOUS-INDUCTION MOTOR is essentially a wound-rotor slip-ring induction motor. It is expensive and is not adapted for power-factor correction.

THE SYNCHRONOUS CONDENSER is an over-excited synchronous motor used at the receiving end of a transmission line to improve the power factor of the line. This reduces the current in the line, regulates the line voltage and decreases the power that has to be delivered by the generators. In new installations static capacitors are largely replacing it.

VERY SMALL CLOCK-OR-TIMING MOTORS run without field excitation as synchronous motors. The telechron motor is shaded-pole having a two-pole laminated frame with an a-c exciting coil. The rotor consists of two or more hardened steel rings with cross bars. It is self starting. Other types consist essentially of a two-pole laminated stator and an inner rotor having an even number of salient poles. These must be spun by hand to get them up to synchronous speed. Many of them operate at subsynchronous speed.

THE SELSYN MOTOR is a small self synchronizing device used in the electrical transmission of angular motion between two or more remote appliances which can not readily be mechanically connected. The system is called a "Synchro-Mechanism." When the "generator" rotor is moved to a certain position with respect to its stator, current is sent through the connecting transmission line and the rotor of the so-called motor moves to the same position with respect to its stator. A Selsyn system indicating the position of the lock gates, fender chains, intake and outlet valves, water level, etc., is used to "lock" the ships through the Panama Canal.

PROBLEMS ON CHAPTER XI

Prob. 23–11. The following data may be taken as typical for a synchronous motor:

Rated capacity	2500 kva.
Normal voltage (between terminals)	2300 volts
Connection	Star
Armature resistance (per phase)	0.022 ohm
Field turns	1800 turns
Normal field current	4.35 amp.
Synchronous impedance per phase	0.92 ohm

Determine the counter e.m.f. at full load, unity power-factor and normal voltage.

Prob. 24–11. Compute the counter emf, field current, and reactive current of the motor in Prob. 23–11 at half load (kva.) and normal voltage, when there is a leading power-factor of 80 per cent at the terminals. Assume the field flux to be proportional to the field current, and the kw. input to be the same in all three cases.

(*b*) When the field current of the motor in (*a*) is increased to 6 amperes, what will be the kilovolt-ampere intake and the power-factor?

(*c*) If the field current of the motor in (*a*) is reduced to 3.5 amperes, what will be the kilovolt-ampere input and the power-factor?

Prob. 25–11. How much reactive power will the motor of Prob. 24(*c*) draw from the line?

Prob. 26–11. (*a*) What must be the field current of the motor in Prob. 24 in order that the motor draw 2000 kva. corrective leading reactive power from the line?

(*b*) How much total effective power (input) is then available for a motor load, when drawing 1000 kw. as in that problem? Note that to load (by power component) while still correcting 2000 kva. will require some additional field excitation.

Prob. 27–11. (*a*) Assuming that the field excitation of the motor of Prob. 23–11 can be increased to 150 per cent of the normal value, what will be the " pull-out " load at this excitation?

(*b*) What percentage is the rated full load of the pull-out load?

Prob. 28–11. What would be the " pull-out " load (output per phase) on a synchronous motor if the field strength were such that the counter e.m.f. per phase was 3000 volts when the impressed e.m.f. was 3300 volts. Armature resistance is 0.15 ohm per phase, synchronous reactance, 7.5 ohms per phase. Plot a curve between output and angle $X°$ (Table A, Chap. XI), using as values of $X°$, 70°, 75°, 80°, 82°, 84°, 86°, 88°, 90°, 92°, 94°, 96°, 98°, 100°, 105°.

Prob. 29–11. What input does each phase of the motor in Prob. 28 take when the power-factor is 90 per cent lagging with the same field excitation as in Prob. 28?

Prob. 30–11. What will be the synchronous position of the rotor of the motor in Prob. 29 under the conditions of that problem?

Prob. 31–11. What would be the pull-out load per phase on the synchronous motor of Prob. 28–11, if the field strength were such that the counter e.m.f. became 2000 volts? Other conditions as in Prob. 28–11.

Prob. 32–11. Repeat Prob. 31–11 for a field strength producing 4400 volts induced e.m.f.

Prob. 33–11. The impressed voltage on the motor of Prob. 23–11 is increased to 3000 volts.

(*a*) What value must the field current have in order that the armature current shall not exceed the normal full-load value, when operating at unity-power factor?

(*b*) What will be the power intake of the motor in kilowatts and in kilovolt-amperes at full load?

CHAPTER XII

SHORT TRANSMISSION AND DISTRIBUTING LINES

The fundamental ideas in the transmission and distribution of electrical power may best be understood by considering an actual installation. Thus let us suppose that a town can utilize 1200 kw at 0.80 power-factor for 3000 hours per year, and that it is situated ten miles from waterpower. The electric generating plant must be located at the waterfall, and electric energy must be transmitted ten miles to the town. Our problem then is: What is the most practicable electric transmission system to install and what are the main characteristics and peculiarities of the system? We will work this out as Example 1.

1–12. Most Economical Size of Wire, Single-phase Line.

1. Voltage. The choice of voltage is more or less arbitrary — the engineer always trying to use as high a voltage as conditions permit. Better methods of insulating both line and machines are continually raising the voltage at which power may be most economically transmitted over given distances. The expense of insulating both the line and the apparatus connected to it increases rapidly as we choose higher voltages. Of course, the cost of the copper in the circuit goes down rapidly at the same time, as the higher voltages enable us to transmit the same power with the same loss over **smaller wires.** Above a certain voltage, the difficulty and expense of insulating increases faster than the cost of the copper decreases, so that the total cost of transmission would be increased by raising the voltage any further. The limiting pressure at which this occurs is being continually raised by improvements in the manufacture of insulation, which make good insulation cheaper, or insulation of a given cost much stronger.

At the end of the year 1948, the highest line voltage was 287,000 volts. This is the potential at the sending end of the 266-mile line from Hoover Dam to Los Angeles. The characteristics of this installation will be discussed in the following chapter on "Long Transmission Lines." There are definite indications however that the transmission voltage may soon go still higher.

In this country research is now being conducted on an experimental alternating-current 500,000-volt line. In Sweden, preliminary engineering is being done on a 350–380 kilovolt line, and reports are coming to us concerning a proposed 600,000 volt project. It is understood that the plan proposed is to generate a-c power. The voltage is to be stepped up, converted to direct voltage and transmitted at 600,000 volts. At the receiving end it will be reconverted to alternating voltage and stepped down for local distribution.

An old " thumb-rule " states that the most economical voltage for transmission is about **" 1000 volts per mile of line."*** Experience has shown this rule to be a surprisingly close approximation. Thus a two-mile line would be constructed to operate at approximately 2000 volts, a ten-mile line at 10,000 volts, etc. The following voltages have become standardized by practice:

2200 to 2400, 6000 to 6900, 11,000, 13,200 to 13,800, 22,000 to 24,000, 33,000, 44,000, 66,000, 88,000, 110,000, 140,000 to 150,000.

Accordingly, we may choose 11,000 volts as a practicable pressure at which to transmit power over the 10 miles required by this problem.

2. Alternating or Direct Current. In America there are practically no industrial direct-current systems of a voltage higher than 600 volts. This fact leads to the choice of alternating current.

3. Frequency. There are at present three standard frequencies in this country, 25, 50 and 60 cycles. Main-line electric railways operate on 25 cycles. For homes and industrial plants 60-cycle power is generally preferred, because a great part of the power in these installations is for illumination in which freedom from flicker is an all-important requirement. Were it not for the fact

* This rule apparently is based on the fact that 1000 volts per mile is the most economical voltage, allowing 2 per cent line drop, 10 per cent interest and depreciation, 3000 hours use per year, and line of copper wire at a cost per pound of 14 times the cost of power per kw-hr. A voltage-distance table of actual installations taken at random is given below.

Distance, miles	Voltage	Distance, miles	Voltage	Distance, miles	Voltage
7	11,000	8	11,000	6	11,000
44	50,000	29	50,000	84	50,000
144	100,000	108	100,000	266	287,000

For a full discussion of the choice of voltage see Still, " Overhead Electric Power Transmission."

that 25-cycle power produces a pronounced flicker in ordinary luminants, probably 25-cycle installations would be the standard of the country.

Twenty-five cycles have the following advantages.

(1) Lower iron losses in generators, motors and transformers, as explained in Chapters VII, VIII and IX

(2) Lower charging-current loss as explained in Chap. XIII

(3) Lower speed for machines having a given number of poles as explained in Chap. VII. In certain installations this later advantage is so important that an auxiliary machine called a " frequency-changer " is often installed to lower the frequency of the power delivered to 25 cycles.

4. Size of Line Wire. Single-phase. Having decided to use an alternating-current system of 60 cycles and 11,000 volts, the size of the conductors will depend upon whether we employ a single-phase or a three-phase system. It will be shown later that the three-phase system possesses a great advantage over the single-phase, and this would probably be chosen. However, for the sake of completeness, we will consider the single-phase first, and then take up the three-phase system and compare the two.

The choice of voltage, frequency and size of wire is determined primarily by the amount of capital that must be invested in the plant to install and to operate it. This is most clearly seen in the method in common use to determine the size of wire. **The conductor should always be of such a size that it results in the lowest total annual expense.** This annual expense consists of two items:

(1) **Fixed charges,** which include

(*a*) Interest on the money invested in the line.

(*b*) Taxes and depreciation in the value of the line.

(2) **Value of Energy lost in line.**

Perhaps the clearest plan is to tabulate, as in Table I–12, the cost of transmitting by several sizes of wire, under identical conditions, and to pick out the size showing the lowest total cost.

Our problem, then, is to select the most economical size of copper wire to transmit 1200 kw, single-phase, 80 per cent power-factor, at 11,000 volts at the receiving end. This power is to be delivered steadily for 3000 hours per year. These figures are fairly representative of modern practice.

The length of the line is 10 miles, the length of wire required, 20 miles.

Let us start with a No. 000 gauge, stranded copper wire, and compute the cost. According to the Wire Table I, Appendix B, 20 miles of this wire would weigh

$$20 \times 2740 = 54{,}800 \text{ lbs.}$$

The cost of wire for a transmission line (except for very large sizes) is proportional to the weight of the conductor. The cost per pound for installed wire would depend upon market price of metals, which fluctuate widely, and also upon cost of transportation and of labor. Assume that conditions are such that 20 cents per pound installed would be a fair average price.

The cost of No. 000 conductor would then be

$$54{,}800 \times \$0.20 = \$10{,}960.$$

On this sum we must allow yearly interest at 5 per cent; in addition, the annual taxes and depreciation would approximate 3.5 per cent.

The **fixed charges** would then amount to $3.5 + 5 = 8.5$ per cent.

$$0.085 \times \$10{,}960 = \$932.00.$$

The power loss in the line would be the I^2R loss of the line.

The apparent power delivered would be

$$\frac{1200}{0.8} = 1500 \text{ kva.}$$

The current (with pressure of 11,000 volts at the load end of the line) would be:

$$\frac{1{,}500{,}000}{11{,}000} = 136.4 \text{ amp.}$$

The resistance (from Table I, Appendix B) of 20 miles of No. 000 wire

$$\begin{aligned} &= 20 \times 0.328 = 6.56 \text{ ohms} \\ \text{Power loss} &= I^2R \\ &= 136.4^2 \times 6.56 \\ &= 121{,}900 \text{ watts} \\ &= 121.9 \text{ kw.} \end{aligned}$$

The total energy lost per year would then amount to

$$121.9 \times 3000 = 366{,}000 \text{ kw-hr.}$$

At a conservative estimate of 1 cent per kw-hr, the **energy lost in the line per year would cost**

$$366{,}000 \times \$0.01 = \$3660.00.$$

The **total cost per year due to the transmission line would be the sum of the fixed charges and the line loss.**

$$\$932.00 + \$3660 = \$4592.00.$$

If we compute in the same way the annual cost of a No. 0000 transmission line, we find (see Table I) that, while the cable costs more and thus the interest, taxes and depreciation are more every year, the cost of the power lost in the line is so much less that the total annual cost falls to $4069.

TABLE I–12

Relative Costs of Transmitting Power over Conductors of Different Sizes, Other Conditions Being the Same

	No. 000 stranded	No. 0000 stranded	250,000 cir. mils.	300,000 cir. mils.	350,000 cir. mils.	400,000 cir. mils.	450,000 cir. mils.
	$	$	$	$	$	$	$
Cost of 20 miles of wire at 20¢ per lb........	10,960	13,880	16,360	19,560	22,960	26,280	29,880
Annual fixed charges at 8.5% of wire cost....	931	1,179	1,390	1,663	1,950	2,233	2,540
Annual cost of energy lost in line at 1¢ per kw-hr.............	3,660	2,890	2,416	2,064	1,774	1,533	1,365
Total annual cost of transmission	4,591	4,069	3,806	3,727	**3,724**	3,766	3,905

As we continue to increase the size of the wire, the fixed charges continue to increase and the cost of power lost in the line continues to decrease. However, the fixed charges increase more rapidly than the cost of line loss decreases, so that the total expense does not continue to decrease indefinitely, but reaches a minimum value at a definite size of wire. The total annual cost of any wire larger than this size will be greater.

According to Table I–12 the wire which shows the smallest annual cost, $3724, is one of 350,000 cir. mils area. The next smaller, 300,000 cir. mils, would cost $3727 per year, while the next size larger, 400,000 cir. mils, would cost $3766 per year.

It will also be seen from a study of the table that the lowest total annual expense always occurs when the fixed charges on the line become most nearly equal to the yearly cost of energy lost in

the line. It will be noted that the fixed charges exceed the cost of energy lost for all sizes larger than the most economical; but for all sizes smaller than the most economical, the cost of lost energy exceeds the fixed charges. By making use of the fact that the fixed charges should about equal the cost of lost energy we may arrive at the most economical size without many trials.

The table shows that there is no practical need of very precise computations for the determination of the most economical size of wire. The choice of any one of two or three sizes in the vicinity of the minimum cost will not result in any appreciable difference in the total cost. However, as we go farther away on either side of the most economical size, the cost begins to increase rapidly. It is worth while, therefore, to make the above approximate calculations. Furthermore, the above solution, however precisely the mathematics may be done, is only approximate, there being other items of cost to be considered in the final selection of size of wire besides the cost of installed cable and the cost of lost energy. There are also the annual charges on the additional cost of generating and transforming equipment required to handle the power that will be lost in the line, and the fixed charges on the additional cost of poles and insulators due to change in the size of wires. The cost per pound of the wire itself often becomes more for sizes larger than 500,000 cir. mils because each cable is split up into two or more parallel cables for convenience in handling and because heavier pole construction may be required.

The **form of the load curve** also should be taken into account when computing the line loss. The energy annually lost in the line is proportional to the "square root of the mean square" of current delivered throughout the year. For the same average power delivered, therefore, the line loss may vary between wide limits, as we change the daily and annual load curves or schedules.*

Prob. 1–12. If it were decided to transmit the power in the above example at 22,000 volts, what would be the most economical size of wire? Copper conductor in place costs 20 cents per pound. Energy costs 1 cent per kw-hr and all other data the same as in the above example.

Prob. 2–12. If the line in Prob. 1 were to be constructed of aluminum, what would be the most economical size? Compare total annual cost of transmission by the aluminum line, with the corresponding cost

* For a method of determining the annual line loss, see "Handbook for Electrical Engineers, Electric Power": John Wiley & Sons.

by the copper line of Prob. 1. Aluminum weighs 0.304 as much as copper, has 1.61 times the mil-foot resistance and costs 1.7 times as much, say 35 cents per pound, installed.

Prob. 3–12. (*a*) What per cent of the power transmitted is the power lost in the line in the example in the text, using the most economical size of conductor?

(*b*) In Prob. 1?

(*c*) In Prob. 2?

2–12. Voltage Drop in Line. Line Regulation. Before we can definitely decide to install a line with conductors of a given size, it is always well to see that the voltage variation at the receiving end is not excessive between a no-load and a full-load condition. When a load is put on a line the voltage across the receiving end falls, on account of the pressure consumed in overcoming the voltage reactions along the line due to the current. It is necessary to define some standard way of stating the magnitude of this loss of voltage. The method is to determine the voltage at the receiving end with non-inductive full load on the line. Then determine the voltage when the load is removed, meanwhile keeping the impressed voltage constant at the sending end of the line. The difference between these voltages is called the **regulation of the line.** The percentage regulation is the percentage which the change in voltage is of the normal rated voltage at the receiving end.

Thus, per cent line regulation

$$= \frac{\text{(No-load volts)} - \text{(full-load volts)}}{\text{(Full-load volts)}} \times 100 \text{ per cent}$$

(at unity power-factor).

Unless otherwise stated, the **load must be non-inductive** and the **voltage at the sending end** of the line must **remain unchanged.**

Good regulation for a power load ranges between 5 and 10 per cent. For a lighting load it should never exceed 5 per cent and does not usually exceed 3 per cent.

Let us test the regulation of the 10-mile line of Example 1, using the most economical copper wire, — 350,000 cir. mils.

If the power were direct-current, the process would be simple, as follows:

$$\text{Voltage at full load} \qquad = 11{,}000 \text{ volts.}$$

$$\text{Full-load current } \frac{1{,}200{,}000}{11{,}000} = 109 \text{ amperes.}$$

Resistance of line $= 3.18$ ohms $= 20 \times 0.159$.
(See Table I, Appendix B.)

Line drop $= 3.18 \times 109$.
$= 347$ volts.

Volts at sending end $= 11{,}000 + 347 = 11{,}347$ volts.

Volts at load end, at no load $= 11{,}347$.

Line regulation for direct current

$$= \frac{11{,}347 - 11{,}000}{11{,}000} = 3.2\%.$$

This line, we see, would have a sufficiently good regulation for a direct-current system.

We will now determine the regulation of the same line when it carries alternating current. Alternating current has to overcome, not only the resistance of the line wires, but also their inductive reactance. Thus the line drop in a circuit carrying alternating current is usually greater than if the same circuit were carrying direct current, and therefore the regulation would be poorer. It would be good practice to string the wires of a 11,000-volt circuit about 30 inches apart.* From Table III, Appendix B, we find that for 350,000 cir. mil cables strung 30 inches apart:

The reactance per mile of single wire $= 0.591$ ohms.

The reactance of 20 miles of cable $= 20 \times 0.591$
$= 11.82$ ohms.

The current for a non-inductive full load $= \dfrac{1{,}200{,}000}{11{,}000}$
$= 109$ amperes.

The reactance drop then $= E_x$
$= 11.82 \times 109$
$= 1290$ volts.

There would thus be a line drop of 347 volts, as in direct-current power, due to the resistance of the wires, and a line drop of 1290 volts peculiar to alternating-current power, due to the reactance of the line. The sending voltage must therefore be great enough to supply these line drops and leave 11,000 volts at the load end. Since the reactance drop differs in phase from the resistance drop

* See Curve (I) in Appendix B for good practice as to distance between wires.

by 90°, it is necessary to add vectorially the load voltage, the resistance drop, and the reactance drop of the line, to find what the sending voltage must be in order to supply them.

Construct the topographic vector diagram of Fig. 1–12.

The voltage vector OE of the load will lie along the current vector OI, since the voltage and current of the load must be in phase, the power-factor being unity. The vector of the resistance drop, ER, must also lie along the current vector OI, as resistance drop is always in phase with the current. The vector RX of the

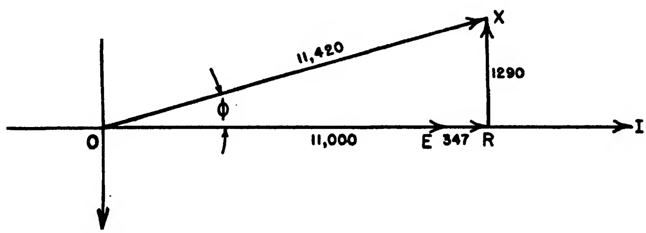


FIG. 1–12. Topographic diagram for finding the generator voltage OX to supply a load voltage of OE over a line having a resistance drop of ER and a reactance drop of RX. The load has unity power-factor.

emf required to overcome the reactance voltage, must lead the current by 90°. The vector sum of these voltages is the line OX, and can be computed from the equation,

$$\begin{aligned}\overline{OX} &= \sqrt{(\overline{OE} + \overline{ER})^2 + \overline{RX}^2} \\ &= \sqrt{11{,}347^2 + 1290^2} \\ &= 11{,}420 \text{ volts.}\end{aligned}$$

The voltage at the sending end must therefore be 11,420 volts and the voltage at the receiving end would of course rise to this value when the load was taken off, there being, at that time, no current and therefore no reacting voltages in the line.

$$\begin{aligned}\text{The regulation} &= \frac{11{,}420 - 11{,}000}{11{,}000} \\ &= 3.82 \text{ per cent.}\end{aligned}$$

This is slightly higher than the value 3.2 per cent, which we obtained for regulation when the line carried direct current, but it is very good regulation for a line carrying alternating current.

3–12. Line Regulation at a Power-factor Less than Unity. But the load which we specified for this line in the former Example

had a power-factor of 0.80 which is about the usual power-factor of an alternating-current load used for industrial purposes.

To find the voltage variation and what is sometimes called the "regulation at 0.80 power-factor" we proceed as follows:

$$\text{The apparent power} = \frac{1200}{0.80} = 1500 \text{ kva.}$$

$$\text{The current (single phase)} = \frac{1{,}500{,}000}{11{,}000} = 136.4 \text{ amp.}$$

$$\text{The resistance drop in the line} = 136.4 \times 3.18 = 434 \text{ volts.}$$

$$\text{The reactance drop in the line} = 136.4 \times 11.82 = 1610 \text{ volts.}$$

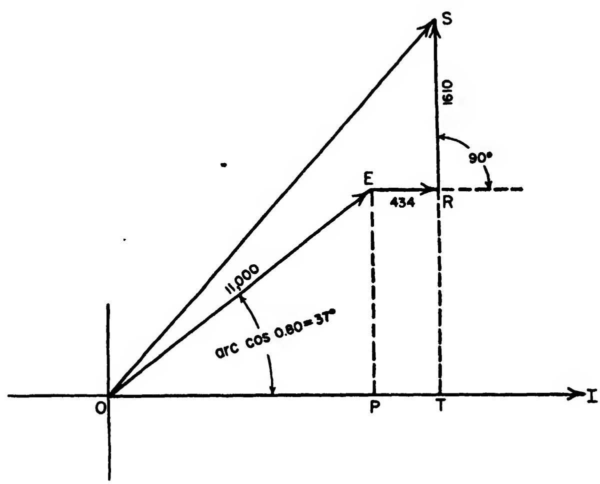


FIG. 2–12. The vector OS represents the sending voltage for a load voltage OE, over a line having a resistance voltage of ER and a reactance voltage RS, when the power-factor of the load is 80 per cent.

Construct the topographic vector diagram Fig. 2–12.

To represent the 11,000 volts at the load draw the vector OE at 37° lead to the current, because on an inductive load of 80 per cent power-factor the voltage leads the current 37°.

To represent the 434 volts which the sending end must supply in order to overcome the resistance of the line, draw the vector ER parallel to the current I, since the voltage consumed in overcoming resistance only is always in phase with the current.

To represent the 1610 volts which the sending end must supply in order to overcome the reactance of the line, draw the vector

RS 90° ahead of ER, because the voltage consumed in overcoming inductive reactance is always 90° ahead of the current.

The total voltage at the sending end of the line must therefore be OS, the vector sum of OE (the voltage at the load), ER (the resistance drop) and RS (the reactance drop).

When the load is thrown off, the voltage at the receiving end will rise to the same value as the voltage at the sending end.

Thus OS also represents the no-load voltage at the receiving end. To find the value of OS, draw the construction lines EP and RT.

$$\begin{aligned} OP &= 11{,}000 \cos 37° = 8800 \\ RT &= 11{,}000 \sin 37° = 6600. \\ OS &= \sqrt{(OP + 434)^2 + (RT + 1610)^2} \\ &= \sqrt{152{,}570{,}000} \\ &= 12{,}351 \text{ volts.} \end{aligned}$$

The no-load voltage thus equals 12,351 volts.

$$\begin{aligned} \text{The regulation at 0.80 power-factor} &= \frac{12{,}351 - 11{,}000}{11{,}000} \\ &= 12.3 \text{ per cent.} \end{aligned}$$

Note that the regulation of the line when the load had the commercial power-factor of 0.80 was very much poorer, being more than three times as great as the regulation at unity power-factor. There are two reasons for this. First, in order to transmit at the same voltage the same quantity of power at a low power-factor as at unity power-factor, a larger current must flow. This means both greater resistance drop and greater reactance drop. In this case the resistance drop was increased from 347 to 434 volts, and the reactance drop from 1290 to 1610 volts.

In the second place, it will be seen from a comparison of Figs. 1–12 and 2–12 that the reactance drop and the voltage of the load are more nearly in phase and therefore their vector sum is more nearly equal to their arithmetical sum when the power-factor is lower. Thus, not only is the reactance drop greater at a low power-factor, but a greater fraction of it is in phase with the line voltage and, therefore, tends to increase it much more than at unity power-factor.

This alone shows the desirability of having a load in which the

current lags as little as possible behind the voltage, especially when the reactance of the line is greater than its resistance. In fact, a slightly leading current is generally advantageous.

Prob. 4–12. Find the regulation of the line in the above example when the power-factor of the load is 70 per cent with the same kilowatt load.

Prob. 5–12. What would be the line regulation of Prob. 4 if the size of the line wire were increased to 450,000 cir. mils?

Prob. 6–12. What would be the line regulation of Prob. 4 if we decreased the size of the conductor to 250,000 cir. mils?

4–12. Three-phase, Three-wire System. Cost. Let us now consider how the cost and the regulation of this 10-mile line would be affected if we installed a three-phase, three-wire system instead of a single-phase system with its two conductors. We will, of course, compute the line cost and regulation on the same basis as for the single-phase line, that is, the transmission of 1200 kw, to a distance of 10 miles, with 11,000 volts between wires and a power-factor of 80 per cent at the end of the line with 3000 hr of use at full load annually.

Apparent power as in single-phase system

$$= \frac{1200}{0.80} = 1500 \text{ kva.}$$

Apparent power in three-phase system is found according to the equation

$$P_a = \sqrt{3}\,EI;$$

where P_a = apparent power in volt-amperes.

E = effective voltage between conductors, in volts.

I = effective current along each conductor, in amperes.

Therefore $1{,}500{,}000 = \sqrt{3} \times 11{,}000 \times I.$

$$I = \frac{1{,}500{,}000}{1.73 \times 11{,}000}$$

$$= 78.8 \text{ amperes.}$$

Or we may use the equation

$$P = \sqrt{3}\,EI \cos\theta;$$

where
$$P = \text{effective power, in watts.}$$
$$E = \text{effective voltage between conductors, in volts.}$$
$$I = \text{effective current along each conductor, in amperes.}$$
$$\cos\theta = \text{power-factor of the load.}$$

Thus
$$1{,}200{,}000 = \sqrt{3} \times 11{,}000 \times I \times 0.80.$$
$$I = \frac{1{,}200{,}000}{1.73 \times 11{,}000 \times 0.80}$$
$$= 78.8 \text{ amperes.}$$

We now compute, as in the case of a single-phase line, the total annual cost of the line when constructed of conductors of different sizes. Starting with No. 0 stranded copper conductor, we find the cost of line as follows:

One mile of No. 0 weighs 1730 lb.

A ten-mile three-wire system would weigh

$$30 \times 1730 = 51{,}900 \text{ lb.}$$

At 20 cents per pound, the line would cost

$$\$0.20 \times 51{,}900 = \$10{,}380.$$

Annual fixed charges at 8.5 per cent would amount to

$$\$10{,}380 \times 0.085 = \$882.$$

The resistance of one conductor of No. 0 wire, as per Table I, Appendix B, would be

$$10 \times 0.518 = 5.18 \text{ ohms.}$$

Each line wire in the three-phase system carries 78.8 amp, therefore the I^2R loss per conductor would be

$$78.8 \times 78.8 \times 5.18 = 32{,}200 \text{ watts.}$$
$$= 32.2 \text{ kw.}$$

The loss in the three wires would be

$$3 \times 32.2 = 96.6 \text{ kw.}$$

For a year of 3000 hours, the total energy loss would be

$$3000 \times 96.6 = 289{,}800 \text{ kw-hr.}$$

At 1 cent per kw-hr, this would cost

$$289{,}800 \times 0.01 = \$2898.$$

Therefore the total yearly cost of the transmission equals

$$\$2898 + \$882 = \$3780.$$

Since the cost of lost energy is greater than the fixed charges, we compute the cost of larger wires in order to find the size of conductor which produces the lowest total annual cost. Setting these down as in Table II–12 we see that the most economical

TABLE II–12 — Relative Cost of Three-phase, Three-wire Transmission for Various Sizes of Conductors

	No. 0 stranded	No. 00	No. 000	No. 0000	250,000 cir. mils.	300,000 cir. mils.	350,000 cir. mils.
	$	$	$	$	$	$	$
Cost of 30 miles of wire at 20¢ per lb........	10,380	13,140	16,440	20,820	24,560	29,350	34,440
Fixed annual charges at 8.5% of wire cost....	882	1,117	1,398	1,770	2,090	2,490	2,930
Annual cost of energy lost in line at 1¢ per kw-hr.............	2,898	2,299	1,834	1,450	1,213	1,033	886
Total annual cost of transmission........	3,780	3,416	3,232	**3,220**	3,303	3,523	3,816

conductor would be No. 0000. The most economical size for a two-wire system under the same conditions was found to be 350,000 cir. mils. (See Table I–12.)

Three conductors of No. 0000 wire at an annual expense of \$3220 will conduct as much power as two conductors of 350,000 cir. mils at a yearly expense of \$3724. There is then a decided yearly saving in using a three-phase system instead of a single-phase.

Prob. 7–12. What would be the most economical size of conductor to use in the three-phase system of the above example if the price of copper were 40 cents per pound installed?

Prob. 8–12. If the cost of energy were 2 cents per kw-hr, what would be the most economical size of conductor to use in the above example?

Prob. 9–12. If a three-phase system were installed in Prob. 1–12 what would be the most economical size of conductor?

Prob. 10–12. Compute Prob. 2–12, using a three-wire three-phase system.

5–12. Regulation of a Three-wire Three-phase System. We should always compute the regulation of a proposed three-phase installation, just as in the case of a single-phase line, in order to see whether or not the most economical wire produces too great a voltage variation at the different loads. For this purpose, it is much simpler always to consider the loads at the receiving end to be star-connected. If the loads are delta-connected, we have merely to consider the voltage between any conductor and an imaginary neutral as explained below. At present let us consider the load as star-connected at *M*, as in Fig. 3–12. The scheme is to

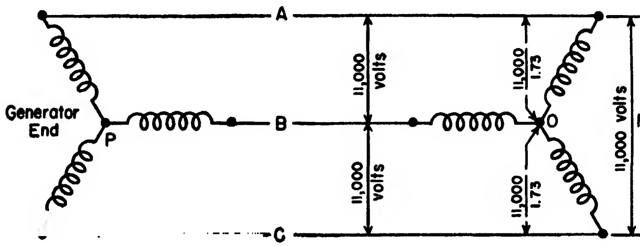


FIG. 3–12. Diagram of a star-connected generator station sending power over a three-wire line to a star-connected load.

compare the change in voltage across one phase (say *OA*) from no load to full load with the voltage at full load across the same phase (*OA*).

The full-load voltage between the line wires at the receiving end being 11,000 volts, the full-load voltage across any phase or coil of a star-connected load would be $\frac{11{,}000}{1.73} = 6360$ volts. Thus the full-load voltage across each of the coils *OA*, *OB*, and *OC* at the receiving end *M* would be 6360 volts.

Let us consider coil *OA* only. We found that the full-load line current at 80 per cent power-factor must be 78.8 amperes. In order to force this full-load current of 78.8 amperes through the coil *OA*, the voltage across the corresponding coil *PA* of the generator must be great enough to overcome the resistance and the reactance of the line wire *A*, and supply the 6360 volts across the coil *OA*. The voltage across the generator coil *PA* is thus the resultant of the voltage across the load coil *OA*, and the resistance voltage and the reactance voltage of the line-wire *A*. The resistance of the 10-mile line-wire *A*, size 0000 B. & S. gauge (see Table I, Appendix B), is 2.59 ohms. The pressure necessary

to overcome the resistance of the line is equal to

$$78.8 \times 2.59 = 204 \text{ volts.}$$

The reactance* of the 10-mile line-wire A with the wires spaced equidistant and 30 inches from one another as in Fig. 4–12 is equal to

$$10 \times 0.621 = 6.21. \quad \text{(See Table III, Appendix B.)}$$

The pressure consumed in overcoming the reactance is

$$78.8 \times 6.21 = 489 \text{ volts.}$$

Construct Fig. 5–12 similar to Fig. 2–12, with the exception that the voltages in Fig. 5–12 are those across coils rather than between line wires.

$$\begin{aligned} OS = E_{PE} &= \sqrt{(VN + 489)^2 + (OT + 204)^2}. \\ VN &= 6360 \sin 37° \\ &= 3816. \\ OT &= 6360 \cos 37° \\ &= 5088. \\ E_{PE} &= \sqrt{4305^2 + 5292} \\ &= 6823. \end{aligned}$$

Therefore E_{PE}, the voltage across the coil PA of the generator, would have to be 6823 volts when the full load is taken from the

* When the wires of a system are not spaced equidistant from one another the reactance of the middle wire will differ from that of the other two. To avoid this unbalanced reactance, the wires are usually **transposed** every five miles on a short line or every 10 to 40 miles on a long line, so that when the whole length of the line is considered, the average distance between them is the same. This average value is used when computing the reactance. Thus three wires might be strung one directly over the other with 36 inches separating each outside wire from the middle one. The average distance would then be

$$\frac{36 + 36 + 72}{3} = 48 \text{ inches,}$$

and this would be the value used in the table or formula for finding the reactance.

When the wires are **not** transposed the **equivalent distance** between them is used. Equivalent distance = $\sqrt[3]{\text{product of the three distances}}$. The equivalent distance in this case would equal $\sqrt[3]{36 \times 36 \times 72}$ or 45. **Unless it is otherwise stated, the lines mentioned in this book are to be considered as transposed.**

receiver end. When the load decreases to zero, the voltage across one phase or coil (as OA) at the receiving end (M) would equal the voltage across the corresponding coil of the generator. The no-load voltage across one phase thus would equal 6823 volts.

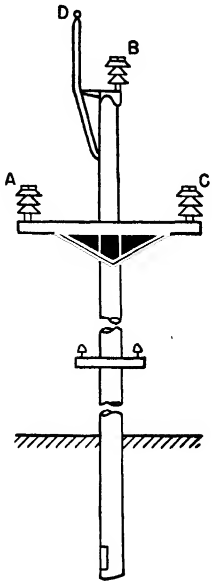


FIG. 4–12. The conductors A, B, and C are placed equidistant from one another. The wire D is placed above B in order to afford protection against lightning.

The change in voltage is equal to

$$6823 - 6360 = 463 \text{ volts.}$$

The regulation at 80 per cent power-factor

$$= \frac{463}{6360} = 7.27 \text{ per cent.}$$

Since the load is assumed to be balanced, the same change takes place across each coil of the load. The change across the line wires will thus be in the same ratio. Consequently the voltage regulation of this three-wire three-phase system is approximately 7 per cent, which is very satisfactory for a load of only 80 per cent power-factor. It will be remembered that the voltage regulation for a single-phase line under exactly similar conditions was 12.7 per cent. Thus the three-phase system is not only more economical but also has better regulation.

If the receiving or the sending end of the line is delta-connected, the method of computing the voltage regulation is similar to the above. Figure 6–12 represents such a system, in which both the receiving end M and the sending end G are delta-connected. We may consider each coil of each end to be made up of two coils of a star connection. Thus the coil TV of the receiving end may be imagined to be made up of the two coils TO and OV connected in star at the neutral point O. The voltage across either of these imaginary coils would be $\frac{11{,}000}{1.73} = 6360$ volts. This is called the "**voltage to neutral**" in a delta-connected arrangement. In determining the voltage regulation of a delta-connected system, the amount which this "voltage to neutral" changes when the load changes from full load to no load is found, and compared with

the "voltage to neutral" at full load, just as in the case of a star-connected arrangement.

The use of the Mershon diagram will save much mathematical work in computing the regulation of transmission lines. This

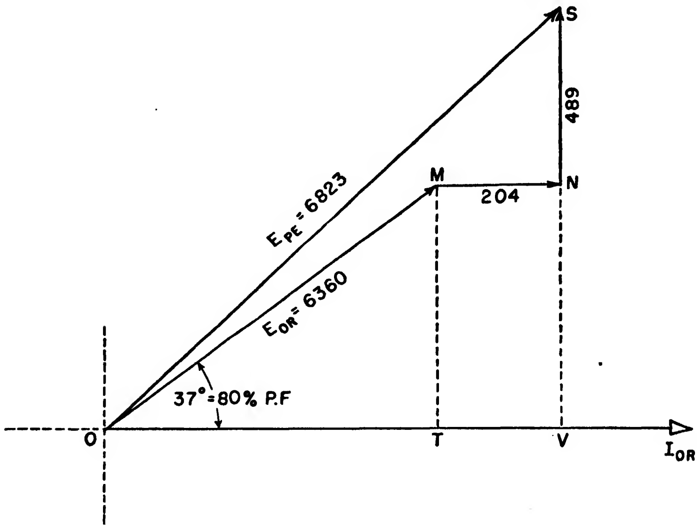


FIG. 5–12. Vector OS represents the voltage across one coil at the sending end of line in Fig. 3–12; OM, the voltage across one coil of the load; MN, the resistance drop of one wire; and NS, the reactance drop of one wire.

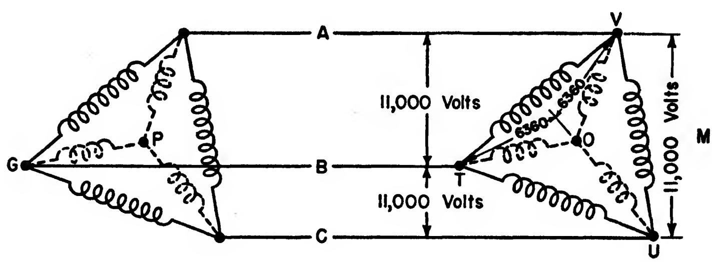


FIG. 6–12. Phase TV of the delta-connected load M may be considered to be made up of coils TO and OV. The phase VU may be considered to be made up of VO and OU, etc. The points P and O are thus the neutral points of star-connected coils.

diagram is equivalent to a large number of vector diagrams like those of Fig. 2–12 and 5–12.*

* See "Handbook for Electrical Engineers: Electric Power." John Wiley & Sons.

Prob. 11–12. Compute the voltage regulation of the three-phase system of Prob. 7–5, assuming the load to be star-connected.

Prob. 12–12. If the load on the system of Prob. 7–12 were delta-connected, what would the regulation be?

Prob. 13–12. Three-phase power is to be transmitted 14 miles. Power to be delivered, 4200 kw. Voltage at load, 33,000 volts; wires arranged in vertical plane, 50 inches apart; frequency, 25 cycles; power-factor of load, 85 per cent; size of wire, No. 0, B. & S. Compute the regulation if the load is delta-connected.

6–12. To Compute the Voltage at the Load. When an induction motor is started, it usually takes a much larger current than the full-load current, and always at a low power-factor. The voltage at the receiving end of a transmission line drops considerably under these conditions. This variation in terminal voltage can be determined as follows:

Example 1. In a three-phase transmission line, each wire of which has 2 ohms resistance and 3 ohms reactance, the full load is 2000 kw at 85 per cent power-factor. The full-load voltage at the receiving end is 6600 volts. Find the voltage of the load when several of the induction motors, which constitute part of the load, are starting simultaneously, and lower the power-factor to 75 per cent, at the same time increasing the load to 2100 kw.

The first step is to find the voltage at the sending end from data of normal conditions. This is done as in all previous examples of this chapter.

$$P = \sqrt{3}\, EI \cos\theta.$$

$$I = \frac{2{,}000{,}000}{1.73 \times 6600 \times 0.85}$$

$$= 206 \text{ amp per line-wire.}$$

$$\text{Resistance drop} = 2 \times 206$$

$$= 412 \text{ volts.}$$

$$\text{Reactance drop} = 3 \times 206$$

$$= 618 \text{ volts.}$$

$$\text{Voltage to neutral} = \frac{6600}{1.73} = 3810 \text{ volts.}$$

Construct a diagram as in Fig. 2–12 and 5–12 and solve for *OS*, the voltage to neutral at the sending end.

$$OS = 4490 \text{ volts.}$$

With the usual method of operation, a voltage to neutral of approximately 4490 volts at the sending end would be maintained under all conditions (unless a feeder voltage regulator is used; see Chap. VIII, Art. 38, 39).

The second step is to find what the voltage to neutral at the sending end would have to be, if the voltage at the receiving end were to remain 6600 volts, or 3810 volts to neutral, under the new conditions of load and power-factor.

$$P = \sqrt{3}\, EI \cos\theta.$$

$$I = \frac{2{,}100{,}000}{1.73 \times 6600 \times 0.75}$$

$$= 246 \text{ amperes.}$$

$$\text{Resistance drop} = 2 \times 246 = 492 \text{ volts.}$$

$$\text{Reactance drop} = 3 \times 246 = 738 \text{ volts.}$$

Construct a diagram as in Fig. 2–12 and 5–12 and find the value of OS, the voltage to neutral at the sending end.

$$OS = 4670 \text{ volts.}$$

Thus the voltage to neutral at the sending end would have to rise to 4670 volts in order to keep the voltage of the load up to 6600 volts (3810 volts to neutral) when the extra load at a low power-factor was thrown on.

But the conditions at the sending end are such that the voltage to neutral remains practically constant, 4490 volts at all loads. Therefore, if the sending voltage remains constant, the voltage at the receiving end must fall on account of the extra line drop between the generator and the load.

As a third step, we may then consider that the voltage to neutral of the sending end drops from 4670 volts to 4490 and compute the corresponding drop in the load voltage, from 6600 to ($\frac{4490}{4670}$ of 6600), or 6360 volts. As a check on this value we have merely to compute what the generator voltage would have to be in order to maintain a voltage of 6360 volts (3670 volts to neutral) at the receiving end when loaded with 2100 kw at 75 per cent power-factor. The check value is 4560 volts to neutral, showing an error of about 1.5 per cent.

The error in this method lies in the fact that in the second step we have used too small a value for the line current. We may, therefore, better our result by repeating the second and third steps using a value for the line current, which we now know is more precise, as found by the equation

$$I = \frac{2{,}100{,}000}{1.73 \times 6360 \times 0.75}$$

$$= 255 \text{ amperes.}$$

By using 255 instead of 246 amperes for the line current, we find that it would require a generator voltage to neutral of 4710 volts to maintain 6600 volts between terminals at the load. With the generator voltage to neutral remaining 4490, the load voltage would be $\frac{4490}{4710} \times 6600$, or 6290 volts, which checks to within less than 1 per cent.

By repeating steps two and three a number of times, each time using a more precise value for the line current, it is possible to obtain the load

voltage to any desired degree of precision. For most practical work one such repetition is sufficient.

As a matter of fact, on most modern short lines, the terminal voltage at the load ends of a transmission system is usually maintained constant under load changes which normally would result in a 10 per cent drop or rise in voltage. This result is accomplished by means of tap-changing transformers or by automatic feeder voltage regulators, as explained in Chap. VIII, Art. 38 and 39.

Prob. 14–12. A 200-h.p. 2300-volt three-phase induction motor of 93 per cent efficiency has a power-factor of 90 per cent at full load. The distributing circuit has a resistance of 0.4 ohm and a reactance of 0.32 ohm per wire. (*a*) What must be the voltage at the generator end of the distributing line to operate the motor at rated load and at its rated voltage?

(*b*) What is the voltage regulation, assuming a constant voltage at the generator end of the distributing line?

Prob. 15–12. In starting, the induction motor of Prob. 14 takes three times normal current and the power-factor drops to 60 per cent. Assuming that the voltage at the generator end of the distributing line remains constant, what is the voltage across the motor on starting?

Prob. 16–12. If the generator voltage in Prob. 13–12 remains constant, what will the voltage at the load become when the load consists of 2000 kw at 95 per cent power-factor? Check and show per cent error.

Prob. 17–12. What will be the voltage of the load in line of Prob. 13–12 when only 500 kw are being delivered at unity power-factor? Compute per cent error due to this method.

SUMMARY OF CHAPTER XII

THE VOLTAGE at which electrical power is transmitted is usually about 1000 volts per mile.

IN AMERICA, ALTERNATING CURRENT is used in practically all industrial installations, the potential of which is above 550 volts.

THE FREQUENCIES in greatest use are 25 cycles for railway and power work and 60 cycles for lighting.

THE MOST ECONOMICAL SIZE OF TRANSMISSION WIRE is that size which results in the smallest sum total of annual fixed charges (such as interest in money invested, taxes, repairs and depreciation) and annual cost of energy lost in the line. The sum of these items becomes a minimum when the fixed charges equal the cost of lost energy.

THE VOLTAGE REGULATION OF THE LINE must also be taken into consideration when determining the size of wire to be used for transmission unless proper regulation is accomplished by feeder voltage regulators. The voltage regulation of a line equals

$$\frac{\text{(No-load voltage)} - \text{(Full-load voltage)}}{\text{(Full-load voltage)}}.$$

These voltages must be measured at the load end of the line. Good regulation for power service ranges between 5 and 10 per cent. For lighting service it should never exceed 5 per cent.

THE INDUCTIVE REACTANCE of a line causes voltage to be consumed in the line when an alternating current is sent over the line. This reactance drop leads the resistance drop by 90°.

THE GENERATOR VOLTAGE equals the line drop plus the no-load voltage at the load end of a short line, and can be found by adding vectorially any reactance drop and resistance drop to the voltage at the load end.

A THREE-PHASE TRANSMISSION LINE is more economical than a single-phase line for transmitting a given amount of power a given distance, and has better regulation under the same conditions. In a balanced three-phase system the voltage to neutral and the resistance and reactance drop in one line conductor are always used in computing the regulation.

TO COMPUTE THE VOLTAGE AT THE LOAD END of a transmission line corresponding to any amount of load and power-factor:

FIRST STEP. Compute the voltage at the generator end of line from data under normal conditions.

SECOND STEP. Compute what the voltage at the generator end would have to be in order to maintain the same constant voltage at load end under new conditions of load.

THIRD STEP. The voltage at the load end under new conditions is practically the same fraction of the voltage under normal conditions, that the generator voltage computed by the "first step" is of that computed by the "second step." This method is an approximation which is precise enough for all practical conditions.

FEEDER VOLTAGE REGULATORS are used to maintain a constant voltage at the load when power-factor or power is varied within prescribed limits. Such regulators may be operated automatically by a motor controlled by a voltage regulating relay.

PROBLEMS ON CHAPTER XII

Prob. 18–12. The 11,000-volt 3-phase 8-mile pole line of the Southern Power Co., running from Catawba to Pineville, is strung with aluminum stranded cable, the resistance of which is equivalent to the resistance of No. 2 copper wire. The load at Pineville consists of three 37.5-kva and three 125-kva transformers. Power-factor of load equals 80 per cent; frequency, 60 cycles. Compute the voltage regulation of this line. (Elec. Jour., Vol. VIII.)

Prob. 19–12. Assuming the voltage at Catawba to remain constant at the value found in Prob. 18, compute the voltage at Pineville when only 150 kw at 95 per cent power-factor are being used there.

Prob. 20–12. Calculate the most economical size of copper wire for the three-phase distributing system of Prob. 18. Estimate copper

conductor at 19 cents per pound; fixed charges at 9 per cent of line cost; electric energy at 4 mills per kw-hr, 3000 hr at full load per year.

Prob. 21–12. A distributing system arranged as in Fig. 7–12 is delivering 400 kw at 6600 volts and 80 per cent power-factor to transformer at sub-station *A*, and 250 kw, with 90 per cent power-factor to the sub-station *B*, which is near the main line. The conductors from *A* to *B* are No. 1 solid copper, and from *B* to the generating station *S* are stranded copper No. 00. The conductors are arranged as in Fig. 4–12, 30-inch spacing throughout. (*a*) Compute the voltage at *B* and at the station *S*. (*b*) What is the line regulation at *A* and at *B*?

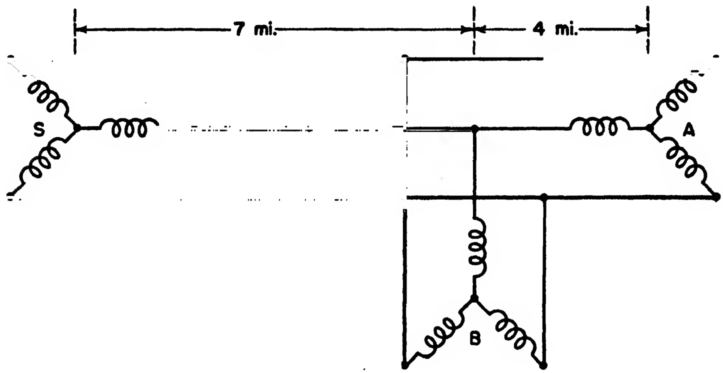


FIG. 7–12. Power is distributed over the three-wire line to sub-station *A* and to sub-station *B*, which is on a spur line.

Prob. 22–12. If the station at *B* is on a spur line 5 miles from the main line, compute (*a*) and (*b*) of Prob. 21. The wires from main line to *B* are of No. 2 solid copper.

Prob. 23–12. Assuming the rate of interest and depreciation, the cost per pound of installed copper cable, hours of use, etc., to be as in Example 1, deduce an expression relating the total annual fixed charges in dollars, to the size of conductor denoted by the symbol *r*, representing the resistance in ohms per mile of conductor. At 20° C commercial annealed copper wire has a weight of 0.3195 pounds per cubic inch and resistivity of 872.5 ohms per mile-pound.

Prob. 24–12. Assuming data as in Example 1 and Prob. 23, deduce an expression relating the annual value (dollars) of energy lost in the line conductors, to the size of conductor denoted by symbol *r* representing its resistance in ohms per mile.

Prob. 25–12. If the variation of all items in the first cost of a transmission line which depends upon the size of conductor were to be in direct proportion to the weight per mile of the conductor, the lowest total annual cost of transmission would be attained when the conductor is of such size that the annual fixed charges are exactly equal to the

annual value of energy lost in the line. Assuming this to be true (which it is, nearly enough for rough, practical calculations), calculate the exact size in ohms per mile and in circular mils of the most economical conductor in Example 1.

Prob. 26–12. To be ready for growth of load expected in the near future, the size of wire installed in a transmission line may be made larger than that calculated to be the most economical size for the present loading. Under the conditions of Example 1 and by the methods outlined in Prob. 23, 24 and 25, calculate how much (per cent) greater than the least total cost of transmission, the annual cost would be if the line wire were made larger than the most economical size by (*a*) 50 per cent; (*b*) 100 per cent; (*c*) 200 per cent.

Prob. 27–12. The transmission specified in Example 1 is to be installed under the condition that it is for temporary service only, and will be dismantled at the end of four years with a scrap value equal to 40 per cent of the initial cost. Assume that the money put aside for depreciation charges does not earn interest, and that the tax rate, cost of copper and value of energy, etc., are as in Example 1. Calculate the most economic size of conductor in this case: (*a*) In ohms per mile. (*b*) In circular mils.

Prob. 28–12. Other conditions being as specified in Example 1, calculate what percentage decrease in the value per kw-hr of energy would justify a saving of 25 per cent in the amount of conductor material used.

Prob. 29–12. A rule for rough calculations of transmission line is to allow in the conductors a power loss equal to approximately 10 per cent of the power delivered. Other conditions being as in Example 1, calculate what relation of the cost per pound of copper installed to the value of a kilowatt-hour will make it permissible to use this rule. The equations and methods suggested in Prob. 23, 24 and 25 may be used here to advantage.

Prob. 30–12. Good voltage regulation (without the aid of voltage regulators) on the short transmission in Example 1, Table I, demands that the resistance drop be not greater than 3 per cent of the voltage delivered at the load (11,000 volts). Calculate how much greater than the least value must be the annual cost of this transmission, in order to accomplish such regulation without a feeder voltage regulator. Power-factor, 80%.

Prob. 31–12. Show, for the general case, that when the most economical size of conductor is chosen, the voltage drop per mile of conductor **due to resistance** is dependent only upon the material and cost per pound of conductor, the percentage of fixed charges on this cost, hours of use, and the value of a kilowatt-hour saved from the line losses; that it does not depend upon the amount of power transmitted, the total distance or length of transmission line, voltage **between conductors, or any other factor.**

Prob. 32–12. Show that the most economical size of conductor under any given conditions requires the line to be proportioned on the basis of a certain number of circular mils of sectional area per ampere of current transmitted, and that this number depends upon the same factors as stated in Prob. 31 for the resistance drop per mile of conductor with most economical size.

Prob. 33–12. Calculate the percentage regulation at unity power-factor for the line of Example 1, when the frequency is (*a*) 25 cycles per second. (*b*) 133 cycles per second.

Prob. 34–12. Calculate the percentage regulation at 80 per cent power-factor for the line of Example 1, when the frequency is (*a*) 25 cycles per second. (*b*) 133 cycles per second.

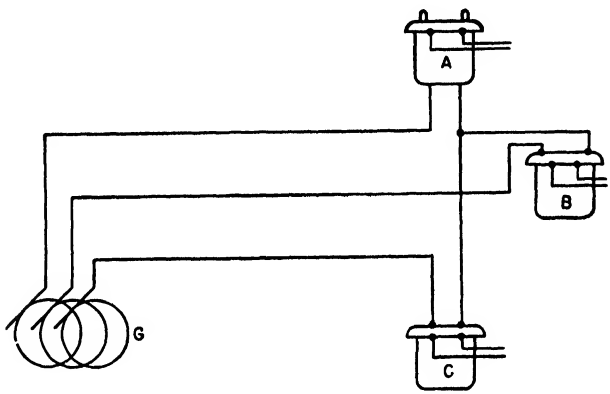


Fig. 8–12. Diagram of a system for transmitting the power of the station G to the three single-phase transformers A, B, and C.

Prob. 35–12. An eleven-mile three-phase line with 16,000 volts, 60 cycles, at the sending end is to supply power at 92 per cent power-factor and 8 per cent regulation. The line consists of stranded aluminum cables spaced 36 inches equidistant and of a size equivalent in resistance to 250,000 cir. mil copper. How much power can it deliver under these specifications?

Prob. 36–12. In the system shown in Fig. 8–12, the distance between the generating station G and the receiving end is 8 miles. The voltage at the receiving end is 11,000 volts, 60 cycles, when each transformer has its full load of 600 kw at 90 per cent power-factor. The line wires consist of 400,000 cir. mil stranded copper spaced 3 feet apart, as in Fig. 4–12. Compute the voltage at the generating station.

Prob. 37–12. The transformers of Prob. 36 become loaded as follows: Each transformer has a load of 500 kw at 80 per cent power-factor. Assuming that the voltage at generator station remains as in

Prob. 36, what will the voltage across each load transformer become? Check your computed voltages and state per cent error.

Prob. 38–12. What emf will be obtained between line wires at the load end of a three-phase three-wire line 10 miles long, of No. 000 copper with wires spaced as in Fig. 4–12 and 30 inches apart (constants as in Tables I and III of Appendix B), carrying a balanced 60-cycle load of 80 amperes per wire from a generator whose emf is 11,000 volts between any two terminals? Power-factor at generator is 80 per cent.

Prob. 39–12. While the line specified in Prob. 38 is delivering 60 amperes per wire at 87 per cent power-factor to a balanced load at the end of the line, another balanced three-phase load of 40 amperes per wire at 50 per cent power-factor is tapped from the middle point of the line. Calculate the voltage between line wires and the kilovolt-amperes and kilowatt output at each load. The station pressure is 11,000 volts.

Prob. 40–12. Calculate the amperes per wire and the power-factor at the station for Prob. 39. Calculate also the total kilovolt-amperes and kilowatt output at the station, and the efficiency of transmission.

Prob. 41–12. If the voltage impressed upon the sending (station) end of the line of Prob. 38 is 11,000, what will be the voltage between wires at the receiving end where a balanced load of 870 kw at 87 per cent power-factor is being consumed? What will be the current per wire and the station power-factor under this condition?

CHAPTER XIII

LONG TRANSMISSION LINES. CAPACITIVE REACTANCE

The city of Oakland, Cal., is supplied with electric power from the Big Bend power plant 154 miles away. A map showing the location of the line is shown in Fig. 1–13. Two three-phase lines are run on the same towers in the manner shown in Fig. 2–13. Each conductor is a seven-strand copper cable, No. 000 B. & S.

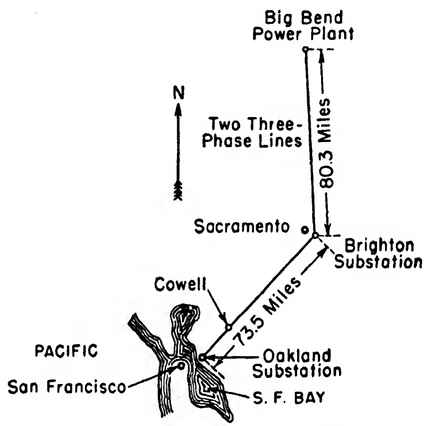


Fig. 1–13. Map showing the route of the 100,000-volt line from Big Bend to Oakland, Cal. (*Faccioli in the Trans. AIEE.*)

gauge. A three-phase generator by means of transformers can supply 10,000 kva at 100,000 volts, 60 cycles, to the power-plant end of the line.

Here is a long high-voltage transmission line, the characteristics of which are essentially different from the short lines which we studied in the previous chapter. Many facts come to light which are startling, when we first perceive them. For instance, when the Oakland end of the line was open, an ammeter inserted in the line wire near the power plant showed that a current of 48 amperes was flowing along the conductors at the power-house end. The voltage between the conductors at the open Oakland end was found to be 111,000 volts, while at the power plant end it was only

89,600 volts. Why should this large alternating current flow into the line conductors when the receiving end is open? Why, under these conditions, should the voltage at the **receiving** end be higher than the voltage at the **sending** end?

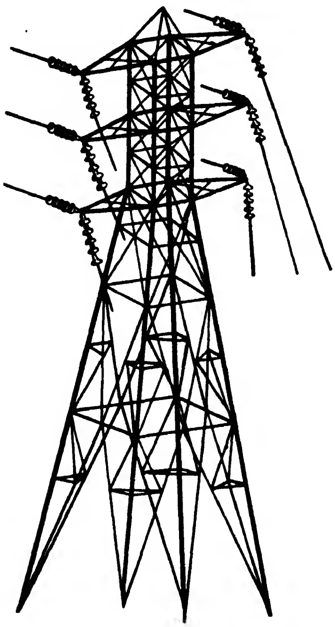


FIG. 2-13. Type of transmission line used on the Big Bend-Oakland Line. (*Faccioli in the Trans. AIEE.*)

1-13. Capacitance. Capacitive Reactance. The answer to these questions involves a study of the characteristics peculiar to long transmission lines. The line current of 48 amperes, for instance, is due to the fact that these wires, 154 miles long, offer large surfaces which must be covered by opposite electric charges every time the voltage changes. Thus when the voltage is rising to its maximum positive value, a charge of electricity is forced out over the line wire to fill it up with electricity. Then as the pressure dies out, this charge flows back, there being no pressure to keep it forced out along the wire. In fact each pair of conductors of a transmission line constitutes a condenser which, on a 60-cycle line, must go through the cycle of being positively charged, discharged, negatively charged and discharged 60 times each second.

We have seen in Chap. XVI, Vol. I, that where a condenser consists of two parallel **plates**, the capacitance can be computed from the equation

$$C = \frac{8.84KA}{10^8 l}, \tag{1-13}$$

in which C = capacitance in microfarads

K = relative dielectric constant (for air $K = 1$)

A = area (one side) in square centimeters of all the dielectric material actually between the plates

l = thickness of dielectric in centimeters

Example 1–13. What is the capacitance of the condenser in Fig. 3–13, formed by the two rectangular parallel metal sheets separated by 0.20 cm of air?

Substituting in Eq. 1–13, we have

$$C = \frac{8.84 \times 1 \times 8 \times 12}{10^8 \times 0.2}$$
$$= 42.4 \times 10^6 \ \mu f$$
$$= 42.4 \text{ micro-microfarads}$$

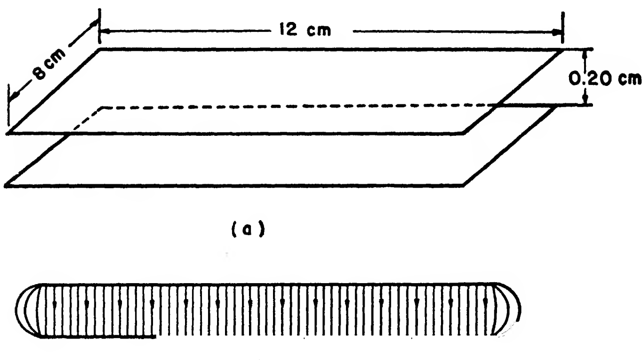


FIG. 3–13. (*a*) A parallel plate condenser. (*b*) Shows that the electrostatic field between the plates is practically of uniform density.

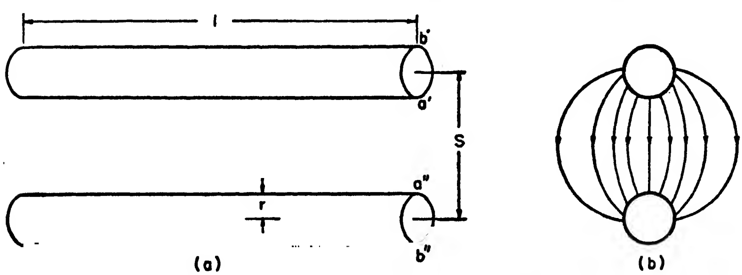


FIG. 4–13. The two wires in (*a*) constitute a condenser. Note in (*b*) that the electrostatic field between the wires is not of uniform density.

But a condenser formed by two parallel plates does not have the same shape as that formed by two parallel round wires, as shown by Fig. 4–13. For instance, the plates of Fig. 3–13 are so thin that the distance from the top of the upper plate to the bottom of the lower plate, is practically the same as the distance between the inner surfaces. Thus the shape of the electrostatic field in the air

between the plates is essentially that of a rectangular block, as shown in Fig. 3–13(b). Since, the capacitance of a condenser is directly proportional to the cross-section area of the electrostatic field and inversely proportional to its length, Eq. 1–13 for the capacitance of Fig. 3–13 is easily derived.

Note, however, that the cross-section of the electrostatic field between two wires, as shown in Fig. 4–13(b), varies greatly throughout its length. And even the length of the field varies from that of the curved lines from b' to b'' to the distance between a' and a''. Because of the differences in the shapes of these two fields, we should not expect Eq. 1–13 to apply to both. As a matter of fact, deriving the equation for the capacitance of a pair of transmission cables is a tedious process, so we shall omit the derivation and state the result in the following equation.*

$$C = \frac{0.0194l}{\log \frac{s}{r}}, \tag{2–13}$$

in which C = capacitance of line (2 wires) in **microfarads**
l = length of line in **miles**
s = distance between wire centers in **inches**
r = radius of each wire in **inches**

Example 2–13. What is the capacitance of a 40-mile 2-wire line, if the conductors are No. 00 solid wires spaced 4 feet apart?

By Equation 2–13,

$$C = \frac{0.0194l}{\log \frac{s}{r}}$$

$$= \frac{0.0194 \times 40}{\log \frac{48}{0.182}}$$

$$= 0.321\ \mu f$$

2–13. Capacitance-to-Neutral of Transmission Lines. Most long transmission lines consist of at least three conductors. Rather than to use several equations when computing the capacitances of lines having a different number of lines arranged in differing

* For the derivation of this equation, see Lawrence's, "Principles of Alternating Currents." McGraw-Hill.

patterns, it is much easier to use one equation which fits all cases. Also, we can then make out tables of capacitance, the values in which apply to lines with any number of cables arranged in any fashion. For this reason the capacitance of a **pair of cables** is seldom computed. Instead, we use an equation for the capacitance of one wire to an imaginary plane called the **neutral plane.** The capacitance thus computed is called the **capacitance-to-neutral.**

Consider the cross-section of the field of the two-wire line of Fig. 5–13. The line C represents the neutral plane half way between the two wires A and B. We can consider the capacitance of A to B made up of the series arrangement of the capacitance from A to C and that of C to B. We have seen in Chap. XVI,

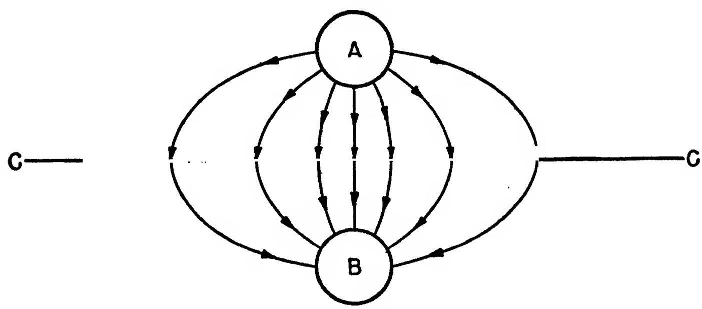


Fig. 5–13. The electrostatic field between the wire A and the neutral plane C, and between the wire B and the neutral plane C.

Vol. I, that the capacitance of two equal condensers connected in series is **half** that of one of the condensers alone. In other words connecting condensers in series is **not** like connecting resistances in series. And this is logical, because the capacitance of a condenser is inversely proportional to the average distance between the plates; — the greater the average distance, the less the capacitance. The average distance between the condenser made up of A and B is just **twice** the average distances from A to C and from B to C. Therefore the capacitance of A to B is just **half** that of A to C or of B to C.

Thus the capacitance from A to C, or from B to C is the **capacitance-to-neutral** of the line and must be **twice** that of the line A to B. Our equation for capacitance-to-neutral of a wire must therefore be **twice** that between the two sides of the line. Thus we merely double the constant 0.0194 in Eq. 2–13 and obtain as

our capacitance-to-neutral for one wire

$$C_0 = \frac{0.0388l}{\log \frac{s}{r}}, \tag{3-13}$$

in which

C_0 = capacitance-to-neutral of one wire in microfarads.
s = distance between centers of wires, in inches.
r = radius of each wire in inches.
l = length of one wire in miles.

This is the formula in general use for computing the capacitance of all types of overhead transmission regardless of the number of wires used.

Example 3-13. How great a charge will 2000 volts, direct current, force upon a 120-mile circuit consisting of two No. 000 stranded conductors strung 18 inches apart?

Capacitance to neutral of each conductor is found as follows:

Outside diameter No. 000 bare copper cable is 0.470 inch.

$$\begin{aligned} C_0 &= \frac{0.0388l}{\log \frac{s}{r}} \\ &= \frac{0.0388 \times 120}{\log\left(\frac{18}{0.235}\right)} \\ &= 2.47\mu f \end{aligned}$$

Since this is the **capacitance of one wire to neutral** we must use the **voltage from one wire to neutral** in computing the charge.

Voltage between wires	= 2000 volts.
Voltage from one wire to neutral	= $\frac{2000}{2}$ = 1000 volts.
Volts (to neutral) × capacitance (to neutral)	= charge.
1000 × 0.000002472	= 0.00247 coulomb.

This method produces the same result as though we considered the capacitance of the two real line wires as being ½ of 2.472 = 1.236 mf. To find the charge on the line we would multiply this **capacitance** between the two wires by the **voltage** between the wires.

$$2000 \times 0.000001236 = 0.00247 \text{ coulomb, as before.}$$

Prob. 1-13. What is the capacitance of one wire to neutral, for the transmission line from Big Bend to Oakland described at the be-

ginning of this chapter. The distance between wires is 10 feet and the conductors are not transposed.

Prob. 2–13. The transmission line from Great Falls to Greenville, S. C., is 96 miles long and consists of two sets of three No. 00 stranded copper cables. Each set is strung in horizontal plane 8 feet apart and is transposed. Considering each set separately, what is the capacitance from each line conductor to neutral?

Prob. 3–13. What direct-current pressure would be required between any two adjacent wires in the same set of the line in Prob. 2 to put a charge on them of 0.002 coulomb?

Prob. 4–13. In Table IV, Appendix B, is given a table of values of the capacitance to neutral for standard solid conductors and standard spacings. Check values for one mile of

(*a*) No. 2 wire with 24-inch spacing.
(*b*) No. 00 wire with 8-foot spacing.

Prob. 5–13. In Table VI, Appendix B, is a table of capacitance to neutral for standard strands. Check value of one of

(*a*) 500,000 cir. mils spaced 30 inches.
(*b*) 750,000 cir. mils spaced 15 feet.

Prob. 6–13. According to Table VI, Appendix B, what should be the capacitance of one conductor to neutral of the transmission line from Big Bend to Oakland? Compare with result calculated by formula, in Prob. 1–13.

3–13. Charging Current. Capacitive Reactance. Since a long transmission line acts as a condenser, an alternating capacitive current, or charging current, flows in it when an alternating emf is applied to it. The value of this charging current can be computed as follows:

The opposition which the alternating emf has to overcome in setting up a current in a circuit containing capacitance only is called the **Capacitive Reactance** of the circuit, and is measured in **Ohms** just as the Resistance and Inductive Reactance are measured in **Ohms.** And just as the current in a circuit containing resistance only can be found by dividing the emf by the **resistance,** and the current in a circuit containing inductance only can be found by dividing the emf by the **inductive reactance,** so the current in a circuit containing capacitance only can be found by dividing the emf by the **capacitive reactance.**

The value of the capacitive reactance, as explained in Chap. IV, is found by the equation

$$X_C = \frac{1}{2\pi f C},$$

in which

X_C = the capacitive reactance in ohms
f = frequency in cycles per second
C = capacitance in farads

The value of the charging current in a circuit containing capacitive reactance only is therefore given by the equation

$$I_C = \frac{E_C}{X_C} = \frac{E_C}{\dfrac{1}{2\pi fC}} = E_C(2\pi fC) \qquad (4\text{–}13)$$

in which

I_C = effective charging current at sending end.
E_C = effective voltage at sending end.

An unloaded transmission line is a close approximation to a circuit containing capacitance only. Of course such a line also has both resistance and inductive reactance. The values of these, however, are so small and their effects so related vectorially to the effect of the capacitive reactance as to produce only a minor effect of a few per cent on the value of the charging current.

Example 4–13. The capacitance-to-neutral of one wire of a certain 50,000-volt single-phase transmission line is $0.758\mu f$. What is the value of the charging current which will flow into the sending end of each conductor when the receiving end is open?

$$I_C = \frac{E_C}{X_C}$$

$$X_C = \frac{1}{2\pi fC}$$

$$= \frac{}{6.28 \times 60 \times 0.758 \times 10^{-6}}$$

$$= 3500 \text{ ohms}$$

Thus reactance-to-neutral = 3500 ohms

and voltage-to-neutral $= \frac{50{,}000}{2} = 25{,}000$ volts.

Therefore

$$I_C = \frac{25{,}000}{3500}$$

$$= 7.14 \text{ amperes.}$$

This charging current of 7.14 amperes will lead the voltage by 90° as shown in Fig. 6–13.

Prob. 7–13. Draw the vector diagram for above example with voltage 15° after it has passed up through its zero value. Find instantaneous values of current and voltage.

Prob. 8–13. Effective alternating voltage of 220 volts (frequency 60) is impressed upon a circuit containing a condenser only. If the current is 2 amperes, what is the capacitive reactance of the condenser?

Prob. 9–13. When the instantaneous voltage of Prob. 8 is 120 volts, positive and increasing, what instantaneous value will the current have?

Prob. 10–13. What voltage is necessary to force a maximum current of 20 amperes through a circuit containing 50 ohms of capacitive reactance?

Prob. 11–13. (*a*) Draw vector diagram and determine instantaneous value of voltage when instantaneous current in Prob. 9 is 5 amperes, positive and decreasing. (*b*) How many electrical degrees will the voltage have passed through by that time?

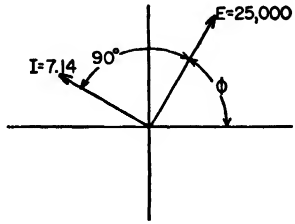


FIG. 6–13. The vector I representing a charging current of 7.14 amps leads by 90° the vector E representing the impressed voltage 25,000 volts.

4–13. Capacitive Reactance of Long Transmission Lines. The capacitance, and consequently the capacitive reactance, of a transmission line is in reality distributed along a line uniformly from end to end. In some problems, in order to obtain precise information, it is necessary to use this true distribution. In Example 4–13 we considered all the capacitance to be concentrated at the sending end of the line. This does not lead to very precise results in long lines, because it assumes that the a-c voltage is the same all along the wire, and we have definitely stated that on an unloaded long line, the alternating voltage is higher at the receiving end than at the sending end. However, if we use two condensers, each one-half the capacitance of the line, and place one at the sending end and the other at the receiving end, we usually get sufficiently precise results. Note carefully, however, that a condenser of **one-half** capacitance will have **twice** the capacitive reactance. This is evident from the equation $X_C = \dfrac{1}{2\pi fC}$, which shows that the capacitive reactance varies **inversely** with the capacitance.

The following examples will show the advantages of considering the capacitance of a long line to be concentrated in two capacitors

each one-half the capacitance of the line, and situated, one at each end of the line. We will first compute the charging current of the 100,000-volt 3-phase line from Big Bend to Oakland, Cal., described at the beginning of this chapter, using only one condenser placed at the sending end, and equal to the whole capacitance of the line.

We will then compute the charging current by the two-capacitor method.

Example 5-13. The capacitance-tó-neutral of the Big Bend-Oakland 154-mile 100,000-volt 3-phase 60-cycle line is 2.20 μf. If the whole capacitance is considered to be concentrated in one condenser at the sending end of the line, what value will be obtained for the charging current I_C, when the line is unloaded?

Solution by the one-condenser method. The line-to-line voltage at the sending end is given as 89,600 volts.

The line-to-neutral voltage of the 3-phase line equals

$$V_0 = \frac{89{,}600}{1.73} = 51{,}800 \text{ volts}$$

The capacitive reactance is found by

$$\begin{aligned} X_C &= \frac{1}{2\pi f C} \\ &= \frac{1}{6.28 \times 60 \times 2.20 \times 10^{-6}} \\ &= 1210 \text{ ohms} \\ I_C &= \frac{51{,}800}{1210} = 43 \text{ amperes.} \end{aligned}$$

By test, the charging current was found to be 48 amp. The discrepancy is due to the fact that the line-to-line voltage was not 89,600 volts throughout the length of the line. In fact, the measured voltage at the receiving end, strange as it may seem was 110,000 volts, nearly 22,000 volts higher than at the sending end.

Note in the following solution how the two-condenser diminishes this discrepancy.

Solution by the two-condenser method. In Fig. 7-13, the capacitive reactance of each of the two end condensers must be 2420 ohms, (twice that of a single condenser because they are in parallel and must have the same combined reactance as a single condenser representing the capacitive reactance of the line.

The charging current entering the line at the sending end must be the sum of the currents in A and B.

Charging current in A,

$$I_A = \frac{51{,}800}{2420} = 21.4 \text{ amp}$$

Charging current in B

$$I_B = \frac{63,600}{2420} = 26.3$$

Total charging current $= I_A + I_B = 21.4 + 26.3 = 47.7$ amp. Note that this is within 1 per cent of the measured value 48 amp.

Prob. 12–13. From "Charging current" table in Appendix B, compute the capacitive reactance to neutral of 140 miles of No. 000 solid conductor spaced 3 feet from the other similar line wire, if the system is single-phase 60-cycle.

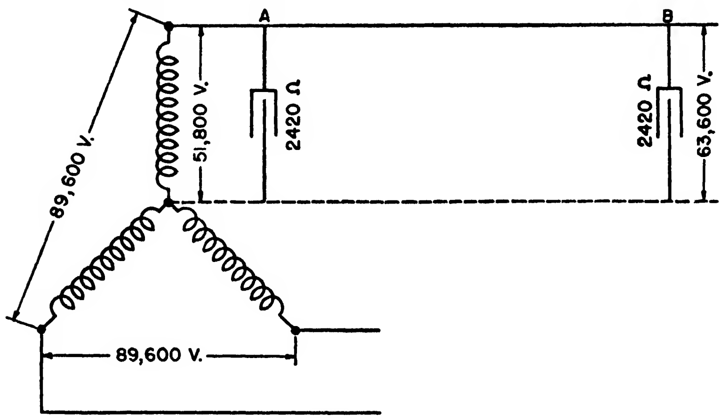


FIG. 7–13. An unloaded three-phase transmission line. The two condensers represent the capacitive reactance-to-neutral of one conductor.

Prob. 13–13. What is the capacity reactance of one wire to neutral for a three-phase line, strung with 350,000 cir. mils stranded cables, spaced 15 feet apart? Length of line, 80 miles; frequency, 60 cycles. See Appendix B.

Prob. 14–13. What charging current will flow in each wire of Prob. 13, if the open circuit voltage at the sending end is 95,000 and at the receiving end is 114,000? Frequency, 60 cycles.

Prob. 15–13. What would be the charging current of Prob. 14 if the frequency were 25 cycles?

Prob. 16–13. A 350-mile, three-phase, 60,000-volt transmission system, operating at 60 cycles, uses 2800 reactive kva in charging the line.

(*a*) What is the charging current?

(*b*) What is the capacity reactance of the line, one wire to neutral?

(*c*) What is the charging current per mile per 1,000,000 volts to neutral?

5–13. Why the Voltage is Sometimes Higher at the Receiving End than at the Sending End. We have noticed that on a long unloaded a-c transmission line the voltage at the receiving end is higher than at the sending end. In explaining the reason for this, let us take as an example the transmission line of the Great Falls Power Company, which transmits 15,000 kw to a distance of 130 miles from Great Falls to Butte, Montana, at a pressure of 100,000 volts.

Two three-phase lines are run on separate towers. The conductors of each line are No. 0 stranded hard copper cable with hemp centers, outside diameter 0.398 inch, and are strung in a

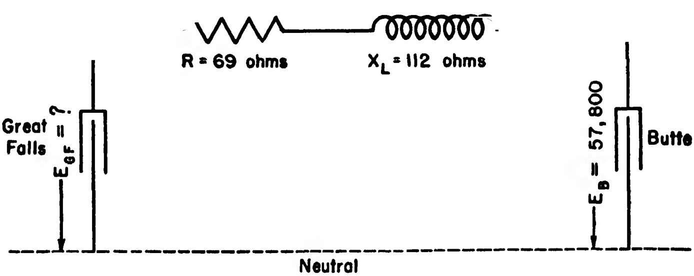


Fig. 8–13. Diagrammatic representation of the resistance and the inductive and capacitive reactance of a single conductor of the Great Falls-Butte line.

horizontal plane and spaced 10 feet 4 inches, with no transpositions. The charging current at a generator voltage of 100,000 volts and 60 cycles was measured when the receiving end was open and was found to be 39 amperes per wire. This checks closely the computed value, as will be seen in Prob. 17–13.

Figure 8–13 represents one cable of this line and shows the capacitive reactance represented by two condensers one at each end of the line. As in the previous example of course the capacitive reactance of each of these must be double the capacitive reactance-to-neutral, although in this problem their numerical value is not required, since we are given the charging current. The values of the inductive reactance and the resistance indicated in the figure are found as follows.

By reference to Table III in Appendix B and using "equivalent" spacing of 13.0 feet, we find that the inductive reactance per mile of each wire of this line must be 0.860 ohm, approximately.

The inductive reactance-to-neutral of one linewire equals

$$X_L = 130 \times 0.860$$
$$= 112 \text{ ohms.}$$

The resistance of each wire must be

$$R = 130 \times 0.530$$
$$= 68.9 \text{ ohms.}$$

Let us examine the conditions in the line when it is open at the receiving end at Butte, and with enough voltage at the Great Falls end to supply the charging current of 39 amperes. We will assume that the voltage at Butte is 100,000 volts between conductors. We must then compute what voltage is required at the sending end to produce a charging current of 39 amperes in a line of 69 ohms resistance and 112 ohms inductive reactance, while maintaining 100,000 volts at the receiving end.

The voltage between conductors at the receiving end at Butte is to be 100,000 volts, therefore the voltage-to-neutral at Butte is equal to

$$E_B = \frac{100{,}000}{\sqrt{3}} = 57{,}800 \text{ volts.}$$

Draw vector OE_B, Fig. 9–13, to represent this voltage at the open receiving end.

The line OI_C, drawn at right angles to OE_B, will then represent the average charging current of 19.5 amperes which traverses the full length of the line and which leads the voltage producing it by 90°. But the generator has to maintain not only the voltage OE_B, but also the voltage to overcome the resistance and the inductive reactance of the line.

The voltage to overcome the resistance is equal to

$$E_R = 68.9 \times 19.5$$
$$= 1344 \text{ volts.}$$

Since this voltage must be in phase with the current, we draw the vector E_R from the end of E_B and at right angles to E_B (parallel to the current vector I_C).

The voltage to overcome the inductive reactance must equal

$$E_x = 112 \times 19.5$$
$$= 2184 \text{ volts.}$$

Since this voltage must lead the current 90°, and therefore must lead E_R by 90°, we draw E_x 90° ahead of E_R.

The vector $E_{G,F}$, joining O to the end of E_x, will then represent the voltage at the sending end, because it is the resultant of the series combination of E_B (the voltage at the receiving end), E_R (the voltage to overcome resistance of line) and E_x (the voltage to overcome the inductive reactance of the line).

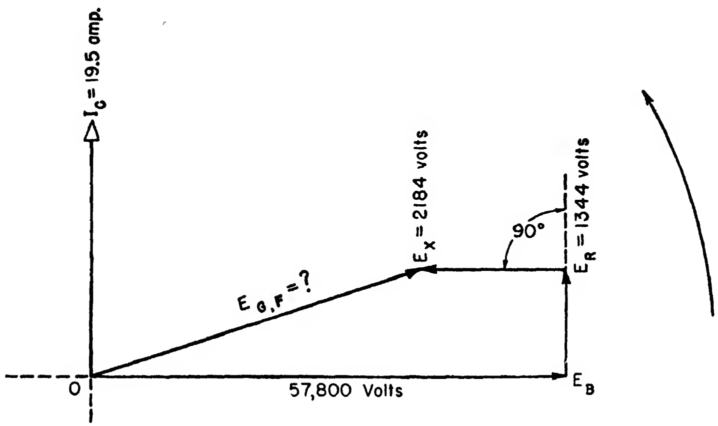


FIG. 9–13. The vector diagram of the voltage relations in an unloaded long transmission line. The voltage $E_{G,F}$ at the sending end is less than the voltage E_B at the receiving end.

The numerical value of the voltage at the generator end equals

$$E_{G,F} = \sqrt{(57{,}800 - 2184)^2 + 1344^2}$$
$$= 55{,}634 \text{ volts.}$$

The voltage between conductors at the generator end must equal

$$1.732 \times 55{,}634 = 96{,}458 \text{ volts.}$$

This value is distinctly lower than the 100,000 volts which is the pressure between conductors at the receiving end.

Note that this decrease in voltage at the sending end is due to the presence of both capacitive and inductive reactance in the circuit. One tends to neutralize the effect of the other. Thus the voltage E_x used in overcoming the inductive reactance is in the opposite direction E_B, the voltage used in overcoming the capacitive reactance of the line. Since the resistance drop E_R is at right angles to the terminal voltage E_B, it does not affect, to any ap-

preciable extent, the value of the voltage $E_{G,F}$ which is needed to send the charging current over the line.

Prob. 17–13. Compute from data of size, spacing, etc., in text above, the capacitance of one wire to neutral of the Great Falls-Butte line. From this value of capacitance and the voltages in the above text compute the charging current per wire and check against value taken from tables in Appendix B.

Prob. 18–13. Compute the open-line voltage at the generator end of the line from Great Falls to Greenville, S. C., if the voltage at Greenville is 100,000 volts. Frequency = 25 cycles. For remaining data see Prob. 2–13. Consider that the two sets of conductors are so far apart that they do not affect each other.

Prob. 19–13. How many kilovolt-amperes are used in charging the three wires of the Great Falls-Butte line, if the voltage at the open receiving end is 100,000 between conductors?

6–13. Regulation of Transmission Line Containing Capacitance. The presence of capacitance in a line is often advantageous to the system, especially when the load has a lagging power-factor. To show this, we merely have to note the effect of capacitance upon the regulation of a given line for a given load and power-factor.

The full load of the Great Falls-Butte transmission is 15,000 kw at 85 per cent power-factor distributed equally between the two lines, or 7500 kw on each. A line-to-line voltage of 100,000 volts is maintained at the load.

To find the load current per conductor:

$$P = \sqrt{3}\, IE \cos\theta$$

$$I_{\text{load}} = \frac{P}{\sqrt{3}\, E \cos\theta}$$

$$\frac{7{,}500{,}000}{1.73 \times 100{,}000 \times 0.85}$$

$$= 51 \text{ amp. per conductor.}$$

At a power-factor of 0.85 the current lags practically 32° behind the voltage. Draw the vector E_{load}, Fig. 10–13, to represent the voltage across the receiving end, 32° ahead of the vector I_{load}, which represents the current of 51 amperes taken by the load. The charging current of 19.5 amperes is then represented by the vector I_C, which is 90° ahead of the voltage vector E_{load}, and consequently 90° + 32° or 122° ahead of the I_{load}. The current which the line carries must be the combination of the load current

and the capacitive current. This is represented by the vector I_{line} which is the resultant of the vectors I_{load} and I_C.

The value of I_{line} is found by means of the equation for the diagonal of a parallelogram as given in Appendix A, Table II.

$$I_{\text{line}} = \sqrt{I_{\text{load}}^2 + I_C^2 + 2I_C I_{\text{load}} \cos 122^\circ}$$

$$\begin{aligned} I_{\text{line}} &= \sqrt{51^2 + 19.5^2 + 2 \times 19.5 \times 51 \times \cos 122^\circ} \\ &= 43.9 \text{ amp.} \end{aligned}$$

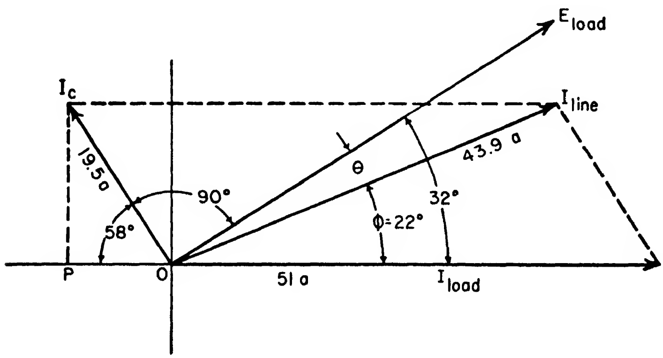


FIG. 10–13. Current and voltage relations at the load end of a long transmission line carrying an inductive load. The line current I_{line} is less than the load current I_{load} because of the capacitive current I_C.

Thus a line current of only 43.9 amperes is able to supply a current of 51 amperes to the load, because of the effect which the leading charging current has when it combines with a lagging load current to produce the total line current.

The angle ϕ by which this line current I_{line} leads the load current I_{load} can be found as follows:

Drop a perpendicular from end of vector I_C.

$$\begin{aligned} I_C P &= 19.5 \sin 58^\circ \\ &= 16.5. \end{aligned}$$

A perpendicular dropped from the end of vector I_{line} will have the same length, 16.5.

Thus

$$\begin{aligned} \sin \phi &= \frac{16.5}{43.9} \\ &= 0.376 \\ 0.376 &= \sin 22^\circ. \\ \phi &= 22^\circ \text{ approx.} \end{aligned}$$

The angle θ which is the phase difference between the voltage of the load E_{load} and the line current I_{line} can then be found:

$$\theta = 32° - 22°$$
$$= 10°.$$

To find the voltage regulation of the line with this load and power-factor, construct Fig. 11–13, repeating Fig. 10–13 as a basis in order to get the proper phase relations. Draw vector

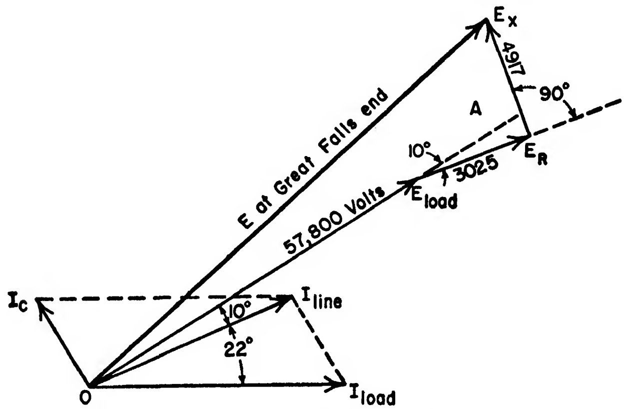


FIG. 11–13. Vector diagram for finding the voltage E at the sending end of a long transmission line which is carrying an inductive load.

E_{load} at an angle of 32° leading the vector I_{load}, and 10° ahead of line current I_{line}.

The voltage necessary to send the line current of 43.9 amperes against a resistance of 70 ohms per wire equals

$$E_R = 70 \times 43.9$$
$$= 3025 \text{ volts.}$$

This voltage is in phase with the line current and is represented by the vector E_R drawn lagging 10° behind the load voltage vector E_{load}.

The voltage necessary to send the line current of 43.9 amperes against the inductive reactance of 112 ohms per line wire equals

$$E_x = 112 \times 43.9$$
$$= 4917 \text{ volts.}$$

This voltage must be 90° ahead of the current and thus the vector E_x representing it is drawn 90° ahead of vector E_R.

The vector E, joining O and the end of the vector E_x, must represent the voltage which is necessary to overcome these three components, and thus represents the voltage at the Great Falls end of the line.

To find the value of E, the voltage at the sending end, extend the line E_{load} until it meets E_x at A. The angle at A equals $90° + 10° = 100°$.

$$
\begin{aligned}
&\text{The amount cut from } E_x = 3025 \tan 10.0° = 533. \\
&\text{The line} \quad AE_x = 4917 - 533 \\
&\qquad = 4384. \\
&\text{The extension of} \quad E_{\text{load}} = \frac{3025}{\cos 10.0°} \\
&\qquad = 3072. \\
&\text{The line} \quad OA = 57{,}800 + 3072 \\
&\qquad = 60{,}870.
\end{aligned}
$$

The triangle OAE_x can be solved as follows:

$$
\begin{aligned}
E &= \sqrt{OA^2 + AE_x^2 - 2\,OA \times AE_x \cos 100°} \\
&= \sqrt{60{,}870^2 + 4384^2 - 2 \times 60{,}870 \times 4384 \times \cos 100°} \\
&= 61{,}780 \text{ volts.}
\end{aligned}
$$

The voltage to neutral at the generator end at full load then equals 61,780 volts.

We have computed that 55,630 volts at the generator end at no-load produces a receiving-end voltage of 57,800 volts. A voltage of 61,780 at the generator end would therefore produce approximately $\frac{61{,}780}{55{,}630} \times 57{,}800$ or 64,200 volts at the receiving end when the line was open.

The voltage regulation at 85 per cent power-factor will therefore be $\frac{64{,}200 - 57{,}800}{57{,}800} = 11.0$ per cent. We thus have a line the receiving end of which has a voltage between conductors of 100,000 at full load, but of 111,000 volts at no-load. (Data from Proc. A.I.E.E.)

In the above examples, the results are only approximations, due to the fact that the impressed voltages do not have a pure sine wave-form. As explained later, ripples, or harmonics, occur (to a slight extent, to be sure) in the wave-form of all commercial

generators. These ripples are greatly magnified by the line capacitance and tend to make the charging current, and the voltage values depending upon it, somewhat larger than the usual computed values. The above method, however, gives values which differ so little from tested values, that it can be used with confidence in all commercial computations.

Prob. 20–13. Compute the voltage regulation of the Great Falls-Butte line at unity power-factor. Load, 7500 kw at 100,000 volts.

Prob. 21–13. (*a*) Compute the voltage regulation of the Great Falls-Butte line at 90 per cent leading power-factor. Load, 7500 kw at 100,000 volts.

(*b*) What effect does the capacity of the line have upon the regulation when the power-factor is (1) Lagging? (2) Unity? (3) Leading?

Prob. 22–13. The three-phase transmission line from Shoshone to Denver, Colorado, is 153.5 miles long. The conductors are arranged in a horizontal plane, 124 inches apart with no transpositions, and consist of No. 0 six-strand hemp-center copper cables. When 100,000 volts at 60 cycles are impressed on the Shoshone end, what current will flow per wire if the Denver end is open?

Prob. 23–13. What will be the voltage at the Denver end of the line in Prob. 22–13?

Prob. 24–13. What voltage at the generator end is necessary to deliver 5000 kw at 100,000 volts at 0.80 power-factor at the Denver end of line in Prob. 22–13?

Prob. 25–13. What is the voltage regulation of the line under the conditions of Prob. 24–13?

Prob. 26–13. What would be the voltage regulation of the line in Prob. 24–13 if the load of 5000 kw had unity power-factor?

7–13. Capacitance of Underground Cables. The capacitance of underground cables is very high in comparison with that of overhead cables because the cables are laid with very little space between them as seen in Fig. 12–13. The insulation material, rubber or impregnated paper, also makes the capacitance from two to four times higher on account of a certain **dielectric power** which it possesses to a much greater degree than air. All of these conditions combine to produce a condenser of large capacitance. Even two and three-tenths of a microfarad per mile are not uncommon values. This, together with the fact that the breakdown strength of the insulation limits the voltage, renders it impracticable to transmit power by alternating current any great

distance underground or by submarine cables. In most large cities cables are laid in underground ducts up to distances of 10 miles and at voltages between 11,000 and 23,000 volts. Of course

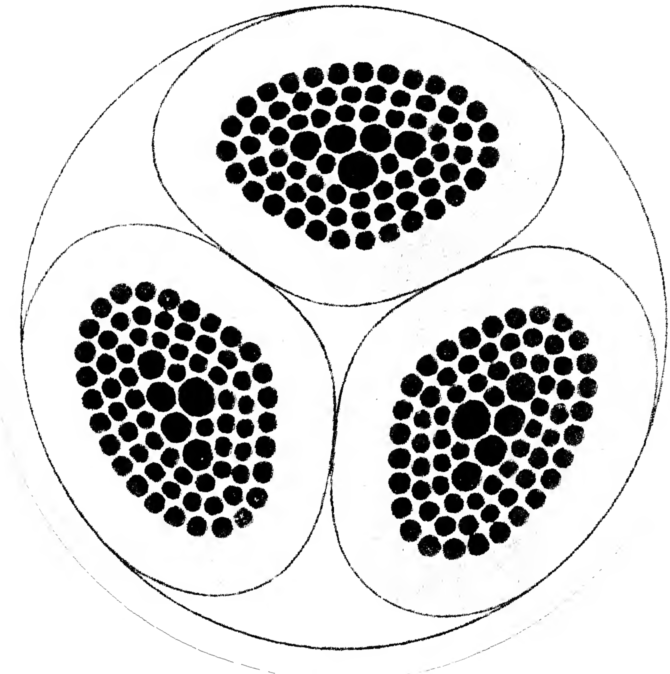


FIG. 12-13. Cross-section of an underground 3-conductor lead-covered cable for operation on a grounded-neutral system, with 15,000 volts between conductors. (*Simplex Wire & Cable Co.*)

this disadvantage does not exist in the transmission of direct-current power.

Data for the following problems were furnished by the Simplex Wire and Cable Co.

Prob. 27-13. The 3-conductor 350,000-cir. mil cable of Fig. 12-13 has 0.185 in. impregnated paper insulation and is lead covered. It is designed to be operated on a grounded-neutral, Y-connected system with 15,000 volts between conductors.

The a-c resistance of each conductor is 0.333 ohm per 1000 ft. at 25° C.

The inductance to ground of each conductor is 0.082 millihenry per 1000 ft.

The capacitance to ground of each conductor is 0.134 microfarad per 1000 ft.

The safe carrying capacity of the cable is 310 amp. per conductor.

What will be the charging current of this cable on a 4-mile line, if the open-circuit 60-cycle voltage at the receiving end is 15,000 volts, line-to-line?

Prob. 28–13. (*a*) What will be the voltage at the sending end in Prob. 27?

(*b*) How much power is lost in charging the open line?

Prob. 29–13. How much 60-cycle power at 75% power factor can this cable deliver to a 15,000-volt load over a 4-mile line?

Prob. 30–13. What would be the answer to Prob. 29 if the power factor were unity?

8–13. Current Surges and Oscillations in Long Lines. A long line is subjected to current surges from two causes, — (*a*) lightning discharges in the vicinity of the line; (*b*) the necessary switching

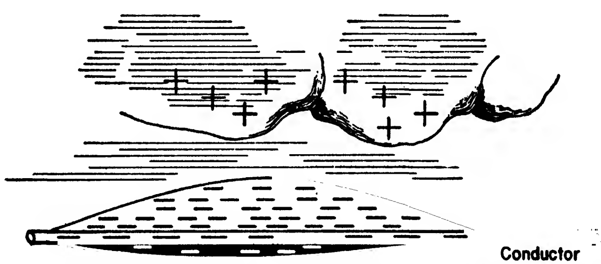


FIG. 13–13. The positive charge on a cloud draws a negative charge to that part of the conductor nearest the cloud.

operations. The more serious of these are likely to be the lightning disturbances. A cloud, generally charged positively as in Fig. 13–13, comes near a portion of the line, and attracts a large negative charge to this part of the conductor. When the cloud is discharged by a lightning-flash either to earth or to another cloud, this large negative charge on the wire is suddenly released and rushes along the wire, just as a flood of water rushes along a narrow valley when the retaining wall of a reservoir at its head suddenly gives way. But the electric wave rushes along the wire at the speed of light.

If the wave-front of this surge or electric flood hits the windings of a transformer or generator, these windings act as a wall acts to the sudden rush of water. The inductance of the windings opposes any sudden passage of electric charge or growth of the

current through them, and the electric charge "piles up" against the transformer. This induces such an excessive pressure between the windings that a charge may be forced through the insulation, and an arc started. While the normal voltage between the turns is never enough to start an arc, once the insulation has been broken down and an arc has been started by a momentary higher voltage, the line voltage is usually sufficient to maintain the arc long enough to severely damage the machine.

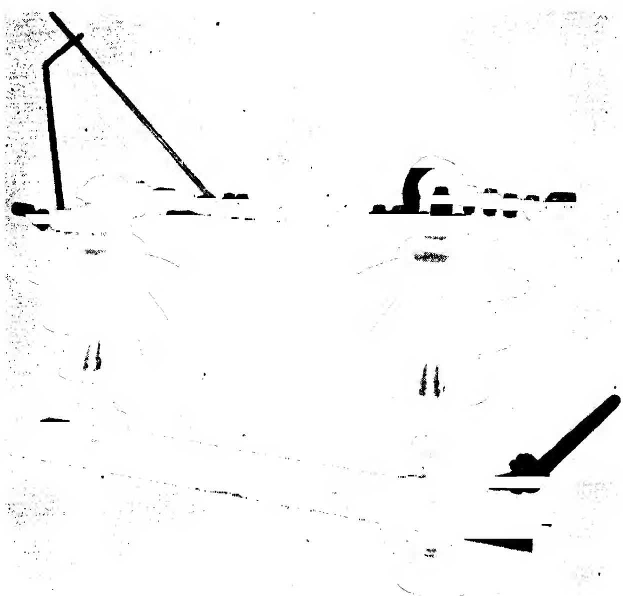


FIG. 14–13. Horn gap for 110,000-volt lightning arrester. One horn is connected to the line and the other to the ground, generally through a Thyrite arrester. (*General Electric Co.*)

In addition to the damage done to the generator or transformer, this arc also sets up very disturbing oscillations in the line, which may damage other machines connected to it.

Similar surges and oscillations may be set up in switching the current on and off the line. The larger the current switched on or off, the greater the disturbance.

As a general rule, in switching on the current it is best to connect the step-down transformers to the receiving end before connecting the step-up transformers to the generator.

9–13. Lightning and Surge Protection. The devices used to protect the line and machines from damage by surges in the line, due either to switching or lightning disturbances, are called lightning arresters. These are connected through horn-gaps to the line at points as near as possible to the transformers. One side of the horn shown in Fig 14–13, is connected to line; the other to one terminal of some form of lightning arrester, the other terminal of which is thoroughly grounded. The length of the gap is adjusted

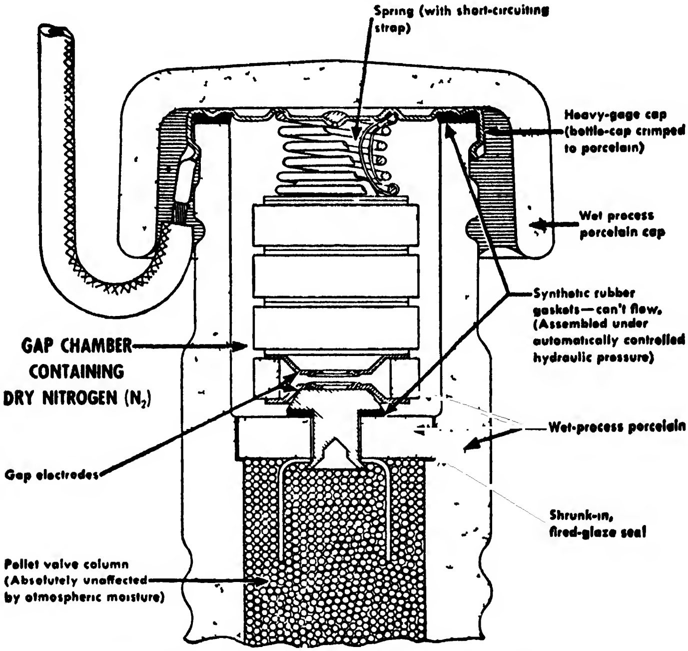


FIG. 15–13 A 9000-volt Pellet lightning arrester. Cutaway view. (*General Electric Co*)

so that the ordinary line voltage is not enough to cause an arc across the gap, but a dangerously high voltage will break down the air insulation at the smallest space and form an arc. The heated air around the arc, and the magnetic effect of the arc cause the arc to travel up the gap. The horns are so constructed that the distance between them gradually increases toward the top. Thus as the arc travels up, it soon reaches a place where the distance is too great for the voltage to maintain the arc, and it is thus extinguished. The excess charge on the conductor is thus

harmlessly conducted through the arresters to the ground instead of being sent back over the line.

In case the arc is not extinguished, one of the best features of a good arrester is that it hinders a surge from returning to the line when the voltage across the horn gap reverses. There are several types of lightning arresters in general use, all of which differ in details but operate on about the same principles.

For low voltages, up to about 70,000 volts, the pellet type shown in Fig. 15–13 is often used. It consists of a small insulating tube filled with small lead peroxide pellets which have been coated with an insulating powder. When an electric discharge enters the tube it breaks down the insulating coating and is conducted to the ground. The coating, however, immediately resumes its insulating property and prevents a return of a surge to the horn.

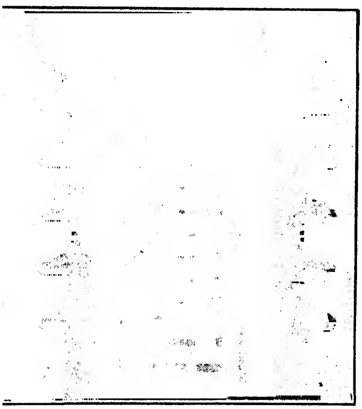


Fig. 16–13. An 11,500-volt Thyrite lightning arrester. Cutaway view. (*General Electric Co.*)

For higher voltages, the more expensive Thyrite arrester, shown in Fig. 16–13, is extensively used. It consists of a pile of disks made of an inorganic ceramic compound, which has the property of changing from an insulating material to a good conductor when a sufficiently high voltage is applied to it. It operates equally well on both d-c and a-c voltages, and can be immersed in oil without disturbing its action.

10–13. Corona Loss. When the voltage between two conductors is raised beyond a certain value, a faint glow of violet light appears in the air at the surface of the conductors. This phenomenon is called **Corona,** and results in a loss of power from the conductors into the surrounding air. This action is best explained by the electron theory of electricity, which was taken up briefly in the first chapter of Vol. I. It was stated there that the atoms of every substance are composed of a positive nucleus and negative electrons which rotate in space around the nucleus. When the intensity of the voltage at the surface of a conductor becomes great enough it will tear some of the electrons away from

the nuclei of the atoms in the air and cause them to collide with other atoms. If the electrons unite with these other atoms they will add energy to the atoms which is dissipated in light and produces the glow in the air around the wire. This tearing apart of the atoms is called **ionization.** The nucleus from which an electron has been torn, becomes deficient in negative electricity and is called a **positive ion.** An atom containing a surplus of electrons is called a negative ion. All ions tend to become neutral; the positive ions by attracting enough free electrons, and the negative ions by giving up the surplus electrons.

This process of ionization of air at nominal temperature and pressure requires somewhat greater electric intensity than 30,000 volts per cm in the air. This is called the "disruptive critical voltage." It is usually only in the air at the surface of the wire that this intensity reaches this figure, so that the glow extends but a very short distance out from the wire. However if the voltage between conductors is sufficient to produce this voltage intensity of 30,000 volts per cm throughout enough of the space between the wires, a spark will pass and a **flash-over** may result.

Two common precautions are employed to lessen the corona loss, and avoid the danger of flash-over. One is to space the wires as far apart as is economically possible, because for a given voltage between two wires, the less the voltage per inch will be. The other is to use as large a wire as is practicable, because the larger the surface of a wire, the lower the intensity of the voltage.

The following factors also effect the corona loss.

1. The condition of the surface of the wire. Tests have proven that a clean smooth surface on a conductor will result in the lowest corona loss. Thus any scratches on the wire caused by dragging it over the ground, and even the grease from the dies will increase the loss by corona. These defects usually become less after the wire has been up a few months and becomes " weathered," so that the voltage at which corona appears is but a few per cent lower than it is for polished conductors.

2. Weather conditions. Rain, snow or sleet and even high humidity greatly increase the loss. In fact under severe conditions of this kind it may become 40 or 50 times as large as under normal conditions. However such conditions are usually of short duration and rarely extend over much of the line.

Approximate Equations for Corona Loss. From data taken on test lines, equations have been developed which can be used to

approximate the corona loss which may be expected on a given transmission line under set conditions. Much more experimenting must be done however before these equations can be applied with confidence in the design of lines which are to operate at very high voltages.

The following equation* developed by F. W. Peek, Jr., has been found in recent texts on two-phase and symmetrical three-phase lines to be a good approximation at the higher voltages.

$$P_m = \frac{390}{d}(f+25)\sqrt{\frac{r}{D}}(E_n - E_0)^2 10^{-5} \text{ kw loss per mile per conductor.} \tag{4-13}$$

In the above equation the value of d (the relative density of the air) and of E_0 the disruptive critical voltage must first be obtained from the following equations

$$d = \frac{3.92b}{273 + T} \tag{5-13}$$

in which

d = relative density of the air
b = barometer height in centimeters of mercury
T = temperature of the conductor in degrees C.

Under standard conditions

$$b = 76.0 \quad \text{and} \quad T = 25 \text{ C}$$

$$\text{and } d = \frac{3.92 \times 76}{273 + 25}$$

$$= 1$$

The value of E_0, the disruptive critical voltage, is found by the equation

$$E_0 = 123\, mrd \log_{10} \frac{D}{r} \text{ effective kilovolts to neutral} \tag{6-13}$$

in which

E_0 = effective kv to neutral necessary to start corona
m = relative condition of surface of wire; varies from 1.0 for polished surface to 0.80 for roughened surface or stranded cables

* Proc. AIEE, 1912.

r = radius of wire in inches

d = relative density of air as found in Eq. 5–13

D = distance between conductors in inches

Example 6–13. A 150-mile 3-phase transmission line is to operate at 144,000 volts, 60 cycles. The cables are No. 000-stranded copper and are equilaterly spaced 12 ft 2 in. What corona loss might be expected in fair weather after the cable has been installed for about 6 months?

Solution. Under these conditions conservative values for

$$d = 1$$

and $$E_0 = 123\, mrd \log \frac{D}{r} \text{ effective kilovolts to neutral} \qquad (6\text{–}13)$$

where $m = 0.93$ (a conservative value)

$$r = \frac{.470}{2} = 0.235 \text{ in. (Table VI, App. B.)}$$

Thus $$E_0 = 123 \times 0.93 \times 0.235 \log_{10} \frac{146}{0.235}$$
$$= 75.0 \text{ kilovolts to neutral}$$

Substituting these values in Eq. 4–13

$$P_m = \frac{390}{1}(60 + 25)\sqrt{\frac{0.235}{146}}\left(\frac{144}{1.73} - 75.0\right)^2 10^{-5} \text{ kw per mile per wire.}$$
$$= 390 \times 85 \times 0.040 \times 68.3 \times 10^{-5} = 0.91 \text{ kw per mile per wire}$$

or 2.7 kw per mile of line. This solution assumes no voltage drop along the line.

It should be remembered that the formula used in the above example only approximates the value of corona loss under the given conditions. Much experimental work is being done on this phenomenon to obtain more precise equations, for various conditions of the cable and the weather.*

Prob. 31–13. If the transmission line of Example 6 is carrying 110 amperes per conductor, what is the power lost due to the resistance of the conductors?

Prob. 32–13. (*a*) What would be the loss in the line of Example 6 if the pressure between conductors were lowered to 110,000 volts?

(*b*) Compare ratio of loss with ratio of voltages.

Prob. 33–13. Show that for pressures of 44,000 volts and under, the corona losses are negligible on a line of standard spacing and commercial frequencies.

Prob. 34–13. What would be the corona loss in Example 6 if for the copper line conductors aluminum conductors of equivalent conductivity were substituted?

* See Wm. S. Peterson's discussion, AIEE Transactions 1933, Page 62.

11–13. Efficiency of Long Transmission Lines. By efficiency of transmission lines is meant the efficiency of the conductors only. The transformers or other apparatus are not to be included as part of the line. This efficiency must be measured under standard conditions, with a non-inductive load at the receiving end, with voltage of rated value and rated frequency and of sine wave-form. Since a line rarely operates under standard conditions, it is often desirable to find the efficiency under given conditions. But if no conditions are specified as to power-factor, sine wave-form, etc., standard conditions are understood to be meant. In computing the efficiency of the line, therefore, we have merely to divide the kilowatts **delivered by the line wires** to the apparatus at the receiving end by the kilowatts **received by the line wires** at the generator end under standard conditions.

The values are most easily arrived at by the following means:

$$\left.\begin{array}{l}\text{Power}\\\text{received by}\\\text{line wires}\end{array}\right\}\text{ must equal }\left\{\begin{array}{l}\text{Power}\\\text{delivered by}\\\text{line wires}\end{array}\right\}+\left\{\begin{array}{l}\text{Power lost in}\\\text{line wires.}\end{array}\right.$$

$$\left.\begin{array}{l}\text{Power}\\\text{lost in line}\\\text{wires}\end{array}\right\}=\left\{\begin{array}{l}I^2R\text{ loss}\\\text{in line}\\\text{wires}\end{array}\right\}+\left\{\begin{array}{l}\text{Corona loss}\\\text{in line wires}\end{array}\right\}+\left\{\begin{array}{l}\text{Leakage loss}\\\text{in line wires.}\end{array}\right.$$

In a well-constructed power transmission line the "leakage loss" is negligible, on account of the relatively small number of points of support where leakage may occur, therefore:

Efficiency of line equals

$$\frac{\text{Power delivered (by line wires)}}{(\text{Power delivered}) + (I^2R\text{ loss}) + (\text{Corona loss})}. \qquad (7\text{–}13)$$

Example 7–13. The following data for a typical three-phase transmission line are adapted from the *Electric Journal.*

Length of line	200 miles.
Frequency	60 cycles.
Load delivered to step-down transformers at load end of line	11,250 kw.
Power-factor lagging, at high-tension terminals of step-down transformers	85 per cent.
Voltage between conductors at receiving end of line	108,000 volts.
Conductors, copper cables	250,000 cir. mils.
Mean spacing of conductors	12.6 feet.

Resistance of transformers and protective coils at each end referred to high-tension side................................	4.1 ohms.
Reactance of transformers and protective coils at each end referred to high-tension side................................	64.5 ohms.

Find the efficiency of the line under these conditions. Note that the conditions are not quite standard, in that the power-factor is less than unity.

$$\text{Volts to neutral} = \frac{108{,}000}{1.73} = 62{,}400 \text{ volts.}$$

The resistance of each conductor of the line, from table, equals

$$200 \times 0.2165 = 43.3 \text{ ohms.}$$

The reactance of each line conductor equals

$$200 \times 0.804 = 161 \text{ ohms.}$$

From Table 7, Appendix B, we find that the charging current of each conductor equals

$$200 \times 5.41 \times \frac{62{,}400}{1{,}000{,}000} = 67.5 \text{ amp.}$$

Power taken by each step-down transformer equals

$$\frac{11{,}250}{3} = 3750 \text{ kw.}$$

Current taken by each transformer equals

$$I = \frac{3{,}750{,}000}{0.85 \times 62{,}400} = 70.6 \text{ amp.}$$

Figure 17–13 shows the high-tension side of the transformers at each end of the line. The resistance and reactance of the transformers referred to the high-tension side are represented, and also the resistance and inductive reactance of each line wire are represented. Let us consider line wire AB only.

The current in AB is a combination of the average charging current, $\frac{67.5}{2}$ or 33.8 amperes, and the transformer current of 70.6 amperes, and may be found by constructing the vector diagram of Fig. 18–13.

$$\begin{aligned} I_{\text{line}} &= \sqrt{70.6^2 + 33.8^2 + 2 \times 70.6 \times 33.8 \cos 122^\circ} \\ &= 60.0 \text{ amp.} \end{aligned}$$

The I^2R loss in one line wire equals

$$60.0^2 \times 43.3 = 156 \text{ kw.}$$

The I^2R loss in the three conductors equals

$$3 \times 156 = 468 \text{ kw.}$$

The disruptive critical voltage equals

$$E_0 = 123\ mrd \log_{10} \frac{D}{r} \text{ kv.} \qquad m = 0.9,\ d = 1$$

$$E_0 = 123 \times 0.9 \times 0.288 \times 1 \times \log \frac{151}{0.288}$$

$$= 86.6 \text{ kv.}$$

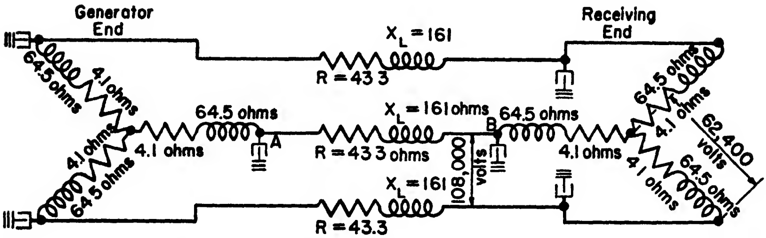


FIG. 17–13. Diagram of a transmission line showing the values and relative arrangements of the resistance and reactance of line and transformers.

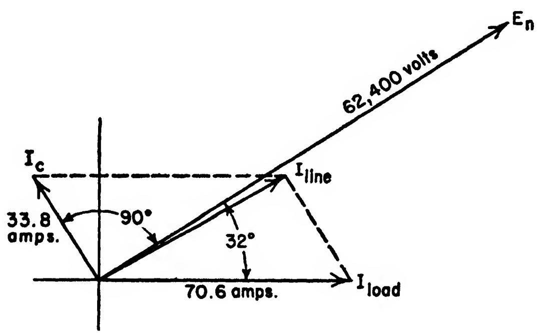


FIG. 18–13. The line current, I_{line}, is the resultant of the load current, I_{load}, and the charging current I_C.

Since the voltage to neutral is only $\frac{108{,}000}{\sqrt{3}}$ or 62,400 volts, it is less than the critical disruptive voltage, and there would be no corona loss in fair weather.

The corona loss = 0.

Total loss in line wires therefore equals the I^2R loss

= 468 kw.

Total input into line is equal to:

Load delivered to step-down transformers	=	11,250 kw.
I^2R loss in line	=	468 kw.
Total input into line	=	11,718 kw.

$$\text{Efficiency of transmission line} = \frac{11{,}250}{11{,}718} = 96.0 \text{ per cent.}$$

Note that this is the line efficiency under the special condition of a load with a lagging power-factor of 85 per cent.

Prob. 35–13. Find the efficiency of the line in Example 7–13, if the voltage at the receiving end were raised to 150,000 volts. Amount of power delivered, power-factor, and all other conditions the same as in the example.

12–13. Over-all Efficiency of Transmission. It is often necessary to compute the efficiency of transmission from the generator terminals to the load terminals, and to determine the power-factor at the generator. In this case it is necessary to add the losses in the transformers, feeder regulators, current limiting reactances and choke coils as well as the line losses. The simplest way to arrive at the total amount of power delivered by the generator is to combine all the power quantities taken by the several parts of the system.

First. Resolve the power taken in each part of the system into two components at 90° to each other, namely: the **Effective Power,** and the **Reactive Power.**

Second. Add all the effective power quantities together to obtain the total effective power, and all the reactive power quantities to obtain the total reactive power.

Third. Total apparent power delivered by the generator equals the square root of the sum of the squares of the total effective and the total reactive power.

Fourth. The power-factor of the generator equals the ratio of the effective power to the apparent power, delivered by the generator.

Example 8–13. Find the over-all efficiency of transmission in Example 7.

First. The power delivered to each step-down transformer equals 3750 kw at 85 per cent power-factor. By constructing Fig. 19–13, we see that this produces an apparent power of $\frac{3750}{0.85} = 4410$ kva, of which $\sqrt{4410^2 - 3750^2}$ (or 4410 sin 32°), equal to 2326 kva, is inductive reactive power.

The effective power consumed by each line wire equals $60.0^2 \times 43.3 =$ 156 kw.

Construct Fig. 20–13, adding the 156 kw in phase with the effective power of 3750 kw delivered to one of the step-down transformers by the line wire connected to it. The effective power consumed by each step-

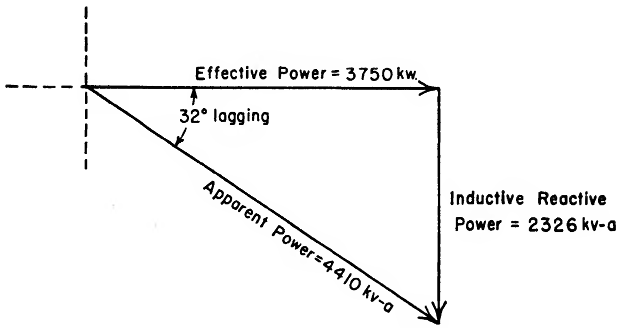


FIG. 19–13. The apparent power 4410 kva is the resultant of the inductive reactive power 2326 kvar and the effective power 3750 kw.

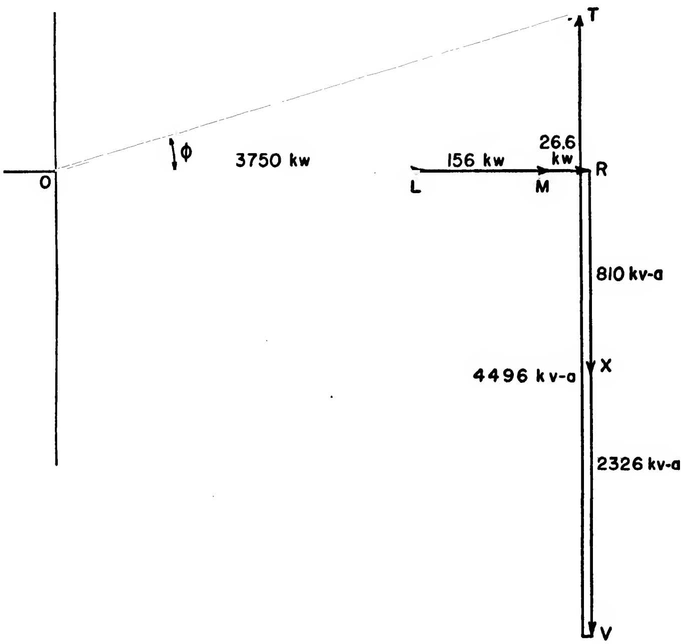


FIG. 20–13. Total apparent power OT delivered to the step-up transformers is the resultant of the total reactive and the total effective power delivered.

up transformer with accompanying current-limiting reactance coils is composed of the I^2R loss and the core losses. The core losses in a well-designed transformer of this size average about 80 per cent of the I^2R loss. Total effective power loss of one step-up transformer equals

$$1.80 \times (60.0^2 \times 4.1) = 26.6 \text{ kw.}$$

Add this in Fig. 20–13 to the lines representing the previously determined effective power.

The inductive reactive power taken by one line conductor and one step-up transformer equals

$$(64.5 + 161)60.0^2 = 810 \text{ kva.}$$

Draw, in Fig. 20–13, the line RX at right angles (lagging) to the line OR, to scale representing this 810 kva inductive reactive power. Add to this line the line XV representing the inductive reactive power delivered to each step-down transformer, namely 2326 kva. By means of a diagram similar to Fig. 11–13, the equivalent voltage to neutral at the sending end is found to equal approximately 66,600 volts.

The capacitive reactive power in each phase equals approximately

$$67.5 \times 66{,}600 = 4496 \text{ kva.}$$

Draw the vector VT in the opposite direction to the inductive-reactive-power vectors. This completes the effective and reactive power which must be supplied to each step-up transformer by the generator. The resultant vector OT will thus represent the power supplied by the generator.

$$OT = \sqrt{RT^2 + OR^2}$$

$$OT = \sqrt{(3750 + 156 + 26.6)^2 + (4496 - 810 - 2326)^2}$$

$$= 4160 \text{ kva.}$$

$$\text{Power-factor of generator} = \cos \phi = \frac{OR}{OT} = \frac{3750 + 156 + 26.6}{4160}$$

$$= 94.6 \text{ per cent leading.}$$

Total power delivered to the three step-up transformers

$$4160 \times 3 = 12{,}480 \text{ kva, representing}$$

$$3 \times (3750 + 156 + 26.6) = 11{,}800 \text{ kw at } 94.6\% \text{ power-factor.}$$

We have only to find the power delivered by the step-down transformer to the load in order to determine the over-all efficiency of transmission.

Effective power consumed by each step-down transformer:

$$I^2R \text{ loss} = 70.6^2 \times 4.1 = 20.42 \text{ kw.}$$

$$I^2R + \text{core loss} = 1.8 \times 20.4 = 36.8 \text{ kw.}$$

Inductive reactive power taken by each step-down transformer:

$$70.6^2 \times 64.5 = 321 \text{ kva.}$$

Referring to Fig. 19–13, which represents the total power delivered to the step-down transformer, we see that the effective power delivered to each phase of the load equals

$$3750 - 37 = 3713 \text{ kw.}$$

The inductive reactive power delivered to each phase of the load equals

$$2326 - 321 = 2005 \text{ kva.}$$

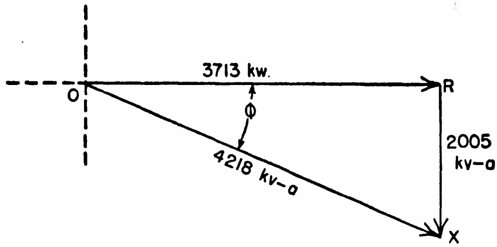


FIG. 21–13. The total effective power delivered by the step-down transformers to the load is represented by the vector OR. Reactive power delivered equals RX. Effective power equals OX.

From Fig. 21–13, constructed from these values, we find total apparent power delivered to load by each step-down transformer equals

$$\sqrt{3713^2 + 2005^2} = 4218 \text{ kva.}$$

$$\text{Power-factor of load} = \cos\theta = \frac{3713}{4218} = 88.0 \text{ per cent.}$$

Total effective power delivered to load equals

$$3713 \times 3 = 11{,}139 \text{ kw.}$$

$$\text{Over-all efficiency of transmission} = \frac{11{,}139}{11{,}800} = 94.4 \text{ per cent.}$$

Prob. 36–13. Find the over-all efficiency of transmission of Example 8–13 under standard conditions when delivering the same power to the load transformers.

Prob. 37–13. What would be the over-all efficiency of transmission in Example 8–13 if the voltage were raised to 150,000 volts, all other conditions remaining as in Example 8–13? Assume that the transformer and reactance coils are rewound so that they have the same losses as before, when transforming the same kilovolt-amperes.

13–13. Hoover Dam–Los Angeles Transmission System. At present the 266-mile 3-phase transmission line from Hoover Dam to Los Angeles, Cal., is the longest in this country and its voltage

the highest, being 287,500 volts between conductors at the dam and 275,000 volts at Los Angeles. It has a normal operating capacity of from 235,000 to 240,000 kw with a full-load efficiency of about 92 per cent.

An interesting feature of this system is that synchronous condensers are installed at the receiving end to maintain a constant voltage there from no-load to full-load. As explained in Chap. XI, by adjusting the field currents of these condensers they can be made to draw a definite amount of either a leading or a lagging current from the line. The following examples will serve to illustrate this and to show approximate methods of determining the amount of current required.

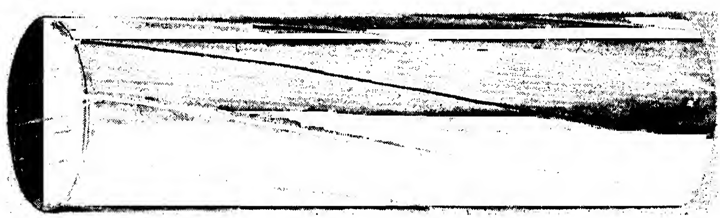


FIG. 22–13. A flexible hollow conductor.

Example 9–13. The voltage at the sending end of the Hoover Dam–Los Angeles line is maintained at 287,500 volts between conductors. What inductive current per conductor must be taken by the synchronous condensers at the receiving end to maintain the voltage there at 275,000 volts between conductors at no load?

Solution. The line consists of two 3-phase circuits operated in parallel. Each conductor is a flexible hollow tube, as shown in Fig. 22–13, constructed of hard-drawn copper 1.40 in. in diameter and having a cross section area of 512,000 cir. mils.

The resistance per mile.

$$R \text{ per mile at } 20^\circ \text{ C} = \frac{10.7 \times 5280}{512,000} = 0.1105 \text{ ohms}$$

$$R \text{ per mile at } 25^\circ \text{ C} = 0.1105 \times \frac{234.5 + 25}{234.5 + 20} = \mathbf{0.112 \text{ ohms}} \text{ per mile}$$

Resistance of 266-mile conductor $= 0.112 \times 266 = 30$ ohms.

The inductive reactance-to-neutral per mile. Since we can not use the tables in the Appendix for this value as we did for short lines we will use the following equation for inductance.*

$$L_m = .000741 \log_{10}\left(2.568\frac{S}{d}\right) \text{ per mile} \tag{8-13}$$

L_m = inductance-to-neutral of one conductor
S = average distance between centers of conductors, **in inches.**
d = diameter of conductors in **inches.**

For 225 miles out from the generating station, the line consists of two parallel 3-phase circuits strung on 2 parallel rows of steel towers. The three conductors on each tower are spaced 32.5 ft apart in the horizontal plane, an average spacing of 43.3 ft.

The inductance-to-neutral per mile is approximately

$$\begin{aligned} L_m &= .000741 \times \log_{10}\left(\frac{2.568 \times 43.3 \times 12}{1.40}\right) \\ &= .000741 \log_{10} 950 \\ &= .00221 \text{ henry per mile} \end{aligned}$$

The inductive reactance per mile is

$$X_L = 2\pi f L_m = 6.28 \times 60 \times .00221 = 0.83 \text{ ohm per mile}$$
$$X_L \text{ for 225 miles} = 0.83 \times 225 = 187 \text{ ohms}$$

For the remaining 41 miles which are through territory where land is more valuable, the six conductors are strung on a single tower. Here the 3 conductors constituting one circuit are spaced vertically 24.5 feet apart.

The inductance-to-neutral per mile for 41 miles is

$$L_m = .000741 \log_{10}\left(\frac{2.568 \times 32.7 \times 12}{1.4}\right) = 0.00221 \text{ henry per mile}$$

$$X_L \text{ for 41 mi.} = 60 \times 6.28 \times .00221 \times 41 = 33 \text{ ohms}$$

Total inductive reactance-to-neutral of one conductor for 266 miles is

$$X_L = 33 + 187 = 220 \text{ ohms}$$

* See Still's "Overhead Electric Power Transmission," McGraw-Hill Book Co.

The capacitance-to-neutral per mile.

$$C_m = \frac{0.0388l}{\log_{10} \frac{s}{r}} \tag{9-13}$$

in which

C = capacitance-to-neutral in **microfarads**
s = average distance between centers of wires in **inches**
r = radius of wire in **inches**
l = length of wire in **miles**

For 225 miles,

The average spacing for 225 miles is $\dfrac{32.5 + 32.5 + 65}{3} = 43.3$ ft

$$C_{225} = \frac{0.0388 \times 225}{\log_{10} \dfrac{43.3 \times 12}{0.70}}$$

$$= \frac{8.73}{\log 742}$$

$$= \frac{8.73}{2.87} = 3.04\ \mu\text{f}$$

For 41 miles,

The average spacing is $\dfrac{24.5 + 24.5 + 49}{3} = 32.7$ ft

$$C_{41} = \frac{0.0388 \times 41}{\log_{10} \dfrac{32.7 \times 12}{0.70}}$$

$$= \frac{1.593}{\log 560}$$

$$= \frac{1.593}{2.748} = 0.580\ \mu\text{f}$$

Total capacitance of 266 miles,

$$0.580 + 3.04 = 3.62\ \mu\text{f}$$

Total capacitive reactance to neutral equals

$$X_C = \frac{1}{60 \times 6.28 \times 3.62 \times 10^{-6}} = 730 \text{ ohms to neutral}$$

Line-to-neutral constants

$$R = 30 \text{ ohms per conductor}$$
$$X_L = 220 \text{ ohms to neutral}$$
$$X_C = 730 \text{ ohms to neutral.}$$

Our problem is to compute the inductive current which the synchronous condensers at the receiving load end must draw in order to maintain 275,000 volts per phase, or 159,000 volts to neutral, at the receiving end when it is unloaded. The voltage at the sending end is kept constant at 287,500 volts per phase, or 166,000 volts to neutral.

Figure 23–13 represents the circuit diagram of one conductor with no synchronous condensers added. Note that capacitive

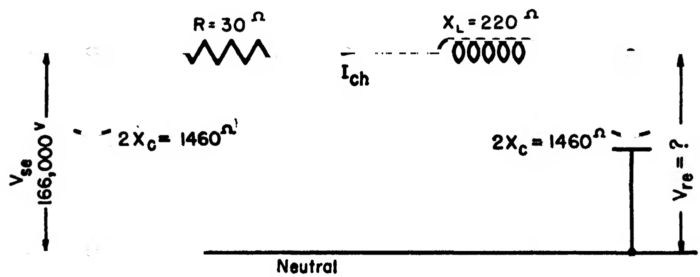


FIG. 23–13. The voltage V_{re} at the receiving end of the long unloaded transmission line will be higher than the voltage V_{se} at the sending end.

reactance is represented by two condensers each having twice the reactance of the line since they are in parallel.

The charging current per conductor is found as follows.

Since the effect of the resistance 30 ohms is practically zero,

$$I_{ch} = \frac{166{,}000}{1460 - 220} = \frac{166{,}000}{1240} = 134^{a} \text{ leading the voltage by } 90^\circ$$

The voltage drop in the line is

$$I_{ch}R \oplus I_{ch}X_L = 134 \times 30 \oplus 134 \times 220$$

The resistance drop $I_{ch}R = 4020$ in phase with I_{ch}.

The inductive reactance drop $I_{ch}X_L = 134 \times 220 = 29{,}500^{v}$ leading by 90°.

V at the receiving end must equal $V_{se} \ominus (I_{ch}R \oplus I_{ch}X_L)$.

In Fig. 24–13 $I_{ch}R$ is drawn in phase with I_{ch}, and $I_{ch}X_L$ is drawn leading the current by 90°. Note that $I_{ch}R$ is practically

90° to E_{se} and is so small that $I_{ch}X_L$ is practically in the opposite direction to V_{se}, thus

$$V_{re} = V_{se} - (-I_C X_L) = V_{se} + I_C X_L$$

Thus $V_{re} = 166{,}000 + 29{,}500 = 195{,}500$ volts.

Check. The voltage at the receiving end should equal the capacitive reactance of the condenser multiplied by the charging current

$$V_{re} = 2X_C I_C = 1460 \times 134 = 195{,}500 \text{ within } \tfrac{1}{2} \text{ of 1 per cent.}$$

This is the voltage which would appear at the receiving end if no synchronous condenser were connected there, and is 195,500 × 1.73 or 338,000 volts between lines. This is about 63,000 volts between conductors higher than the 275,000 volts which is desired.

This situation is corrected by connecting two synchronous condensers to the receiving end of each three-phase circuit. By regulating the field currents of these they draw just the amount of reactive current to produce the line drop needed to maintain the terminal voltage at 159,000^v volts to neutral.

The amount of this reactive current can be found as follows.

Since we can neglect the effect of the small line resistance, the drop of voltage from 166,000 volts at the sending end to 159,000 volts at the receiving end must be due to the inductive reactance of the line of 220 ohms per conductor.

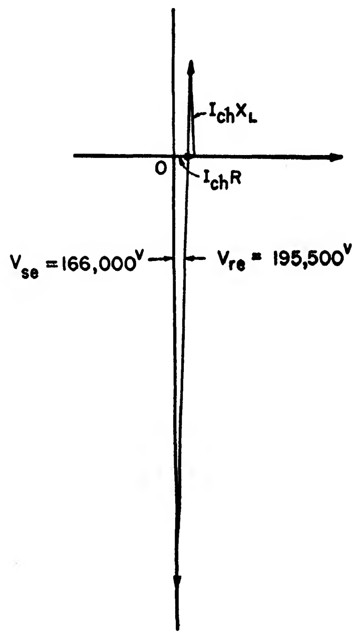


FIG. 24–13. Vector diagram showing why the voltage V_{re} at the receiving end of a long transmission line is higher than the voltage V_{se} at the sending end.

The line current to cause this drop of 166,000 − 159,000 or 7000 volts equals $\frac{7000}{220}$ or 32 amperes.

This current must lag 90° behind the 166,000 volts, since the line is practically wholly reactive.

If the voltage at the receiving end is to remain at 159,000 volts, the condenser assumed to be at this end, as shown in Fig. 23–13, must draw $\frac{159,000}{1460}$ or 109 amperes leading current.

But if the condenser draws **any** current whatever from the line, it will increase the line current above the 32 amperes required, and the terminal voltage across the condenser will drop below 159,000 volts. Thus the synchronous condensers must draw 109 lagging amperes from the line, in order to neutralize the 109 leading amperes of the line.

It is well at this point to remember that there is no actual condenser at the end of the line, but the condenser merely represents one-half the capacitance of the line assumed to be concentrated at the end of the line. The 109 leading amperes are really the average charging current in the line, and the 109 lagging amperes must also come from the line, in order to neutralize this leading current. Similarly the 32 lagging amperes producing the line drop must also come from the line. Thus the synchronous condensers must draw 109 + 32 or 141 lagging amperes from each of the six line wires. This means that the unloaded line requires an apparent power load on the synchronous condensers of 141 $\times$ 159 $\times$ 6 or 135,000 kva to maintain a voltage of 275 kilovolts between conductors.

Example 10–13. Maintaining constant voltage when the line is loaded. The system has a reliable operating capacity of from 235,000 to 240,000 kilowatts. We will assume a load of 216,000 kw at 90 per cent power factor at the receiving end. This is divided between two circuits so each circuit is carrying 108,000 kw, or $\frac{108,000}{0.90} = 120,000$ kva.

The voltage between conductors is 275,000 volts, so the load current each conductor is carrying equals $\frac{120,000,000}{1.73 \times 275,000}$ or 252[a].

Figure 25–13 shows the circuit of one conductor and neutral, the voltage-to-neutral at the receiving end being $\frac{275,000}{1.73}$ or 159,000 volts.

The load of 120,000 kva is paralleled by a synchronous condenser which is to draw from the line any reactive current which is needed to maintain the receiving voltage at 159,000 volts.

Let us first determine what the current in the line would be before adding the condenser. Figure 26–13 shows that there would be a current of 227 amp in phase with the terminal voltage and a

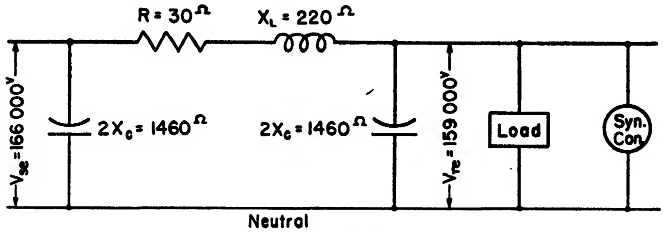


FIG. 25–13. The transmission line of Fig. 23–13 showing a synchronous condenser attached in parallel with the load.

reactive current of 110 amp lagging 90° behind the voltage and also a charging current of $\frac{159{,}000}{1460}$ or 109 amp leading the terminal

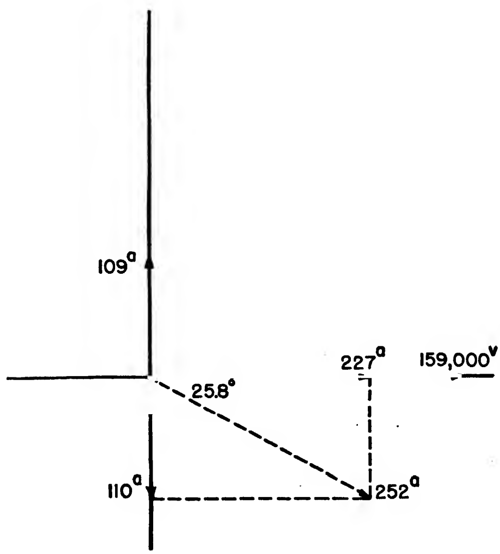


FIG. 26–13. The vector diagram of the currents drawn by charging current and the load alone on the line of Fig. 25–13.

voltage by 90°. By combining the reactive currents we have the results in Fig. 27–13, showing that without the synchronous condenser, there would be an in-phase current of 227 amp and a lagging current of 110–109 or 1.00 amp.

Instead of using this diagram to determine what reactive current must be taken by the condenser to maintain constant voltage, it is better to determine the total reactive current needed and add it vectorially to the 1.00 amp already flowing.

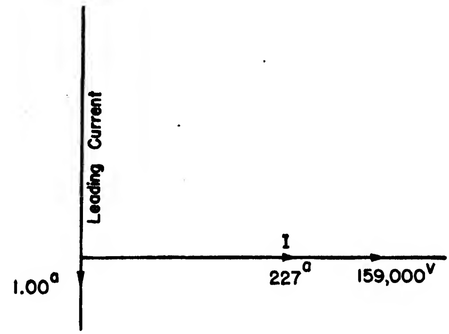


FIG. 27–13. Vector diagram showing the three currents of Fig. 26–13 combined into two currents.

Figure 28–13 will enable us to determine the line drops caused by the load current I and to write an equation for the total reactive current I' required to maintain 159,000 volts at the terminals.

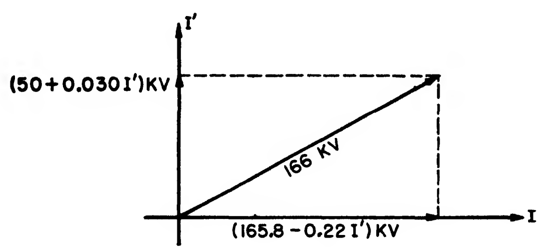


FIG. 28–13. Diagram for determining the total reactive current needed to maintain constant voltage at the receiving end. Enough current must be supplied by the synchronous condensers to make up for any reactive current not supplied by the charging current and the reactive current of the load.

The in-phase load-current of 227 amp flowing in the line needs 227×30 or 6800 volts which is in phase with the terminal voltage and makes a pressure $159{,}000 + 6800$ or 165,800 in-phase volts which the generator must have. This current also flows through the line inductive reactance and causes a drop of 220×227 or 50.0 kv which of course leads the current by 90° and is shown in an upward direction in the figure. But the unknown

reactive current I' also causes voltage drops in the line. The drop $30I'$ must be in the same direction as I'. Thus the drop at 90° to the current must be $50 + .030I'$ kilovolts. The unknown reactive current also causes a drop in the line inductive reactance of $220I'$ volts and must lead I' by 90°, which puts it directly in opposition to the line voltage and must therefore be subtracted from it. This makes the total in-phase voltage across the generator $165.8 - 0.22I'$ kilovolts.

The vector sum of the in-phase voltage and the 90° voltage must equal the 166 kilovolts at the generating end of the line. Thus

$$(165.8 - 0.22I')^2 + (50 + 0.030I')^2 = 166^2$$

Solving this equation, we obtain 36 amp for I', the amount of leading reactive current which must be drawn from the line to maintain the receiving voltage at 159,000 volts when the sending voltage is 166,000 volts.

Since there already is 1.00 ampere lagging current in the line, the synchronous condensers are called on to draw $36 + 1.00$ or 37 amperes per conductor. There are six conductors in the line; so the condensers must take $6 \times 37 \times 159{,}000$ or 35,000,000 voltamperes or 35,000 kva.

Prob. 38–13. How many kilovoltamperes (leading or lagging?) must the synchronous condensers draw from the above transmission line when 235,000 kilowatts at unity power-factor are being taken by the load? The line-to-line voltage at the sending end is 287.5 kv and at the receiving end, 275 kv.

Prob. 39–13. Compute the approximate corona loss for the 225-mile section of line in Prob. 38–13.

14–13. Effect of Irregular Forms of EMF Wave upon the Charging Current. Owing to the difficulties in properly proportioning the pole tips of a generator and in distributing the armature windings, the wave-form of the emf produced is rarely a true sine curve. It usually contains more or less well-defined ripples. The greatest cause of these ripples is the fact that the wave-form produced does not consist of a simple wave but is usually made up of several waves. There is not only the emf wave of a given frequency and given effective value which the machine was designed to produce, but there are also other waves of greater frequencies and usually of much smaller effective values. Each of these waves

approximates a true sine wave in form and the resultant wave-form of emf is merely a combination of them all.

The wave-form which the machine was designed to produce is called the **fundamental** or **primary harmonic**; the others are called the **minor harmonics.** The minor harmonics produced in the line by the modern generator usually consist of very small waves which have a frequency of three, five, or seven times the frequency of the fundamental wave. Even higher frequency waves are sometimes produced, and like the lower frequencies, their relation to the fundamental frequency is always an odd number.

Figure 29–13 shows the fundamental and a third harmonic of much smaller effective value. Each has its own sine wave-form.

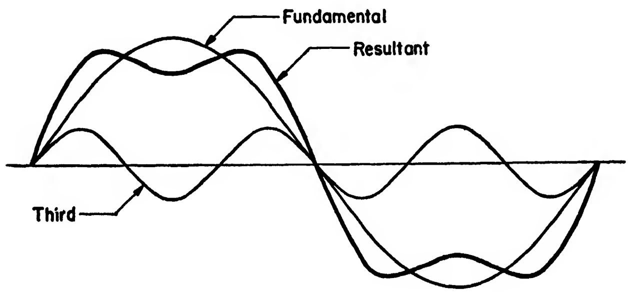


FIG. 29–13. A wave-form produced by a fundamental sine wave and a third harmonic in phase with the fundamental.

Note how they combine to produce a wave which has well-defined ripples. The third harmonic is made much larger in proportion to the fundamental than would exist in a properly designed machine. But a third harmonic of a smaller effective value would distort the fundamental in the same way, only to a less degree. Figure 30–13 shows the same fundamental and third harmonic at a different phase with each other, and the resulting wave-form which they produce. In Fig. 31–13, a fifth harmonic combines with the fundamental to produce the resultant shown, and in Fig. 32–13, both a third and a fifth of the same effective values combine with the fundamental to produce still another wave-form. The reason for this irregularity in the current curve in a circuit possessing large capacitance can be seen from the following example.

Example 11–13. The emf E, Fig. 32–13, is the resultant of a fundamental sine wave of 100 volts (effective), a third harmonic of 10 volts (effective), and a fifth harmonic of 10 volts (effective).

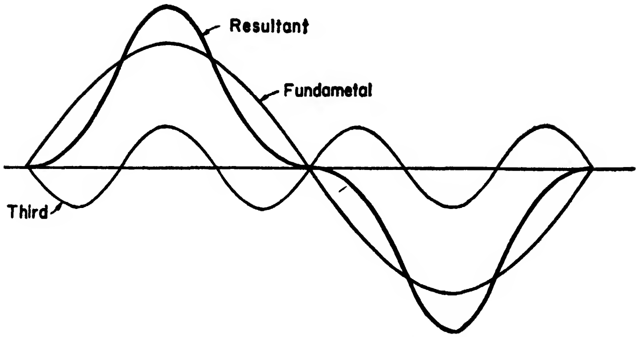


FIG. 30–13. A wave-form produced by a fundamental and a third harmonic which has a different phase relation than that of Fig. 29–13.

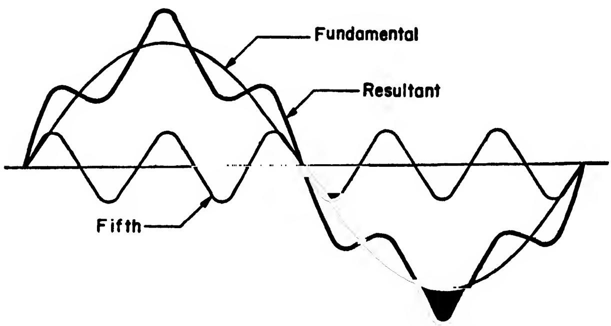


FIG. 31–13. The wave-form produced by the combination of a fifth harmonic with the fundamental.

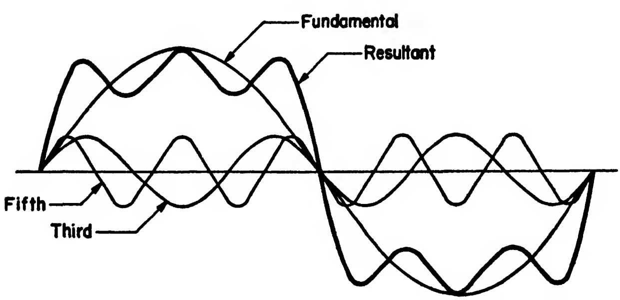


FIG. 32–13. The wave-form produced by the combination of both a third and a fifth harmonic with the fundamental.

What are the component parts of the resultant curve of current when this emf is impressed upon a circuit containing:
(*a*) Resistance only, of 5 ohms?
(*b*) Inductive reactance only, 5 ohms (at 60 cycles)?
(*c*) Capacitive reactance only, 5 ohms (at 60 cycles)?

(a) Circuit containing resistance only.

Since the resistance of the circuit would not change to any appreciable extent with the frequency of the impressed voltage, the line would offer practically the same resistance to the currents set up by each component of the emf.

$$\text{Fundamental current} = \frac{E}{R} = \frac{100}{5} = 20 \text{ amperes.}$$

$$\text{Third-harmonic current} = \frac{E_3}{R} = \frac{10}{5} = 2.0 \text{ amperes.}$$

$$\text{Fifth-harmonic current} = \frac{E_5}{R} = \frac{10}{5} = 2.0 \text{ amperes.}$$

Thus the **minor** current holds the same relation to the fundamental current that the minor emf's hold to the fundamental emf. The ripples in the current curve would, therefore, be neither more nor less pronounced than those in the emf curve.

(b) Circuit containing inductive reactance only.

The inductive reactance to the fundamental current is 5 ohms. Since the inductive reactance of a circuit is proportional to the frequency (being equal to $2\pi fL$), the inductive reactance to the third-harmonic current will be 3×5, or 15 ohms, because the frequency is three times as great; and the inductive reactance to the fifth-harmonic current will be 5×5, or 25 ohms.

$$\text{Fundamental current} = \frac{100}{5} = 20 \text{ amperes.}$$

$$\text{Third-harmonic current} = \frac{10}{15} = 0.67 \text{ ampere.}$$

$$\text{Fifth-harmonic current} = \frac{10}{25} = 0.40 \text{ ampere.}$$

The minor components of the current are much smaller parts of the fundamental current than the minor components of the emf are of the fundamental emf. Thus in an inductive circuit, the current curve is smoother than the irregular emf curve which produces it

(c) Circuit containing capacitive reactance only.

Since the capacitive reactance of a circuit is inversely proportional to the frequency of the impressed emf $\left(\text{being equal to } \frac{1}{2\pi fC}\right)$, the

capacitive reactance offered to the third-harmonic current equals $\frac{5}{3}$ or 1.67 ohms; and the reactance to the fifth-harmonic current equals $\frac{5}{5}$ or 1 ohm.

$$\text{Fundamental current} = \frac{100}{5} = 20 \text{ amperes.}$$

$$\text{Third-harmonic current} = \frac{10}{1.67} = 6 \text{ amperes.}$$

$$\text{Fifth-harmonic current} = \frac{10}{1} = 10 \text{ amperes.}$$

The minor-harmonic currents have thus become much greater in proportion to the fundamental current than the minor harmonic emf's are to the fundamental emf. The irregularities of the current wave in a circuit containing capacitive reactance only would thus be much greater than the irregularities in the emf curve producing the current. The following problems bring out the effect of resistance, inductive and capacitive reactance on the effective value of the current produced by an emf of irregular wave-form.

Prob. 40–13. An emf with an irregular shaped wave-form, produced by a fundamental and a fifth harmonic, is impressed upon a circuit containing resistance only, of 4 ohms. The maximum value of the fundamental harmonic of emf is 100 volts; the maximum value of the fifth harmonic is 20 volts.

(*a*) Plot to large scale one cycle of the component and resultant emf's with the fifth harmonic holding the same phase relation to the fundamental as in Fig. 31–13.

(*b*) Plot the component and resultant curves of current.

(*c*) Plot the squared values of one-half loop of the resultant current curve and find the effective current (root-mean-square value). If convenient, use a planimeter for finding area under the squared curve.

Prob. 41–13. Assume that the emf of Prob. 40 is impressed upon a circuit containing inductive reactance only, of 4 ohms at the frequency of the fundamental. Complete (*b*) and (*c*) of Prob. 40–13, and compare this value for effective current with that of Prob. 40–13.

Prob. 42–13. Assume that the emf of Prob. 40 is impressed on a circuit containing capacitive reactance only, of 4 ohms at the frequency of the fundamental. Complete (*b*) and (*c*) of Prob. 40, and compare this value of effective current with that of Prob. 40.

SUMMARY TO CHAPTER XIII

LONG TRANSMISSION LINES have capacitive reactance as well as resistance and inductive reactance.

THE CAPACITANCE-TO-NEUTRAL of one conductor of a trans-

mission line is found from the equation,

$$C_0 = \frac{0.03881}{\log \frac{s}{r}},$$

where C_0 = capacitance-to-neutral of one wire in microfarads
s = distance between centers of wires in inches
r = radius of each wire in inches
l = length of one wire in miles

THE CAPACITIVE REACTANCE-TO-NEUTRAL of one wire can be found from the equation

$$X_C = \frac{1}{2\pi f C_0}$$

in which X_C = the capacitive reactance-to-neutral of one wire in ohms
f = frequency in cycles-per-second
C_0 = capacitance-to-neutral of one wire in microfarads

A CHARGING CURRENT FLOWS back and forth when an alternating voltage is applied to a line, even when the far end is open.

The equation for the charging current in amperes is

$$I_C = \frac{E_n}{X_C}$$

THE VOLTAGE AT THE FAR END of an unloaded long transmission line is higher than the voltage at the sending end. The reason for this is that the inductive reactance drop is nearly opposite the generator voltage drop, while the resistance drop is nearly 90° to the generator voltage drop.

LIGHTNING ARRESTERS are attached through horn-gaps to long lines to protect the lines and connected machines from damage by lightning discharges. They are usually effective against surges set up by lightning discharges near the line but are not so effective against direct strokes. The pellet type may be used for voltage up to 70 kv and the Thyrite for voltage above that value.

CORONA LOSS from the conductor is the result of peak voltage reaching a value high enough to ionize the air about the wire and cause electricity to discharge in the air. It is accompanied by a glow about the wire. The power lost by corona can be found approximately by the equation,

$$P_m = \frac{390}{d}(f + 25)\sqrt{\frac{r}{D}}(E_n - E_0)^2 \times 10^{-5} \text{ kw per mile per conductor}$$

in which

$$d = \text{air density} = \frac{3.92b}{273 + T}$$

b = barometer height in cms of mercury
T = temperature in degrees C

$$E_0 = 123\ \mathrm{mdr}\ \log_{10}\frac{D}{r}\ \text{effective kilovolts to neutral}$$

E_0 = voltage to start corona in effective kilovolts to neutral
m = varies from 1.00 for polished surface of wire to 0.80 for roughened surface or stranded cables
r = radius of conductor in inches
D = distance between conductors in inches
E_n = applied voltage to neutral in effective kilovolts

THE HOOVER DAM-LOS ANGELES line operates at 287,500 volts at the sending end and 275,000 volts at the receiving end, over a distance of 226 miles. These voltages are maintained constant by synchronous condensers installed at the receiving end.

UNDERGROUND OR SUBMARINE CABLES have lower inductance but higher capacitance. They require costly insulation and at present their use is limited to about 23,000 volts.

THE PRESENCE OF HIGHER HARMONICS in the wave-form of alternating emf impressed upon a circuit of large capacitance causes the charging current to be much more irregular in form than the emf curve. The higher the frequency of the harmonics the greater the distortion of the current wave in a capacitive circuit.

PROBLEMS ON CHAPTER XIII

Prob. 43–13. Power is transmitted from Meppen, Ill., to Alton, Ill., a distance of 28.7 miles, at 66,000 volts, three-phase, 25 cycles. The line consists of No. 2, stranded copper, strung in horizontal plane, $7\frac{1}{2}$ feet apart, with no transpositions. Compute the charging current of this line.

Prob. 44–13. What is the voltage in the generator end of line in Prob. 43, when the line is open at the receiving end and the voltage there is 66,000 volts?

Prob. 45–13. The Sierra and San Francisco Power Co. transmit 34,000 kw from Stanislaus to San Francisco, a distance of 138 miles, by means of two three-phase circuits at a pressure of 104,000 volts. Frequency, 60 cycles. Conductors are No. 00, copper, six-strand, hemp-center, arranged in vertical plane, spaced 96 inches apart. Compute, by formulas, and check from tables in Appendix B:

(*a*) Capacitance of line per conductor to neutral.
(*b*) Inductive reactance of line per conductor to neutral.

Prob. 46–13. What is the charging current of the Stanislaus-San Francisco line? Assume 104,000 volts at sending end.

Prob. 47-13. When the line is open at the San Francisco end, and the pressure there is 104,000 volts, what is the pressure at the Stanislaus end?

Prob. 48-13. Compute the regulation of the Stanislaus-San Francisco line with full load of 85 per cent power-factor lagging.

Prob. 49-13. What is the voltage at the Stanislaus end at half-load, 0.95 power-factor lagging? Assume voltage at San Francisco to be maintained constant at 104,000 volts.

Prob. 50-13. If the voltage at the Stanislaus end of the line should become 118,000 volts when San Francisco is taking a load of 10,000 kw at 0.80 lagging power-factor, what will be the voltage at San Francisco?

Prob. 51-13. Compute the fair-weather corona loss of the line in Prob. 45.

Prob. 52-13. What is the efficiency of transmission of the Stanislaus-San Francisco line at

(*a*) Full load, unity power-factor?

(*b*) Full load, 85 per cent lagging power-factor?

Prob. 53-13. An electric power company is planning to transmit 22,500 kw at 110,000 volts, three-phase, 60-cycles, over a distance of 200 miles. Assume energy to cost 8 mills per kw-hr, and estimate interest, depreciation, taxes, etc., at 10 per cent. If there are two lines per tower, operated in parallel, and the line carries full load 16 hours per day and half load 8 hours per day every day of the year, unity power factor;

(*a*) What size aluminum cable would you advise be used at 35 cents per pound in place?

(*b*) What size copper cable at 20 cents per pound in place?

Prob. 54-13. (*a*) What spacing of conductors would you advise be used on the line in Prob. 53?

(*b*) What will be the charging current if copper cables are used? Neglect line drop.

Prob. 55-13. What will be the fair-weather corona loss, neglecting line drop, on the line if installed as in Prob. 54?

Prob. 56-13. Compute the regulation of the line in Prob. 53, at 0.80 lagging power-factor.

Prob. 57-13. What is the efficiency of the transmission line and the power-factor at the generator under the conditions in Prob. 56?

Prob. 58-13. The "Electrical World," gives the following data on the Cheat Haven-Butler, Pa., transmission line: the line is 106 miles long and operates at 125,000 volts, three-phase, 60-cycles, 2 lines per tower. Conductors No. 0, copper, six-strand, spaced in vertical plane, 60 inches apart. Compute the fair-weather corona loss using these data.

Prob. 59–13. What is the charging current on the Cheat Haven-Butler line?

Prob. 60–13. What is the line regulation of line in Prob. 58–13 when transmitting full load of 32,000 kw at 80 per cent lagging power-factor?

Prob. 61–13. Compute the efficiency of transmission of the line in Prob. 60, neglecting the transformers.

Prob. 62–13. What is the power-factor at the generators of the Stanislaus-San Francisco line when full load at 85 per cent power-factor and 104,000 volts is being taken from the receiving end? Neglect the transformers.

Prob. 63–13. What regulation will the line have and what power-factor will the generators have in the project of Prob. 53–13 as you have planned it, using aluminum cables, when the full load has a power-factor of 80 per cent?

Prob. 64–13. With the transmission line of Example 10–13 operated as in that example, check the sending end voltage needed to maintain 159,000 volts to neutral at Los Angeles.

Prob. 65–13. What is the greatest kilowatt load which can be taken from the Los Angeles end of the line in Example 10–13, if 166,000 volts to neutral are maintained at the sending end and 159,000 volts to neutral at the receiving end?

CHAPTER XIV

CONVERTERS AND RECTIFIERS

In many cities and towns electric systems were installed long before alternating-current machinery had been developed. The equipment of such places necessarily consists of direct-current machines. Even at the present time direct-current motors are better adapted to certain kinds of work — wherever adjustable speed is desired, for instance. Some commercial processes, such as electroplating and the charging of storage batteries, must be done by means of direct currents. If it is desired to take the power for any of these installations from an alternating-current system, it is necessary first to convert the alternating currents to direct currents.

The modern device for converting large quantities of a-c power to d-c power is the **ignitron,** which is essentially a large steel electronic tube containing mercury vapor under very low pressure. Until recent years, however, motor-generators and synchronous converters were the only machines available for this purpose. The fact that for years to come, millions of kilowatts will be converted to d-c power by these machines, makes it necessary for us to understand their operating characteristics, as well as those of the newer devices.

1–14. The Motor-generator Converter. This device consists of two separate machines, — an alternating-current motor and a direct-current generator, usually direct-connected to each other. The motor used for large sets, say over 100 kva, is generally the synchronous motor, on account of its constant speed and adjustable power-factor. This is connected to a compound direct-current generator, as shown in Fig. 1–14.

By varying the field current of the motor, the power factor of the motor may be adjusted without affecting the voltage of the generator. By varying the field of the generator, its voltage may be adjusted without disturbing the power-factor of the motor. For smaller sets, the power-factor of which is of much less importance, an induction motor is generally used, because it can be started much more easily than the synchronous motor.

2–14. The Synchronous Converter. Instead of using two separate machines for motor and generator, it was more general practice to make one machine, a synchronous motor of the revolving-armature type, perform the function of both motor and generator. The alternating current enters the closed winding of the armature through the collecting rings and causes the rotor to turn in synchronism with the alternations of the current, as explained in Chapter XI for the revolving armature type. We thus have a revolving armature with alternating currents surging back and forth through the windings. We are already familiar* with

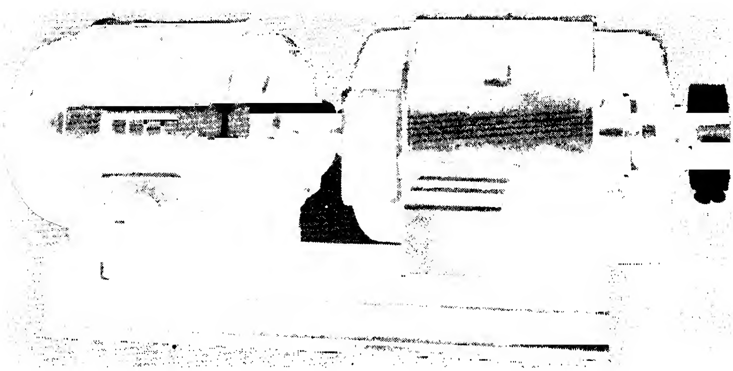


FIG. 1–14. 150-kw synchronous-motor-generator set. (*General Electric Co.*)

the facts that the armature windings of a direct-current generator always carry such alternating currents, and that a commutator properly connected to the windings is all that is necessary to cause direct current to be delivered to a set of brushes. Thus we have only to tap the windings at the proper points and connect these taps to the proper segments of a commutator in order to deliver direct current to a set of brushes bearing on the commutator.

Thus the same armature fitted with collecting-rings and a commutator, revolving in a field separately excited from a source of direct-current supply, receives alternating current at the rings and delivers direct current at the commutator. Such a machine is called a **synchronous converter** or a **rotary converter** and is shown in Fig. 2–14.

* See Vol. 1, Chap. X, Art. 5, 6, 7.

3-14. The Synchronous Converter versus the Motor-generator. The synchronous converter has the following advantages over a motor-generator converter:

First: The synchronous converter has a higher efficiency than a motor-generator of the same rating. The synchronous converter has but one field, thus the field loss is smaller. It has but one armature, thus the core loss is smaller. The currents in the armature windings are the resultants of the direct currents and the alternating currents which most of the time are opposing each other. Thus the I^2R loss in the armature of the synchronous converter is smaller than it would be in either a motor or a generator handling the same amount of power. Accordingly the I^2R loss is **much** less than the I^2R loss in a motor-generator which has two armatures, each with an I^2R loss higher than that of the synchronous converter.

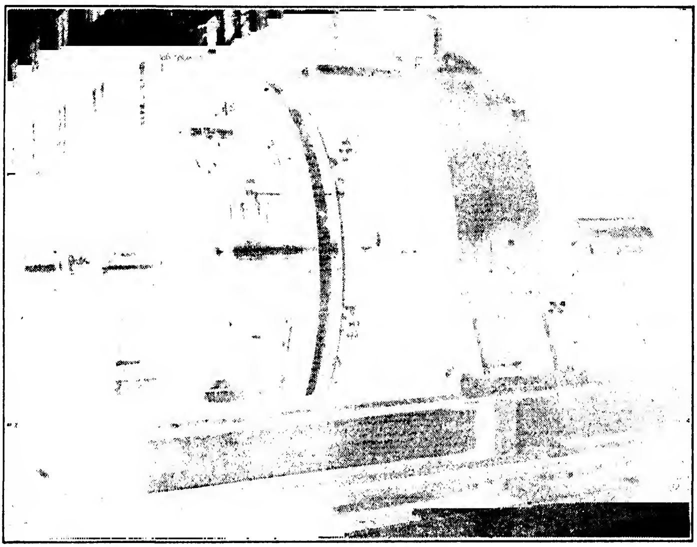


FIG. 2-14. 1000-kw 6-phase synchronous converter. (*General Electric Co.*)

Second: The synchronous converter weighs less and occupies less space per kilovolt-ampere capacity.

Third: If no large amount of auxiliary apparatus is required, the synchronous converter costs from 25 to 50 per cent less than the motor-generator set.

The motor-generator converter has the following advantages over the synchronous converter:

First: By using induction motors for the drive, motor-generator converters of small capacity (less than 100 kw) start easily and operate satisfactorily. They are much used in central stations for the purpose of furnishing the direct current for the fields of the alternators. Small rotary converters are difficult to start and have the instability of small synchronous motors. Thus they do not operate well on lines where sudden changes in load occur.

Second: The voltage of the generator of a motor-generator can be controlled and regulated by compounding the field so that constant voltage may be maintained across the direct-current brushes, even though the alternating pressure varies through wide ranges, as it may at the end of long lines. The voltage across the direct-current brushes of a synchronous converter holds practically a fixed ratio to the alternating voltage across the rings. Thus transformers are usually necessary with synchronous converters in order to lower the alternating voltage so as to bring the direct voltage down to commercial value. Any change in the alternating voltage produces a corresponding change in the direct voltage. Changing the field excitation of a synchronous converter produces almost no change in the voltage across the direct-current brushes. It merely changes the angle of lead or lag of the alternating current taken by the machine, as explained in Chapter XI in the case of synchronous motors. Thus wherever the direct-current voltage must be made to vary through wide ranges, a motor-generator converter is usually preferred.

There are various devices for regulating the direct voltage of a synchronous converter and maintaining it constant throughout a limited amount of change in the alternating voltage, but they all mean extra expense and many require expert attendants.

Third: The synchronous motor-generator can be used to improve the power-factor of the load on a system. The direct-current generator may be run idle and the total kilovolt-ampere capacity of the synchronous motor may be used to supply only reactive power to the line, or part of its load may consist of effective power. A synchronous motor can in this way supply 70 per cent of its rated kilovolt-ampere capacity as reactive power, and still be able to supply at the same time another 70 per cent of real power (kilowatts) to the generator to be converted into direct-current power.

While it is true that the power-factor of the power taken by a synchronous converter can be varied just as that of a synchronous

motor by means of changing the field current, still the kilovolt-ampere capacity of the converter is lowered rapidly as its power-factor drops much below unity — a power-factor of 90 per cent lowering the capacity to about 70 per cent of its unity power-factor rating. It is thus undesirable to depend upon the synchronous converter to supply much reactive power to the line load.

Example 1–14. A power station is supplying 7500 kw at 80 per cent lagging power-factor, which puts the rated full load (in kilovolt-amperes) upon the generators.

(*a*) How many kilowatts can be added to the above load by the use of a synchronous motor if it is over-excited enough to produce unity power-factor in the load on the generators?

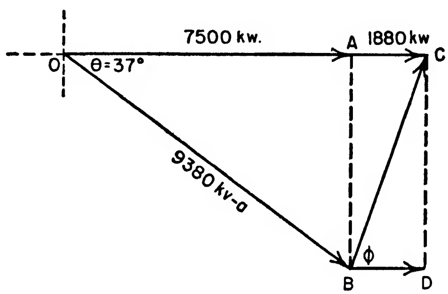


FIG. 3–14. By adding an overexcited synchronous motor to supply the leading reactive kva, the same a-c line current can supply an extra 1880 kw or 2260 hp to the receiving station.

(*b*) At what power-factor must the synchronous motor be run?

(*c*) Assuming an efficiency of 90 per cent at this load, how large horse-power mechanical load can the motor carry in addition to supplying the necessary reactive load?

Construct Fig. 3–14, drawing the vector OA to represent the 7500 kw, or effective power. Draw the vector OB at an angle of 37° (arc cos 0.80) to OA to represent the total apparent power which the generators must produce in order to deliver 7500 kw at 80 per cent power-factor.

$$OB = \frac{7500}{0.80} = 9380 \text{ kva.}$$

If the generators were delivering 9380 kva at unity power-factor they would be delivering 9380 kw effective power. They would thus be delivering 9380 − 7500 or 1880 kw more of real power.

Add vector AC, Fig. 3–14, to vector OA to represent this added effective power. Vector OC would then represent the same kva at unity power-factor on the generators that vector OB does at 0.80 power-factor. Or OC represents the resultant load when a load of 9380 kva

at 0.80 lagging power-factor is combined with **some other load** to produce 9380 kva at unity power-factor. This **other load,** which combined with the 9380 kva at a lagging 0.80 power-factor will produce 9380 kw, must be represented by the vector BC, since OC is merely the resultant of OB and BC.

$$\begin{aligned} BC &= \sqrt{BA^2 + AC^2}. \\ BA &= 9380 \sin 37° \\ &= 5630. \\ AC &= 1880. \\ BC &= \sqrt{5630^2 + 1880^2} \\ &= 5940 \text{ kva.} \end{aligned}$$

(*a*) Thus the synchronous motor may take 5940 kva from the line and still not increase the load on the generators. But note that of this load, 5630 kva (represented by the line BA) must be a leading reactive load, to counterbalance the 5630 kva lagging reactive load already on the line. The effective part of the added load is 1880 kva at unity power-factor. The power-factor of the synchronous motor load is thus

$$(b) \qquad \frac{\text{effective power}}{\text{apparent power}} = \frac{1880}{5940} = 0.316 \text{ leading.}$$

At 90 per cent efficiency, the motor could supply a mechanical load of,

$$(c) \qquad 0.90 \times 1880 \times \frac{1}{.746} = 2260 \text{ hp.}$$

Thus by using an over-excited synchronous motor, 2260 more horsepower can be taken from the line without adding to the load on the generators.

Prob. 1–14. If the motor of Example 1 were direct-connected to a direct-current generator having 92 per cent efficiency, how many kilowatts could it convert to direct-current power under the conditions of Example 1?

Prob. 2–14. (*a*) How many kilowatts, direct-current, could be delivered by the motor-generator converter of Prob. 1, if the motor is over-excited enough to raise the power-factor of the system of Example 1 to 0.90 lagging, and not add any kva load to the generators? Generator efficiency, 92 per cent. Motor efficiency, 90 per cent. (*b*) At what power-factor must the synchronous motor operate in this case?

Prob. 3–14. (*a*) If a synchronous converter operating at 90 per cent leading power-factor were used to supply the same direct-current power as in Prob. 2, how many kilovolt-amperes would be added to the load on the generators? Assume an efficiency of 95 per cent for the converter. (*b*) At what power-factor will the generators now operate?

Prob. 4–14. It is fair to assume that the synchronous converter of Prob. 3, when operating at 90 per cent power-factor, can deliver

not more than 70 per cent of its rated load (i.e., its load at unity power-factor). What must be the full-load rating (unity power-factor) of the converter of Prob. 3?

Prob. 5–14. How many revolutions per minute will the motor-generator converter of Fig. 1–14 make when operating on a 60-cycle system? The motor has 10 poles.

Prob. 6–14. How many poles would the motor of Prob. 5 have if it were intended to operate at 750 rpm on a 25-cycle system?

4–14. Ratio of the Alternating EMF to the Direct EMF in a Synchronous Converter.

Single-phase. Consider the diagram of a simple single-phase synchronous converter shown in Fig. 4–14. The poles N and S

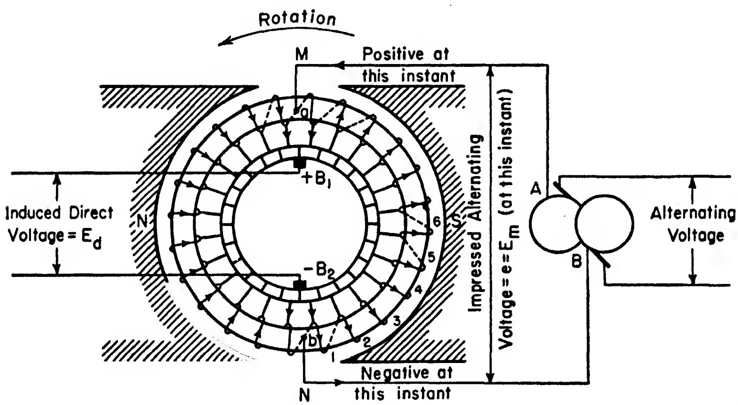


FIG. 4–14. Diagram of the armature windings and the connections of a single-phase synchronous converter. At this instant the maximum value of the alternating emf is being delivered through the rings to the armature at the tapping points a and b, causing it to rotate as marked. The induced emf marked on the armature windings is being delivered to the brushes B_1 and B_2.

are excited by direct current from an outside source. Alternating-current power is delivered to the collecting-rings A and B from an outside source. From these rings the lead wires M and N deliver the alternating current to the armature winding at the two tapping points a and b, situated 180 electrical degrees apart from each other. At the instant shown in Fig. 4–14 the alternating emf and current, at unity power-factor, would be at a maximum, and, considering the lead M positive at this instant, the armature current would cause the armature to rotate counter-clockwise as indicated. There would then be set up in the armature windings **an induced emf** as marked in the coils. Note carefully that what-

ever current the alternating emf may force through the armature windings at this instant must be forced **against** the induced emf and must then produce a motor effect tending to turn the armature in a counter-clockwise direction.

Note also that an alternating current at this instant can flow directly from the wires M and N through the neutral coils to the direct-current brushes without going through the armature. Any appliance attached to the brushes B_1 and B_2 would at this instant receive all its power directly from the a-c line.

Let us assume, for the sake of simplicity, that the armature resistance and reactance are negligibly small, and that the losses

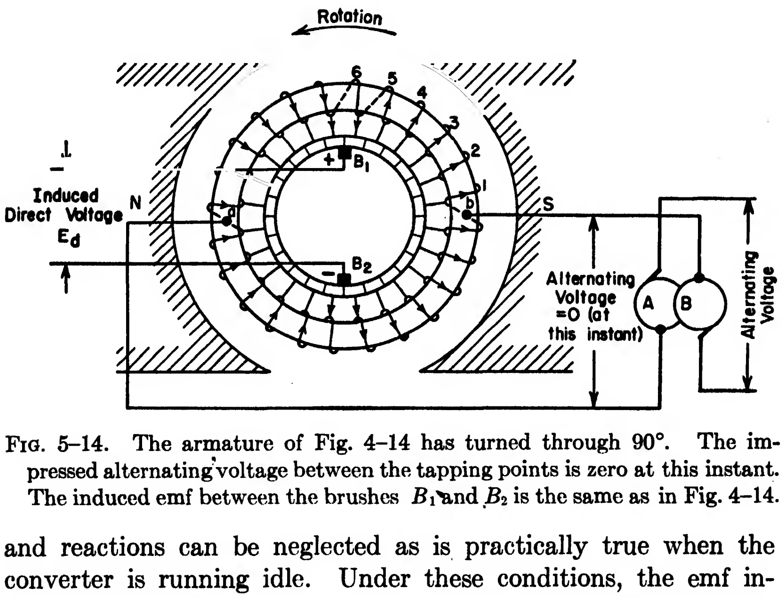


FIG. 5–14. The armature of Fig. 4–14 has turned through 90°. The impressed alternating voltage between the tapping points is zero at this instant. The induced emf between the brushes B_1 and B_2 is the same as in Fig. 4–14.

and reactions can be neglected as is practically true when the converter is running idle. Under these conditions, the emf induced in the windings is practically equal to the impressed emf. Now the induced emf is the emf which is delivered by the armature to the direct-current brushes, and the impressed emf is the maximum instantaneous value of the impressed alternating emf.

When the armature has turned through 90°, as in Fig. 5–14, the induced emf between the taps a and b becomes zero. But since the machine is in synchronism and in phase with the line voltage, the impressed emf between the rings AB at this instant has also become zero. There is thus no current in the wires M and N. The direct emf across the brushes, B_1 and B_2, however, will be the same as before.

We thus have a direct emf at the brushes which is equal to the maximum value of the alternating emf at the rings, and we may write the equation

$$E_d = E_m,$$

in which

E_d = direct induced emf.

E_m = maximum value of impressed alternating emf.

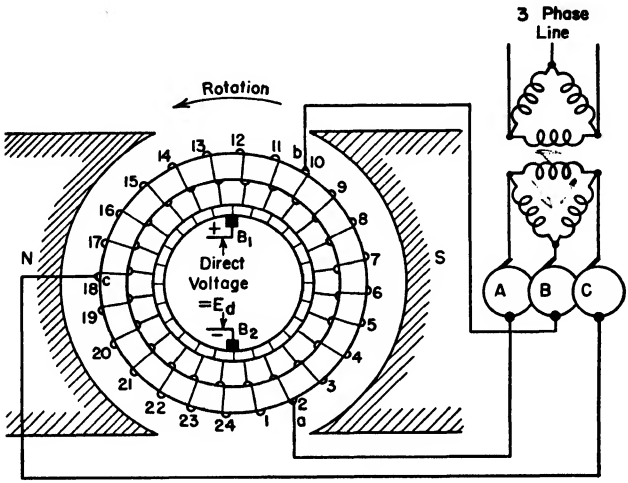


Fig. 6–14. Diagram of a three-phase three-ring converter. The impressed three-phase alternating voltage is brought to the three equidistant tapping points a, b and c on the armature. The induced voltage is delivered as before to the brushes B_1 and B_2.

Since the alternating emf is harmonic,

$$E = 0.707E_m.$$

Therefore

$$E = 0.707E_d;$$

where E = effective value of impressed alternating emf.

E_d = direct induced emf.

Thus **for a single-phase converter, we may say that at no load the alternating voltage is 0.707 of the direct voltage.**

Three-phase. For the voltage relations in a three-phase converter, consider Fig. 6–14 and 7–14. The three lead wires are tapped into the armature windings at three equidistant points a, b and c (that is, with eight coils between any two taps), and

brought out to their respective slip rings *A*, *B* and *C*. The alternating voltage impressed on the rings is thus applied to the armature at these three points and causes the armature to rotate as a synchronous motor. This induces an emf in the armature windings of practically a sine-wave form.

Let the vector 1 in Fig. 7–14 represent the maximum emf induced in coil 1, and vector 2 the maximum emf induced in coil 2,

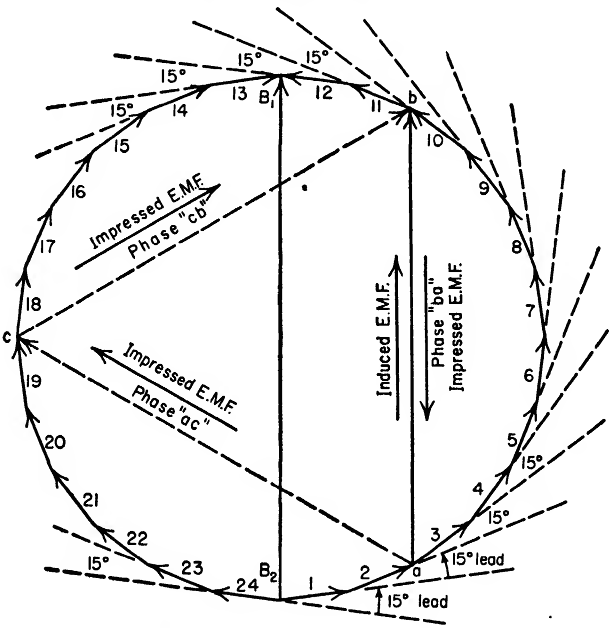


Fig. 7–14. Vector diagram of the induced and impressed voltage in the armature winding of a three-phase converter. The maximum induced voltage between the points *a*, *b* and *c* is equal and opposite to the impressed voltage at all times.

etc. Note that the emf's differ in phase with one another by $\frac{360°}{24}$ or 15°. Thus the emf in coil 2 is 15° ahead of the emf in coil 1, the emf of coil 3 is 15° ahead of the emf in coil 2, etc. The maximum induced emf through the twelve coils on one side or path of the armature, coils 1 to 12, and 24 to 13 inclusive, equals the vector sum of the maximum voltages across each coil. We have seen that this maximum emf across twelve coils is the voltage between the direct-current brushes.

The maximum induced emf between any two of the alternating-

current taps equals the vector sum of the maximum emf's in the eight coils composing the phase. Thus the vector *ab*, Fig. 7–14, is the vector sum of the maximum emf's in coils 3 to 10, inclusive, and represents the maximum value of the induced emf's across the phase *ab*, which is composed of these coils. This maximum value of the induced emf will take place when the phase *ab* is in the position shown in Fig. 6–14 and 7–14 because at that instant the instantaneous values of the voltages in the several coils composing the phase are nearest the maximum value. Note that in Fig. 6–14 at this instant the coils 3 to 10 are cutting lines of flux at the angle most nearly equal to a right angle and therefore at the greatest rate. Figure 7–14 shows the same fact, in that the vectors 3 to 10 are so situated that their resultant *ab* makes a right angle with the horizontal, — this being the position of a vector corresponding to the maximum instantaneous value.

Assuming the armature resistance, reactance, etc., to be negligible, as they practically are at no load, the impressed alternating emf equals the induced alternating emf in each phase, at all instants. If the maximum induced emf *ab* occurs at the instant shown in Fig. 6–14 and 7–14, then the maximum impressed emf *ba* should occur at the same instant. Otherwise there would be instants when the induced emf was higher than the impressed emf and currents would be sent back from the armature windings into the source of supply. Thus the vector *ab* represents the maximum induced emf in the phase *ab* and also the maximum emf impressed on the phase *ba*, which of course is in the opposite direction to the induced emf.

Accordingly, the ratio of the vector *ba* to the vector B_2B_1 equals the ratio of the maximum impressed alternating emf to the direct emf, of a three-phase converter. This ratio is merely the ratio of a side of an equilateral triangle to the diameter of a circle circumscribed about the triangle. Thus in Fig. 8–14, the ratio

$$\frac{AB}{AD} = \frac{\text{maximum impressed alternating emf, three-phase}}{\text{direct emf}}.$$

Since the angle *ABD*, Fig. 8–14, is a right angle,* and the angle

* An angle inscribed in a semicircle is a right angle.

Any angle at the circumference of a circle is equal to one-half of the angle at the center whose sides cut out the same arc on the circumference. The arc *AB* is 120°, therefore the angle *ADB* which subtends this arc is $\frac{120°}{2} = 60°$. Similarly the angle *DBA* is a right angle.

ADB is 60°,

$$\frac{AB}{AD} = \sin 60° = 0.866.$$

Thus the maximum value of the alternating emf is 0.866 of the direct emf.

But the effective value of alternating emf is the value generally used and this is 0.707 of the maximum value when the emf has a sine wave-form.

Therefore the effective value of the impressed alternating emf is 0.707 × 0.866 or 0.612 of the direct emf.

The actual values of the ratio of alternating voltage to direct voltage varies but little from the ideal values of this article. Any variation may be due to any of the following causes:

(*a*) The wave-form of the emf induced in the windings may not be that of a sine curve and thus the effective value would not be exactly 0.707 of the maximum. This depends largely upon what fraction of the pole pitch is covered by the pole-shoe. It has been found that if about 70 per cent of the pole pitch is covered by the pole-shoe, then the induced emf will have practically a sine wave-form. As modern converters are designed very closely to this specification, the form of the emf is very close to that of a sine wave.

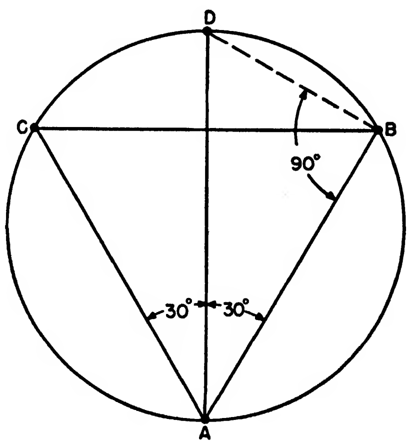


Fig. 8–14. The line *AD* represents the value of the voltage across the direct-current brushes. The lines *AB*, *BC* and *CA* represent the maximum value of the impressed alternating voltage.

$$\frac{AC}{AD} = 0.866.$$

The wave-form of the impressed emf also must be approximately a sine wave in order not to affect the ratio of voltages. It is especially important that the wave-form of impressed emf shall coincide exactly with that of the induced emf, otherwise equalizing currents of considerable magnitude may circulate in the armature windings.

(*b*) The direct-current brushes may not be set on the neutral

axis and accordingly the direct voltage would not be the maximum induced voltage.

(*c*) The resistance of the armature must be overcome whenever the machine is delivering current. Voltage across the direct-current brushes would be a little lower than the induced emf when the machine is converting alternating current to direct current. The ratio of alternating voltage to direct voltage would thus be a little larger at full load than at no load. For the same reasons, if the machine is used to convert direct current to alternating current the ratio would be lower at full load. The converter is then said to be **inverted.** The following table is based on data published by the General Electric Co.

TABLE I-14

VOLTAGE RATIOS OF CONVERTERS

Ratio of Alternating EMF to Direct EMF

Number of phases	Under ideal conditions	Under actual conditions	
		Full-load, straight	Full-load, inverted
One, two, six (diametral)	0.707	0.71	0.675
Three or six (double-delta)	0.612	0.62	0.580

Example 2-14. It is desired to maintain 550 volts between the direct-current brushes of a three-phase synchronous converter. What alternating emf must be applied between the rings?

For a three-ring converter,

$$\begin{aligned} E &= 0.62E_d \\ &= 0.62 \times 550 \\ &= 341 \text{ volts (at full load).} \end{aligned}$$

5-14. Ratio of Alternating Current per Ring to Direct Current. If we neglect the losses in the converter, which are actually very small, the total alternating-current power put in at the rings equals the total direct-current power taken out at the brushes.

Three-phase Converter. The alternating-current power received at the rings of a three-phase converter running at unity power-factor, if we neglect the slight losses in the machine, would be equal to the direct-current power delivered at the direct-current brushes.

$$1.73E_3I_3 \cos\theta = E_dI_d,$$

where

E_3 = effective voltage between alternating-current rings.
I_3 = effective current delivered to each ring.

But $\cos\theta = 1.0$

and $E_3 = 0.612\,E_d.$

Thus we may write

$$1.73 \times 0.612 E_d I_3 \times 1.0 = E_d I_d,$$

$$I_3 = \frac{I_d}{1.73 \times 0.612}$$
$$= 0.943 I_d.$$

As three-phase converters always operate at less than 100 per cent efficiency and sometimes at less than unity power-factor, the alternating current per ring is always somewhat greater than 0.943 of the direct current delivered by the machine.

Prob. 7–14. If the emf of a 3-phase 2300-volt transmission line were applied directly to the rings of a 3-ring converter, what would be the voltage across the direct-current brushes?

Prob. 8–14. Derive the ratio of the effective alternating emf impressed across adjacent rings of a six-phase converter to the emf across the direct-current brushes.

(It is customary to state as the alternating emf of a six-phase converter, either the voltage between rings which are tapped to points 180 electrical degrees apart on the armature, or the voltage between rings tapped to points which are 120 electrical degrees apart. This makes the emf ratios of a six-phase converter the same as that of a single-phase or that of a three-phase. See Arts. 4–14 and 5–14.)

Prob. 9–14. A three-phase three-ring converter is taking 1000 amperes per ring at a voltage of 700 between rings. If it were operating at 100 per cent efficiency and unity power-factor, what direct current would it be delivering, and at what voltage?

Prob. 10–14. What would be the direct current and voltage, if the converter of Prob. 9 were running with same alternating current and voltage at the rings, but were operating under practical conditions of 98 per cent power-factor and 92 per cent efficiency?

Prob. 11–14. Determine the ratio of the alternating current per ring of a six-ring converter to the direct current at the brushes for the ideal conditions of 100 per cent efficiency and unity power-factor.

Prob. 12–14. What would be the alternating current per ring and the alternating voltage between adjacent rings in a six-phase converter

operating at unity power-factor and 93 per cent efficiency, if it were delivering 2000 amperes direct current at 550 volts?

6–14. The Dobrowolsky Three-Wire Generator. When a synchronous converter is supplying a three-wire direct-current system, the neutral wire is brought back to the neutral point of the transformer secondaries. In this way, the unbalanced current carried by the neutral wire returns to the system at the neutral point and enters the armature winding through the collecting-rings.

An arrangement similar to this is often used in connection with the direct-current Dobrowolsky generator, in order to get the advantage of three-wire distribution.

For more detailed information concerning this generator and the construction, operation and maintenance of synchronous converters, see "Alternating-Current Electricity. Second Course," by Timbie and Higbie.

7–14. The Ignitron. Electric Conduction through Vapors. We have stated at the beginning of this chapter that within the last few years installations of machines for converting large quantities of a-c power to d-c power have consisted almost entirely of an electronic device called an **ignitron.** In fact, while many millions of kilowatts of power are still converted by synchronous converters, one manufacturing company alone has already installed ignitrons which are converting over a million kilowatts. It is necessary, therefore, that we understand the principle upon which this machine operates and become familiar with the more important details of its construction and operation.

In the first chapter of Vol. I we saw that all matter consists of different combinations of atoms. Each atom consists of a central nucleus made up of protons consisting of positive charges of electricity, and neutrons consisting of equal charges of both positive and negative electricity. Revolving about this nucleus are negative charges of electricity called electrons, each atom having as many electrons as there are protons in the nucleus.

In solid materials, an electric current consists of a flow of negative electrons which an impressed voltage has torn away from their nuclei. The electrons are so light in weight and small in size that they can easily be moved along in the spaces between the atoms and the molecules of the solid substance. The nucleus, however, is so large and heavy that it hardly drifts at all in a circuit of solid material.

The current in a solid conductor, therefore, consists of a flow of negative electrons through the circuit in a direction which is opposite to that which unfortunately was chosen as positive.

In Chapter XIV of Vol. I we found that in a liquid when electrons were torn from an atom by an electric voltage, they progressed through the liquid in the negative direction just as they do in a solid conductor. In a liquid, however, the nucleus is also much more free to move than in a solid. Thus the nucleus, having lost one or more electrons, and thus being positively charged, becomes an ion and moves slowly along through the liquid in a positive direction. In such a liquid, therefore, the electric current will consist of the sum of these two currents, since the flow of negative charges in the negative direction is the equivalent of a flow of positive charges in the positive direction.

It is readily seen that in a highly rarefied vapor a comparatively low voltage should easily maintain a large current consisting of both electrons and ions. The fact that the ignitron efficiently and quietly accomplishes this result is largely responsible for its superceding synchronous converters in supplying the heaviest d-c power requirements of industry.

8–14. How the Ignitron Converts A-C to D-C. The simple elements of a heavy-power ignitron, shown in Fig. 9–14, are inclosed in a steel tank, having double walls so that the inner walls may be water cooled. The element *A* is called the **anode** because it always has a **positive** voltage potential, when a current is flowing through the tank. The anode is connected to the terminal *b* of the secondary coil of the transformer. The element *c* is called the **cathode** because it must always be **negative,** when current is flowing, and is connected to one terminal of the d-c, or load, line. The other side of the d-c line is connected to the terminal *a* of the transformer secondary. The element *I* consists of a carborundum or a boron-carbide rod and is called an **ignitor.** This is insulated from the tank and is connected to the transformer secondary through auxiliary devices which operate the switch *S*. The pressure in the inner tank is reduced and maintained at less than one-half a millionth of atmospheric pressure.

During the half-cycle when the terminal *b* is negative, the switch *S* remains open. As soon, however, as *b* becomes positive, the switch *S* closes and a small current passes from the ignitor *I* to the mercury pool, and produces intense heat and electric arcs. These vaporize some of the mercury, the atoms of which are

separated into electrons which are negative and ions which are positive, being nuclëi which have lost at least one of their (negative) electrons. The electrons thrown off in great numbers and at a high speed fly toward the (positive) anode knocking off quantities of electrons from the mercury vapor with which they collide on the way. All these electrons are strongly attracted to

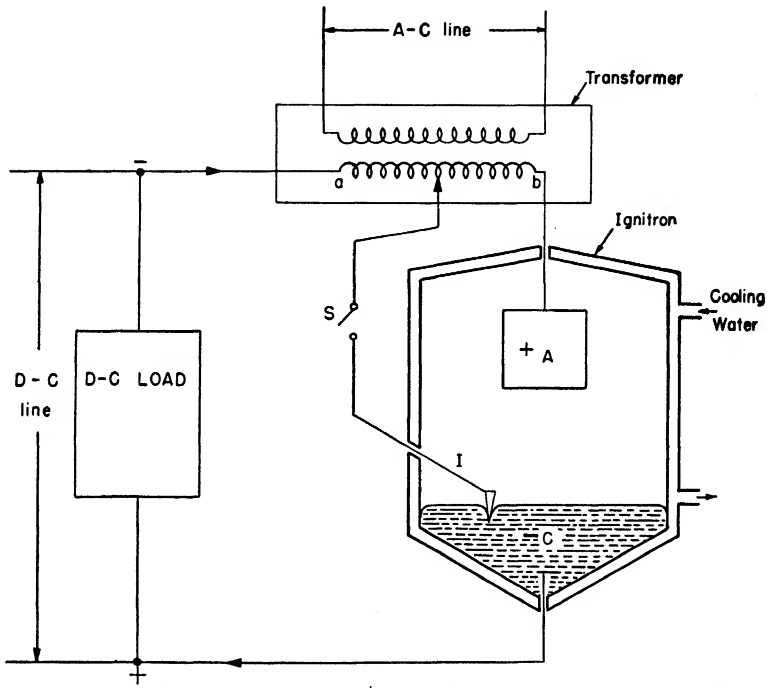


Fig. 9–14. Diagram of a circuit containing an ignitron for rectifying a-c to d-c power.

the (positive) anode, which they enter, and flow in a negative direction through the transformer secondary to the d-c load. We have seen that this negative flow of (negative) electrons constitutes what we call a positive flow of electricity. In the meantime the (positive) ions of mercury vapor are attracted to the negative mercury pool where they simmer over the surface and create many so-called " cathode spots " and give up their positive charges to the mercury pool, from which they go to the positive terminal of the d-c line. These "spots" emit still more electrons from the mercury which are attracted to the anode.

The result is that electrons pass in such great numbers from the cathode to the anode that they often constitute currents averaging many hundreds of amperes.

This action continues until just before the voltage of the terminal *b* becomes zero, at which instant the switch *S* opens and disconnects the ignitor and no more mercury is being ionized and the

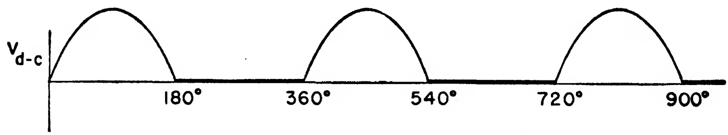


FIG. 10–14. Form of the d-c voltage produced by a single-anode ignitron.

current rapidly decreases to zero. The anode has now become negative but it does not emit electrons so that during the negative half cycle no current flows in either direction.

The curve of d-c voltage delivered by a single tube is shown in Fig. 10–14. The average value of the d-c voltage having a curve of this character would be about one third the maximum value of one of the loops. By paralleling two ignitrons and timing the firing of both, the d-c voltage can be made to have the shape shown in Fig. 11–14. Note that the loops are made to overlap

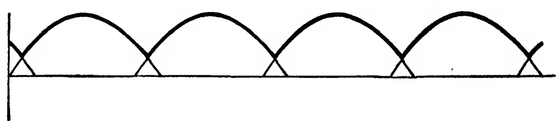


FIG. 11–14. Form of the d-c voltage when two single-anode rectifiers are operated in parallel.

so that the voltage never decreases to zero. Note from Fig. 9–14 that the positive side of the d-c line is the side coming from the cathode or negative terminal of the ignitron. The maximum d-c voltage is the maximum voltage across the transformer secondary *ab*, minus about 35 volts. In other words there is a drop of from 20 to 25 volts (effective value) lost in the ignitron, regardless of the value of the current going through it. This makes the efficiency very high when operating at high voltages and large currents.

9–14. Refinements in Ignitrons of Large Sizes. In ignitrons of large current capacity it has been found necessary to install a baffle plate above the mercury pool as shown in Fig. 12–14(*a*). This plate is insulated from the mercury and deflects any mercury

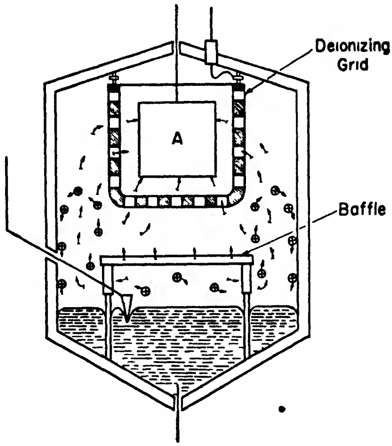


FIG. 12–14(*a*) A large ignitron with baffle and deionizing grid.

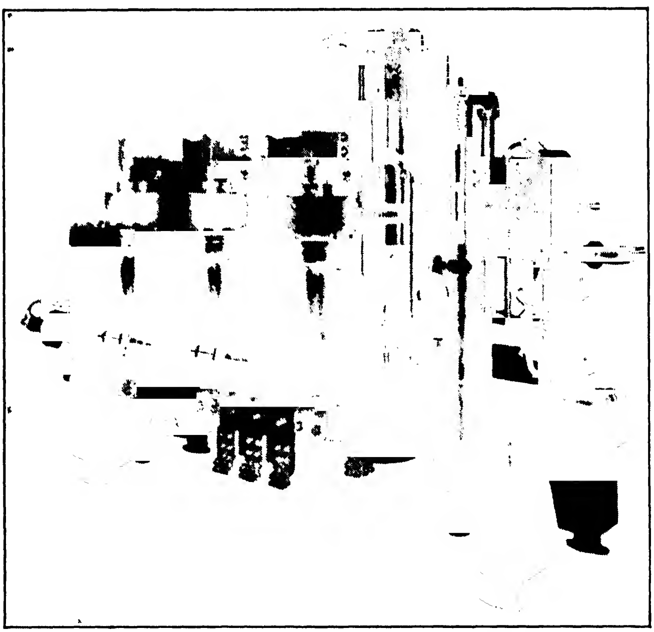


FIG. 12–14(*b*). A 6-phase ignitron (*General Electric Co.*).

from splashing on the anode and causing flash backs. The anode is also completely surrounded with a deionizing mesh grid which is positively charged. This adds to the speed of the (negative) electrons leaving the mercury pool. Of course many of the electrons hit the mesh but most of them easily pass through the open spaces in the mesh and reach the anode. This positive mesh also strongly repels the (positive) ionized mercury vapor and drives it to the wall of the container where it forms into drops which flow back into the mercury pool.

The high vacuum is maintained in the large ignitrons by a mercury pump. The smaller ones are hermetically sealed. Figure 12–14(*b*) shows a six-phase ignitron.

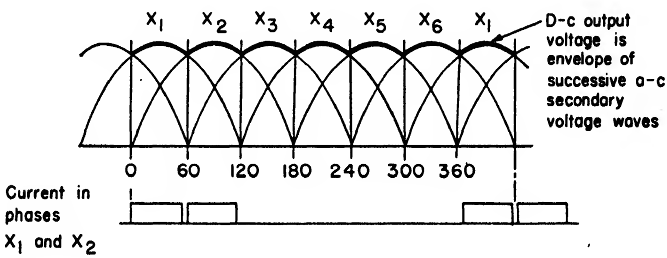


FIG. 13–14. Form of d-c voltage produced by a six-anode ignitron. (*General Electric Co.*)

In very large sizes, six anodes are placed in the same tank and are excited positively in successive intervals of 60 electrical degrees. Thus the d-c voltage reaches a maximum six times during each complete cycle, resulting in the curve of Fig. 13–14. The practical d-c curve is not quite so smooth.

The connection diagram for the six-phase ignitron is shown in Fig. 14–14, the anodes for convenience being represented as though they were in separate tanks.

This type of ignitron is started by dipping the positively charged ignitor into the mercury pool and drawing it out again to start small arcs which create the first electrons and ionized particles of mercury vapor. Since, at every instant, two of the anodes are positive, the ionization of the mercury continues without any further need for an ignitor.

The six-anode and twelve-anode ignitrons are capable of carrying from 100 to 3000 amperes at from 1000 to 5000 volts.

Among the advantages ignitrons offer for rectifying a-c power

are: No large moving parts, quiet operation, high efficiency, easy installation and no necessity for heavy foundations. They cannot, however, be used for power-factor correction.

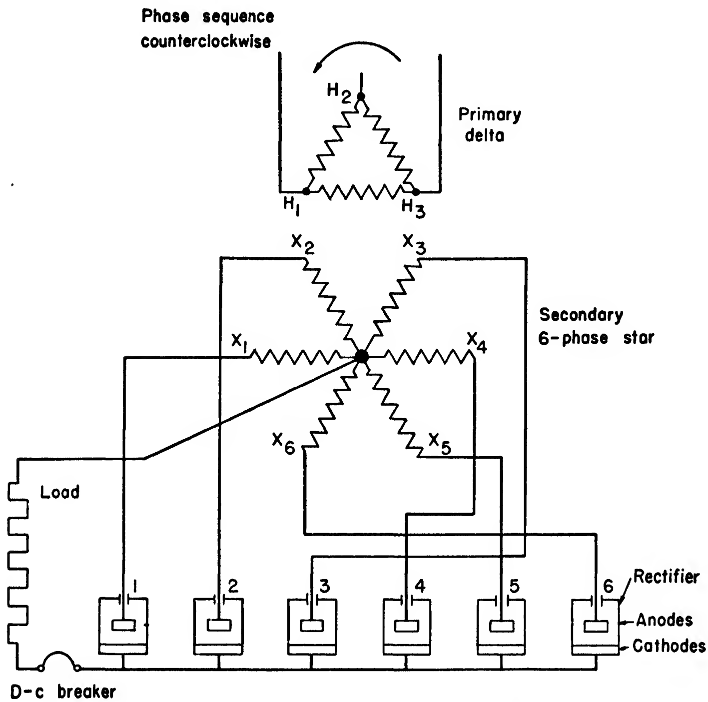


FIG. 14–14. Diagram of a circuit using a six-anode ignitron for converting 3-phase a-c to direct-current power. (*General Electric Co.*)

Prob. 13–14. Show how the secondary coils S_1, S_2, S_3, etc., of Fig. 15–14 should be connected to produce the 6-phase star of Fig. 14–14, with a 60° phase difference between phases.

SMALLER RECTIFIERS

For changing alternating-current power in small quantities into direct-current power there are several devices much less expensive than the motor-generator or the synchronous converter. These are called rectifiers and are of three types, (*a*) the mercury arc, (*b*) the electrolytic and (*c*) dry-plate. Since the direct current from these three types consists of a series of pulses, they cannot be used on inductive circuits.

There is a fourth type, the vacuum-tube rectifier, used in radio. By means of combinations of capacitors and inductors, this type delivers the fairly smooth but very small direct currents used in communication appliances.

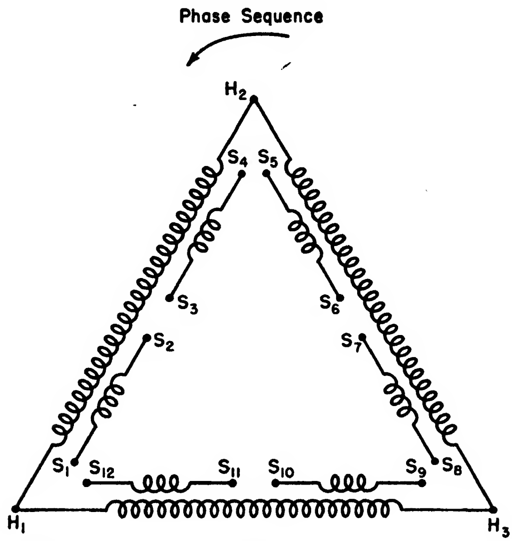


FIG. 15–14. Diagram of a three-phase transformer with the primary coils connected in delta, and the secondary coils left open to be connected into six-phase star having a 60° phase difference between phases.

10–14. Mercury-arc Rectifier. By far the most common rectifier is the mercury arc, which is widely used to rectify alternating currents for the purpose of charging storage batteries. A connection-diagram of a common type of mercury-arc rectifier is shown in Fig. 16–14.

The glass tube containing the mercury is exhausted until a very low pressure is obtained. There are two wells, B and X, which contain mercury, and two positive graphite electrodes A and A', generally called the anodes.

The anodes A and A' are connected to opposite sides of the line from the transformer. A coil of high reactance but of low resistance ($T_1 - T_2$) is also connected across the transformer. The negative side of the battery to be charged is connected to the middle point C of the reactance coil, and the positive side to the large mercury well at B. The small mercury pool at X, which is merely used to start the arc, is connected through a resistance R to

one side of the transformer line. There is such a high resistance offered by the gap between the mercury wells that it would take many thousand volts to start a current through it, so a starting device is necessary. The tube is tilted until a bridge of mercury is formed across the space between B and X. This offers a path from wire E through resistance R, from X to B, through the battery to C, through half the reactance coil T_1, to the other side of the circuit D. An alternating current would therefore flow through this path. If the tube is now tilted back, an arc is formed which vaporizes some of the mercury and so charges it with electricity that the resistance is cut down between the points A' and B and between the points A and B. Now mercury vapor possesses the quality in common with almost all metallic vapors of allowing a current to pass easily in one direction and hardly at all in the other direction. Thus the current can now easily pass from either A or A' to B, depending upon whether A or A' happens to be positive at this instant. If A' happens to be positive, a current is immediately set up through the vapor between A' and B, and flows from A' to B, through the battery to C, through T_1 to the other side (D) of the transformer. At the next instant A' has become negative and A positive. Practically no current can flow back from B to A', but since A is now positive, a current flows from A to B through the vapor, then through the battery to C, through T_2 to the side E of the transformer. Thus the current through the battery is always in one direction.

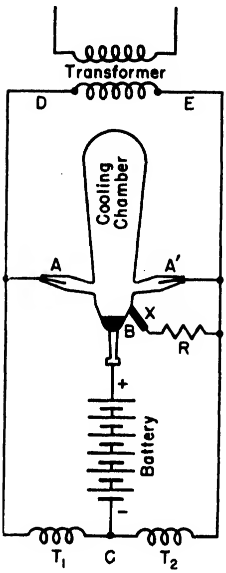


FIG. 16–14. Diagram of the connections for the single-phase mercury-arc rectifier.

But the mercury arc has the properties of any other arc, — it requires a voltage to maintain it. Now when E is changing from positive to negative, or vice versa, there is an instant when the voltage is zero, and at this instant the arc tends to go out. The inductive reactance of the coils T_1 and T_2 is used for the purpose of preserving the arc. For, as we have seen, a current is set up in T_1 before the voltage from A' to B dies out. This current, during the decay of the voltage from A' to B, tends to keep up its strength because of the inductance of the coil T_1 and thus jumps through

the vapor from A to B forming a local circuit, — from A to B, through the battery, through T_1 to A again. So, even before A becomes positive on account of the secondary transformer voltage, a current is already flowing from A to B, and it is merely increased by the rising positive voltage from A to B. Thus the currents overlap one another, as it were, and maintain a resultant current flowing through the tube continuously. The direct current is not smooth however, but consists of a series of positive pulses, although it never falls to zero.

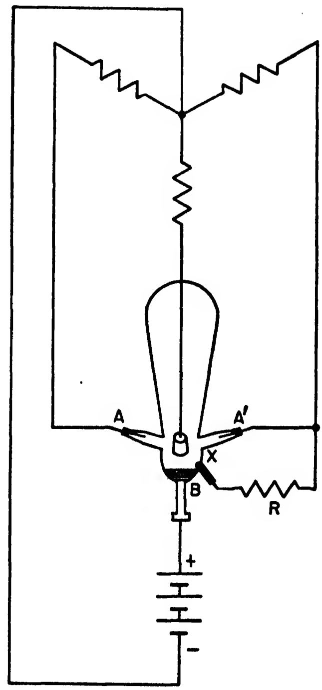


FIG. 17–14. Diagram of connections for a three-phase mercury-arc rectifier.

The current in the opposite direction (from mercury well to carbon anode) is not entirely shut off. A small "inverse" current will always flow in this direction, and it may at any instant become large enough to cause a short circuit, — for instance, when the tube gets old and the vacuum falls off. This allows the inverse current to assume very great proportions and destroys the rectifying action. In fact any cause which lowers the vacuum will allow the inverse current to be set up. Mercury will sometimes condense on the side of the tube and drop on the red hot carbon anodes. This vaporizes the drop of mercury and instantly lowers the vacuum. The tube thus practically forms a short circuit on the alternating-current line, since the current can flow both ways through it with very little resistance. Precautions are therefore taken in designing the shape of the tube to prevent mercury globules from coming into contact with the carbon anodes.

The reactance coils T_1 and T_2 may be omitted if a special transformer with large leakage reactance is used, having the secondary divided into two equal coils. The negative of the batteries is then brought to the juncture of these two secondary windings, which perform the duties of both reactance coils T_1 and T_2 and also of the secondary winding DE.

A tube constructed for use on a three-phase circuit works even

better than a single-phase tube. Figure 17–14 shows the connection for a three-phase tube. Note that the return from the battery is brought back to the neutral juncture of the three Y-connected transformer coils.

11–14. Rating and Efficiency of Mercury-arc Rectifiers. The most common size of tube for charging storage batteries is built of glass with a maximum capacity of 30 amperes direct current, and with a minimum current of 5 or 6 amperes. If the current in these tubes is raised much above 30 amperes, the glass becomes too hot and soon breaks down by taking on a coating of mercury which short-circuits the terminals. If the current drops below 5 or 6 amperes, the arc "goes out" and the tube must be tilted up and started again. In other words, it takes 5 or 6 amperes to keep the mercury sufficiently vaporized and ionized to maintain an arc.

Other sizes in glass are rated at 10, 20, 40 and 50 amperes maximum. The drop across the tube is always 14 volts, regardless of what the impressed voltage is. The efficiency then depends entirely upon the voltage at which the tube is operated. In practice, the combined efficiency of the transformers and tubes, etc., ranges from 80 per cent to 92 per cent. The power-factor is about 90 per cent. The regulation is excellent, depending entirely upon the drop in the transformers and reactance coils, since the drop in the bulb does not change at all with the load. The life depends upon the temperature at which the bulb is run, — a low temperature resulting in an indefinitely long life.

12–14. Electrolytic Rectifier. Many metals immersed in some solution offer a high resistance to the passage of an electric current when it is flowing **from** the metal to the solution, but yet offer a very low resistance when current flows from the liquid to the metal. In the case of aluminum, it is practical to make commercial use of this property for rectifying alternating-current power in relatively small amounts.

A plate of aluminum and a plate of lead are placed in an electrolyte, generally a solution of neutral ammonium sulphate. The aluminum will allow a current to flow from the lead through the solution to the aluminum plate with not much resistance, but offers a high resistance to the flow from the aluminum through the electrolyte to the lead plate. Such a cell is called an **electrolytic rectifier.**

An arrangement of four cells is generally used when an alternating current is to be rectified for charging storage batteries,

as in Fig. 18–14. When the side B of the transformer is positive, the current can flow through cell I from the lead to the aluminum, but not much current can get through cell II as it would have to pass from the aluminum to the lead. Therefore, it is forced through the battery to point D. From here, it can flow through cell IV, from lead to aluminum again, and reach the other side A of the transformer. Similarly, when A becomes positive, the current flows from A through cell II, through the battery and cell III to the side B of the transformer.

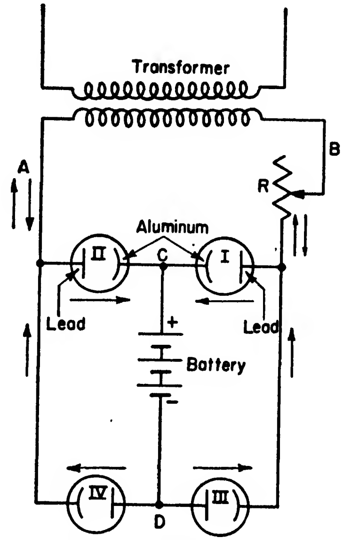


FIG. 18–14. Diagram of connection of four aluminum cells to be used as an electrolytic rectifier.

The efficiency of this arrangement depends upon the temperature of the electrolyte; being about 30 per cent for an equipment for charging a 6-volt battery, if the temperature does not rise above 30° C, but becoming very small as the aluminum cells get hot. In order to operate at the proper temperature, the aluminum plates should each have an area of about 2 square inches for each ampere which is delivered to the battery. Thus to charge a battery at the rate of 10 amperes, each aluminum cell should have an aluminum plate of about 20 square inches and a lead plate of about the same area, and should hold nearly a quart of electrolyte. The amount of current is regulated by the resistance R.

13–14. Dry-plate Rectifiers. The copper-oxide rectifier is a true dry-plate rectifier. It is composed of a stack of copper disks one surface of which is coated with copper oxide. The copper disks are separated from one another by thin lead disks or lead foil. The whole stack is pressed tightly together. An electric current will pass easily from the oxide to the copper, but hardly at all from copper to copper oxide. However, it will pass from the lead disks to oxide. Such a stack makes a satisfactory converter from a-c to d-c power for relatively small currents and is much used in a-c measuring instruments and in telephone appliances.

Most of the other so-called "dry-plate" rectifiers consist of plates of different metals separated by paper sheets soaked in an electrolyte. They depend upon chemical reaction which is similar to that of the aluminum rectifier described in Art. 12–14.

14–14. Vacuum Tube Rectifiers, for Radio Service. The circuit shown in Fig. 19–14 illustrates the fundamental principle

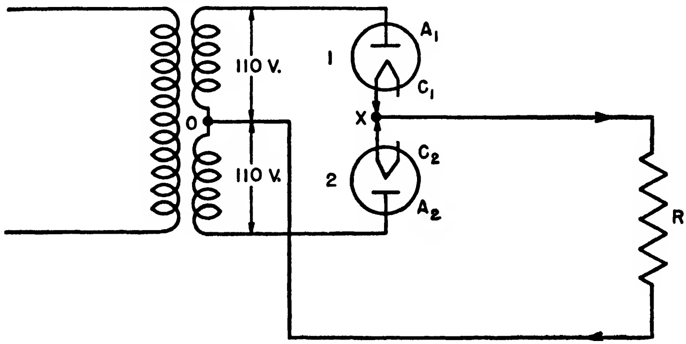


FIG. 19–14. Vacuum-tube full-wave rectifying circuit.

of a full-wave tube rectifier circuit. The tubes 1 and 2, called diodes, are highly evacuated. The temperature of cathodes C_1 and C_2 is raised by means of a heater circuit (not shown) until they emit electrons during the half circle in which they are maintained

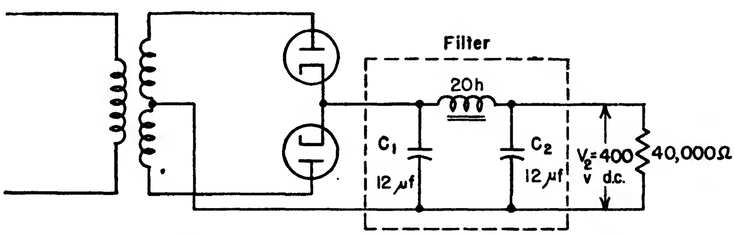


FIG. 20–14. The filter reduces the fluctuations in the direct-current circuit.

negative by voltage from the transformer coils. The electrons freed from the hot cathodes, migrate to the cold positive anodes; those leaving C_1 go to A_1 to O, through R to X and back through the tube 1. The electrons emitted by C_2 go to A_2, to O, through R to X and back through tube 2. Note that the electrons from both tubes flow in the same direction through R, and that this electron flow is in the opposite direction to the conventional positive direction marked on the diagram. We have learned, however, that a flow of negative electricity (electrons) in one

direction is the equivalent to a flow of positive electricity in the opposite direction.

Since the transformer makes C_1 and C_2 negative during alternate half cycles, the current through R consists of two positive loops each cycle, and the circuit is therefore a full-wave rectifier.

However, a direct current consisting of two positive loops and varying twice each cycle from zero to maximum is far from the steady direct current needed in most radio work. Accordingly a smoothing circuit is put in between the tubes and the resistance R. In Fig. 20–14 is shown a filter or smoothing circuit which reduces a 120-cycle fluctuation of the direct current in the 40,000-ohm load down to a point that is less than $\frac{1}{100}$ of what it would be in Fig. 19–14. There are many other combinations of inductors and capacitors used for suppressing disturbing fluctuations in direct-current circuits.

SUMMARY OF CHAPTER XIV

A SYNCHRONOUS CONVERTER is a synchronous motor the revolving armature of which is fitted with both a commutator and collecting-rings. Alternating-current power supplied to the rings drives the machine as a synchronous motor and direct-current power can be taken from the armature by means of brushes bearing on the commutator. This machine has been largely superceded by the ignitron.

THE RATIO OF THE ALTERNATING EMF to the direct emf of a synchronous converter is practically fixed, as follows:

Number of phases	Under ideal conditions	Under actual conditions	
		Full load, straight	Full load, inverted
One, two, six (diametral)	0.707	0.71	0.675
Three or six (double delta)	0.612	0.62	0.580

THE RATIO OF THE ALTERNATING CURRENT PER RING TO THE DIRECT CURRENT is as follows for unity power-factor and 100 per cent efficiency. Correction can easily be made for other power-factors and efficiencies.

Single-phase (two rings)	$I_1 = \mathbf{1.41}\, I_d$
Two-phase (four rings)	$I_2 = \mathbf{0.707}\, I_d$
Three-phase (three rings)	$I_3 = \mathbf{0.943}\, I_d$
Six-phase (six rings)	$I_6 = \mathbf{0.471}\, I_d$

I_d = **direct current per main lead.**

IGNITRONS have largely superseded rotary converters in heavy power installations and where large direct currents are required. These machines can be furnished to supply from 100 to 3000 amperes at from 1000 to 5000 volts. The ignitron consists of a highly evacuated steel tank in which electrons and ions in large quantities are forced from a negative pool of mercury by an electric arc. The electrons rush to the positive terminal, called the anode, while the positive ions in the mercury vapor carry their positive charge back to the negative mercury pool, called the cathode. A carborundum or boron-carbide rod inserted into the mercury pool supplies the vaporizing arc. A switching device maintains the arc only during the half cycle in which the pool is negative, so that little reverse current flows through the circuit, thus producing a direct current composed of positive loops.

By connecting several tanks in parallel, or putting into one tank six or twelve anodes which can be made positive in a half cycle, one after the other, the resulting direct current will consist of positive loops differing in phase from each other by 60° or 30°. The result of the overlapping of these loops is that the current has only very minor fluctuations.

THE CHIEF ADVANTAGES OF THE IGNITRON are: No large moving parts, quiet operation, high efficiency, easy installation and no necessity for heavy foundations.

RECTIFIERS are used to convert small amounts of a-c power to d-c power. Among the common types in use are: the mercury arc, the electrolytic and the dry plate.

THE MERCURY ARC RECTIFIER is the more common type and is used generally to charge storage batteries, when from 5 to 30 amperes are required. It consists of a highly evacuated glass tube, with two terminals, to which alternating voltage is applied by the secondary of a transformer, across which a reactance coil is also connected. At the middle point of the reactance coil is connected the negative terminal of the battery to be charged, the positive terminal being connected to a mercury pool in the base of the tube. An arc is started by tilting the tube so that the mercury pool comes into contact with a third terminal which applies a high voltage to the mercury pool. This changes some of the mercury into vapor, which allows a stream of (negative) electrons to pass from the pool to each of the two main terminals alternately during the half cycle when they are positive. Thus a flow of negative electrons takes place from negative to positive through the battery. Since this is the equivalent of a flow of positive charges from positive to negative, the battery is being charged. If the tube is kept from getting too hot, the rarefied vapor will allow almost no reverse current to pass. The upper part of the tube consists of a large cooling chamber in order to cool the vapor.

ELECTROLYTIC RECTIFIERS ARE MADE by immersing two plates of different metals in some electrolytic solution which offers a high resistance to the passage of electric current in one direction. Lead and aluminum plates immersed in ammonium sulfate offer a high resistance to a current from the aluminum to the lead, but allow the

passage of a current of about $\frac{1}{2}$ ampere per square inch of aluminum in the reverse direction.

A DRY PLATE RECTIFIER consists of a stack of copper plates, one surface of each of which has been coated with copper oxide. A current will pass from the oxide to the copper, but only slightly from the copper to the oxide. However, current passes in both directions to the interleaving lead disk. This rectifier has found much use in radio and in telephone circuits.

VACUUM TUBE RECTIFIERS are widely used in radio circuits to supply d-c power. A full-wave rectifier consists of two highly evacuated tubes in which a current of electrons driven off from a heated cathode flow to a cold positive anode and thence through the load. When the cathode of one tube becomes positive, the tube will not conduct. But the cathode of the second tube now becomes negative and supplies the other half cycle positive loop. In most circuits a filter is added which can reduce a fluctuation of any particular frequency to a negligible value.

PROBLEMS ON CHAPTER XIV

Prob. 14–14. In a certain manufacturing plant the average power-factor of the load on the generators was 0.50 lagging. A motor-generator converter set was added having a 1100 kva synchronous motor. When this set is running at full load with 0.20 leading power-factor the power-factor of the generators becomes 98 per cent.

(*a*) What was the total kilovolt-ampere output of the generators before the motor-generator converter was added?

(*b*) What was the effective power output of the generators before the converter was added?

Prob. 15–14. After the motor-generator converter of Prob. 14 was added to the plant:

(*a*) What was the apparent output (kva) of the generators?

(*b*) What was the effective power output (kw) of the generators?

Prob. 16–14. At an efficiency of 80 per cent for the motor-generator converter of Prob. 14, what direct-current power was delivered by the set?

Prob. 17–14. A 60-cycle synchronous converter is to be used to maintain constant voltage at the load-end of a short three-phase transmission line, from no-load to a full-load of 6000 kva per phase at 75 per cent power-factor lagging. The transformers at both ends of the line are alike; both being Y-connected, and having an equivalent high-side resistance of 1.5 ohms per phase and an equivalent high-side inductive reactance of 9 ohms per phase. Each line conductor has a resistance of 10.73 ohms, and an inductive reactance of 19.55 ohms. The high-side voltage-to-neutral of the load-end transformers is to be maintained at 30,000 volts per phase. The converter must be field-adjusted to deliver either lagging or leading kva.

What kva, both leading and lagging, must the converter be able to deliver to the line?

The capacitance of the line is negligible.

APPENDIX A

TABLE I

Natural sines, cosines, tangents and cotangents for decimal fractions of a degree

Deg.	Sin	Cos	Tan	Cot	Deg.
0.0	0.00000	1.0000	0.00000	∞	**90.0**
.1	.00175	1.0000	.00175	573.0	.9
.2	.00349	1.0000	.00349	286.5	.8
.3	.00524	1.0000	.00524	191.0	.7
.4	.00698	1.0000	.00698	143.24	.6
.5	.00873	1.0000	.00873	114.59	.5
.6	.01047	0.9999	.01047	95.49	.4
.7	.01222	.9999	.01222	81.85	.3
.8	.01396	.9999	.01396	71.62	.2
.9	.01571	.9999	.01571	63.66	.1
1.0	0.01745	0.9998	0.01746	57.29	**89.0**
.1	.01920	.9998	.01920	52.08	.9
.2	.02094	.9998	.02095	47.74	.8
.3	.02269	.9997	.02269	44.07	.7
.4	.02443	.9997	.02444	40.92	.6
.5	.02618	.9997	.02619	38.19	.5
.6	.02792	.9996	.02793	35.80	.4
.7	.02967	.9996	.02968	33.69	.3
.8	.03141	.9995	.03143	31.82	.2
.9	.03316	.9995	.03317	30.14	.1
2.0	0.03490	0.9994	0.03492	28.64	**88.0**
.1	.03664	.9993	.03667	27.27	.9
.2	.03839	.9993	.03842	26.03	.8
.3	.04013	.9992	.04016	24.90	.7
.4	.04188	.9991	.04191	23.86	.6
.5	.04362	.9990	.04366	22.90	.5
.6	.04536	.9990	.04541	22.02	.4
.7	.04711	.9989	.04716	21.20	.3
.8	.04885	.9988	.04891	20.45	.2
.9	.05059	.9987	.05066	19.74	.1
3.0	0.05234	0.9986	0.05241	19.081	**87.0**
.1	.05408	.9985	.05416	18.464	.9
.2	.05582	.9984	.05591	17.886	.8
.3	.05756	.9983	.05766	17.343	.7
.4	.05931	.9982	.05941	16.832	.6
.5	.06105	.9981	.06116	16.350	.5
.6	.06279	.9980	.06291	15.895	.4
.7	.06453	.9979	.06467	15.464	.3
.8	.06627	.9978	.06642	15.056	.2
.9	.06802	.9977	.06817	14.669	.1
4.0	0.06976	0.9976	0.06993	14.301	**86.0**
Deg.	Cos	Sin	Cot	Tan	Deg.

Deg.	Sin	Cos	Tan	Cot	Deg.
4.0	0.06976	0.9976	0.06993	14.301	**86.0**
.1	.07150	.9974	.07168	13.951	.9
.2	.07324	.9973	.07344	13.617	.8
.3	.07498	.9972	.07519	13.300	.7
.4	.07672	.9971	.07695	12.996	.6
.5	.07846	.9969	.07870	12.706	.5
.6	.08020	.9968	.08046	12.429	.4
.7	.08194	.9966	.08221	12.163	.3
.8	.08368	.9965	.08397	11.909	.2
.9	.08542	.9963	.08573	11.664	.1
5.0	0.08716	0.9962	0.08749	11.430	**85.0**
.2	.09063	.9959	.09101	10.988	.8
.4	.09411	.9956	.09453	10.579	.6
.6	.09758	.9952	.09805	10.199	.4
.8	.10106	.9949	.10158	9.845	.2
6.0	0.10453	0.9945	0.10510	9.514	**84.0**
.2	.10800	.9942	.10863	9.205	.8
.4	.11147	.9938	.11217	8.915	.6
.6	.11494	.9934	.11570	8.643	.4
.8	.11840	.9930	.11924	8.386	.2
7.0	0.12187	0.9925	0.12278	8.144	**83.0**
.2	.12533	.9921	.12633	7.916	.8
.4	.12880	.9917	.12988	7.700	.6
.6	.13226	.9912	.13343	7.495	.4
.8	.13572	.9907	.13698	7.300	.2
8.0	0.13917	0.9903	0.14054	7.115	**82.0**
.2	.14263	.9898	.14410	6.940	.8
.4	.14608	.9893	.14767	6.772	.6
.6	.14954	.9888	.15124	6.612	.4
.8	.15299	.9882	.15481	6.460	.2
9.0	0.15643	0.9877	0.15838	6.314	**81.0**
.2	.15988	.9871	.16196	6.174	.8
.4	.16333	.9866	.16555	6.041	.6
.6	.16677	.9860	.16914	5.912	.4
.8	.17021	.9854	.17273	5.789	.2
10.0	0.1736	0.9848	0.1763	5.671	**80.0**
.2	.1771	.9842	.1799	5.558	.8
.4	.1805	.9836	.1835	5.449	.6
.6	.1840	.9829	.1871	5.343	.4
.8	.1874	.9823	.1908	5.242	.2
11.0	0.1908	0.9816	0.1944	5.145	**79.0**
Deg.	Cos	Sin	Cot	Tan	Deg.

TABLE I—*Continued*

Natural sines, cosines, tangents and cotangents for decimal fractions of a degree

Deg.	Sin	Cos	Tan	Cot	Deg.
11.0	0.1908	0.9816	0.1944	5.145	**79.0**
.2	.1942	.9810	.1980	5.050	.8
.4	.1977	.9803	.2016	4.959	.6
.6	.2011	.9796	.2053	4.872	.4
.8	.2045	.9789	.2089	4.787	.2
12.0	0.2079	0.9781	0.2126	4.705	**78.0**
.2	.2113	.9774	.2162	4.625	.8
.4	.2147	.9767	.2199	4.548	.6
.6	.2181	.9759	.2235	4.474	.4
.8	.2215	.9751	.2272	4.402	.2
13.0	0.2250	0.9744	0.2309	4.331	**77.0**
.2	.2284	.9736	.2345	4.264	.8
.4	.2317	.9728	.2382	4.198	.6
.6	.2351	.9720	.2419	4.134	.4
.8	.2385	.9711	.2456	4.071	.2
14.0	0.2419	0.9703	0.2493	4.011	**76.0**
.2	.2453	.9694	.2530	3.952	.8
.4	.2487	.9686	.2568	3.895	.6
.6	.2521	.9677	.2605	3.839	.4
.8	.2554	.9668	.2642	3.785	.2
15.0	0.2588	0.9659	0.2679	3.732	**75.0**
.2	.2622	.9650	.2717	3.681	.8
.4	.2656	.9641	.2754	3.630	.6
.6	.2689	.9632	.2792	3.582	.4
.8	.2723	.9622	.2830	3.534	.2
16.0	0.2756	0.9613	0.2867	3.487	**74.0**
.2	.2790	.9603	.2905	3.442	.8
.4	.2823	.9593	.2943	3.398	.6
.6	.2857	.9583	.2981	3.354	.4
.8	.2890	.9573	.3019	3.312	.2
17.0	0.2924	0.9563	0.3057	3.271	**73.0**
.2	.2957	.9553	.3096	3.230	.8
.4	.2990	.9542	.3134	3.191	.6
.6	.3024	.9532	.3172	3.152	.4
.8	.3057	.9521	.3211	3.115	.2
18.0	0.3090	0.9511	0.3249	3.078	**72.0**
.2	.3123	.9500	.3288	3.042	.8
.4	.3156	.9489	.3327	3.006	.6
.6	.3190	.9478	.3365	2.971	.4
.8	.3223	.9466	.3404	2.937	.2
19.0	0.3256	0.9455	0.3443	2.904	**71.0**
.2	.3289	.9444	.3482	2.872	.8
.4	.3322	.9432	.3522	2.840	.6
.6	.3355	.9421	.3561	2.808	.4
.8	.3387	.9409	.3600	2.778	.2
20.0	0.3420	0.9397	0.3640	2.747	**70.0**
Deg.	Cos	Sin	Cot	Tan	Deg.

Deg.	Sin	Cos	Tan	Cot	Deg.
20.0	0.3420	0.9397	0.3640	2.747	**70.0**
.2	.3453	.9385	.3679	2.718	.8
.4	.3486	.9373	.3719	2.689	.6
.6	.3518	.9361	.3759	2.660	.4
.8	.3551	.9348	.3799	2.633	.2
21.0	0.3584	0.9336	0.3839	2.605	**69.0**
.2	.3616	.9323	.3879	2.578	.8
.4	.3649	.9311	.3919	2.552	.6
.6	.3681	.9298	.3959	2.526	.4
.8	.3714	.9285	.4000	2.500	.2
22.0	0.3746	0.9272	0.4040	2.475	**68.0**
.2	.3778	.9259	.4081	2.450	.8
.4	.3811	.9245	.4122	2.426	.6
.6	.3843	.9232	.4163	2.402	.4
.8	.3875	.9219	.4204	2.379	.2
23.0	0.3907	0.9205	0.4245	2.356	**67.0**
.2	.3939	.9191	.4286	2.333	.8
.4	.3971	.9178	.4327	2.311	.6
.6	.4003	.9164	.4369	2.289	.4
.8	.4035	.9150	.4411	2.267	.2
24.0	0.4067	0.9135	0.4452	2.246	**66.0**
.2	.4099	.9121	.4494	2.225	.8
.4	.4131	.9107	.4536	2.204	.6
.6	.4163	.9092	.4578	2.184	.4
.8	.4195	.9078	.4621	2.164	.2
25.0	0.4226	0.9063	0.4663	2.145	**65.0**
.2	.4258	.9048	.4706	2.125	.8
.4	.4289	.9033	.4748	2.106	.6
.6	.4321	.9018	.4791	2.087	.4
.8	.4352	.9003	.4834	2.069	.2
26.0	0.4384	0.8988	0.4877	2.050	**64.0**
.2	.4415	.8973	.4921	2.032	.8
.4	.4446	.8957	.4964	2.014	.6
.6	.4478	.8942	.5008	1.997	.4
.8	.4509	.8926	.5051	1.980	.2
27.0	0.4540	0.8910	0.5095	1.963	**63.0**
.2	.4571	.8894	.5139	1.946	.8
.4	.4602	.8878	.5184	1.929	.6
.6	.4633	.8862	.5228	1.913	.4
.8	.4664	.8846	.5272	1.897	.2
28.0	0.4695	0.8829	0.5317	1.881	**62.0**
.2	.4726	.8813	.5362	1.865	.8
.4	.4756	.8796	.5407	1.849	.6
.6	.4787	.8780	.5452	1.834	.4
.8	.4818	.8763	.5498	1.819	.2
29.0	0.4848	0.8746	0.5543	1.804	**61.0**
Deg.	Cos	Sin	Cot	Tan	Deg.

TABLE I—*Continued*

Natural sines, cosines, tangents and cotangents for decimal fractions of a degree

Deg.	Sin	Cos	Tan	Cot	Deg.
29.0	0.4848	0.8746	0.5543	1.804	**61.0**
.2	.4879	.8729	.5589	1.789	.8
.4	.4909	.8712	.5635	1.775	.6
.6	.4939	.8695	.5681	1.760	.4
.8	.4970	.8678	.5727	1.746	.2
30.0	0.5000	0.8660	0.5774	1.732	**60.0**
.2	.5030	.8643	.5820	1.7182	.8
.4	.5060	.8625	.5867	1.7045	.6
.6	.5090	.8607	.5914	1.6909	.4
.8	.5120	.8590	.5961	1.6775	.2
31.0	0.5150	0.8572	0.6009	1.6643	**59.0**
.2	.5180	.8554	.6056	1.6512	.8
.4	.5210	.8536	.6104	1.6383	.6
.6	.5240	.8517	.6152	1.6255	.4
.8	.5270	.8499	.6200	1.6128	.2
32.0	0.5299	0.8480	0.6249	1.6003	**58.0**
.2	.5329	.8462	.6297	1.5880	.8
.4	.5358	.8443	.6346	1.5757	.6
.6	.5388	.8425	.6395	1.5637	.4
.8	.5417	.8406	.6445	1.5517	.2
33.0	0.5446	0.8387	0.6494	1.5399	**57.0**
.2	.5476	.8368	.6544	1.5282	.8
.4	.5505	.8348	.6594	1.5166	.6
.6	.5534	.8329	.6644	1.5051	.4
.8	.5563	.8310	.6694	1.4938	.2
34.0	0.5592	0.8290	0.6745	1.4826	**56.0**
.2	.5621	.8271	.6796	1.4715	.8
.4	.5650	.8251	.6847	1.4605	.6
.6	.5678	.8231	.6899	1.4496	.4
.8	.5707	.8211	.6950	1.4388	.2
35.0	0.5736	0.8192	0.7002	1.4281	**55.0**
.2	.5764	.8171	.7054	1.4176	.8
.4	.5793	.8151	.7107	1.4071	.6
.6	.5821	.8131	.7159	1.3968	.4
.8	.5850	.8111	.7212	1.3865	.2
36.0	0.5878	0.8090	0.7265	1.3764	**54.0**
.2	.5906	.8070	.7319	1.3663	.8
.4	.5934	.8049	.7373	1.3564	.6
.6	.5962	.8028	.7427	1.3465	.4
.8	.5990	.8007	.7481	1.3367	.2
37.0	0.6018	0.7986	0.7536	1.3270	**53.0**
Deg.	Cos	Sin	Cot	Tan	Deg.

Deg.	Sin	Cos	Tan	Cot	Deg.
37.0	0.6018	0.7986	0.7536	1.3270	**53.0**
.2	.6046	.7965	.7590	1.3175	.8
.4	.6074	.7944	.7646	1.3079	.6
.6	.6101	.7923	.7701	1.2985	.4
.8	.6129	.7902	.7757	1.2892	.2
38.0	0.6157	0.7880	0.7813	1.2799	**52.0**
.2	.6184	.7859	.7869	1.2708	.8
.4	.6211	.7837	.7926	1.2617	.6
.6	.6239	.7815	.7983	1.2527	.4
.8	.6266	.7793	.8040	1.2437	.2
39.0	0.6293	0.7771	0.8098	1.2349	**51.0**
.2	.6320	.7749	.8156	1.2261	.8
.4	.6347	.7727	.8214	1.2174	.6
.6	.6374	.7705	.8273	1.2088	.4
.8	.6401	.7683	.8332	1.2002	.2
40.0	0.6428	0.7660	0.8391	1.1918	**50.0**
.2	.6455	.7638	.8451	1.1833	.8
.4	.6481	.7615	.8511	1.1750	.6
.6	.6508	.7593	.8571	1.1667	.4
.8	.6534	.7570	.8632	1.1585	.2
41.0	0.6561	0.7547	0.8693	1.1504	**49.0**
.2	.6587	.7524	.8754	1.1423	.8
.4	.6613	.7501	.8816	1.1343	.6
.6	.6639	.7478	.8878	1.1263	.4
.8	.6665	.7455	.8941	1.1184	.2
42.0	0.6691	0.7431	0.9004	1.1106	**48.0**
.2	.6717	.7408	.9067	1.1028	.8
.4	.6743	.7385	.9131	1.0951	.6
.6	.6769	.7361	.9195	1.0875	.4
.8	.6794	.7337	.9260	1.0799	.2
43.0	0.6820	0.7314	0.9325	1.0724	**47.0**
.2	.6845	.7290	.9391	1.0649	.8
.4	.6871	.7266	.9457	1.0575	.6
.6	.6896	.7242	.9523	1.0501	.4
.8	.6921	.7218	.9590	1.0428	.2
44.0	0.6947	0.7193	0.9657	1.0355	**46.0**
.2	.6972	.7169	.9725	1.0283	.8
.4	.6997	.7145	.9793	1.0212	.6
.6	.7022	.7120	.9861	1.0141	.4
.8	.7046	.7096	.9930	1.0070	.2
45.0	0.7071	0.7071	1.0000	1.0000	**45.0**
Deg.	Cos	Sin	Cot	Tan	Deg.

Trigonometric Equations

The right-angle triangle

Consider the right-angle triangle of Figs. 1 and 2 in which the sides a and b are at right angles to each other. The side h is called the hypothenuse.

In any right-angle triangle,

$$\sin \alpha = \frac{\text{side opposite } \alpha}{\text{hypothenuse}} = \frac{a}{h}$$

thus $$a = h \sin \alpha$$

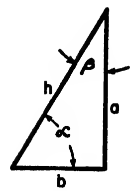


FIG. 1. A right-angle triangle. $h^2 = a^2 + b^2$.

Example 1. If $h = 10$

and $\alpha = 60°$

$a = 10 \sin 60°$

From table, $\sin 60° = 0.866$

Thus $a = 10 \times 0.866 = 8.66$

$$\text{Cosine } \alpha = \frac{\text{side adjacent } \alpha}{\text{hypothenuse}} = \frac{b}{h} \quad \text{or} \quad \cos \alpha = \frac{b}{h}$$

Thus $$b = h \cos \alpha$$

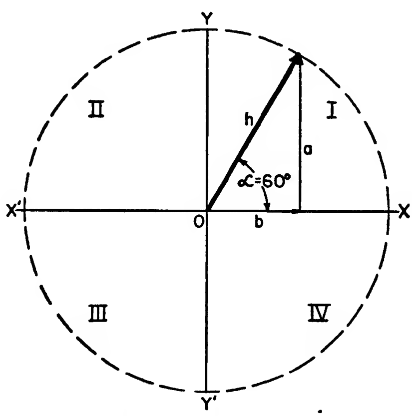


FIG. 2. Here the right-angle of Fig. 1 has been formed by rotating the hypothenuse h counter-clockwise 60° from the axis OX, and dropping the line a perpendicular to OX.

Example 2. $b = 10 \cos 60°$

From Table I, $\cos 60° = 0.500$

So $b = 10 \times 0.500$ or 5.00

$$\text{Tangent } \alpha = \frac{\text{side opposite } \alpha}{\text{side adjacent } \alpha} \quad \text{or} \quad \frac{\sin \alpha}{\cos \alpha} = \frac{a}{b} \quad \text{or} \quad \tan \alpha = \frac{a}{b}$$

Thus $\qquad a = b \tan \alpha$

Example 3. From Table I,

$$\tan 60° = 1.73$$

So $$a = 5.00 \times 1.732 = 8.66$$

which checks with Example 1.

$$\text{Cotangent } \alpha = \frac{1}{\tan \alpha} = \frac{\text{side adjacent } \alpha}{\text{side opposite } \alpha} = \frac{\cos \alpha}{\sin \alpha} = \frac{b}{a} \text{ or } \cot \alpha = \frac{b}{a}$$

Thus $\qquad b = a \cot \alpha$

Example 4. From Table I,

$$\cot 60° = 0.577$$

So $$b = 8.66 \times .577 = 5.00$$

which checks with Example 2.

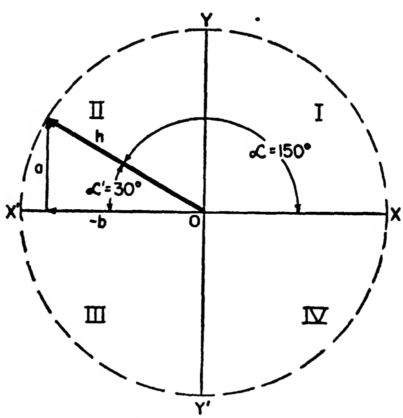


FIG. 3. The right-angle triangle has been formed by rotating h 150° from OX.

$$\sin 150° = \sin 30°.$$
$$\cos 150° = -\cos 30°.$$

In alternating-current work we often use expressions like $h \sin 150°$ and $h \cos 150°$. These expressions merely mean that the hypothenuse h of Figs. 1 and 2 has swung around in the counter-clockwise direction to the position shown in Fig. 3.

Note from Fig. 3 that $\qquad \sin 150° = \frac{a}{h}.$

and also that $$\sin 30^\circ = \frac{a}{h}$$

Thus $\sin 150^\circ = \sin(180^\circ - 150^\circ)$ or $\sin 30^\circ$

or $\sin \alpha = \sin(180^\circ - \alpha)$ or $\sin \alpha'$

Thus $h \sin \alpha = h \sin(180^\circ - \alpha) = h \sin \alpha'$

This is true whenever h lies in the quadrant marked II. Similarly $\cos 150^\circ = \frac{b}{h} = \cos 30^\circ$ (numerically). But note that in Fig. 3

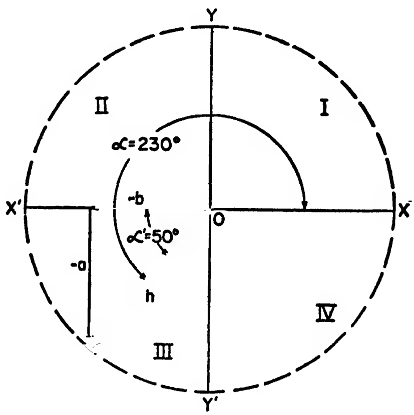


FIG. 4. The right-angle triangle is formed by rotating h 230° from OX.
$\sin 230^\circ = -\sin 50^\circ$.
$\cos 230^\circ = -\cos 50^\circ$.

the line b is **negative** since it is drawn in the negative direction from the point 0. The hypothenuse h is always positive.

Thus $$\cos 150^\circ = \frac{-b}{h} = -\cos 30^\circ$$

or $$h \cos \alpha = -h \cos(180 - \alpha) = -h \cos \alpha'$$

Similarly, $$\tan 150^\circ = \frac{a}{-b} = -\tan 30^\circ$$

Note that when h has swung around 230° into quadrant III, as in Fig. 4, both a and b are negative and that

$$\sin 230^\circ = \frac{-a}{h} = -\sin(230^\circ - 180^\circ) = -\sin 50^\circ$$

and $$\cos 230° = \frac{-b}{h} = -\cos(230° - 180°) = -\cos 50°$$

$$\tan 230° = \frac{-a}{-b} = \tan 50°$$

In quadrant IV in Fig. 5

$$\sin 300° = \frac{-a}{h} = -\sin(360° - 300°) = -\sin 60°$$

$$\cos 300° = \frac{b}{h} = \cos(360° - 300°) = \cos 60°$$

$$\tan 300° = \frac{-a}{b} = -\tan 60°$$

In solving alternating-current problems, do not try to remember these trigonometric equations. By drawing a rough sketch of

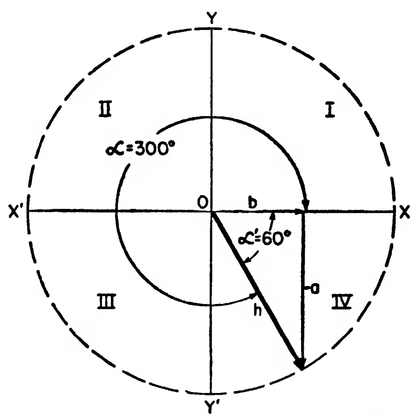


FIG. 5. The vector h has now been rotated 300° from OX.

$\sin 300° = -\sin 60°$.
$\cos 300° = +\cos 60°$.

each diagram it is easy to obtain the correct value and sign of the desired quantity.

Useful Trigonometric Equations

The sum of the angles in any triangle equals 180°. In a right-angle triangle, (see Fig. 1),

$$h^2 = a^2 + b^2$$

In any triangle (see Fig. 6).

$$A^2 = B^2 + C^2 - 2BC \cos a$$

This equation is particularly useful in finding the angles when the sides only are given. Note that the angle (a) must always be that opposite the side (A).

Law of sines

In any triangle (see Fig. 6), the **ratio** of the **sines** of any two angles equals the **ratio** of the **sides** opposite the respective angles.

$$\frac{\sin a}{\sin b} = \frac{A}{B} \quad \text{and} \quad \frac{\sin b}{\sin c} = \frac{B}{C} \quad \text{and} \quad \frac{\sin c}{\sin a} = \frac{C}{A}$$

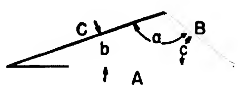


FIG. 6. The triangle ABC represents any triangle, and

$$A^2 = B^2 + C^2 - 2BC \cos a.$$

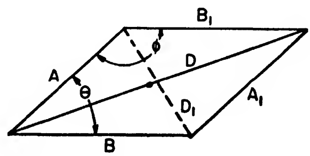


FIG. 7. The diagonals of any parallelogram can be computed from the equations

$$D^2 = A^2 + B^2 + 2AB \cos \theta$$
$$D_1^2 = A^2 + B_1^2 + 2AB_1 \cos \phi.$$

Cosine of sum of two angles, $\cos (a + b) = \cos a \cos b - \sin a \sin b$.

Cosine of difference of two angles, $\cos (a - b) = \cos a \cos b + \sin a \sin b$.

The diagonal of any parallelogram (see Fig. 7) can be found from the equation

$$D^2 = A^2 + B^2 + 2AB \cos \theta$$

Note that the angle θ must be the angle between sides A and B from whose intersection the diagonal D starts.

APPENDIX B

TABLE I

Properties of Annealed Copper Wire

B. & S. gage.	Area in cir. mils. d^2	Diameter in mils. d		Number of strands in cable.	Resistance per mile at 20° C or 68° F (approx.).	Weight per mile in pounds (approx.).	
		Solid.	Stranded.			Solid.	Stranded.
14	4,107	64 05	73	7	13 3	65 6	70
12	6,530	80 81	92	7	8 40	104	108
10	10,380	101 9	116	7	5 27	166	172
8	16,510	128 5	146	7	3 31	264	269
6	26,250	162 0	184	7	2 08	420	428
5	33,100	181 9	206	7	1 65	530	544
4	41,740	204 3	232	7	1 31	667	682
3	52,630	229 4	260	7	1 04	841	866
2	66,370	257 6	292	7	0 824	1062	1,087
1	83,690	289 3	333	7	0 656	1337	1,368
0	105,540	324 9	375	7	0 518	1687	1,730
00	133,080	364 8	419	7	0 412	2127	2,190
000	167,810	409 6	470	7	0 328	2682	2,740
0000	211,600	460 0	528	19	0 259	3381	3,470
	250,000	500 0	575	19	0 217	..	4,090
	300,000	547 7	630	19	0 185	...	4,890
	350,000	591 6	681	19	0 159	....	5,740
	400,000	632 5	729	37	0 137	....	6,570
	450,000	670 8	773	37	0 122	..	7,470
	500,000	707 1	815	37	0 111	..	8,210
	550,000	741 6	855	37	0 100	.	9,020
	600,000	774 6	893	37	0 0898	..	9,850
	650,000	806 2	929	37	0 0845	..	10,660
	700,000	836 7	964	37	0 0793	...	11,480
	750,000	866 0	998	61	0 0739	..	12,320
	800,000	894 4	1031	61	0 0687	...	13,130
	900,000	948 7	1093	61	0 0633	..	14,850
	1,000,000	1000	1151	61	0 0528	...	16,420
	1,250,000	1118	1289	61	0.0438	....	20,300
	1,500,000	1225	1413	91	0 0304	....	24,600
	1,750,000	1323	1526	91	0 0311	....	28,700
	2,000,000	1414	1631	127	0 0275	....	32,800

TABLE II

Properties of Aluminum Wire*

Aluminum Cables

The commercial sizes of stranded aluminum cables, made by the Aluminum Company of America, are not even circular-mil and B. & S. sizes, but are of such cross sections as to give the same conductivity as even circular-mil and B. & S. sizes of copper cables of 97 per cent conductivity. In the following table, the first four columns are taken from a pamphlet entitled "Instructions for Installation and Maintenance of Aluminum Electrical Conductors," issued by the Aluminum Company of America.

B. & S. gage or circular mils.		Usual number of strands.	Diameter of bare cable, inches.	Ohms per mile at 20° C or 68° F		Weight in pounds per mile.	
Copper (97 per cent) equivalent.	Aluminum 61 per cent.			Solid.	Stranded.	Solid.	Stranded.
6	41,740	7	$\frac{15}{64}$	2.147	2.194	200	203.3
5	52,630	7	$\frac{17}{64}$	1.703	1.740	253	256.1
4	66,370	7	$\frac{19}{64}$	1.350	1.380	319	323.1
3	83,640	7	$\frac{21}{64}$	1.071	1.094	402	406.6
2	105,530	7	$\frac{23}{64}$	0.8486	0.868	507	513.2
1	133,220	7	$\frac{13}{32}$	0.6720	0.689	640	647.3
0	167,800	7	$\frac{15}{32}$	0.5342	0.546	805	818.4
00	211,950	7	$\frac{33}{64}$	0.4229	0.433	1017	1030
000	266,800	7	$\frac{37}{64}$	0.3360	0.343	1281	1297
0000	336,420	7	$\frac{21}{32}$	0.2667	0.272	1617	1638
250,000	397,500	19	$\frac{23}{32}$	0.2253	0.2306	1907	1927
300,000	477,000	19	$\frac{25}{32}$	0.1879	0.1921	2290	2318
350,000	556,500	19	$\frac{55}{64}$	0.1611	0.1645	2670	2703
400,000	636,000	37	$\frac{29}{32}$	0.1409	0.1440	3050	3089
450,000	715,500	37	$\frac{31}{32}$	0.1254	0.1280	3440	3474
500,000	795,000	37	$1\frac{1}{64}$	0.1127	0.1152	3820	3865
550,000	874,500	37	$1\frac{1}{16}$	.....	0.1048	....	4250
600,000	954,000	37	$1\frac{7}{64}$	.....	0.0959	....	4631
650,000	1,033,500	37	$1\frac{5}{32}$	.....	0.0886	....	5016
700,000	1,113,000	37	$1\frac{13}{64}$	.....	0.0823	....	5412
750,000	1,192,500	37	$1\frac{1}{4}$	.....	0.0768	....	5797
800,000	1,272,000	61	$1\frac{9}{32}$	.....	0.0720	....	6189
850,000	1,351,500	61	$1\frac{21}{64}$	.....	0.0678	....	6563
900,000	1,431,000	61	$1\frac{23}{64}$	.....	0.0640	....	6954
950,000	1,515,000	61	$1\frac{13}{32}$	.....	0.0607	....	7355
1,000,000	1,590,000	61	$1\frac{7}{16}$	.....	0.0577	....	7719

* From "Handbook for Electrical Engineers: Electric Power," Second Edition. John Wiley & Sons, Inc.

TABLE III

60-Cycle Inductive Reactance of Solid Non-magnetic Wires*

Ohms per Mile of Each Wire of a Single-phase or of a Symmetrical Three-phase Line

Size of wire, cir. mils. or A.W.G.	Diam. of wire, inches.	Inches between wires, center to center.							
		1	3	6	9	12	18	24	30
1,000,000	1.0000	0.1145	0.2478	0.3319	0.3811	0.4158	0.4652	0.5003	0.5270
750,000	0.8660	0.1319	0.2652	0.3493	0.3985	0.4336	0.4826	0.5176	0.5448
500,000	0.7071	0.1565	0.2898	0.3739	0.4230	0.4581	0.5074	0.5421	0.5693
350,000	0.5916	0.1782	0.3115	0.3955	0.4449	0.4795	0.5289	0.5640	0.5908
250,000	0.5000	0.1986	0.3319	0.4158	0.4652	0.5003	0.5493	0.5844	0.6115
0000	0.4600	0.2087	0.3420	0.4260	0.4754	0.5101	0.5595	0.5945	0.6213
000	0.4096	0.2228	0.3561	0.4403	0.4893	0.5244	0.5734	0.6085	0.6356
00	0.3648	0.2368	0.3701	0.4543	0.5033	0.5384	0.5877	0.6224	0.6496
0	0.3249	0.2509	0.3842	0.4682	0.5176	0.5523	0.6017	0.6364	0.6635
1	0.2893	0.2650	0.3985	0.4826	0.5316	0.5666	0.6156	0.6507	0.6778
2	0.2576	0.2791	0.4124	0.4965	0.5455	0.5806	0.6300	0.6647	0.6918
4	0.2043	0.3072	0.4403	0.5248	0.5738	0.6099	0.6579	0.6929	0.7201
6	0.1620	0.3353	0.4686	0.5527	0.6021	0.6368	0.6861	0.7208	0.7480
8	0.1285	0.3635	0.4969	0.5810	0.6600	0.6650	0.7140	0.7491	0.7762
10	0.1019	0.3917	0.5248	0.6089	0.6382	0.6933	0.7423	0.7774	0.8045
12	0.08081	0.4196	0.5531	0.6371	0.6865	0.7212	0.7706	0.8053	0.8324
14	0.06408	0.4479	0.5813	0.6654	0.7144	0.7495	0.7985	0.8335	0.8607
16	0.05082	0.4762	0.6092	0.6933	0.7427	0.7774	0.8268	0.8618	0.8886

Size of wire, cir. mils. or A.W.G.	Feet between wires, center to center.								
	3	4	5	6	8	10	15	20	25
1,000,000	0.5493	0.5844	0.6115	0.6334	0.6684	0.6956	0.7446	0.7796	0.8068
750,000	0.5666	0.6017	0.6288	0.6507	0.6858	0.7129	0.7619	0.7970	0.8241
500,000	0.5915	0.6262	0.6533	0.6756	0.7103	0.7374	0.7868	0.8215	0.8486
350,000	0.6130	0.6481	0.6752	0.6971	0.7321	0.7593	0.8083	0.8433	0.8705
250,000	0.6334	0.6684	0.6956	0.7174	0.7525	0.7796	0.8286	0.8637	0.8909
0000	0.6435	0.6786	0.7057	0.7276	0.7627	0.7898	0.8388	0.8739	0.9010
000	0.6575	0.6925	0.7196	0.7416	0.7766	0.8038	0.8528	0.8878	0.9150
00	0.6718	0.7065	0.7336	0.7559	0.7906	0.8177	0.8671	0.9018	0.9289
0	0.6858	0.7204	0.7476	0.7698	0.8049	0.8317	0.8810	0.9161	0.9429
1	0.6997	0.7348	0.7619	0.7838	0.8188	0.8460	0.8950	0.9301	0.9572
2	0.7140	0.7487	0.7759	0.7981	0.8328	0.8599	0.9093	0.9440	0.9712
4	0.7419	0.7770	0.8041	0.8260	0.8611	0.8882	0.9372	0.9723	0.9994
6	0.7702	0.8049	0.8320	0.8543	0.8893	0.9161	0.9655	1.001	1.027
8	0.7985	0.8332	0.8603	0.8826	0.9172	0.9444	0.9938	1.028	1.056
10	0.8264	0.8614	0.8886	0.9105	0.9455	0.9727	1.022	1.057	1.084
12	0.8547	0.8893	0.9165	0.9387	0.9734	1.001	1.050	1.085	1.112
14	0.8826	0.9176	0.9448	0.9670	1.002	1.029	1.078	1.113	1.140
16	0.9108	0.9459	0.9730	0.9949	1.030	1.057	1.106	1.141	1.168

The reactances given in this table also apply, with a practically negligible error, to ordinary stranded wires of the same cross-section.

For 25-cycle reactance multiply values in table by $\frac{25}{60}$ or 0.417.

* "Handbook for Electrical Engineers: Electric Power." John Wiley & Sons, Inc.

TABLE IV

CAPACITANCE TO NEUTRAL OF SMOOTH ROUND WIRES*

Microfarads per Mile of Each Wire of a Single-phase or of a Symmetrical Three-phase Line

Size of wire, A.W.G.	Diam. of wire, inches	Inches between wires, center to center.							
		1	3	6	9	12	18	24	30
0000	0.4600	0.06332	0.03490	0.02741	0.02438	0.02261	0.02051	0.01924	0.01836
000	0.4096	0.05802	0.03336	0.02647	0.02364	0.02197	0.01998	0.01877	0.01793
00	0.3648	0.05366	0.03198	0.02559	0.02293	0.02136	0.01947	0.01832	0.01752
0	0.3249	0.04995	0.03069	0.02477	0.02227	0.02078	0.01899	0.01790	0.01713
1	0.2893	0.04676	0.02951	0.02400	0.02165	0.02024	0.01854	0.01749	0.01676
2	0.2576	0.04400	0.02842	0.02328	0.02106	0.01972	0.01810	0.01710	0.01640
4	0.2043	0.03937	0.02645	0.02195	0.01997	0.01876	0.01729	0.01638	0.01573
6	0.1620	0.03566	0.02475	0.02077	0.01898	0.01789	0.01655	0.01571	0.01512
8	0.1285	0.03262	0.02326	0.01971	0.01809	0.01710	0.01587	0.01510	0.01455
10	0.1019	0.03006	0.02194	0.01875	0.01728	0.01637	0.01524	0.01453	0.01402
12	0.08081	0.02787	0.02076	0.01788	0.01654	0.01570	0.01466	0.01400	0.01353
14	0.06408	0.02599	0.01970	0.01709	0.01586	0.01509	0.01412	0.01351	0.01307
16	0.05082	0.02434	0.01874	0.01636	0.01523	0.01452	0.01362	0.01305	0.01264

Size of wire, A.W.G.	Feet between wires, center to center.								
	3	4	5	6	8	10	15	20	25
0000	0.01769	0.01674	0.01607	0.01556	0.01482	0.01429	0.01342	0.01286	0.01246
000	0.01730	0.01639	0.01574	0.01525	0.01454	0.01403	0.01319	0.01265	0.01227
00	0.01692	0.01604	0.01543	0.01496	0.01427	0.01378	0.01297	0.01245	0.01207
0	0.01656	0.01572	0.01512	0.01467	0.01401	0.01354	0.01275	0.01225	0.01189
1	0.01621	0.01540	0.01483	0.01440	0.01376	0.01330	0.01255	0.01206	0.01171
2	0.01587	0.01510	0.01455	0.01413	0.01352	0.01308	0.01235	0.01187	0.01153
4	0.01525	0.01453	0.01402	0.01363	0.01306	0.01265	0.01196	0.01152	0.01120
6	0.01467	0.01400	0.01353	0.01317	0.01263	0.01225	0.01160	0.01118	0.01088
8	0.01413	0.01351	0.01307	0.01273	0.01223	0.01187	0.01126	0.01087	0.01058
10	0.01363	0.01306	0.01264	0.01233	0.01186	0.01152	0.01094	0.01057	0.01030
12	0.01316	0.01263	0.01224	0.01194	0.01150	0.01118	0.01064	0.01029	0.01003
14	0.01273	0.01223	0.01187	0.01159	0.01117	0.01087	0.01036	0.01002	0.009777
16	0.01232	0.01185	0.01151	0.01125	0.01085	0.01057	0.01008	0.009768	0.009536

The capacitance *between* wires equals one-half the values given in this table.

* "Handbook for Electrical Engineers: Electric Power," Second Edition. John Wiley & Sons, Inc.

TABLE V

60-Cycle Charging Current in Line of Smooth Round Wires*

Charging current on each line wire, in amperes per mile = (current from table) $\times \dfrac{(\textit{volts to neutral})}{1{,}000{,}000}$.

Amperes in Each Wire of Line *One Mile Long*, for Each 1,000,000 Volts to Neutral

Size of wire, A.W.G.	Diam. of wire, inches.	Inches between wires, center to center.							
		1	3	6	9	12	18	24	30
0000	0.4600	23.87	13.16	10.33	9.191	8.524	7.732	7.253	6.922
000	0.4096	21.87	12.58	9.979	8.912	8.283	7.532	7.076	6.760
00	0.3648	20.23	12.06	9.647	8.645	8.053	7.340	6.907	6.605
0	0.3249	18.83	11.57	9.338	8.396	7.834	7.159	6.748	6.458
1	0.2893	17.63	11.13	9.048	8.162	7.630	6.990	6.594	6.319
2	0.2576	16.59	10.71	8.777	7.940	7.434	6.824	6.447	6.183
4	0.2043	14.84	9.972	8.275	7.529	7.073	6.518	6.175	5.930
6	0.1620	13.44	9.331	7.830	7.155	6.745	6.239	5.923	5.700
8	0.1285	12.30	8.769	7.430	6.820	6.447	5.983	5.693	5.485
10	0.1019	11.33	8.271	7.069	6.515	6.171	5.745	5.478	5.286
12	0.08081	10.51	7.827	6.741	6.236	5.919	5.527	5.278	5.101
14	0.06408	9.798	7.427	6.443	5.979	5.689	5.323	5.093	4.927
16	0.05082	9.176	7.065	6.168	5.742	5.474	5.135	4.920	4.765

Size of wire, A.W.G.	Feet between wires, center to center.								
	3	4	5	6	8	10	15	20	25
0000	6.669	6.311	6.058	5.866	5.587	5.387	5.059	4.848	4.697
000	6.522	6.179	5.934	5.749	5.482	5.289	4.973	4.769	4.626
00	6.379	6.047	5.817	5.640	5.380	5.195	4.890	4.694	4.550
0	6.243	5.926	5.700	5.531	5.282	5.105	4.807	4.618	4.483
1	6.111	5.806	5.591	5.429	5.188	5.014	4.731	4.547	4.415
2	5.983	5.693	5.485	5.327	5.097	4.931	4.656	4.475	4.347
4	5.749	5.478	5.286	5.139	4.924	4.769	4.509	4.343	4.222
6	5.531	5.278	5.101	4.965	4.762	4.618	4.373	4.215	4.102
8	5.327	5.093	4.927	4.799	4.611	4.475	4.245	4.098	3.989
10	5.139	4.924	4.765	4.648	4.471	4.343	4.124	3.985	3.883
12	4.961	4.762	4.614	4.501	4.336	4.215	4.011	3.879	3.781
14	4.799	4.611	4.475	4.369	4.211	4.098	3.906	3.778	3.686
16	4.645	4.467	4.339	4.241	4.090	3.985	3.800	3.683	3.595

For 25 cycles the charging current equals $\frac{25}{60}$ or 0.417 of the values in this table.

* From "Handbook for Electrical Engineers: Electric Power," Second Edition. John Wiley & Sons, Inc.

TABLE VI

Capacitance to Neutral of Standard Strands*

Microfarads per Mile of each Conductor of a Single-phase or of a Symmetrical Three-phase Line

Size of cable, C.M. or A.W.G.	Diam. of strand, inches.	Inches between conductors, center to center.							
		1	3	6	9	12	18	24	30
1,000,000	1.152	......	0.0554	0.0383	0.0325	0.0294	0.0260	0.0240	0.0226
750,000	0.998	......	0.0506	0.0361	0.0309	0.0281	0.0249	0.0231	0.0218
500,000	0.814	0.134	0.0452	0.0333	0.0289	0.0264	0.0236	0.0219	0.0208
350,000	0.681	0.0955	0.0413	0.0312	0.0273	0.0251	0.0225	0.0210	0.0200
250,000	0.575	0.0776	0.0383	0.0295	0.0260	0.0240	0.0216	0.0202	0.0192
0000	0.528	0.0713	0.0369	0.0286	0.0253	0.0234	0.0212	0.0198	0.0189
000	0.470	0.0644	0.0352	0.0276	0.0245	0.0227	0.0206	0.0193	0.0184
00	0.418	0.0590	0.0336	0.0266	0.0238	0.0221	0.0201	0.0189	0.0180
0	0.373	0.0544	0.0322	0.0258	0.0231	0.0214	0.0196	0.0184	0.0176
1	0.332	0.0506	0.0309	0.0249	0.0224	0.0209	0.0191	0.0180	0.0172
2	0.292	0.0470	0.0296	0.0241	0.0217	0.0203	0.0186	0.0175	0.0168
4	0.232	0.0417	0.0275	0.0227	0.0205	0.0193	0.0177	0.0168	0 0161
6	0.184	0.0376	0.0256	0.0214	0.0195	0.0184	0.0169	0.0161	0.0154

Size of cable, C.M. or A.W.G.	Feet between conductors, center to center.								
	3	4	5	6	8	10	15	20	25
1,000,000	0.0216	0.0202	0.0193	0.0185	0.0175	0.0168	0.0156	0.0148	0.0143
750,000	0.0209	0.0196	0.0187	0.0180	0.0170	0.0163	0.0152	0.0145	0.0140
500,000	0.0200	0.0188	0.0179	0.0173	0.0164	0.0157	0.0147	0.0140	0.0135
350,000	0.0192	0.0181	0.0173	0.0167	0.0159	0.0153	0.0143	0.0136	0.0132
250,000	0.0185	0.0175	0.0168	0.0162	0.0154	0.0148	0.0139	0.0133	0.0129
0000	0.0182	0.0172	0.0165	0.0160	0.0152	0.0146	0.0137	0.0131	0.0127
000	0.0178	0.0168	0.0161	0.0156	0.0149	0.0143	0.0135	0.0129	0.0125
00	0.0174	0.0165	0.0158	0.0153	0.0146	0.0141	0.0132	0.0127	0.0123
0	0.0170	0.0161	0.0155	0.0150	0.0143	0.0138	0.0130	0.0125	0.0121
1	0.0166	0.0158	0.0152	0.0147	0.0141	0.0136	0.0128	0.0123	0.0119
2	0.0162	0.0154	0.0149	0.0144	0.0138	0.0133	0.0126	0.0121	0.0117
4	0.0156	0.0148	0.0143	0.0139	0.0135	0.0129	0.0122	0.0117	0.0114
6	0.0150	0.0143	0.0138	0.0134	0.0129	0.0125	0.0118	0.0114	0.0111

The capacitance *between* conductors equals one-half the values given in this table.

* "Handbook for Electrical Engineers: Electric Power," Second Edition. John Wiley & Sons, Inc.

TABLE VII

60-Cycle Charging Current in Line of Standard Strands*

Charging current on each line wire, in amperes per mile = (current from table) $\times \frac{(volts\ to\ neutral)}{1{,}000{,}000}$

Approximate Amperes in Each Wire of One-mile Line for Each 1,000,000 volts to neutral

Size of cable, C.M. or A.W.G.	Diam. of strand, inches.	Inches between cables, center to center.							
		1	3	6	9	12	18	24	30
1,000,000	1.152	....	20.9	14.4	12.3	11.1	9.80	9.05	8.52
750,000	0.998	....	19.1	13.6	11.6	10.6	9.39	8.71	8.22
500,000	0.814	50.5	17.0	12.6	10.9	9.95	8.90	8.26	7.84
350,000	0.681	36.0	15.6	11.8	10.3	9.46	8.48	7.92	7.54
250,000	0.575	29.3	14.4	11.1	9.80	9.05	8.14	7.62	7.24
0000	0.528	26.9	13.9	10.8	9.54	8.82	7.99	7.46	7.13
000	0.470	24.3	13.3	10.4	9.24	8.56	7.77	7.28	6.94
00	0.418	22.2	12.7	10.0	8.97	8.33	7.58	7.13	6.79
0	0.373	20.5	12.1	9.73	8.71	8.07	7.39	6.94	6.63
1	0.332	19.1	11.6	9.39	8.44	7.88	7.20	6.79	6.48
2	0.292	17.7	11.2	9.09	8.18	7.65	7.01	6.60	6.33
4	0.232	15.7	10.4	8.56	7.73	7.28	6.67	6.33	6.07
6	0.184	14.2	9.65	8.07	7.35	6.94	6.37	6.07	5.81

Size of cable, C.M. or A.W.G.	Feet between cables, center to center.								
	3	4	5	6	8	10	15	20	25
1,000,000	8.14	7.62	7.28	6.97	6.60	6.33	5.88	5.58	5.39
750,000	7.88	7.39	7.05	6.79	6.41	6.15	5.73	5.47	5.28
500,000	7.54	7.09	6.75	6.52	6.18	5.92	5.54	5.28	5.09
350,000	7.24	6.82	6.52	6.30	5.99	5.77	5.39	5.13	4.98
250,000	6.97	6.60	6.33	6.11	5.81	5.58	5.24	5.01	4.86
0000	6.86	6.48	6.22	6.03	5.73	5.50	5.16	4.94	4.79
000	6.71	6.33	6.07	5.88	5.62	5.39	5.09	4.86	4.71
00	6.56	6.22	5.96	5.77	5.50	5.32	4.98	4.79	4.64
0	6.40	6.07	5.84	5.66	5.39	5.20	4.90	4.71	4.56
1	6.26	5.96	5.73	5.54	5.32	5.13	4.83	4.64	4.49
2	6.11	5.81	5.62	5.43	5.20	5.01	4.75	4.56	4.41
4	5.88	5.58	5.39	5.24	5.01	4.86	4.60	4.41	4.30
6	5.66	5.39	5.20	5.05	4.86	4.71	4.45	4.30	4.18

For 25 cycles the charging current equals $\frac{25}{60}$ or 0.417 of the values in this table.

* "Handbook for Electrical Engineers: Electric Power," Second Edition. John Wiley & Sons, Inc.

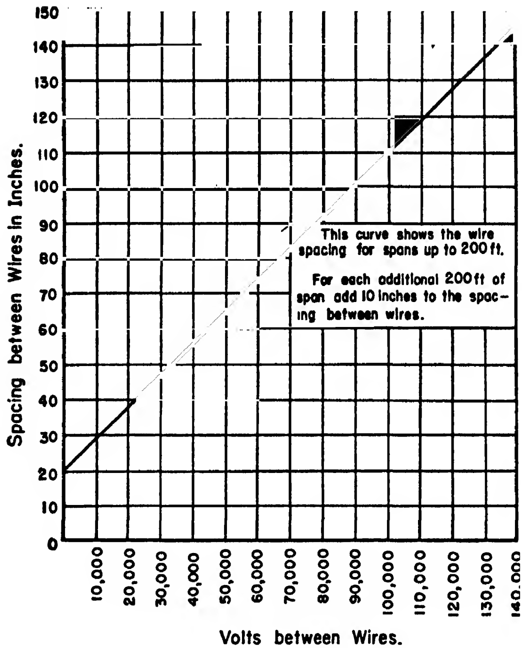


FIG. 8. Spacing of conductors for spans up to 200 ft. *From Still's "Overhead Electric Power Transmission." McGraw-Hill Book Co.*

TABLE VIII

HEMP-CENTER COPPER CABLE*

Made of 6 wires around a hemp center
(*The American Brass Co.*)

Size of cable, A.W.G.	Diameter of wire, inch.	Outside diameter, inch.
0000	0.1879	0.564
000	0.1672	0.502
00	0.1489	0.447
0	0.1326	0.398
1	0.1181	0.354
2	0.1052	0.316
3	0.0937	0.281
4	0 0835	0 250

* Standard Handbook for Electrical Engineers. McGraw-Hill Book Co.

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